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**Appendix:**
**Chapterwise Solved January 2019 JEE Main Questions (All Sets)** **A.1–A.5**

# 1 Geometrical Optics: Part 1

## INTRODUCTION

Light is a form of radiant energy, that is, energy emitted by excited atoms or molecules which can cause the sensation of vision in a normal human eye.

The branch of physics which deals with the phenomena of light is called optics. The two branches of Optics are as follows:

**Geometrical optics:** This deals with the study of light in which light is considered as moving along a straight line as a ray. A ray of light gives the direction of propagation of light. When light meets a surface which separates two media, reflection and refraction take place. An image or an array of images may be formed due to this.

**Physical optics:** It deals with the theories regarding the nature of light and provides an explanation for the different phenomena in light, such as reflection, refraction, interference, diffraction, polarization, and rectilinear propagation.

## NATURE OF OBJECTS AND IMAGES

Ray optics primarily deals with determining the position and nature of the image formed when an object is placed in front of an optical element. Before we proceed further, let us clearly define what is an object and what is an image.

### TYPES OF OBJECTS

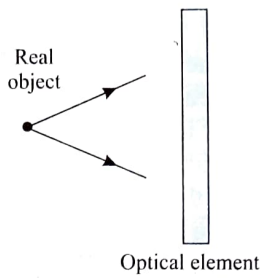
An object is a source of light rays that are incident on an optical element. An object may be a point object or an extended object. Since extended objects can be modeled as a collection of points, we only need to study point objects. Objects are of two kinds: real objects and virtual objects.

#### Real Object

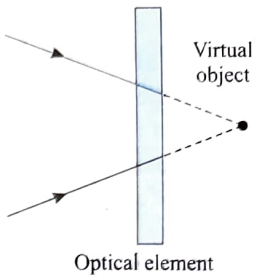
An object is real if two or more incident rays actually emanate or seem to emanate from a point. Figure (a) is a typical example of a real object. In this case, two rays emanate from the object and are incident on the optical element and the object is actually present. Hence, it is called a real object.

#### Virtual Object

Now, consider a converging set of rays as shown in Fig. (b). If not intercepted the rays will meet at a point. However, if the rays are intercepted by an optical element placed as shown in the figure, then the point of convergence is a virtual point behind the optical element. This point is called the virtual object for the optical element.




(a) A real object placed in front of an optical element




(b) A virtual object that appears to be behind the optical element

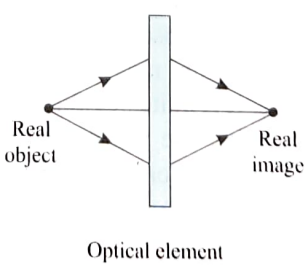
So, an object is real when two rays emanate or diverge from a point, and an object is virtual when two incident rays seem to converge to that point.

### TYPES OF IMAGES

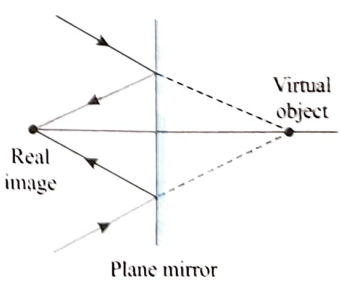
An image is the point of convergence or apparent point of divergence of rays after they interact with a given optical element. An object provides rays that will be incident on an optical element. The optical element reflects or refracts the incident light rays which then meet at a point to form an image. As in the case of objects, images too can be real or virtual.

#### Real Image

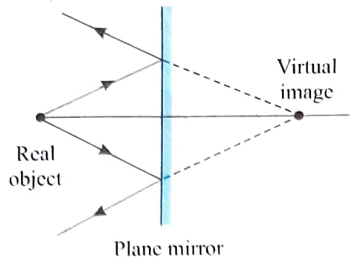
Real images are formed when the reflected or refracted rays actually meet or converge to a point. If a screen is placed at that point, a bright spot will be visible on the screen. Thus, a real image can be captured on a screen. Examples of real images are shown in Figs. (a) and (b). Note that in the former the object is real while in the latter the object is virtual. Thus, both real and virtual objects can form real images.




(a) A real image formed by a real object placed in front of the convex lens




(b) A real image formed by a virtual object that appears to be behind the mirror




(c) A virtual image formed by a real object in front of the mirror

### Virtual Image

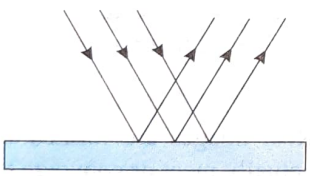
When light rays, after interacting with the optical element, actually meet at a point the image formed is a real image. However, if the rays do not meet at a point but appear to emanate from a point, then a virtual image is formed. Consider the case of an object placed in front of a plane mirror. Here, the two reflected rays will never meet at any point. However, if we extend the reflected rays backwards, they appear to emanate from a point. This point is the virtual image of the object. As we will see later in the section on lenses, it is possible for a virtual image to be formed by a virtual object as well. Thus, we can conclude that both virtual and real images can be formed by either real or virtual objects depending on the optical element.

## REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called **reflection of light**.

### REGULAR REFLECTION

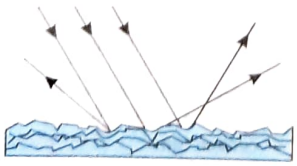
When the reflection takes place from a perfect plane surface it is called **regular reflection** [see Fig. (a)]. In this case, the reflected light has large intensity in one direction and negligibly small intensity in other directions.



(a) Regular reflection

### DIFFUSED REFLECTION

When the surface is rough, we do not get a regular behaviour of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface. It is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position. Such a reflection is called as **diffused reflection** [see Fig. (b)]. For example, reflection from a wall, from a newspaper, etc. This is why you cannot see your face in the newspaper and on the wall.

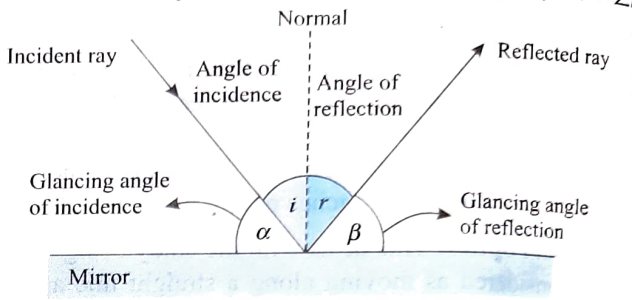


(b) Diffused reflection

## LAWS OF REFLECTION

Reflection of light is the process of deflecting a beam of light in the same medium. It has been found experimentally that rays undergoing reflection follow two laws called the Laws of reflection:

1. The incident ray, the reflected ray, and the normal at the point of incidence lie in the same plane. This plane is called the **plane of incidence (or plane of reflection)**.
2. The **angle of incidence** (the angle between normal and the incident ray) and the **angle of reflection** (the angle between the reflected ray and the normal) are equal, i.e., $\angle i = \angle r$.

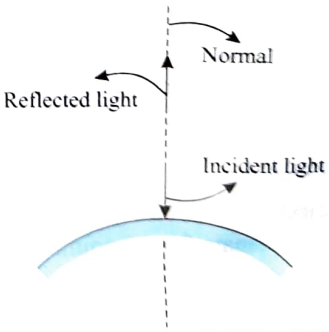



**Important Points:**

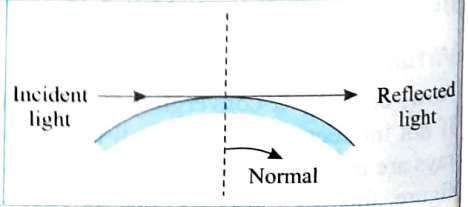
- The angle between an incident ray (on a plane of surface) and the surface is called the **glancing angle of incidence**. Simply it is the angle made by the incident ray with the plane of the surface (on which the light is incident)
- The angle between a surface and reflected ray is called **glancing angle of reflection**. Simply it is the angle made by the reflected light with the plane of the surface (from which the light is reflected)
- In case of reflection,
  **Glancing angle of incidence ($\alpha$) = Glancing angle of reflection ($\beta$)**

**Special Cases**

**Normal incidence:**
In case light is incident normally
$i = r = 0$




**Grazing incidence:**
In case light strikes the reflecting surface tangentially
$i = r = 90°$



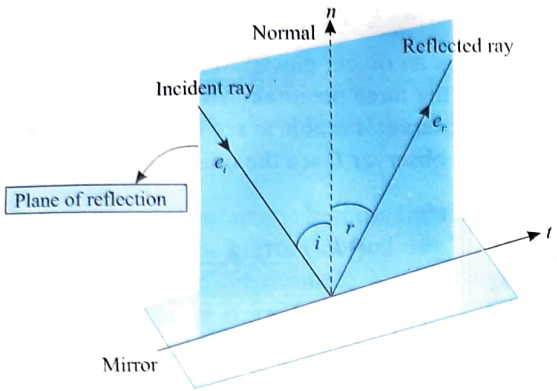
## VECTOR FORM OF THE LAW OF REFLECTION

Let, $\hat{e}_i$ = unit vector along the incident ray,

$\hat{e}_r$ = unit vector along the reflected ray,

$\hat{t}$ = unit vector along tangential direction,

$\hat{n}$ = unit vector along outward normal




Let, $\angle i = \angle r = \angle\theta$

$$\hat{e}_i = \sin\theta\,\hat{t} - \cos\theta\,\hat{n} \quad \ldots(i)$$

$$\hat{e}_r = \sin\theta\,\hat{t} + \cos\theta\,\hat{n} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\hat{e}_r - \hat{e}_i = 2\cos\theta\,\hat{n} \quad \ldots(iii)$$

Also, $\hat{e}_i \cdot \hat{n} = (1)(1)\cos(\pi - \theta) = -\cos\theta$

By putting the value of cos θ in Eq. (iii),

$$\hat{e}_r - \hat{e}_i = -2(\hat{e}_i \cdot \hat{n})\hat{n}$$

The unit vector along the reflected ray, $\hat{e}_r = \hat{e}_i - 2(\hat{e}_i \cdot \hat{n})\hat{n}$

It is the vector form of the law of reflection.

### ILLUSTRATION 1.1

A ray of light is incident on a plane mirror along a vector $a\hat{i} + b\hat{j} - c\hat{k}$. The normal on incidence point is along $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$. Find a unit vector along the reflected ray.

**Sol.** Unit vector in the direction of incident ray $\hat{e}_i = \frac{a\hat{i} + b\hat{j} - c\hat{k}}{\sqrt{a^2+b^2+c^2}}$ and we are given the unit vector in the direction of normal $\hat{n} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$.

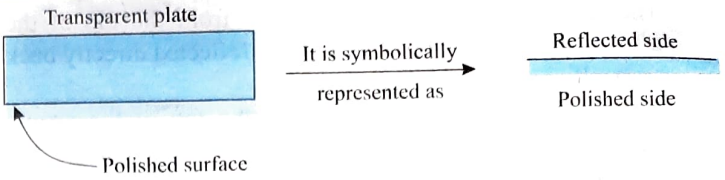
The unit vector along the reflected ray, $\hat{e}_r = \hat{e}_i - 2(\hat{e}_i \cdot \hat{n})\hat{n}$

$$\therefore \quad \hat{e}_r = \frac{a\hat{i} + b\hat{j} - c\hat{k}}{\sqrt{a^2+b^2+c^2}} - 2\left[\frac{a+b}{\sqrt{a^2+b^2+c^2}\sqrt{2}}\right]\frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

$$\Rightarrow \quad \hat{e}_r = \frac{1}{\sqrt{a^2+b^2+c^2}}[-b\hat{i} - a\hat{j} - c\hat{k}]$$

# REFLECTION FROM A PLANE SURFACE: PLANE MIRROR

A plane mirror is formed by polishing one surface of a plane thin glass plate (see figure). It is also said to be silvered on one side.

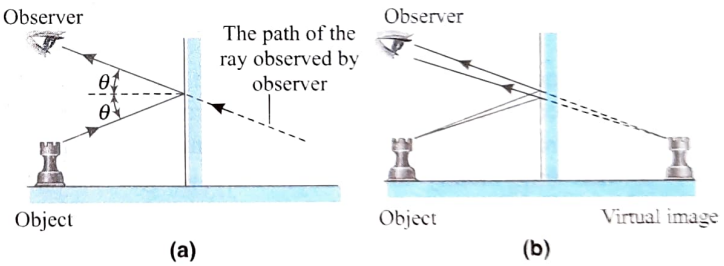



A beam of parallel rays of light, incident on a plane mirror, will get reflected as a beam of parallel reflected rays.

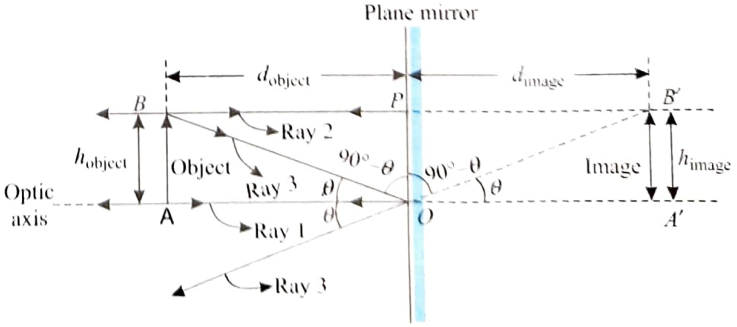
## IMAGE FORMATION FROM PLAIN MIRROR

If you stand in front of a plain mirror, you will see an image of yourself that appears to be behind the mirror. This perception occurs because your brain assumes that light rays are reaching to your eyes have traveled in straight lines with no change in direction. Thus, if light rays appear to originate at a point (say behind the mirror), your eye observes a source of light at that point, whether or not an actual light source is there. This type of image, from which the light does not emanate, is referred to as a **virtual image.** By their nature, virtual images cannot be displayed on a screen.

Now let us observe, why an image appears to originate from behind a plane mirror. In Fig. (a), you can observe a light ray travelling from the top of an object. After reflection from the mirror (angle of reflection equals angle of incidence) the ray enters the eye of the observer. To the observer, it appears that the ray originates from behind the mirror, somewhere back along the dashed line. In fact, here the rays going in all directions leave each point on the object, but only a small bundle of such rays is intercepted by the eye. In Fig. (b), you can observe a bundle of two rays leaving the top of the object. All the rays that leave a given point on the object, no matter what angle $\theta$ they have when they strike the mirror, appear to originate from a corresponding point on the image behind the mirror (see the dashed lines in Fig. (b).




For image construction, let us place an object (an arrow) with height $h_{object}$ a distance $d_{object}$ from the mirror. The object is oriented so that the tail of the arrow is on the **optic axis**, which is defined as a normal to the plane of the mirror.



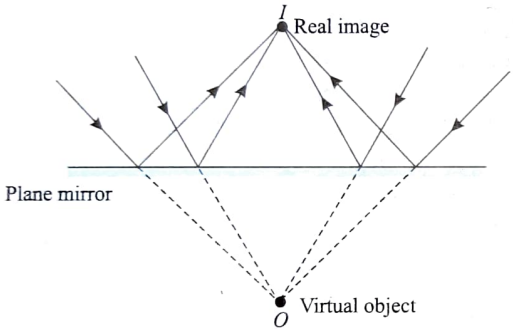

1. The first light ray (Ray-1), starts from the bottom of the arrow along the optic axis. This ray is reflected directly back on itself.

2. The second light ray (Ray-2), starts from the top of the arrow parallel to the optic axis and is reflected directly back on itself.
3. The third ray (Ray-3), starts from the top of the arrow, strikes the mirror where the optic axis intersects the mirror, and is reflected with an angle equal to its angle of incidence.

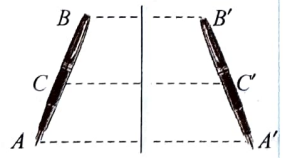
The extrapolations of ray-2 and ray-1 intersect at the point where the top of the image forms (see figure). It turns out that all rays from the arrow tip that hit the mirror—not just the two rays shown here—extrapolate such that they intersect at the image. Thus, the mirror produces a virtual image on the opposite side of the mirror. This image has a height $h_{image}$ and is located a distance $d_{image}$ to the right of the mirror. In figure the triangle $BOP$ is congruent with the triangle $B'OP$. Thus, $h_{image} = h_{object}$ i.e. the size of the image is the same as that of the object and $|d_{image}| = |d_{object}|$ i.e. distance of object from mirror is equal to the distance of image from the mirror.

**Important Points:**

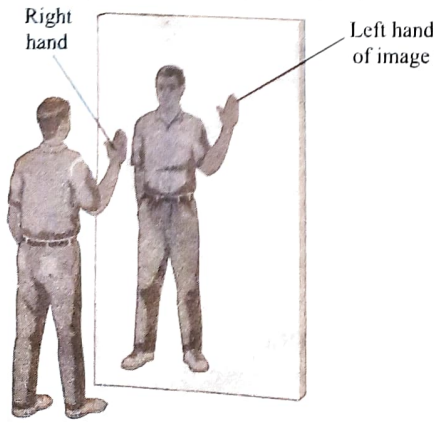
- For a real object the image is virtual and for a virtual object the image is real.



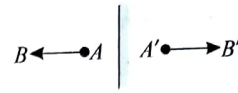

- An extended object like $AB$ shown in figure can be considered as a combination of infinite number of point objects from $A$ to $B$. Image of every point object will be formed individually and thus infinite images will be formed. $A'$ will be image of $A$, $C'$ will be image of $C$, $B'$ will be image of $B$, etc. All point images together form an extended image. Thus, extended image is formed of an extended object.




- The image is upright, if the extended object is placed parallel to the plane mirror (see figure).

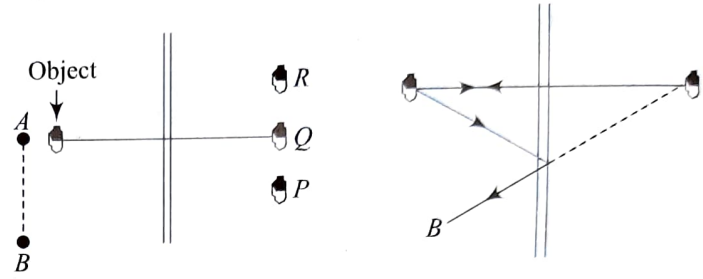



- The image is inverted, if the extended object lies perpendicular to the plane mirror (see figure).




## ILLUSTRATION 1.2

Figure (a) shows an object placed in front of a plane mirror. $P$, $Q$ and $R$ are the three positions where the image of object may be seen. Observer $A$ is able to see the image at position $Q$. Where does the observer $B$ see the image of the object?

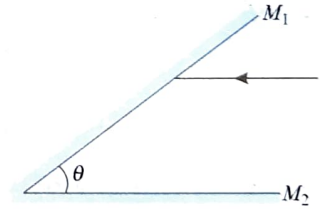



(a) (b)

**Sol.** Observer $B$ also observes the image of the object at the position $Q$ which is explained by the ray diagram in Fig. (b). The position of image will be independent of the position of the observer.

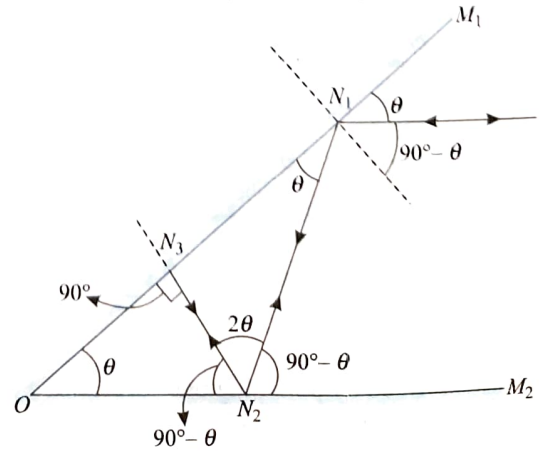
## ILLUSTRATION 1.3

Two plane mirrors $M_1$ and $M_2$ are inclined at angle $\theta$. A light ray incident travelling parallel to the surface of mirror $M_2$ and incident on mirror $M_1$. What should be the value of $\theta$ so that light ray retraces its path after third reflection.




**Sol.** If light ray retraces its path, it should strike one of the mirrors normally. In the question it is given that after third reflection the ray retraces its path. It means, the ray should incident on mirror $M_1$ normally after reflection from mirror $M_2$.

Initially the ray strikes the mirror $M_1$ at point $N_1$. Then after Ist reflection it strikes mirror $M_2$ at point $N_2$.



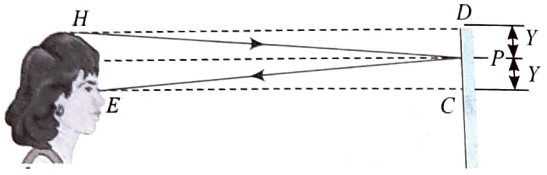
At point $N_2$, the ray reflects again and finally it strikes again the mirror $M_1$ normally at point $N_3$.

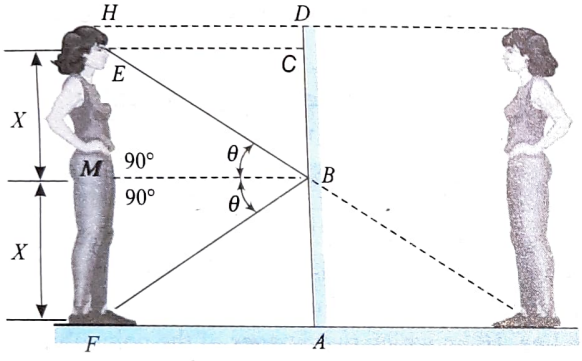
From simple geometry we can find the angle $ON_2N_3 = 90° - \theta$. It means $\angle N_1N_2M_2$ should also be $90° - \theta$, as glancing angles to the plane mirror should be equal. Hence the angle $N_1N_2N_3 = 2\theta$. In $\Delta N_1N_2N_3$: $\theta + 90° + 2\theta = 180°$ which gives $\theta = 30°$.

### ILLUSTRATION 1.4

A woman is standing in front of a plane mirror. What is the minimum mirror height that is necessary for her to see her full image.

**Sol.** The woman sees her image because light emanating from her body is reflected by the mirror (labeled *ABCD* in figure) and enters her eyes. Consider, for example, a ray of light from her foot *F*. This ray strikes the mirror at *B* and enters her eyes at *E*. According to the law of reflection, the angles of incidence and reflection both are $\theta$.

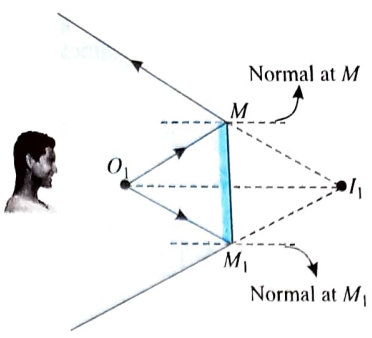


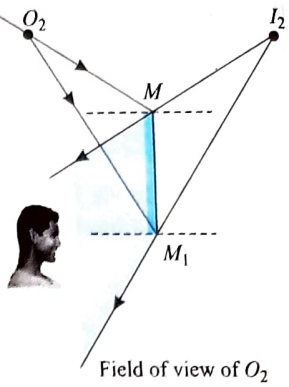

The section *AB* of the mirror is not necessary in order for the woman to see her full image. The section *BC* of the mirror that produces the image is one-half the woman's height between *F* and *E*. This follows because the right triangles *FBM* and *EBM* are identical. The blowup in figure illustrates a similar line of reasoning, starting with a ray from the woman's head at *H*. This ray is reflected from the mirror at *P* and enters her eyes. The top mirror section *PD* may be removed without disturbing this reflection. The necessary section *CP* is one-half the woman's height between her head at *H* and her eyes at *E*. We find, then, that only the sections *BC* and *CP* are needed for the woman to see her full length. The height of section *BC* plus section *CP* is exactly one-half the woman's height. The conclusions here are valid regardless of how far the person stands from the mirror. Thus, to view one's full length in a mirror, only a half-length mirror is needed.

## FIELD OF VIEW

The field of view is said to be the area visible from the perspective of the observer through the mirror. It is the region where diverging rays from object or image are present. If our eyes are present in field of view and receive diverging rays, then only we can see image. The field of view of image is decided by drawing extreme reflected or refracted rays.




Field of view of $O_1$

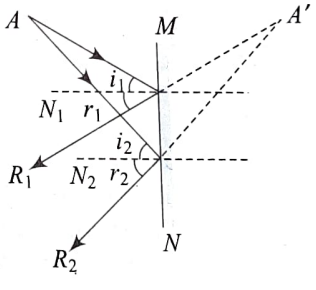


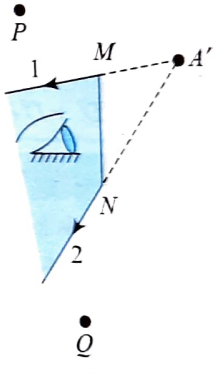

Field of view of $O_2$

### ILLUSTRATION 1.5

Figure shows a point object *A* and a plane mirror *MN*. Find the position of the image of object *A*, in mirror *MN*, by drawing ray diagram. Indicate the region in which the observer's eye must be present in order to view the image. (This region is called field of view.)

**Sol.** Consider any two rays emanating from the object [see Fig. (a)]. $N_1$ and $N_2$ are normals; $i_1 = r_1$ and $i_2 = r_2$.



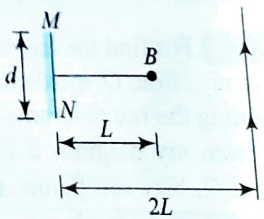




**(a)** **(b)**

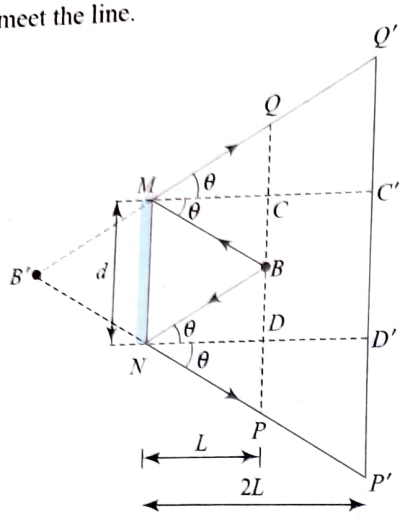
The meeting point of reflected rays $R_1$ and $R_2$ is image $A'$. Though only two rays are considered it must be understood that all rays from *A* reflect from mirror *MN* such that their meeting point is $A'$. To obtain the region in which reflected rays are present, join $A'$ with the ends of mirror and extend. Figure (b) shows this region as shaded. In the figure, there are no reflected rays beyond the rays 1 and 2, therefore the observers *P* and *Q* will not be able to see the image because they do not receive any reflected ray.

### ILLUSTRATION 1.6

A point source of light *B* is placed at a distance *L* in front of the centre of a mirror of width *d* hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2*L* from it, as shown in figure. Find the greatest distance over which he can see the image of the light source from the mirror.



**Sol.** Draw the rays $BM$ and $BN$ incident on the mirror edges, and draw the reflected rays (see figure). From the figure, it is clear that the man can observe in the angular region $P'B'Q'$. The observer can see through the distance $P'Q'$, where reflected ray from $B$ can meet the line.




From geometry, $CD = d = C'D'$

$BD = DP = d/2$; $CB = QC = d/2$

$P'Q' = C'D' + P'D' + Q'C'$

$$\Rightarrow \quad \frac{ND}{DP} = \frac{ND'}{D'P'} \Rightarrow \frac{L}{d/2} = \frac{2L}{D'P'} \Rightarrow D'P' = d$$

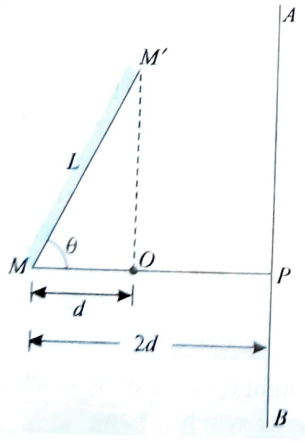
Similarly, $Q'C' = d$

Hence, distance through which the man can observe the image is

$$Q'C' + C'D' + D'P' = d + d + d = 3d$$

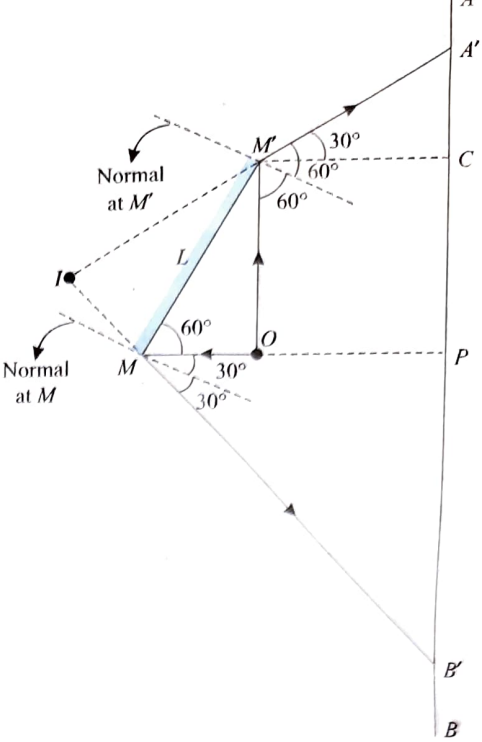
## ILLUSTRATION 1.7

A point object $O$ is kept in front of a plane mirror $MM'$. The line $MOP$ makes an angle $\theta = 60°$ with the mirror. An observer is walking along the line $APB$ (perpendicular to $MOP$. Find the length of his path along $APB$ in which he can see the image of the object. Given $MO = d$, $MP = 2d$ and length of the mirror $L = 2d$.




**Sol.** For find the length of path on the line $AB$, let us consider two rays from $O$, incident on the extreme ends of the mirror. Now plotting the ray diagram.

From ray diagram it is clear that one can see the image of point $O$, between points $A'$ and $B'$ on the line $APB$. Let us now calculate this length.




In $\Delta MPB'$, $PB' = MP \tan 60° = 2d \times \sqrt{3} = 2\sqrt{3}\, d$

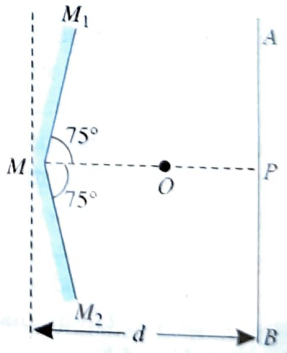
The length of section $A'P = A'C + CD$

$$A'P = d \tan 30° + L \sin 60° = d \times \frac{1}{\sqrt{3}} + 2d \times \frac{\sqrt{3}}{2} = \frac{4}{\sqrt{3}} d$$

Hence $A'B' = A'P + PB' = \frac{4}{\sqrt{3}} d + 2\sqrt{3}\, d = \frac{10}{\sqrt{3}} d$

## ILLUSTRATION 1.8

Two large plane mirrors $MM_1$ and $MM_2$ are arranged at 150° as shown in the figure. $O$ is a point object and $AB$ is a long line perpendicular to the line $OP$. Find the length of the part of the line $AB$ on which two images of the point object $O$ can be seen.




**Sol.** The images of the point object $O$, formed by both the mirrors can be seen where the field of view of both the images overlap.

From ray diagram it is clear that overlapping region for field of view should be between $A''B'$.

Hence length of section $A''B' = 2PB'$

In $\Delta MPB'$ : $\frac{PB'}{MP} = \tan 30° \Rightarrow PB' = MP \tan 30°$

or $\quad PB' = d \times \frac{1}{\sqrt{3}} = \frac{d}{\sqrt{3}}$

Hence $A''B' = 2 \times \frac{d}{\sqrt{3}} = \frac{2d}{\sqrt{3}}$

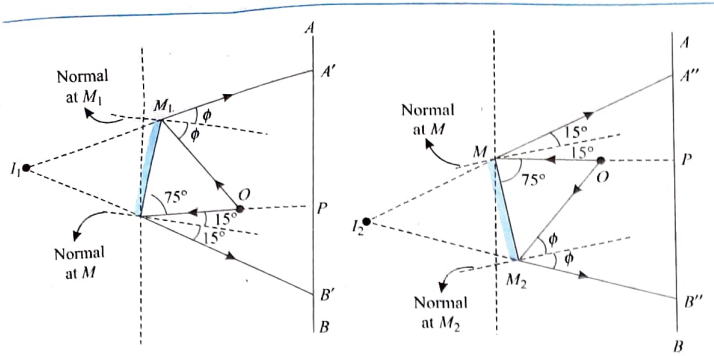
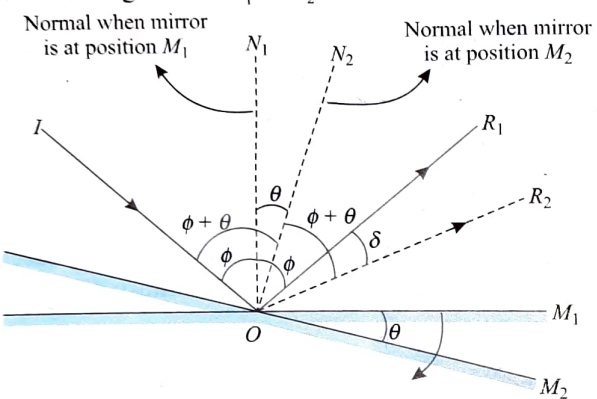



Image of $O$ formed by mirror $MM_1$ Image of $O$ formed by mirror $MM_2$

## Effect of Rotation of Plane Mirror

Direction of incident ray $I$ is kept fixed while mirror $M_1$ is rotated by angle $\theta$ to position $M_2$. Reflected ray $R_1$ changes to position $R_2$ and normal changes from $N_1$ to $N_2$.




From figure it is clear that $\angle ABC = 2\phi + \delta = 2(\phi + \theta)$

or $\delta = 2\theta$

That means when incident ray is fixed, and mirror rotates through an angle $\theta$:

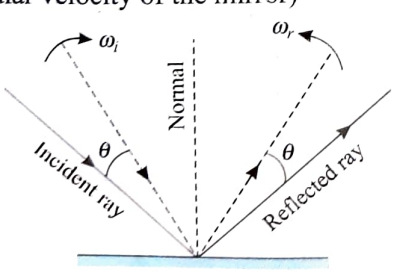
(a) Then reflected ray rotates through the angle $2\theta$ in the same sense as the mirror rotates.

(b) The angular velocity and angular acceleration of new reflected ray becomes twice as that of mirror.

$\omega_r = 2\omega_m$ and $\alpha_r = 2\alpha_m$

($\omega_r$ = angular velocity of reflected ray

$\omega_m$ = angular velocity of the mirror)




**Note:** If mirror is fixed and incident ray rotates:

$\omega_i$ = angular velocity of the incident ray

$\omega_r$ = angular velocity of reflected ray

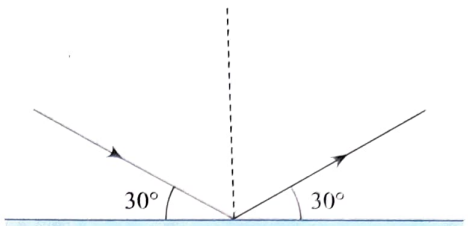
Then, $\omega_i = -\omega_r$

Negative sign represents that the direction of angular velocity is possible to each other.

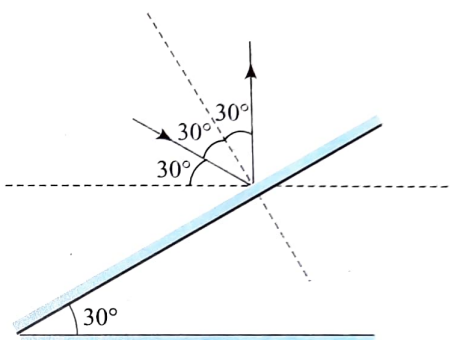
### Illustration 1.9

A light ray is incident on a plane mirror at an angle of 30° with the horizontal. At what angle with horizontal must a plane mirror be placed in its path so that it becomes vertically upwards after reflection?

**Sol.** To make the reflected ray vertical, it should be rotated through an angle of 60°. So, the mirror should be tilted by 60°/2 = 30°, as shown in Fig. (b).




(a)




(b)

## Angle of Deviation

The angle of deviation is the angle made by the reflected ray with the direction of incident ray. Let us learn calculation of deviation angle through an illustration.

### Illustration 1.10

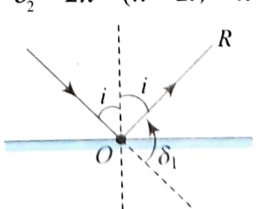
Show that for a light ray incident at an angle $i$ on getting reflected the angle of deviation is $\delta = \pi - 2i$ or $\pi + 2i$.

**Sol.** From Fig. (b), it is clear that light ray bends either by $\delta_1$ anticlockwise or by $\delta_2$ $(= 2\pi - \delta_1)$ clockwise.

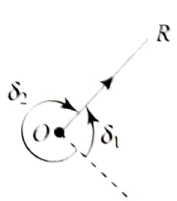
From Fig. (a),

$\delta_1 = \pi - 2i$

$\therefore$ $\delta_2 = 2\pi - (\pi - 2i) = \pi + 2i$




(a) Original direction of propagation




(b) Original direction of propagation

## Finding Angle of Deviation in Case of Several Mirrors

If several plane mirrors are placed on the path of a light ray, each mirror deviates the ray which can be clockwise or anticlockwise. Net deviation is the algebraic sum of the deviations due to the individual mirrors. In such cases, clockwise and anticlockwise deviations are given opposite signs. We take anticlockwise deviations as positive and clockwise deviations is taken as negative.

### Illustration 1.11

Two plane mirrors are inclined to each other at an angle θ. A ray of light is reflected first at one mirror and then at the other. Find the total deviation of the ray.

**Sol.** Let,

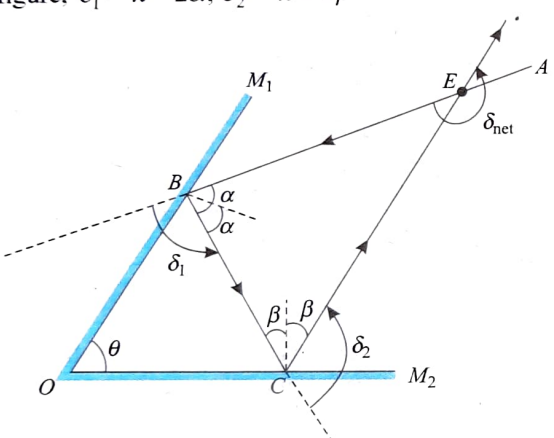
$\alpha$ = Angle of incidence for $M_1$

$\beta$ = Angle of incidence for $M_2$

$\delta_1$ = Deviation due to $M_1$

$\delta_2$ = Deviation due to $M_2$

From figure, $\delta_1 = \pi - 2\alpha$, $\delta_2 = \pi - 2\beta$




Both deviations are in same sense i.e., anticlockwise

$\delta_{net}$ = Total deviation $= \delta_1 + \delta_2 = 2\pi - 2(\alpha + \beta)$

Now in $\Delta OBC$,

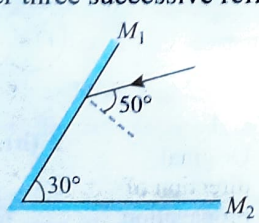
$(90° - \alpha) + (90° - \beta) + \theta = 180°$

or $\alpha + \beta = \theta$

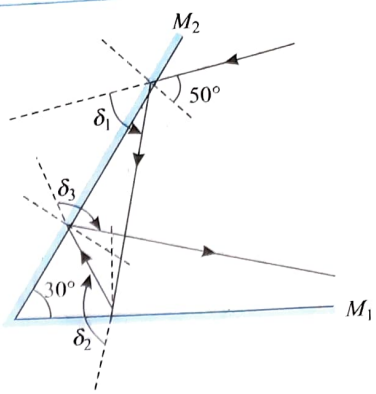
Hence, $\delta_{net} = 2\pi - 2\theta$ in anticlockwise sense and $2\theta$ in clockwise sense. which evidently depends only on $\theta$, the angle between the two mirrors.

### Illustration 1.12

Calculate deviation suffered by incident ray in situation as shown in figure after three successive reflections.




**Sol.** Net deviation is the algebraic sum of the deviations due to the individual mirrors. From figure,




$\delta_1 = 180° - 2 \times 50° = 80°$ anticlockwise

$\delta_2 = 180° - 2 \times 20° = 140°$ clockwise

$\delta_3 = 180° - 2 \times 10° = 160°$ clockwise

Total deviation $\delta = \delta_1 + \delta_2 + \delta_3$. Taking anticlockwise sense of rotation positive and clockwise rotation as negative, we get

$\delta = 80° - 140° - 160° = -220°$

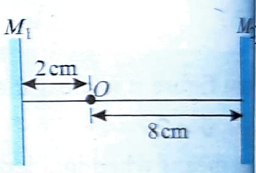
220° clockwise or 140° anticlockwise.

## Images Formed by Two Plane Mirrors

If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will act as an object for second mirror, and this process will continue for every successive reflection. Let us understand this with the illustration discussed below.

### Illustration 1.13

Figure shows a point object placed between two parallel mirrors. Its distance from $M_1$ is 2 cm and that from $M_2$ is 8 cm. Find the distance of images from the two mirrors considering reflection on mirror $M_1$ first.



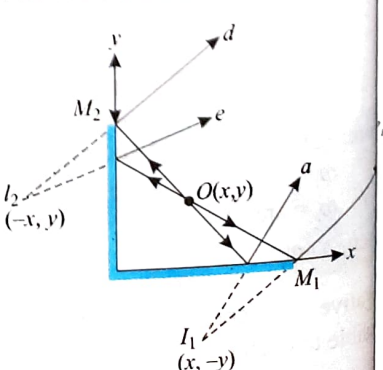

**Sol.** The image formed from the object will act as an object for next reflection. This process will continue again and again up to infinite reflection. Total number of images = ∞.

### Illustration 1.14

Consider two perpendicular mirrors $M_1$ and $M_2$ and a point object $O$. Taking origin at the point of intersection of the mirrors and the coordinates of object as $(x, y)$, find the position and number of images.

**Sol.** We will first consider reflection from mirror-1 ($M_1$).

Let two rays $a$ and $b$ strike mirror $M_1$ and these rays will form image $I_1$ at $(x, -y)$, such that $O$ and $I_1$ are equidistant from mirror $M_1$. These rays do not form further image because they do not strike any mirror again. Now consider rays '$d$' and '$e$' strike mirror $M_2$ only and

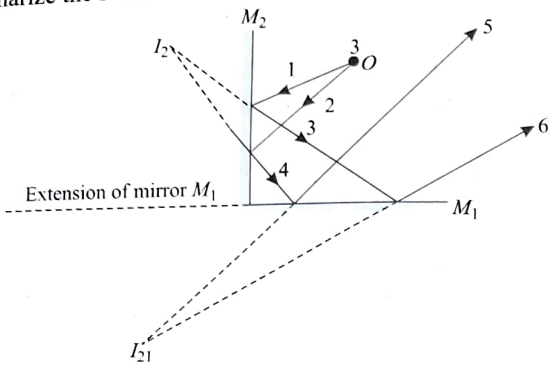


these rays will form image $I_2$ at $(-x, y)$, such that $O$ and $I_2$ are equidistant from mirror $M_2$.

Now, let us consider those rays which strike mirror $M_2$ first and then mirror $M_1$. For incident rays 1 and 2 object is $O$, and the reflected rays 3 and 4 form image $I_2$.

Now these reflected rays 3 and 4 incident on $M$, for which the object is $I_2$, reflect as rays 5 and 6 and form image $I_{21}$. Rays 5 and 6 do not strike any mirror further, so image formation stops.

The images $I_2$ and $I_{21}$ are equidistant from $M_1$. Let us now summarize the situation in a single figure.




For rays reflecting first from $M_1$ and then from $M_2$, first image $I_1$ (at $(x, -y)$) will be formed. $I_1$ will act as object for mirror $M_2$ and then its image $I_1$ (at $(-x, -y)$) will be formed. $I_{12}$ and $I_{21}$ coincide. Hence, three images are formed.

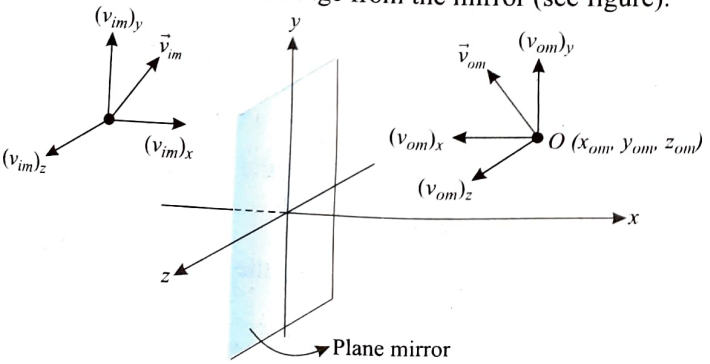
**Important Points:**

**Number of images formed by two inclined mirrors:**

1. If $\frac{360^\circ}{\theta}$ = even number; number of images $= \frac{360^\circ}{\theta} - 1$
2. If $\frac{360^\circ}{\theta}$ = odd number; number of images $= \frac{360^\circ}{\theta} - 1$, if the object is placed on the angle bisector.
3. If $\frac{360^\circ}{\theta}$ = odd number; number of images $= \frac{360^\circ}{\theta}$, if the object is not placed on the angle bisector.
4. If $\frac{360^\circ}{\theta} \neq$ integer, then count the number of images as explained above.

## Relation Between Velocity of Object and Image

In case of plane mirror, the distance of the object from the mirror is equal to the distance of image from the mirror (see figure).




Hence, from the mirror property:

$$x_{im} = -x_{om}, \quad y_{im} = y_{om} \text{ and } z_{im} = z_{om}$$

Here, $x_{im}$ means $x$-coordinate of image with respect to mirror. Similarly, others have also meaning.

Differentiating w.r.t. time, we get

$$\vec{v}_{(im)x} = -\vec{v}_{(om)x}; \quad \vec{v}_{(im)y} = \vec{v}_{(om)y}; \quad \vec{v}_{(im)z} = \vec{v}_{(om)z}$$

Here, $\vec{v}_{(om)}$ = velocity of the object w.r.t. mirror

$\vec{v}_{(im)}$ = velocity of the image w.r.t. mirror

$$\Rightarrow \quad \vec{v}_i - \vec{v}_m = -\left(\vec{v}_0 - \vec{v}_m\right)$$

Here, $\vec{v}_i$ = velocity of image with respect to ground

$\vec{v}_o$ = velocity of object w.r.t. ground

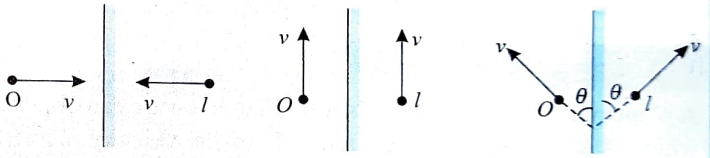
If mirror is at rest $\vec{v}_m = 0 \Rightarrow \vec{v}_i = -\vec{v}_o$

In the direction normal to the mirror = | Relative velocity of image w.r.t. mirror | = | Relative velocity of object w.r.t. mirror |

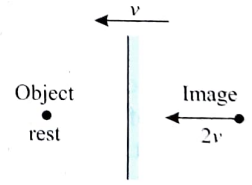
The velocity of object is equal to the velocity of image parallel to the mirror surface.

**Important Points:**

- With respect to plane mirror image speed is equal to the object speed.
- With respect to plane mirror image velocity and object velocity make equal angles from the plane mirror on two opposite sides of the mirror.



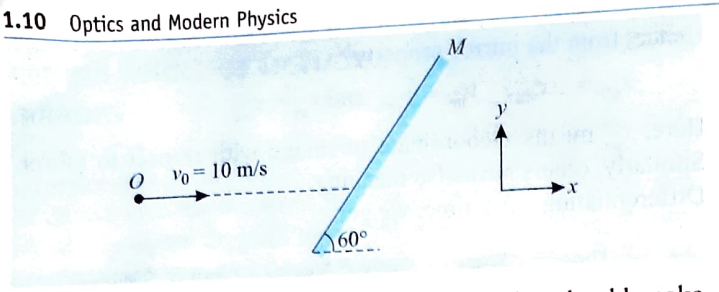

- Components of velocities of the object and image with respect to mirror which are along the mirror are equal.
- Components of velocities of the object and image with respect to mirror which are perpendicular to the mirror are equal and opposite.
- If a plane mirror moves towards a stationary object with speed $v$, the image moves towards the object with speed $2v$.




- If an angle moves towards (or moves away) from a stationary plane mirror with speed $v$, the image will also moves towards (or moves away) with same speed. The relative velocity of the image with respect to object will be $2v$.

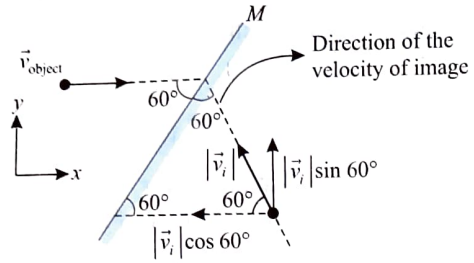
### Illustration 1.15

An object is moving horizontally with velocity 10 m/s towards a fixed plane mirror as shown in figure. Find the image velocity.




**Sol.** The image velocity and object velocity should make equal angle from the plane mirror on two opposite sides of the mirror, and speed of the object and image will be equal i.e.,

$$|\vec{v}_{i,m}| = |\vec{v}_{o,m}|$$




As mirror is at rest hence

$$|\vec{v}_i| = |\vec{v}_o|$$

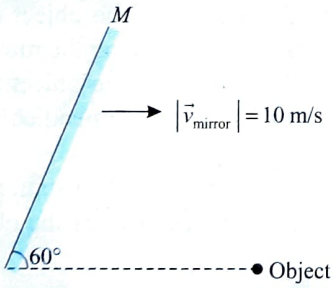
$$\vec{v}_{image} = |\vec{v}_i|\cos 60°(-\hat{i}) + |\vec{v}_i|\sin 60°\,\hat{j}$$

$$= 10\times\frac{1}{2}(-\hat{i}) + 10\times\frac{\sqrt{3}}{2}(\hat{j})\,(\text{m/s})$$

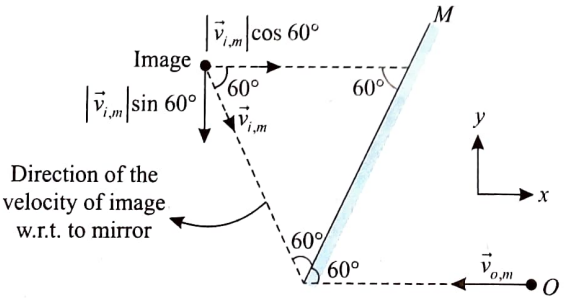
$$\vec{v}_{image} = -5\hat{i} + \sqrt{3}\,\hat{j}\ (\text{m/s})$$

## ILLUSTRATION 1.16

A plane mirror is moving with speed 10 m/s towards an object which is at rest as shown in figure. Find the velocity of the image.




**Sol.** Let us analyse the motion of both the object and image w.r.t. mirror. In mirror frame of reference the object will move with velocity 10 m/s towards left. As we know, the image velocity and object velocity should make equal angle from the surface of the mirror on two opposite sides. The speed of the image with respect to mirror should be equal to the speed of the object with respect to mirror.




Hence $\vec{v}_{i,m} = |\vec{v}_{i,m}|\cos 60°\hat{i} + |\vec{v}_{i,m}|\sin 60(-\hat{j})$

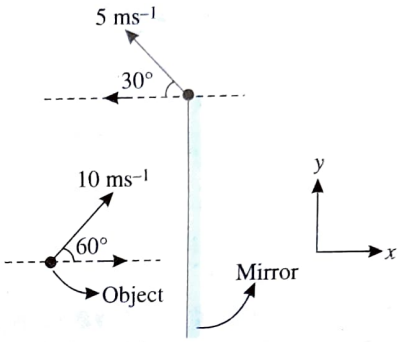
$$= 10\times\frac{1}{2}\hat{i} - 10\times\frac{\sqrt{3}}{2}\hat{j} = \left(5\hat{i} - 5\sqrt{3}\hat{j}\right)\text{m/s}$$

We can write, $\vec{v}_i = \vec{v}_{i,m} + \vec{v}_m = (5\hat{i} - 5\sqrt{3}\hat{j}) + 10\hat{i}$

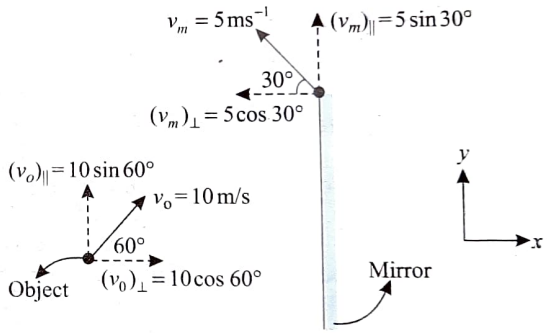
$$\Rightarrow \quad \vec{v}_{image} = (15\hat{i} - 5\sqrt{3}\hat{j})\ \text{m/s}$$

## ILLUSTRATION 1.17

Figure shows a plane mirror and an object that are moving towards each other. Find the velocity of image.




**Sol.** In the direction parallel to the surface of the mirror alo y-axis.




$$(\vec{v}_{o,m})_{\parallel} = (\vec{v}_{i,m})_{\parallel}$$

$(\vec{v}_{o,m})_{\parallel}$ = Relative velocity of the object w.r.t. mirror parall to its surface.

Hence $(\vec{v}_o - \vec{v}_m)_{\parallel} = (\vec{v}_i - \vec{v}_m)_{\parallel}$ or $(\vec{v}_o)_{\parallel} = (\vec{v}_i)_{\parallel}$

Hence velocity of the image parallel to the surface of t mirror should be

$$(\vec{v}_i)_{\parallel} = 10\sin 60°\,\hat{j} = 5\sqrt{3}\,\hat{j}\ (\text{m/s})$$

In the direction normal to the mirror or along $x$-axis.

$$(\vec{v}_{i,m})_{\perp} = -(\vec{v}_{o,m})_{\perp}$$

$(\vec{v}_{i,m})_{\perp}$ = Relative velocity of the image w.r.t. mirr perpendicular to the surface

$(\vec{v}_{o,m})_{\perp}$ = Relative velocity of the object w.r.t. mirr perpendicular to the surface

Hence $(\vec{v}_i - \vec{v}_m)_{\perp} = -(\vec{v}_o - \vec{v}_m)_{\perp} \Rightarrow (\vec{v}_i)_{\perp} = 2(\vec{v}_m)_{\perp} - (\vec{v}_0)_{\perp}$

$$(\vec{v}_i)_\perp = \left[2(-5\cos 30°\hat{i}) - (10\cos 60°\hat{i})\right] (\text{m/s})$$

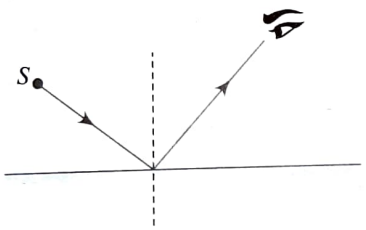
$$= -5(1+\sqrt{3})\hat{i}\ (\text{m/s})$$

Hence velocity of the image $\vec{v}_i = (\vec{v}_i)_\parallel + (\vec{v}_i)_\perp$

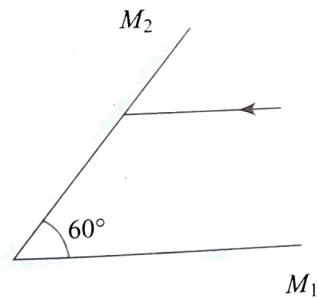
or $\vec{v}_i = -5(1+\sqrt{3})\hat{i} + 5\sqrt{3}\,\hat{j}\ (\text{m/s})$

## CONCEPT APPLICATION EXERCISE 1.1

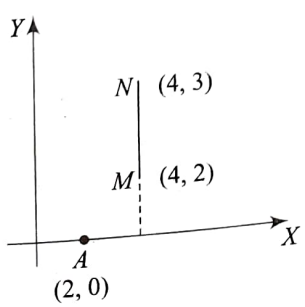
1. There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object, find the distance which the image will move.
2. Two plane mirrors are placed parallel to each other. The distance between the mirrors is 10 cm. An object is placed between the mirrors at a distance of 4 cm from one of them, say $M_1$. What is the distance between the first image formed at $M_1$ and the second image formed at $M_2$?
3. A ray of light travels from a light source $S$ to an observer after reflection from a plane mirror. If the source rotates in the clockwise direction by 10°, by what angle and in what direction must the mirror be rotated so that the light ray still strikes the observer?



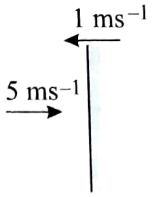

4. Two plane mirrors are inclined at an angle of 60° as shown in figure. A ray of light parallel to $M_1$ strikes $M_2$. At what angle will the ray finally emerge?



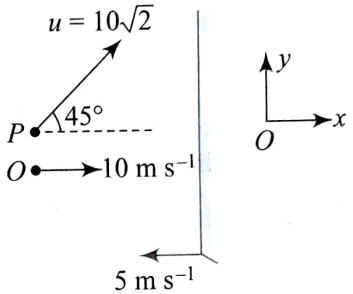

5. A man is standing at distance $x$ from a plane mirror in front of him. He wants to see the entire wall in mirror which is at distance $y$ behind the man. Find the minimum size of the mirror required.
6. Find the region on $Y$-axis in which reflected rays are present. Object is at $A$ (2, 0) and $MN$ is a plane mirror, as shown in figure.




7. An object moves with 5 ms$^{-1}$ toward right while the mirror moves with 1 ms$^{-1}$ toward the left as shown in figure. Find the velocity of image.




8. In figure, a plane mirror is moving with a uniform speed of 5 ms$^{-1}$ along negative $x$-direction and observer $O$ is moving with a velocity of 10 ms$^{-1}$. What is the velocity of image of a particle $P$, moving with a velocity as shown in the figure, as observed by observer $O$? Also find its direction.




### ANSWERS

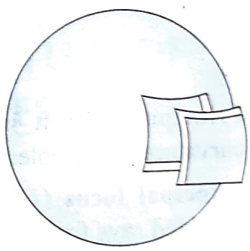
**1.** 20 cm **2.** 28 cm **3.** 5° clockwise **4.** 60°

**5.** $\frac{hx}{y+2x}$ **7.** 7 m/s **8.** $(-30\hat{i}+10\hat{j})\,\text{ms}^{-1}$; $\tan^{-1}(-1/3)$

# REFLECTION FROM A CURVED SURFACE

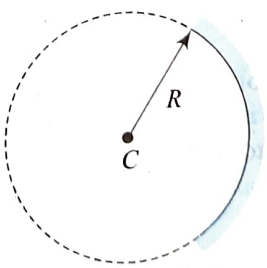
## SPHERICAL MIRRORS

A **spherical mirror** is formed by polishing one surface of a part of sphere. A spherical mirror is a reflecting surface whose shape is a section of a spherical surface (see figure).

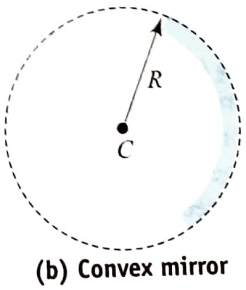


Spherical mirror

Depending upon which part is shining, the spherical mirror is classified as:




(a) Concave mirror



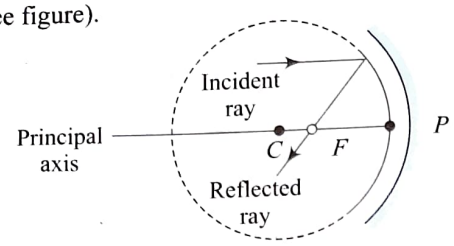

(b) Convex mirror

**Concave mirror:** If the inside surface of the mirror is polished, it is a concave mirror [see Fig. (a)].

**Convex mirror:** If the outside surface of the mirror is polished, it is a convex mirror [see Fig. (b)].

## Important Terms

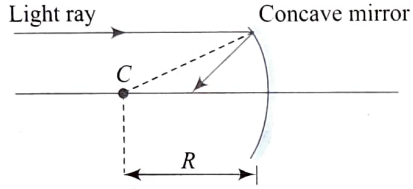
**Pole (*P*):** It is the geometrical centre of the spherical reflecting surface (see figure).



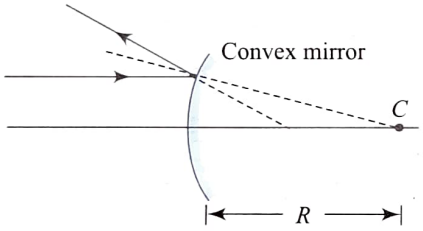

Concave mirror

A point on the surface of the mirror from where the position of the object can be specified easily is called pole. The pole is generally taken at the mid point of reflecting surface.

**Centre of curvature and radius of curvature:** The centre of the sphere of which the mirror is a part, is called **centre of curvature** (*C*) [see Figs. (a) and (b)]. The radius of the sphere of which the mirror is a part is called **radius of curvature** (*R*). A plane mirror can be treated as a special case of a spherical mirror: one which has an **infinite radius of curvature**.




(a)




(b)

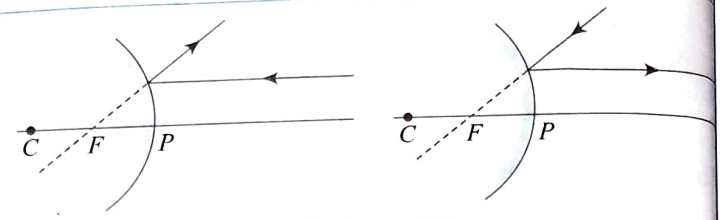
**Principal axis:** It is the straight line joining the centre of curvature to the pole.

**Principal focus (*F*):** It is the point of intersection of all the reflected rays for which the incident rays strike the mirror (with small aperture) parallel to the principal axis. In a concave mirror it is real and in a convex mirror it is virtual. The distance from pole to focus is called **focal length**.

**Aperture** (related to the size of mirror): It is the diameter of the mirror.

**Focus (*F*):** When a narrow beam of rays of light, parallel to the principal axis and close to it, is incident on the surface of a mirror, the reflected beam is found to converge to or appears to diverge from a point on the principal axis. This point is called the focus (see figures).

(a) Concave mirror

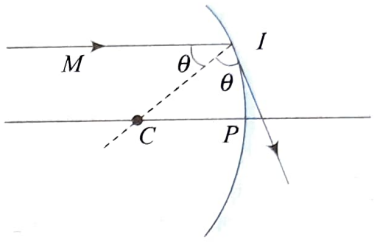



(b) Convex mirror

**Focal length (*f*):** It is the distance between the pole and th principal focus. For spherical mirrors, $F = R/2$.

### ILLUSTRATION 1.18

Find the angle of incidence of the ray shown in figure for which it passes through the pole, given that $MI \parallel CP$.




**Sol.** From the figure, the ray $MI$ is making an angle $\theta$ wit normal $IC$. From law of reflection, $\angle MIC = \angle CIP = \theta$

As $MI \parallel CP \Rightarrow \angle MIC = \angle ICP = \theta$

Now, $CI = CP$

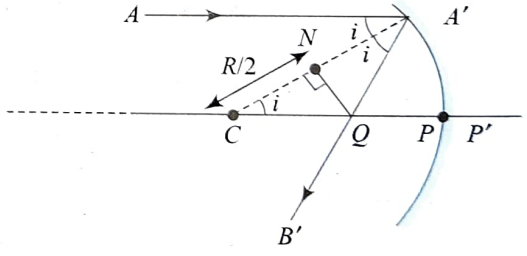
$\Rightarrow \quad \angle CIP = \angle CPI = \theta$

$\therefore \quad$ In $\Delta CIP$, all angles are equal, i.e., $3\theta = 180°$

$\Rightarrow \quad \theta = 60°$

### ILLUSTRATION 1.19

Find the distance $CQ$ if incident light ray parallel to principal axis is incident at an angle $i$. Also, find the distance $CQ$ if $i \to 0$.




**Sol.** In triangle $CNQ$, $\cos i = R/2CQ \Rightarrow CQ = R/2 \cos i$

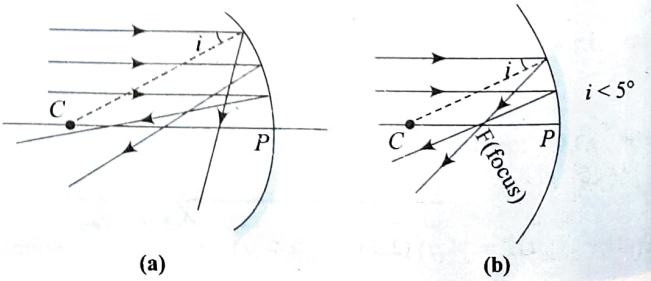
As $i$ increases cos $i$ decreases. Hence, $CQ$ increases.

If $i$ is a small angle, cos $i \approx 1$ [see Fig. (b)].

$\therefore \quad CQ = R/2$

So, paraxial rays meet at a distance equal to $R/2$ from centre o curvature, which is called focus.

**Note:**




(a) (b)

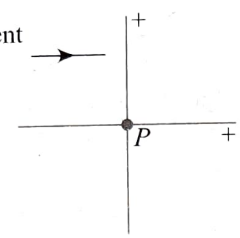
**(a)** If angle of incident $i$ is more, the rays will focus at different points on the principal axis.

**(b)** If angle of incident $i$ is small, the rays will focus at one point on principal axis. This the point is called focus.

## SIGN CONVENTION: CARTESIAN CONVENTION

We will use following sign convention for problem solving in case of reflection as well as refraction.

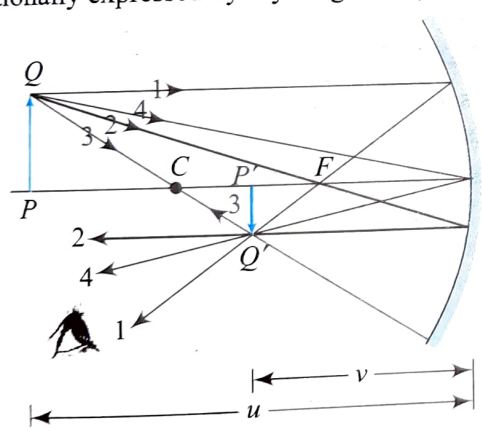
1. All distances are measured from the pole.
2. Distances measured in the direction of incident rays are taken as positive.
3. Distances measured in the direction opposite to that of the incident rays are taken as negative.
4. Distances above the principal axis are taken as positive.
5. Distances below the principal axis are taken as negative.



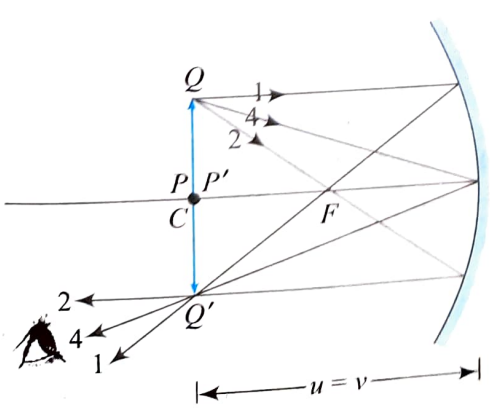

Angles measured from the normal in anticlockwise sense are positive, while that in clockwise sense are negative.

## RULES FOR RAY DIAGRAMS

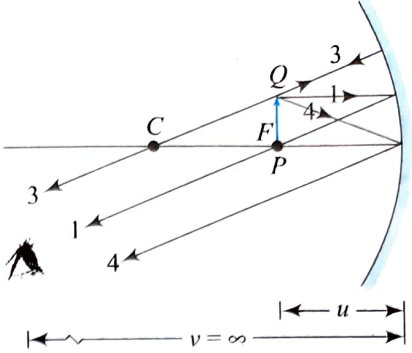
The position, size, and nature of the images formed by mirrors are conventionally expressed by ray diagrams (see figure).



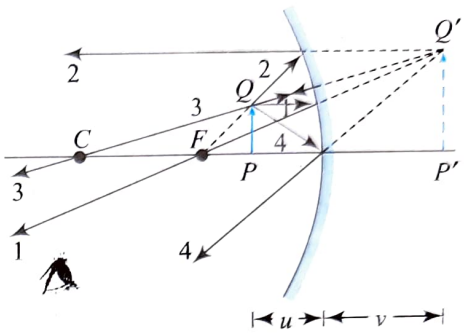

**(a)**




**(b)**




**(c)**




**(d)**

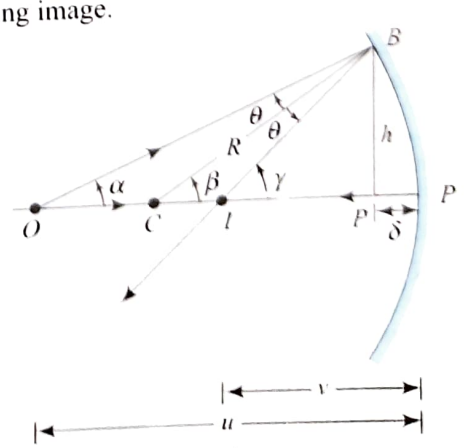
We can locate the image of any extended object graphically by drawing any two of the following four special rays:

1. A ray initially parallel to the principal axis is reflected through the focus of the mirror (1).
2. A ray passing through the centre of curvature is reflected back along itself (3).
3. A ray initially passing through the focus is reflected parallel to the principal axis (2).
4. A ray incident at the pole is reflected symmetrically (4).

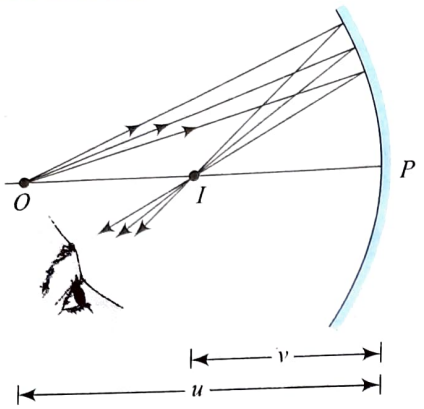
## POSITION, SIZE AND NATURE OF IMAGE FORMED BY SPHERICAL MIRRORS

### MIRROR FORMULA

Consider Fig. (a) where $O$ is a point object and $I$ is the corresponding image.




**(a)**




(b)

$CB$ is normal to the mirror at $B$. By laws of reflection,

$$\angle OBC = \angle CBI = \theta$$

$$\alpha + \theta = \beta, \quad \beta + \theta = \gamma, \quad \alpha + \gamma = 2\beta$$

For small aperture of the mirror, $\alpha, \beta, \gamma$

$$\alpha \approx \tan\alpha, \quad \beta \approx \tan\beta, \quad \gamma \approx \tan\gamma, \quad P' \to P$$

$$\Rightarrow \quad \tan\alpha + \tan\gamma = 2\tan\beta$$

$$\Rightarrow \quad \frac{BP'}{OP'} + \frac{BP'}{IP'} = 2\frac{BP'}{CP'}$$

Applying sign convention,

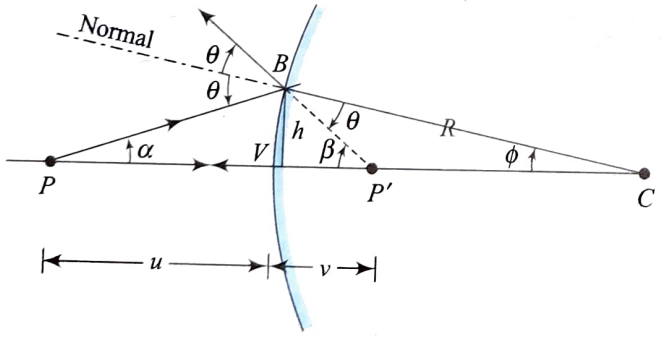
$$u = -OP, \; v = -IP, \; R = -CP$$

$$\Rightarrow \quad -\frac{1}{u} + \left(-\frac{1}{v}\right) = -\frac{2}{R}$$

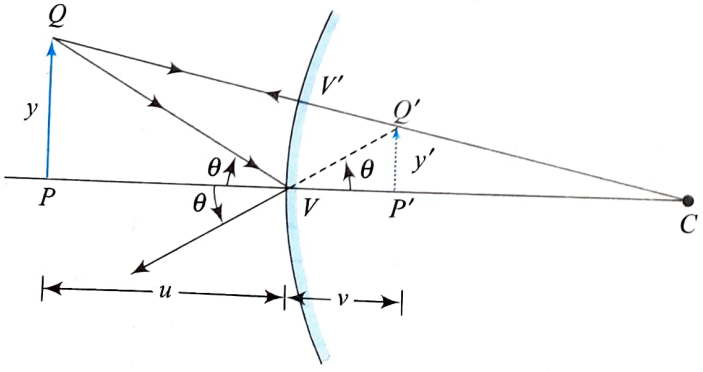
If $u = \infty, \frac{1}{v} = \frac{2}{R}$, but by definition, if $u = \infty$, $v = f$

Hence, $f = \frac{R}{2}$ and $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

For convex mirrors, an exactly similar formula emerges (see figure).




(a)



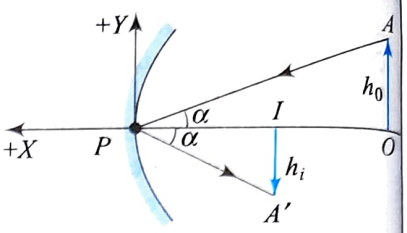

(b)

# MAGNIFICATION

The lateral magnification is defined as the ratio

$$m_v = \frac{\text{height of image}}{\text{height of object}} = \frac{h_i}{h_0}$$

To compute the vertical magnification, consider the extended object $OA$ shown in figure. The base of the object, $O$, will map on to a point $I$ on the principal axis which can be determined from the equation $(1/u) + (1/v) = (1/f)$. The image of the top of the object, $A$, will map on to a point $A'$ that will lie on the perpendicular through $I$. The exact location can be determined by drawing a ray from $A$ passing through the pole and intercepting the line through $I$ at $A'$.




**A real image of an extended object in front of a concave mirror**

Consider the triangles $APO$ and $A'PI$ in the figure. As the two triangles are similar, we get

$$\tan\alpha = \frac{AO}{PO} = \frac{A'I}{PI} \quad \text{or} \quad \frac{A'I}{AO} = \frac{PI}{PO}$$

Applying the sign convention, we get, $u = -PO$

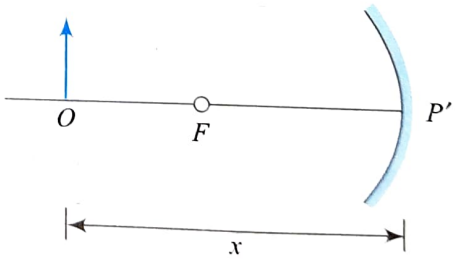
$$v = -PI \quad \Rightarrow \quad h_0 = +AO \quad \Rightarrow \quad h_i = -A'I$$

Therefore, $-\frac{h_i}{h_0} = \frac{v}{u}$ or $m_v = \frac{h_i}{h_0} = -\frac{v}{u}$

## MAGNIFICATION FROM A CONCAVE MIRROR

Figure shows a concave mirror of focal length $f_0$ in front of which an object $O$ is placed at a distance $x$ from the pole $P$. According to Cartesian sign convention, the mirror formula may be modified as $u = -x; f = -f_0$. Thus,

$$\frac{1}{v} + \frac{1}{-x} = \frac{1}{-f_0} \quad \text{or} \quad v = \frac{xf_0}{f_0 - x}$$

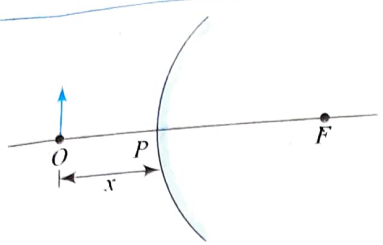



And the magnification formula may be modified as

$$m = \frac{-v}{u} = \frac{y}{x} = \frac{f_0}{f_0 - x}$$

## MAGNIFICATION FROM A CONVEX MIRROR

Figure shows a convex mirror of focal length $f_0$ in front of which an object $O$ is placed at a distance $x$ from the pole $P$. According to Cartesian aign convention, the formulae may be modified as $u = -x$ and $f = +f_0$.



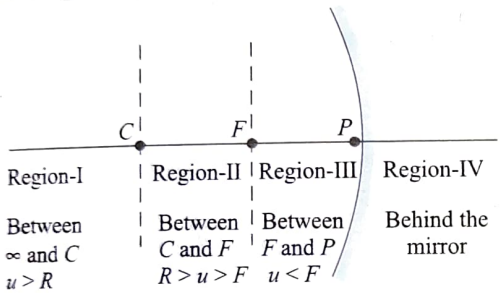

Thus, $v = \dfrac{x f_0}{f_0 + x}$.

The above expression shows that whatever may be the value of $x$ ($|x| > f_0$), $v$ is always positive and its value is always less than or equal to $f_0$. The magnification formula may be modified as $m = f_0/(f_0 + x)$. It explains that $m$ always lies between 0 and +1.

# IMAGE FORMATION

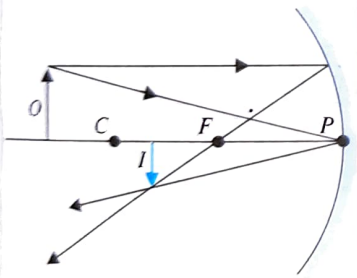
## NATURE OF IMAGE FORMED BY CONCAVE MIRROR

We can divide principal axis in four regions.



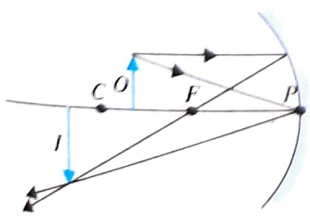

Now we analyze the corresponding image formed.

**Region-I**



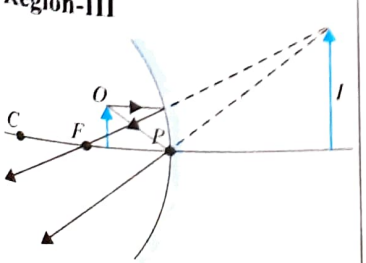

- Object placed between centre of curvature and infinity. Image real, inverted, smaller (diminished)
- If object is moved away from centre of curvature towards infinity, magnification goes on decreasing.

**Region-II**



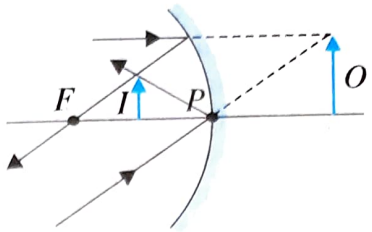

- Object placed between $f$ and centre of curvature. Image is real, inverted, magnified.
- If object is moved from focus towards centre of curvature, magnification goes on decreasing at center of curvature it becomes unity.

**Region-III**




- Object placed between pole and focus. Image is virtual, erect, magnified.
- If object is moved towards pole magnification goes on decreasing at pole magnification $m$ tends to unity.

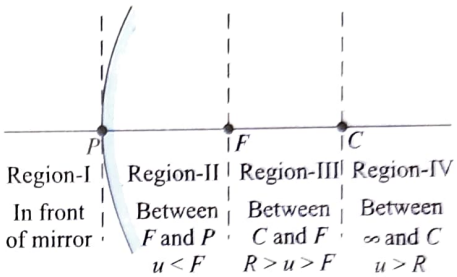
**Region-IV**




- Object is virtual point object. Image real, erect, smaller (diminished)
- As object moves away from pole, magnification goes on decreasing.

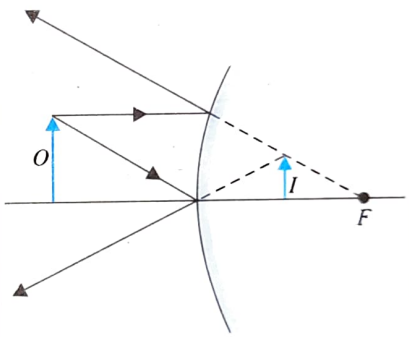
## NATURE OF IMAGE FORMED BY CONVEX MIRROR

Figure shows convex mirror with four different object positions as shown in figure.



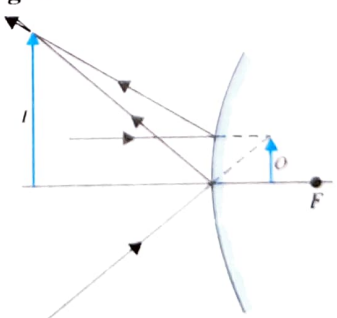

Now we analyze the corresponding image formed.

**Region-I**



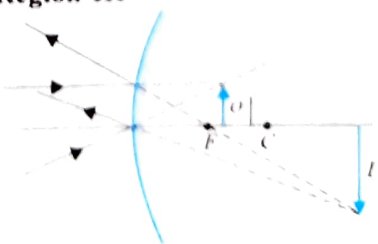

- Object placed in front of mirror. For all positions of object in front of mirror. Image is virtual, erect, smaller in size.
- As object moved towards pole, magnification increases and tends to unity at pole.

**Region-II**

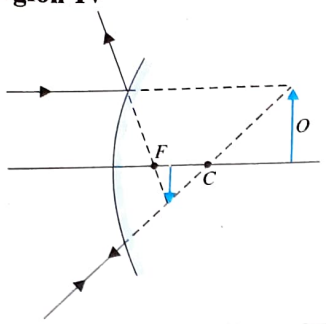



- Virtual object placed between pole and focus. Image formed is real, erect, enlarged.
- As object is moved away from pole, magnification increases.

**Region-III**

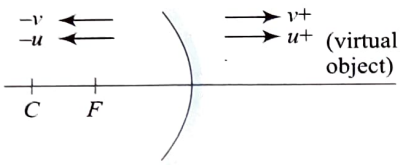



- Virtual object placed between $F$ and $C$. Image formed virtual, inverted, enlarged.
- As object is moved toward $C$ magnification decreases.

| Region-IV | |
|---|---|
|   | • Virtual object is placed beyond $C$. Image is virtual inverted smaller.<br>• As object is moved away from $C$, image size further decrease magnification decreases. |

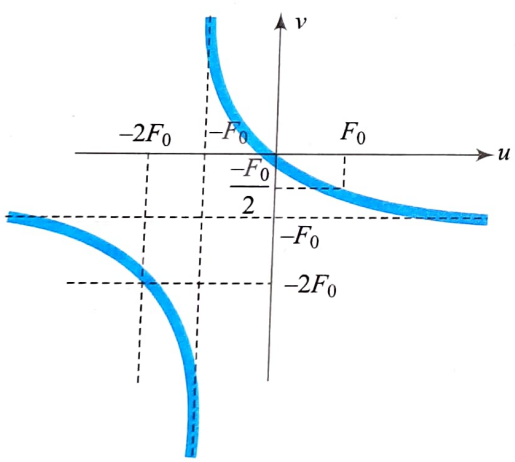
## Object Distance ($u$) vs Image Distance ($v$) Graph

**Concave mirror:** $v = \dfrac{uf}{u-f}$, $f$ is negative. Let $f = -F_0$.



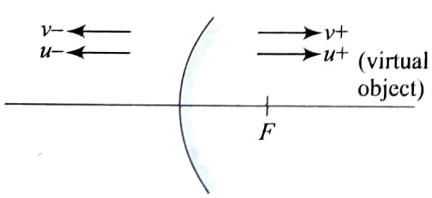

| $u$ | $-\infty$ | $-2F_0$ | $-F_0$ | 0 | $F_0$ | $\infty$ |
|---|---|---|---|---|---|---|
| $v$ | $-F_0$ | $-2F_0$ | $\pm\infty$ | 0 | $\dfrac{-F_0}{2}$ | $F_0$ |

$$v = \frac{-uF_0}{u+F_0}$$

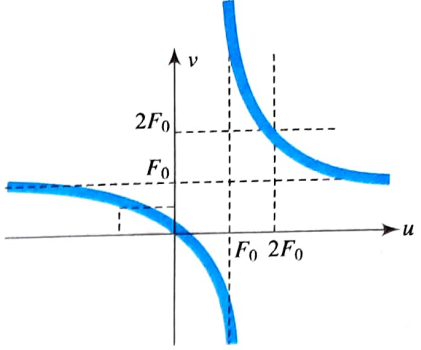



**Convex mirror:** $v = \dfrac{uf}{u-f}$

$f$ is positive. Let $f = F_0 \Rightarrow v = \dfrac{uf_0}{u-f_0}$



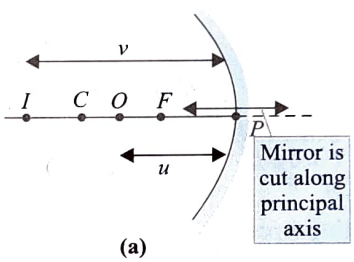

| $u$ | $-\infty$ | $-F_0$ | 0 | $F_0$ | $2F_0$ | $\infty$ |
|---|---|---|---|---|---|---|
| $v$ | $F_0$ | $\dfrac{F_0}{2}$ | 0 | $\pm\infty$ | $2F$ | $F_0$ |



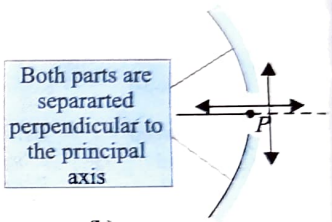

## Cutting of a Mirror

Mirror is cut along its principal axis and two parts are separated perpendicular to the principal axis.

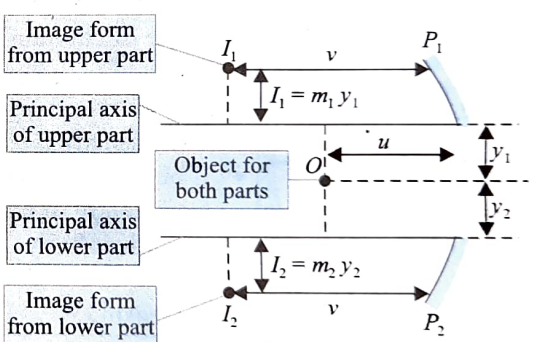
1. For first part the object $O$ will be below principal axis and for second part the object $O$ will be above principal axis.
2. For first part image will form above its principal axis, and for second part the image will form below principal axis at a distance $m_1y_1$ and $m_2y_2$, respectively.




(a)




(b)




(c)

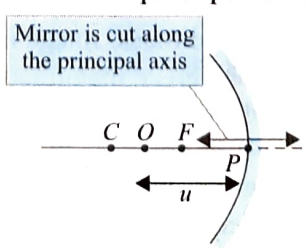
For image partition from principal axis,

$$m = -\frac{v}{u} = \frac{I_1}{O} \quad \Rightarrow \quad |I_1| = my_1$$

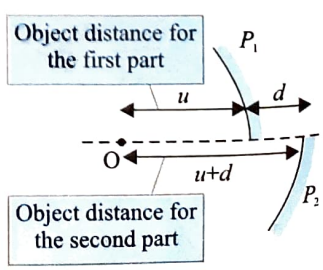
Thus, $s = my_1 + y_1 + my_2 + y_2$; $s = (m+1)(y_1 + y_2)$

If $s$ is the separation distance between two images $I_1$ and $I_2$ of the same object $O$ formed due to two halves of the mirror.

Mirror is cut along its principal axis and two parts are separated parallel to the principal axis.

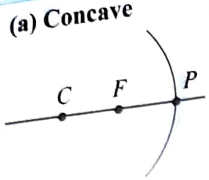
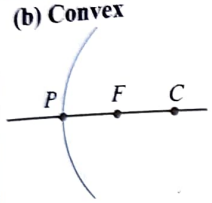



(a)




(b)

## Position, size and nature of image formed by the spherical mirror

| Mirror | Location of the object | Location of the image | Magnification, size of the image | Nature | |
|---|---|---|---|---|---|
| | | | | Real virtual | Erect inverted |
| (a) Concave   | At infinity, i.e., $u = \infty$ | At focus, i.e., $v = f$ | $m << 1$, diminished | Real | inverted |
| | Away from centre of curvature ($u > 2f$) | Between $f$ and $2f$, i.e., $f < v < 2f$ | $m < 1$, diminished | Real | inverted |
| | At centre of curvature $u = 2f$ | At centre of curvature, i.e., $v = 2f$ | $m = 1$, same size as that of the object | Real | inverted |
| | Between centre of curvature and focus: $F < u < 2f$ | Away from the centre of curvature $v > 2f$ | $m > 1$, magnified | Real | inverted |
| | At focus, i.e., $u = f$ | At infinity, i.e., $v = \infty$ | $m = \infty$, magnified | Real | inverted |
| | Between pole and focus $u < f$ | $v > u$ | $m > 1$, magnified | Virtual | erect |
| (b) Convex   | At infinity, i.e., $u = \infty$ | At focus, i.e., $v = f$ | $m < 1$, diminished | Virtual | erect |
| | Anywhere between infinity and pole | Between pole and focus | $m < 1$, diminished | Virtual | erect |

**Note:**

- In case of convex mirrors, as the object moves away from the mirror, the image becomes smaller and moves closer to the focus.
- For convex mirror maximum image distance is its focal length.
- In concave mirror, minimum distance between a real object and its real image is zero (i.e., when $u = v = 2f$).

### Linear magnification

#### Transverse

When an object is placed perpendicular to the principal axis, then linear magnification is called lateral or transverse magnification.

It is given by

$$m = \frac{I}{O} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-v}{f}$$

(*Always use sign convention while solving the problems)

#### Longitudinal

When object lies along the principal axis then its longitudinal magnification

$$m = \frac{I}{O} = \frac{-(v_2 - v_1)}{(u_2 - u_1)}$$

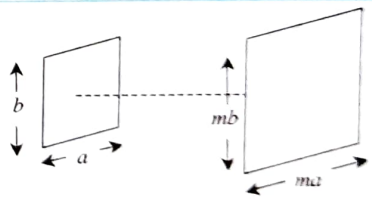
If object is small; $m = -\frac{dv}{du} = \left(\frac{v}{u}\right)^2$

Also length of image

$$= \left(\frac{v}{u}\right)^2 \times \text{Length of object } (L_0)$$

$$(L_i) = \left(\frac{f}{u-f}\right)^2 \cdot L_o$$

### Areal magnification




If a 2D-object is placed with its plane perpendicular to principal axis

Its areal magnification

$$M_s = \frac{\text{Area of image } (A_i)}{\text{Area of object } (A_0)} = \frac{ma \times mb}{ab} = m^2$$

$$\Rightarrow m_s = m^2 = \frac{A_i}{A_0}$$

**Note:**

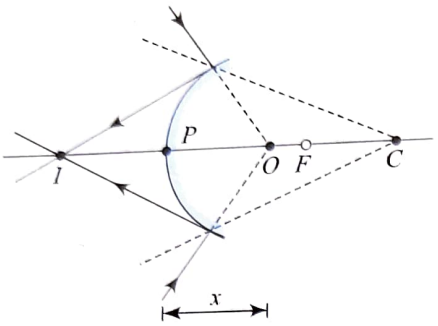
- Do not put the sign of quantity which is to be determined.
- If a spherical mirror produces an image $m$ times the size of the object ($m$ = magnification) then $u$, $v$ and $f$ are given by the following:

$$u = \left(\frac{m-1}{m}\right) f, \quad v = -(m-1) f \quad \text{and} \quad f = \left(\frac{m}{(m-1)}\right) u \text{ (use sign convention)}$$

## ILLUSTRATION 1.20

Can a convex mirror form a real image? Explain.

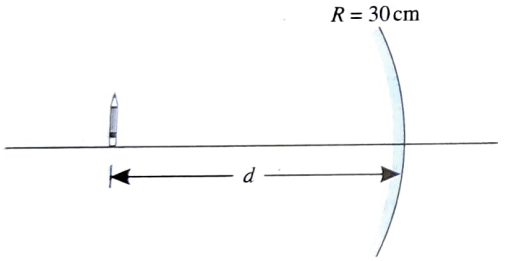
**Sol.** Yes, only when the object is virtual and is placed between $F$ and $P$. Figure shows a convex mirror exposed to a converging beam which converges to a point that lies between $F$ and $P$.




$v = \dfrac{-xf}{f_0 - x}$ ; $v$ becomes negative (real image) only when $x < f_0$.

## ILLUSTRATION 1.21

A concave mirror has radius of curvature $R = 30$ cm. A pencil lies perpendicular to the axis of the mirror at each of the following object distances:




**(a)** $d = 40$ cm **(b)** $d = 30$ cm **(c)** $d = 15$ cm **(d)** $d = 5$ cm

Find the image of the pencil, magnification of each image.

**Sol.** The incident ray is travelling towards right, all the distance measured in rightward direction should be positive,

The focal length of mirror is $f = \dfrac{R}{2} = -15\,\text{cm}$.

From mirror equation, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

**(a)** In this case $d = 40$ cm, the object is placed on left side of centre of curvature and $u = -40$ cm

$$\frac{1}{v} + \frac{1}{(-40)} = \frac{1}{(-15)} \Rightarrow v = -24 \text{ cm}$$

The minus sign shows that the image is in front of the mirror, real.

$$\text{Magnification, } m = -\frac{v}{u} = -\left(\frac{-24}{-40}\right) = -0.6$$

As $m < 1$, the image is diminished in size, minus sign shows that the image is inverted.

**(b)** In this case $d = 30$ cm, the object is placed at the centre of curvature. In this case the image will form at object position. The image is real, formed in front of the mirror.

$$\text{Magnification, } m = -\left(\frac{v}{u}\right) = -\left(\frac{-30}{-30}\right) = -1$$

As $m = -1$, the size of the object and the image are equa minus sign shows that the image is inverted.

**(c)** In this case $d = 15$ cm, the object is placed at the focus. I this case the image will form at infinity.

$$\text{Magnification, } m = -\frac{v}{u} = -\frac{-\infty}{(-15)} = -\infty$$

The image is enlarged real, formed in front of the mirro and will form at infinity.

**(d)** In this case $d = 5$ cm, the object is placed on right side of th focus and $u = -5$ cm

$$\frac{1}{v} + \frac{1}{(-5)} = \frac{1}{(-15)} \Rightarrow \quad = +7.5 \text{ cm}$$

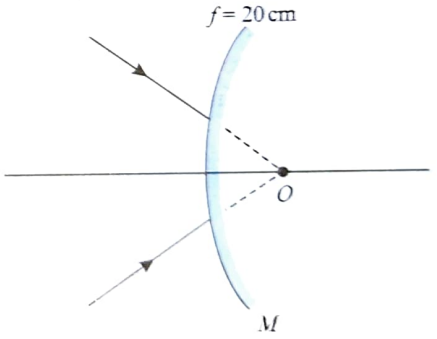
The plus sign shows that the image is behind the mirror virtual. It is a general observation that an object place inside the focal point of a concave mirror produces a virtua image.

$$m = -\left(\frac{v}{u}\right) = -\left(\frac{+7.5}{-5}\right) = +1.5$$

As $m > 1$, the image is magnified; plus, sign shows that it i erect.

## ILLUSTRATION 1.22

A beam of light converges towards a point $O$, behind a convex mirror of focal length 20 cm.



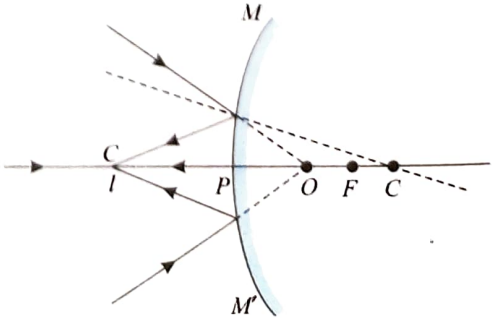

Find the nature and position of image if the point $O$ is

**(a)** 10 cm behind the mirror

**(b)** 30 cm behind the mirror

**Sol.** The incident ray is travelling towards right, all th distance measured in rightward direction should be positive,

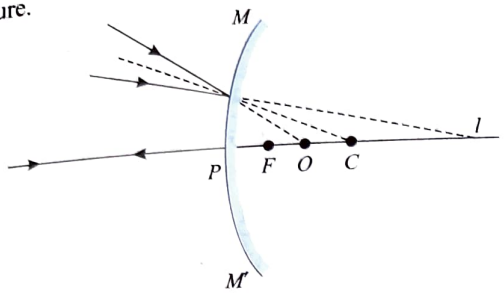
**(a)** The beam of light converges towards $O$, behind the mirro For this situation object will be virtual as shown in figur Here $u = +10$ cm and $f = +20$ cm.



So, $\frac{1}{v}+\frac{1}{+10}=\frac{1}{+20}$ i.e., $v=-20$ cm

i.e., the image will be at a distance of 20 cm in front of the mirror and will be real, erect and enlarged with $m=-(-20/10)=+2$.

(b) For this situation also object will be virtual as shown in figure.




Here, $u=+30$ cm and $f=+20$ cm

So, $\frac{1}{v}+\frac{1}{+30}=\frac{1}{+20}$ i.e., $v=+60$ cm

i.e., the image will be at a distance of 60 cm behind the mirror and will be virtual, inverted and enlarged with $m=-(+60/30)=-2$.

## ILLUSTRATION 1.23

The focal length of a concave mirror is 30 cm. Find the position of the object in front of the mirror, so that the image is three times the size of the object.

**Sol.** As the object is in front of the mirror it is real and for real object the magnified image formed by concave mirror can be inverted (i.e., real) or erect (i.e., virtual), so there are two possibilities:

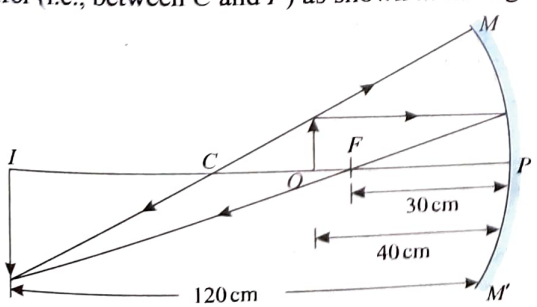
(a) If the image is inverted (i.e., real)

$$m=-\left(\frac{v}{u}\right)=-3 \text{ i.e., } v=3u$$

So from mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

We get, $\frac{1}{3u}+\frac{1}{u}=\frac{1}{(-30)}$ i.e., $u=-40$ cm

i.e., object must be at a distance of 40 cm in front of the mirror (i.e., between $C$ and $F$) as shown in the figure.



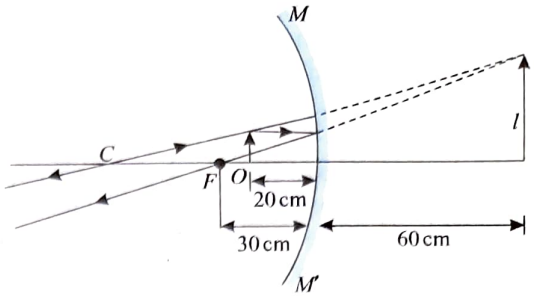

(b) If the image is erect (i.e., virtual)

$m=-(v/u)=3$ i.e., $v=-3u$

So from mirror formula,

$$\frac{1}{(-3u)}+\frac{1}{u}=\frac{1}{(-30)} \text{ i.e., } u=-20 \text{ cm}$$

i.e., object must be at a distance of 20 cm in front of the mirror (i.e., between $F$ and $P$) as shown in the figure).




## ILLUSTRATION 1.24

An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance of 5 cm, then the image becomes triple its size. Find the nature and radius of curvature of the mirror.

**Sol.** Since the image is formed on the screen, it is real. Real object and real image imply concave mirror.

Applying $m=\frac{f}{f-u}$ or $-2=\frac{f}{f-u}$ ...(i)

After shifting, $-3=\frac{f}{f-(u+5)}$ ...(ii)

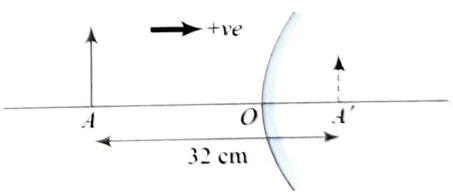
Here, care should be taken that distance of the object becomes $u+5$ not $u-5$. In a concave mirror the size of real image will increase only when the real object is brought closer to the mirror. In doing so, its $x$-coordinate will increase. From Eqs. (i) and (ii), we get

$f=-30$ cm or $R=60$ cm

## ILLUSTRATION 1.25

The distance between a real object and its image in a convex mirror of focal length 12 cm is 32 cm. Find the size of image if the object size is 1 cm.

**Sol.** Let $x$ and $y$ be the magnitudes of object and image distances.




We have $AA'=32$ cm

$\Rightarrow \quad AO+A'O=32$ cm

$\Rightarrow \quad (x+y)=32$ ...(i)

And also, $u=-x$, $v=+y$

$$\frac{1}{-x}+\frac{1}{y}=\frac{1}{+12} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii) simultaneously, we can get $u$ and $v$. The relevant answers are $u=-24$ cm, $v=+8$ cm

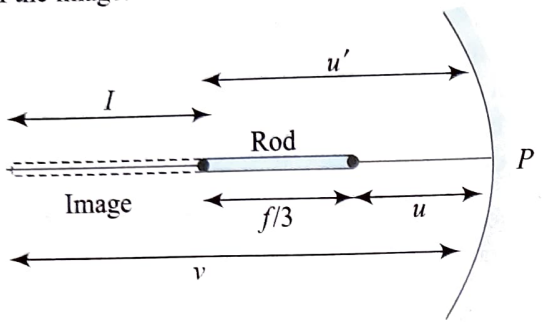
Using $I=-1\left(\frac{+8}{-24}\right)=+\frac{1}{3}$

So, the image size is 1/3 cm.

## ILLUSTRATION 1.26

A thin rod of length $f/3$ is placed along the optical axis of a concave mirror of focal length $f$ such that its image which is real and elongated just touches the rod. Calculate the magnification.

**Sol.** As in question, image touches the rod, i.e., image and object coincide, hence one end of the rod should be at the centre of curvature. It is also written that image is enlarged, it indicates that the orientation of rod should be toward focus then only we can get enlarged image along the principal axis. Let $l$ be the length of the image.




Then, $m = \dfrac{l}{f/3} \Rightarrow l = \dfrac{mf}{3}$

Also, one end of the image coincides with the object, $u' = 2f$.

Now, $u' = u + \dfrac{f}{3} \Rightarrow u = 2f - \dfrac{f}{3} = \dfrac{5f}{3}$

$$v = -\left(u + \frac{f}{3} + \frac{mf}{3}\right)$$

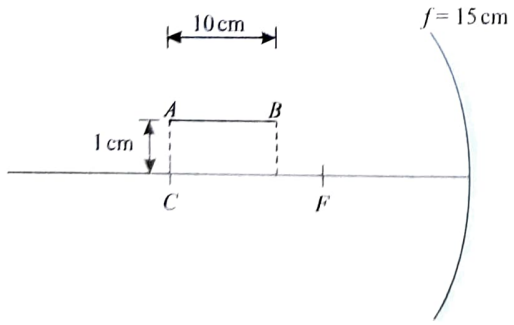
Putting in mirror formula, we get

$$\frac{1}{u + f/3 + mf/3} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{3}{5f + f + mf} + \frac{3}{5f} = \frac{1}{f}$$

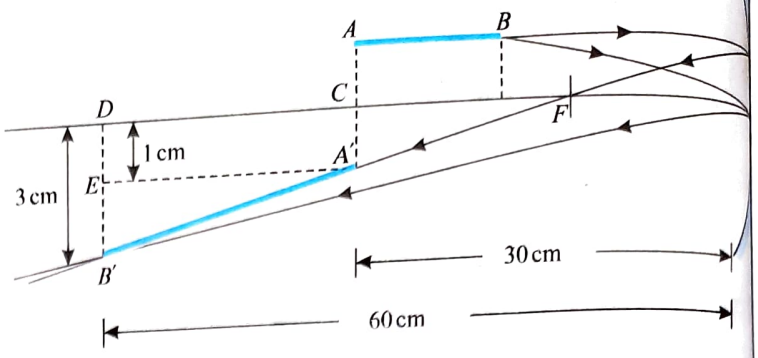
$$\Rightarrow \frac{1}{m+6} = \frac{2}{15} \Rightarrow m = \frac{3}{2}$$

## ILLUSTRATION 1.27

A metal wire $AB$ of length 10 cm is placed in front of a concave mirror of focal length 15 cm as shown in figure. Find the length of the wire.




**Sol.** The image of end $A$ of the wire is placed at a distance $R = 2f = 30$ cm, hence image of $A$ will form at a distance 30 cm in front of the mirror but below the principal axis at a same distance as it is above the principal axis i.e., 1 cm.




Now consider the image of end $B$,

For $B$; $u = -(30-10) = -20$ cm, $f = -15$ cm

Using mirror formula: $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$\frac{1}{v} + \frac{1}{(-20)} = \frac{1}{(-15)} \Rightarrow v = -60 \text{ cm}$$

Now using, $m = \dfrac{I}{O} = -\dfrac{v}{u}$

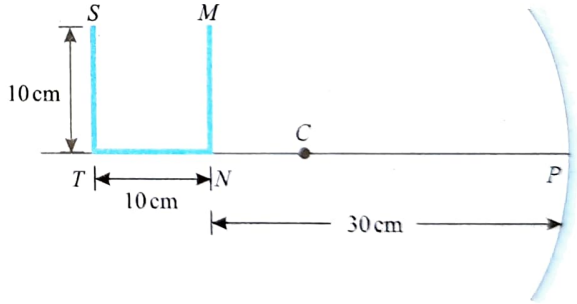
$$\frac{I}{1} = -\frac{(-60)}{(-20)} \Rightarrow I = -3 \text{ cm}$$

Length of the image $A'B' = \sqrt{(A'E)^2 + (B'E)^2}$

$\Rightarrow A'B' = \sqrt{(30)^2 + (2)^2} = \sqrt{904}$ cm or $2\sqrt{226}$ cm

## ILLUSTRATION 1.28

A U-shaped wire is placed before a concave mirror having radius of curvature 20 cm as shown in figure. Find the total length of the image.




**Sol.** The incident ray is travelling towards right, all the distanc measured in rightward direction should be positive, For part $M$

$u = -30$ cm, $f = -10$ cm

Using, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

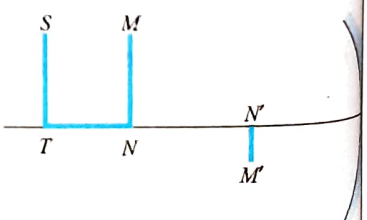
$$\frac{1}{v} + \frac{1}{(-30)} = \frac{1}{(-10)} \Rightarrow v = -15 \text{ cm}$$

The minus sign shows that the image is in front of the mirro real.

Magnification, $m = \dfrac{I}{O} = -\dfrac{v}{u}$

$$\Rightarrow \frac{I}{(+10)} = -\frac{(-15)}{(-30)} = -\frac{1}{2}$$

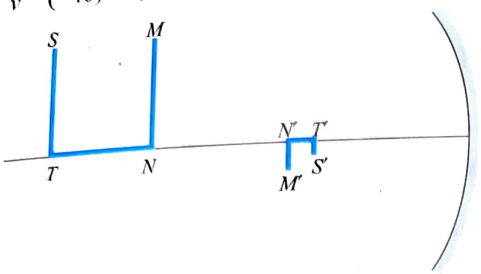
$\Rightarrow I = M'N' = -5$ cm



For image of $ST$, $u = -40$ cm and $f = -10$ cm

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{(-40)}=\frac{1}{(-10)} \Rightarrow v=-\frac{40}{3}\text{cm}$$




For length of image of $ST$, $m=\frac{I}{O}=-\frac{v}{u}$

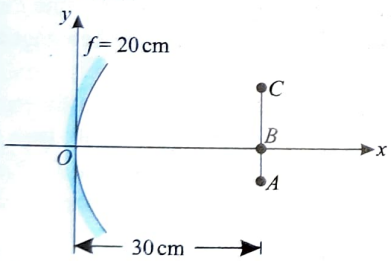
$$\frac{I}{10}=-\left(\frac{-40/3}{-40}\right) \Rightarrow I=S'T'=-\frac{10}{3}\text{cm}$$

Length of image of $NT$, $T'N'=15-\frac{40}{3}=\frac{5}{3}\text{cm}$

Length of wire in image $=5+\frac{10}{3}+\frac{5}{3}=10\text{cm}$

## ILLUSTRATION 1.29

A metal wire $ABC$ of length 3 cm is placed in front of a concave mirror of focal length 20 cm as shown in figure. If $AB = 1$ cm and $BC = 2$ cm, then find the coordinates image of point $A$ and $C$ of the wire.




Focal length of the concave mirror shown in figure is 20 cm.

$ab = 1$ mm and $bc = 2$ mm

For the given situation, make its image.

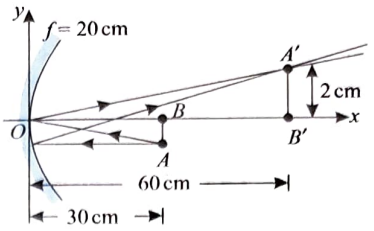
**Sol.** $u = -30$ cm and $f = -20$ cm

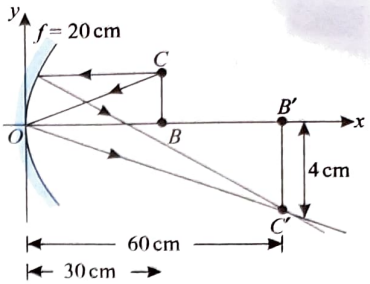
Using the mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we have $\frac{1}{v}+\frac{1}{-30}=\frac{1}{-20}$

Solving this equation, we get $v = -60$ cm

Now, $$m=-\frac{v}{u}=-\frac{(-60)}{(-30)}=-2$$

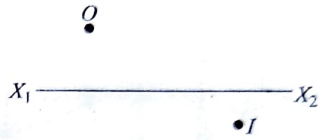
The point $C$ is 2 cm above the principal axis and magnification is –2. Hence, image $C$ will be formed 4 cm below the principal axis. Similarly, point $A$ is 1 cm below the principal axis and value of $m$ is –2. Hence, image of $A$ will be formed 2 cm above the principal axis. Image with ray diagram of $C$ is as shown below.




## ILLUSTRATION 1.30

Point $I$ is the image of a point source of light $O$ in a spherical mirror whose optical axis is $X_1X_2$ (shown in figure). Find by construction the position of the centre of the mirror and its focus.




**Sol.** As the image and object both are on the opposite side of the principal axis, hence the mirror should be concave. For finding the position of the mirror and its focus, let us go step by step.

**Step I:** The line joining object and image position intersect at principal axis at the optical centre $C$ of the mirror. Hence the point of intersection of ray $OI$ with axis $X_1X_2$ is the position of centre of curvature of the mirror ($C$).

**Step II:** We know the ray incident on the mirror at its pole is reflected symmetrically w.r.t. the principal axis, let us plot point $I_1$ symmetrical to $I$ and draw ray $OI_1$ until it intersects the axis at point $P$ (see in given figure). This point should be the pole of the mirror.

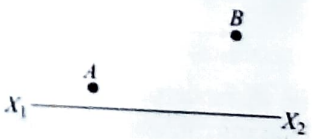
**Step III:** The focus can be found by the usual construction of ray $OM$ parallel to the axis. The reflected ray must pass through focus $F$ (lying on the principal axis of the mirror) and through $I$.

## ILLUSTRATION 1.31

The positions of optical axis $X_1X_2$ of a spherical mirror, the source and the image are known (as shown in figure). Find by construction the positions of the centre of the mirror, its focus, and the pole for the cases

**(a)** $A$– object, $B$– image;

**(b)** $B$– object, $A$– image.



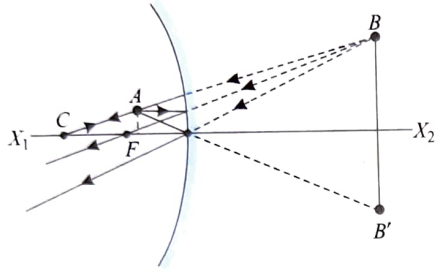
**Sol.** As the image and object both are on the same side of the principal axis, hence the mirror can be concave or convex.

**(a)** For finding the position of the mirror and its focus, let us go step by step.

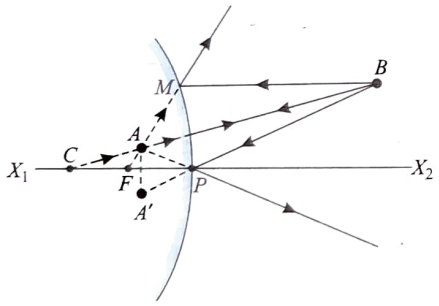
**Step I:** The line joining object position ($A$) and image position ($B$) intersect at principal axis at the optical centre $C$ of the mirror

**Step II:** The position of the pole $P$ can be found by constructing the path of the ray $APA'$ reflected in the pole with the aid of symmetrical point $A'$.

**Step III:** The position of the mirror focus $F$ is determined by means of the usual construction of ray $AMF$ parallel to the axis.



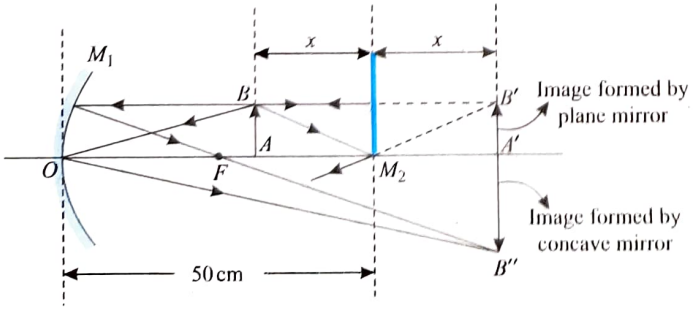

**(b)** This construction can also be used to find the centre $C$ of the mirror and pole $P$ as we have done in case (a) (see figure). The reflected ray $BM$ will pass parallel to the optical axis of the mirror. For this reason, to find the focus.




## ILLUSTRATION 1.32

A plane mirror is placed at a distance of 50 cm from a concave mirror of focal length 16 cm. Where should a short object be placed between the mirrors and facing both the mirrors so that its virtual image in the plane mirror coincides with the real image in concave mirror? What is the ratio of the sizes of the two images?

**Sol.** Let the object be at a distance $x$ from the plane mirror.




The distance of object from concave mirror is $(50 - x)$. For the plane mirror, object and image distances are equal.

$$A'M_2 = A\,M_2 = x$$

Distance of the image from concave mirror,

$$OA' = O\,M_2 + A'\,M_2 = (50 + x)$$

The incident ray direction on concave mirror is from right to left. Taking leftward direction as positive.

For the concave mirror, $u = -(50 - x)$, $v = -(50 + x)$, $f = -16$ cm

Using the mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

we have,

$$\frac{1}{-(50+x)} + \frac{1}{-(50-x)} = \frac{1}{-16} \Rightarrow \quad x = 30 \text{ cm}$$

The object must be placed at a distance of 30 cm from the plane mirror.

The ratio of image sizes $= \dfrac{A'B'}{A'B''} = \dfrac{AB}{A'B''}$

But $AB$ = Size of the object

and $A'B''$ = Size of the image formed by concave mirror

$$|m| = \frac{I}{O} = \frac{v}{u} = \frac{50+x}{50-x} = \frac{50+30}{50-30} = 4$$

The real image formed by the concave mirror is four times the size of virtual image formed by the plane mirror.

## CONCEPT APPLICATION EXERCISE 1.2

**1.** State the following statements as True or False.

**(a)** A convex mirror cannot form a real image for a real object.

**(b)** The image formed by a convex mirror is always diminished and erect.

**(c)** Virtual image formed by a concave mirror is always enlarged.

**(d)** Only in the case of a convex mirror, it may happen that the object and its image move in same direction.

**(e)** In the case of a concave mirror, the image always move faster than the object.

**(f)** If an object is placed in front of a diverging mirror at a distance equal to its focal length, then the height of image formed is half of the height of object.

**(g)** For two positions of an object, a concave mirror can form enlarged image.

**(h)** Concave mirror is used as a rear view mirror in motor vehicles.

**(i)** If some portion of the mirror is covered, then complete image will be formed but of reduced brightness.

**(j)** A plane mirror always forms an erect image of same size as that of the object.

**(k)** The image formed by a plane mirror has left-right reversal.

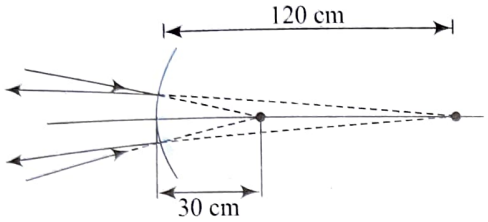
**(l)** A virtual object means a converging beam.

**2.** A particle moves in a circular path of radius 5 cm in a plane perpendicular to the principal axis of a convex mirror with radius of curvature 20 cm. The object is 15 cm in front of the mirror. Calculate the radius of the circular path of the image.

**3.** A body of length 6 cm is placed 10 cm from a concave mirror of focal length 20 cm. Find the position, size, and nature of the image.

**4.** An object is placed 15 cm from a mirror and an image is captured on the screen with magnification 2. Calculate the focal length of the mirror and determine if it is concave or convex.

5. An object is placed 15 cm from a mirror and an erect image of size 5 cm is seen. Determine the focal length and the nature of the mirror. Assume the object size is 15 cm.
6. A beam of light converges to a point on a screen $S$. A mirror is placed in front of the screen at a distance of 10 cm from the screen. It is found that the beam now converges at a point 20 cm in front of the mirror. Find the focal length of the mirror.
7. Converging rays are incident on a convex spherical mirror so that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam so that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.




8. A concave mirror gives a real image magnified 4 times. When the object is moved 3 cm the magnification of the real image is 3 times. Find the focal length of mirror.
9. The image of a real object in a convex mirror is 4 cm from the mirror. If the mirror has a radius of curvature of 24 cm, find the position of object and magnification.
10. When an object is placed at a distance of 25 cm from a mirror, the magnification is $m_1$. The object is moved 15 cm farther away with respect to the earlier position, and the magnification becomes $m_2$. If $m_1/m_2 = 4$, then calculate the focal length of the mirror.
11. A concave mirror forms a real image three times larger than the object on a screen. The object and screen are moved until the image becomes twice the size of the object. If the shift of the object is 6 cm, find the shift of screen.

### ANSWERS

1. (a) True; (b) False; (c) True; (d) False; (e) False; (f) True; (g) True; (h) False; (i) True; (j) True; (k) True; (l) True
2. 2 cm  4. 10 cm (concave)  5. 7.5 cm (concave)
6. 20 cm  7. 24 cm  8. 36 cm  9. 6 cm, $\frac{2}{3}$
10. 20 cm  11. 36 cm

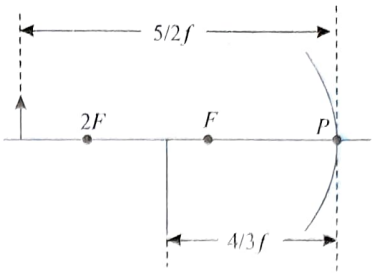
## IMAGES FORMED BY TWO MIRRORS

If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will act as an object for second mirror, and this process will continue for every successive reflection. And the overall lateral magnification is given by $m_1 \times m_2 \times m_3 \times \ldots$, where $m_1$, $m_2$,... etc. are lateral magnifications produced by individual mirrors.

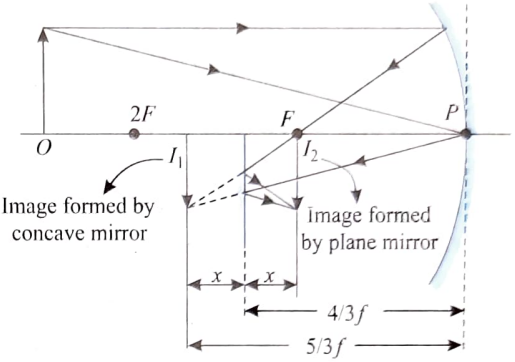
Let us understand this with the illustration discussed below.

### ILLUSTRATION 1.33

A vertically erect object placed on the optic axis at a distance $(5/2)f$ from a concave mirror of focal length $f$ as shown in the figure. If a plane mirror is placed perpendicular to the optic axis at a distance $(4/3)f$ from the pole, facing concave mirror, find the position and nature of the final image formed.




**Sol.** Two rays come from the tip of the object. After reflection from concave mirror, they converge to form a real image at $I_1$.




As the plane mirror is introduced in between, the image $I_1$ acts as virtual object for plane mirror. Hence the reflected rays from the concave mirror get reflected from the plane mirror to form an image $I_2$. In fact $I_2$ is the real image of the image $I_1$ of the object $O$.

The incident ray is travelling towards right, the distance measured from rightward direction should be positive.

For the concave mirror, $u = -\frac{5}{2}f$ and $f = -f$

Using the mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\frac{1}{v}+\frac{1}{\left(-\frac{5}{2}f\right)}=\frac{1}{(-f)} \Rightarrow v=-\frac{5}{3}f$$

(Negative sign indicates the image is left the mirror.)

The image $I_1$ is formed which behaves as a virtual object for the plane mirror and therefore finally a real erect image $I_2$ of same size, will be formed at equal distance $x$ in front of the plane mirror as shown in the figure.

Hence, $I_2$ will be formed at a distance $\left(\frac{5}{3}\right)f - 2x$, from concave mirror, where

$$x=\frac{5}{3}f-\frac{4}{3}f=\frac{1}{3}f$$

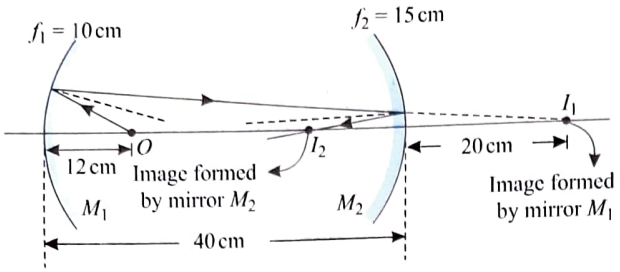
The final image distance $v=\left(\frac{5}{3}\right)f-2\left(\frac{1}{3}f\right)=f$

Hence the final image is formed at the focus of the concave mirror.

### ILLUSTRATION 1.34

Two concave mirrors are placed 40 cm apart and are facing each other. A point object lies between them at a distance of 12 cm from the mirror of focal length 10 cm. The other mirror has a focal length of 15 cm. Find the location of final image formed after two reflections—first at the mirror nearer to the object and second at the other mirror.

**Sol.** The initial incident ray is travelling towards left, all the distance measured from leftward direction should be positive,




**First reflection** from $M_1$: $u = -12$ cm, $f = -10$ cm

Using the mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

we have, $\frac{1}{v}+\frac{1}{(-12)}=\frac{1}{(-10)} \Rightarrow v = -60$ cm

The rays after reflection are going to converge at a point $v = 60$ cm from the first mirror, i.e., $60 - 40 = 20$ cm behind the other mirror.

**Second reflection** from $M_2$: As initial incident rays are directed toward left, all quantities towards right will be taken negative.

The converging rays falling at the second mirror create a virtual object for this mirror at $I_1$ with object distance

$u = -20$ cm and $f = +15$ cm,

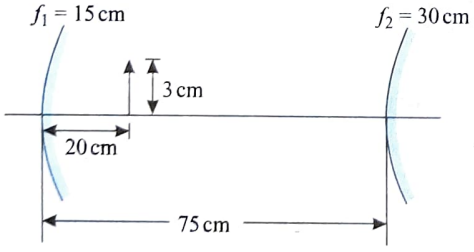
we have, $\frac{1}{v}+\frac{1}{(-20)}=\frac{1}{(+15)} \Rightarrow v = +\frac{60}{7}$ cm

As leftward direction is positive, image will be formed to the left of mirror.

Hence, the final image is formed at a distance of $\frac{60}{7}$ cm from the second mirror and is real.

### ILLUSTRATION 1.35

A concave mirror and a convex mirror of focal lengths 15 cm and 30 cm are placed at a distance of 75 cm. An object of height 3 cm is placed at a distance of 20 cm from the concave mirror. First ray is incident on the concave mirror then on the convex mirror. Find size, position, and nature of the image.




**Sol.** First ray is incident on the concave mirror, it means initial ray travelling towards left. It means all the distance measured from leftward direction should be positive.

Hence for concave mirror, $u = -20$ cm, $f = -15$ cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}; \frac{1}{v}+\frac{1}{(-20)}=\frac{1}{(-15)} \Rightarrow v = -60$ cm

Magnification by concave mirror, $m_1 = -\frac{v}{u} = -\frac{(-60)}{(-20)} = -3$

The image formed by concave mirror will act as object for convex mirror.

For convex mirror, $u = +(75 - 60) = 15$ cm, $f = -30$ cm

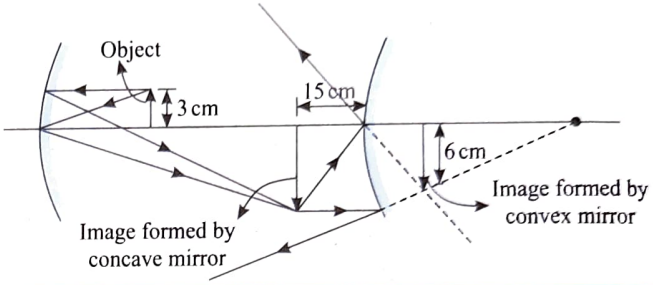
Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}; \frac{1}{v}+\frac{1}{(+15)}=\frac{1}{(-30)} \Rightarrow v = -10$ cm

Magnification by convex mirror, $m_2 = -\frac{v}{u} = -\frac{(-10)}{(+15)} = \frac{2}{3}$

Overall lateral magnification, $m = m_1 m_2 = -3 \times \frac{2}{3} = -2$

Size of image = $-2$ × size of object = $(-2) \times 3 = -6$ cm

Final image will be virtual, inverted, and enlarged.




## RELATION BETWEEN OBJECT AND IMAGE VELOCITY

**Case (i). When object is moving along the principal axis:** Le a point object is moving along the principal axis of a concav mirror of focal length $f$. We can relate the object and imag positions by the mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \ldots(i$$

Differentiate Eq. (i) with respect to time.

$$\Rightarrow \quad -\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0$$

or $$\frac{dv}{dt}=-\left(\frac{v^2}{u^2}\right)\frac{du}{dt} \quad \ldots(ii$$

Here, $\frac{du}{dt}$ is the rate by which $u$ is changing. It is the objec speed with respect to or object velocity when mirror is stationar

Similarly, $\frac{dv}{dt}$ is the rate by which $v$ (distance between imag and mirror) is changing. It is image speed if with respect to o object velocity when mirror is stationary.

Let $\frac{dv}{dt} = V_{i,m}$ = velocity of image w.r.t. mirror

and, $\frac{du}{dt} = V_{o,m}$ = velocity of object w.r.t. mirror

Also, $\frac{v^2}{u^2} = m^2$

Then Eq. (ii) becomes

$$V_{i,m} = -m^2 V_{o,m} \quad \ldots(ii$$

The negative sign shows that if $u$ is decreasing, $v$ will increas i.e., if real object approaches the mirror, its real image will recede from the mirror.

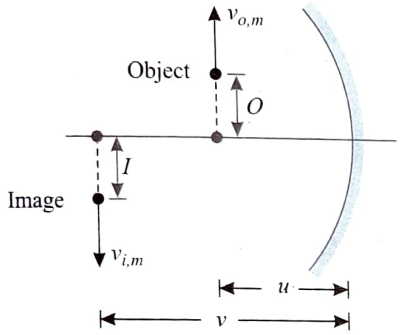
**Important Points:**

- Image speed = $m^2$ × Object speed
- In vector notation, Eq. (iii) can be written as

$$\vec{V}_{i,m} = -m^2 \vec{V}_{o,m} \Rightarrow \left(\vec{V}_i - \vec{V}_m\right) = -m^2\left(\vec{V}_o - \vec{V}_m\right)$$

- Object moves between infinite and centre of curvature
  In this case, $|m| < 1$, hence $V_{i,m} < V_{o,m}$
  The speed of image is less than speed of object
  When the object is at centre of curvature $|m| = 1$, hence $V_{i,m} = V_{o,m}$
- Object moves between centre of curvature and focus.
  In this case, $|m| > 1$, hence $V_{i,m} > V_{o,m}$
  Speed of image is more than speed of object.

**Case (ii). When object is moving normal to the principal axis:** In this case a point object is moving normal to the principal axis of a concave mirror as shown in figure. In this situation the object distance will not change, it makes image distance also constant.




For constant values of v and u the value of transverse magnification also constant or $m = \frac{I}{O} = -\frac{v}{u}$ is a constant. We can write,

$$I = mO \qquad \ldots (i)$$

Differentiate Eq. (i) with respect to time, we get

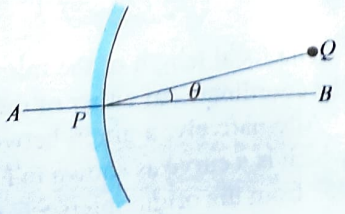
$$\frac{dI}{dt} = m\frac{dO}{dt} \Rightarrow \left(V_{i,m}\right)_y = m\left(V_{o,m}\right)_y$$

Here $\frac{dI}{dt} = \left(V_{i,m}\right)_y$ = velocity of image w.r.t. mirror in the direction perpendicular to principal axis

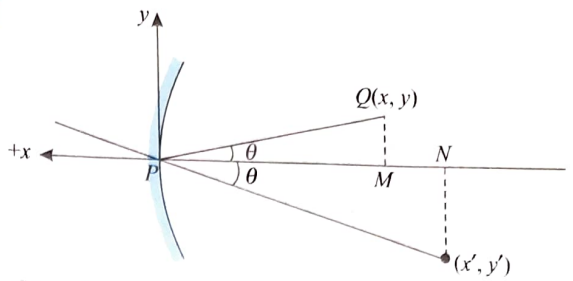
And, $\frac{dO}{dt} = \left(V_{o,m}\right)_y$ = velocity of object w.r.t. mirror in the direction perpendicular to principal axis

## ILLUSTRATION 1.36

In figure, $AB$ is the principal axis of the concave mirror. A point object moves on the line $PQ$ which makes small angle $\theta$ with the principal axis. Show that image also moves in straight line making same angle $\theta$ with principal axis.




**Sol.**




Let $\tan\theta = a$ then equation of the line $PQ$ is

$$y = ax \qquad \ldots (i)$$

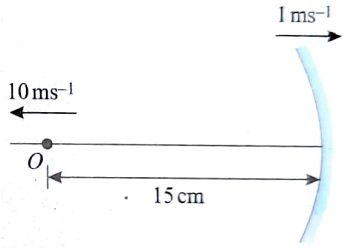
Let $y'$ = $y$-coordinate of the image and $x'$ = $x$-coordinate of image. By using the formula of magnification,

$$\frac{y'}{y} = -\frac{x'}{x} \Rightarrow y' = -\frac{y}{x}x' = -(a)x' = -ax' \Rightarrow y' = -ax'$$

This is an equation of a straight line, making angle $\theta$ with the principal axis. (Hence, proved.)

## ILLUSTRATION 1.37

A concave mirror of focal length 10 cm and a point object both are moving away from each other with velocities 1 $ms^{-1}$ and 10 $ms^{-1}$ respectively as shown in figure. Find the velocity of image when the object is at a distance 15 cm from the mirror.




**Sol.** Using $\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$, we get

$$\frac{1}{v} - \frac{1}{15} = -\frac{1}{10} \Rightarrow v = -30 \text{ cm}$$

Now, using $\vec{V}_{i,m} = -m^2\vec{V}_{o,m}$

Or $\left(\vec{V}_i - \vec{V}_m\right) = -m^2\left(\vec{V}_o - \vec{V}_m\right)$

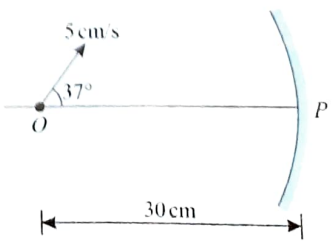
$$\Rightarrow \left[\vec{V}_i - (1)\right] = -\left(\frac{(-30)}{(-15)}\right)^2\left[(-10) - (1)\right]$$

$\Rightarrow V_i = 45$ cm $s^{-1}$ towards right

So, the image will move with velocity 45 cm $s^{-1}$ towards right.

## ILLUSTRATION 1.38

A point object is moving with velocity 5 cm/s at angle 37° towards a stationary concave mirror of focal length 20 cm as shown in figure. Find the image position and its velocity when the object is at a distance 30 cm from the mirror.



**Sol.** We have mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \quad \ldots(i)$$

Substituting the values, $u=-30\,\text{cm}$ and $f=-20$ cm in Eq. (i), we have

$$\frac{1}{v}+\frac{1}{-30}=\frac{1}{-20}$$

Solving this equation, we get $v=-60\,\text{cm}$

Further, $m=-\frac{v}{u}=-\frac{(-60)}{(-30)}=-2$ and $m^2=4$

Let us make the component of the object velocity along the principal axis and along normal to principal axis.

$$\vec{V}_{o,m}=5\cos 37^\circ\hat{i}+5\sin 37^\circ\hat{j}\,(\text{cm/s})$$

$$\Rightarrow \quad \vec{V}_{o,m}=4\hat{i}+3\hat{j}\,(\text{cm/s})$$

Now, using $\vec{V}_{i,m}=-m^2\vec{V}_{o,m}$ for finding the component of the image velocity along principal axis

Or $\left(\vec{V}_i-\vec{V}_m\right)=-m^2\left(\vec{V}_o-\vec{V}_m\right)$

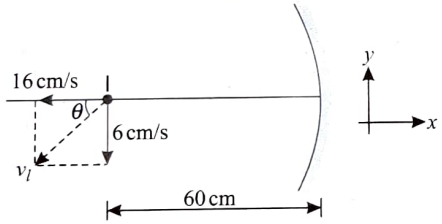
$$\left(\vec{V}_i-0\right)=-4\left(4\hat{i}-0\right)\Rightarrow\left(\vec{V}_i\right)_x=-16\hat{i}\ \text{cm/s}$$

The velocity of image in the direction perpendicular to principal axis,

$$\left(V_{i,m}\right)_y=-m\left(V_{o,m}\right)_y=-4\times 3\hat{j}=-12\hat{j}\ \text{cm/s}$$

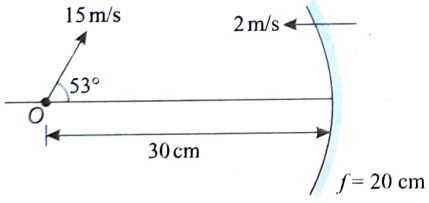
Hence the velocity of image, $\vec{V}_{\text{image}}=-16\hat{i}-12\hat{j}\,(\text{cm/s})$

The position and velocity of image is shown below.




## ILLUSTRATION 1.39

Find the velocity of image in a situation as shown in the figure.




**Sol.** Given: Object distance $u=-30\,\text{cm}, f=-20\,\text{cm}$

Velocity of the object $\vec{v}_o=\left(9\hat{i}+12\hat{j}\right)\text{m/s}$ and velocity of the mirror $\vec{v}_m=-2\hat{i}\ \text{m/s}$

Using mirror formula: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Magnification $m=-\frac{v}{u}=-\frac{(-60)}{(-30)}=-2$

Velocity of the image with respect to mirror in the direction along principal axis ($x$-direction)

$$\left(\vec{V}_{i,m}\right)_x=-m^2\left(\vec{V}_{o,m}\right)_x \Rightarrow \left(\vec{v}_i-\vec{v}_m\right)_x=-m^2\left(\vec{v}_o-\vec{v}_m\right)_x$$

$$\left[\vec{v}_i-(-2\hat{i})\right]=-(-2)^2\left[(9\hat{i})-(-2\hat{i})\right]$$

$$\left(\vec{v}_i\right)_x=-46\hat{i}\ \text{m/s}$$

Velocity of image w.r.t. mirror in $y$-direction

$$\left(\vec{V}_{i,m}\right)_y=m\left(\vec{V}_{o,m}\right)_y \Rightarrow \left(\vec{v}_i-\vec{v}_m\right)_y=m\left(\vec{v}_o-\vec{v}_m\right)_y$$

$$\left[\vec{v}_i-0\right]=(-2)\left[(12\hat{i})-0\right]$$

$$\Rightarrow \quad \left(\vec{v}_i\right)_y=-24\hat{j}\,(\text{m/s})$$

Hence velocity of the image $\vec{v}_i=\left(-46\hat{i}-24\hat{j}\right)\text{m/s}$

## GRAPHICAL METHOD OF DETERMINING THE FOCAL LENGTH OF A CONCAVE MIRROR

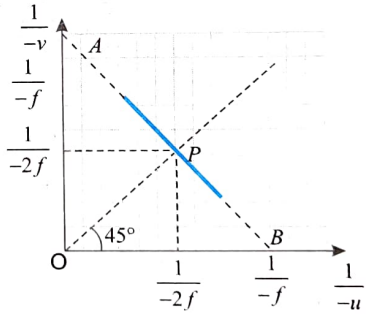
It forms real and inverted image of an object placed beyond its focus. From mirror equation,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

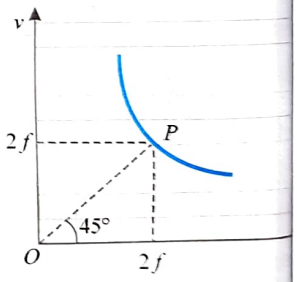
Using Cartesian sign conversion, we have $u=-x$, $v=-y$ and $f=-$

or $$\frac{1}{-y}+\frac{1}{-x}=\frac{1}{-f}$$

A graph between $\frac{1}{-v}$ and $\frac{1}{-u}$ is a straight line, as shown Fig. (a).




(a)




(b)

Note that the slope of the straight line is $-1$ and the intercep on the horizontal and vertical axes are equal. It is equal to $\frac{1}{-f}$

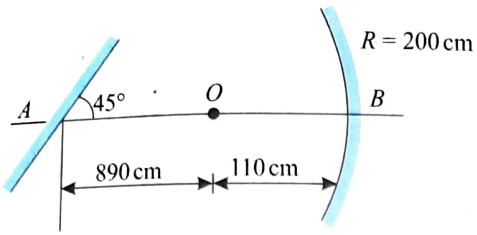
A straight line $OP$ at an angle of 45° with the horizontal axis drawn which intersects the line $AB$ at $P$. The coordinates of t point $P$ are $\left(\frac{1}{-2f},\frac{1}{-2f}\right)$.

The focal length of the mirror can be calculated by measuri the coordinates of either of the points $A$, $B$ or $P$.

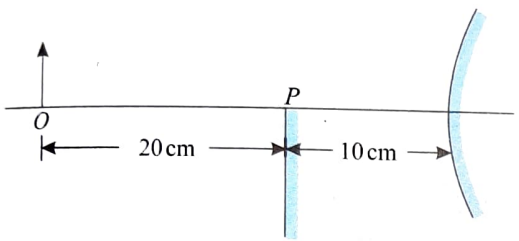
Alternatively, a graph between $-v$ and $-u$ can also be plotte which is a curve as shown in Fig. (b). A line drawn at an angle 45° from the origin intersects it at the point $P$ whose coordinat are $(-2f, -2f)$. By measuring the coordinates of this point, t focal length of the mirror can also be measured.

## CONCEPT APPLICATION EXERCISE 1.3

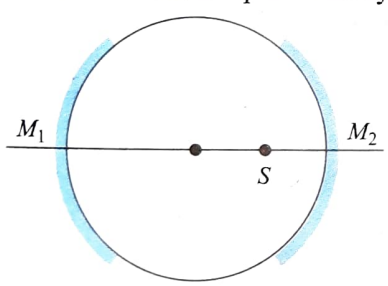
**1.** A plane mirror and a concave mirror are arranged as shown in figure and $O$ is a point object. Find the position of image formed by two reflections, first one taking place at concave mirror.



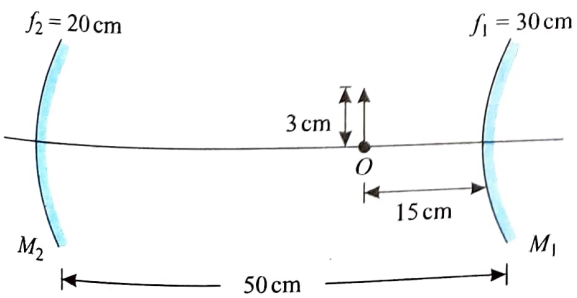

**2.** A small plane mirror strip is kept at a distance of 10 cm in front of a convex mirror, with its plane normal to the principal axis. An object is placed at a distance of 20 cm from the plane mirror, as shown in figure. Calculate the focal length of the convex mirror, if the image formed by the plane mirror and the convex mirror coincide, without parallax.



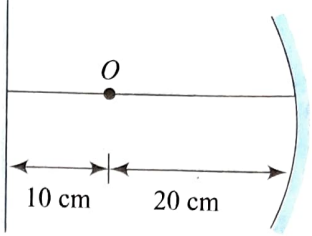

**3.** The inner surface of the wall of a sphere is perfectly reflecting. Radius of the sphere is $R$. A point source $S$ is placed at a distance $R/2$ from the centre of the sphere. Consider the reflection of light from the farthest wall followed by reflection from the nearest wall. Where is the image of the source? Consider paraxial rays only.




**4.** A concave mirror and a convex mirror of focal lengths 20 cm and 30 cm are placed at a distance of 50 cm. An object of height 3 cm is kept at a distance 15 cm from convex mirror. Find the position and height of the image formed by reflection, first at the convex and then at the concave mirror.

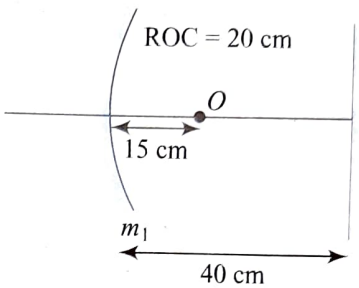



**5.** An object is placed between a plane mirror and a concave mirror of focal length 15 cm as shown in the figure. Find the positions of the images after two reflections.

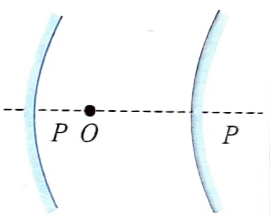



**6.** An object is placed midway between a concave mirror of focal length $f$ and a convex mirror of focal length $f$. The distance between the two mirrors is $6f$. Trace the ray that is first incident on the concave mirror and then the convex mirror.

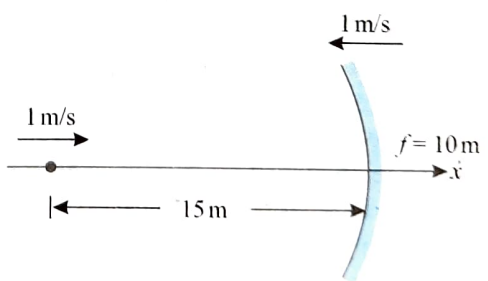
**7.** Find the position of the final image after three successive reflections taking the first reflection on $m_1$.



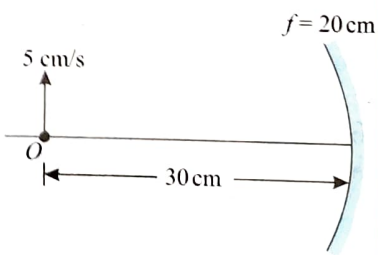

**8.** A concave mirror of focal length 10 cm is placed in front of a convex mirror of focal length 20 cm. The distance between the two mirrors is 20 cm. A point object is placed 5 cm from the concave mirror. Discuss the formation of image.




**9.** A point object moves in $+x$-direction with $v = 1$ m/s along the principal axis of the concave mirror of focal length $f = 10$ m. When the mirror moves with a velocity $\vec{V}_m = -\hat{i}$ m/s and the object is at a distance of 15 m, find the speed of the image.




**10.** Find the velocity of image w.r.t. ground. The mirror is at rest and the object is moving perpendicular to the axis with 5 cm/s as in figure.




### ANSWERS

**2.** 15 cm **3.** $\frac{5R}{6}$ **4.** 30 cm; 1 cm **5.** 24 cm; 60 cm
**7.** 12.5 cm **9.** 9 m/s **10.** 10 cm/s

# REFRACTION OF LIGHT

Deviation or bending of light rays from their original path while passing from one medium to another is called **refraction**. It is due to change in speed of light as light passes from one medium to another medium. If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.

## REFRACTIVE INDEX

Whenever light travels from vacuum to any other medium, its speed slow down. The speed of light is maximum in vacuum.

Why a light wave slows down when it enters an optical medium?

The answer can be found in electromagnetism where we find from Maxwell's equations that the speed of light in vacuum is given by,

$$c_{vac} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

In a medium it is given by

$$c_{med} = \frac{1}{\sqrt{\varepsilon_r \varepsilon_0 \mu_r \mu_0}} = \frac{c_{vac}}{\sqrt{\varepsilon_r \mu_r}}$$

Since $\varepsilon_r$ and $\mu_r$ are both greater than 1, speed of light in a medium will be less than the speed of light in vacuum. The ratio of the speed of light in vacuum to the speed of light in the medium is called the refractive index of the medium, represented by $\mu$.

$$\mu = \frac{c_{vac}}{c_{med}} = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum. More (less) refractive index implies less (more) speed of light in that medium, which therefore is called denser (rarer) medium.

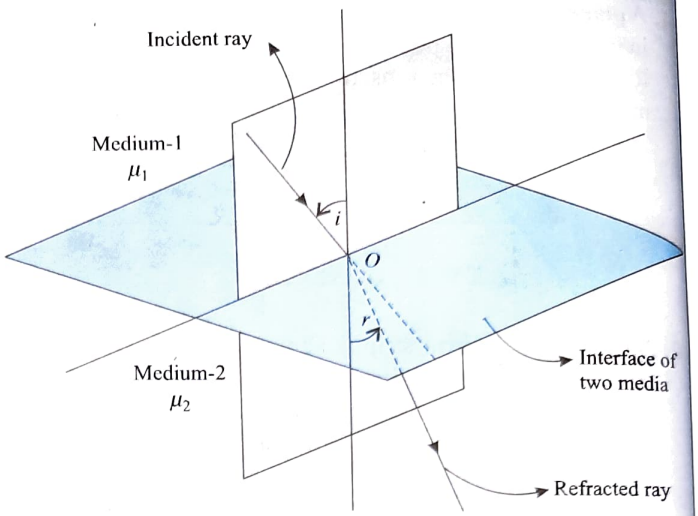
This $\mu$ (refractive index of medium w.r.t. vacuum) is called the **absolute (or standard) refractive index.** Absolute refractive index is commonly referred to simply as refractive index. Further, refractive index is a measure of speed of light in a transparent medium or technically a measure of the **optical density** of the material. For example, the speed of light in water is less than that in air, so water is said to be optically denser. Optical density in general correlates with mass density.

However, in some cases, a material with greater optical density than another can have a lower mass density, e.g., mass density of turpentine oil is less than that of water but its optical density is higher. Thus, greater the refractive index of a material, the greater is the material's optical density and the smaller is the speed of light in that material (as $v = c/\mu$).

# LAWS OF REFRACTION

The bending or the change in direction of propagation of light occurs except when it strikes the interface normally, i.e., when angle of incidence, $i = 0$.

The incident ray, as earlier said, is described by the angle of incidence whereas the refracted ray is described by the angle of refraction ($r$), both angles measured from the normal to the interface at the point of incidence ($O$).




**Law 1:** The incident ray, the normal to the interface at the point of incidence and the refracted ray, lie in the same plane.

**Law 2:** The ratio of the sine of the angle of incidence to the sine of the angle of refraction is always a constant (a different constant for different media),

For any two media the ratio of the sine of the angle of incidence to the sine of the angle of refraction is always constant.

$$\frac{\sin i}{\sin r} = \text{constant} = {}^1\mu_2 = \frac{\mu_2}{\mu_1} \quad \ldots(i)$$

Equation (i) is known as **Snell's law**. The constant ${}^1\mu_2$ (or $\mu_{21}$) is called the refractive index of medium-2 with respect to medium-1 when a ray of light travels from medium-1 to medium-2. In other words, ${}^1\mu_2$ (or $\mu_{21}$) is the refractive index of the medium in which the refracted ray lies w.r.t. the medium in which the incident ray lies.

Equation (i) can be written as $\mu_1 . \sin i = \mu_2 . \sin r$ = constant.

The product of the refractive index and the sine of angle made by the ray with the normal at incident is constant for a given ray in both the media.

## PRINCIPLE OF REVERSIBILITY OF LIGHT RAYS

According to the principle of reversibility of light, if the final path of a ray of light, which has suffered a number of reflections and refractions is reversed, the entire ray retraces its path. Thus if we place a plane mirror at an angle of 90° to the refracted ray then it will become the incident ray and initial ray (incident ray) will be the refracted ray. In such a case, angle of incidence = $r$ and angle of refraction = '$i$'. Thus,

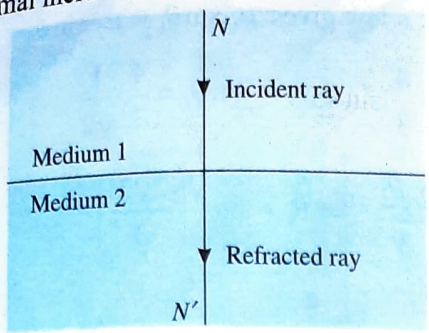
$$\frac{\sin r}{\sin i} = {}^2\mu_1 = \frac{\mu_1}{\mu_2} \quad \ldots(ii)$$

From Eqs. (i) and (ii),

$${}^1\mu_2 = \frac{1}{{}^2\mu_1}$$

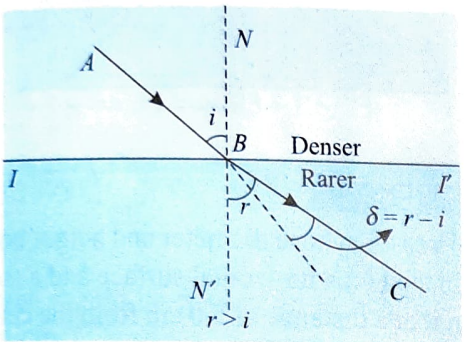
or $${}^2\mu_1 \times {}^1\mu_2 = 1 \quad \ldots(iii)$$

**Case 1.** Normal incidence: $i = 0$

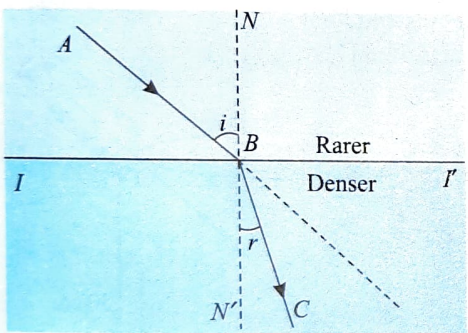



From Snell's law: $r = 0$

**Case 2.** When light moves from denser to rarer medium, it bends away from the normal




**Case 3.** When light moves from rarer to denser medium, it bends towards the normal




## ILLUSTRATION 1.40

Determine the refractive index of glass with respect to water. Given that $\mu_g = 3/2$; $\mu_w = 4/3$.

**Sol.** Refractive index of glass with respect to water or relative refractive index of glass w.r.t. water:

$$^{w}\mu_g = \mu_{gw} = \frac{\text{RI of glass}}{\text{RI of water}} = \frac{\mu_g}{\mu_w} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8} = 1.125$$

## ILLUSTRATION 1.41

Find the speed of light in medium '$a$' if speed of light in medium '$b$' is $c/3$, where $c$ = speed of light in vacuum and light refracts from medium '$a$' to medium '$b$' making 45° and 60°, respectively, with the normal.

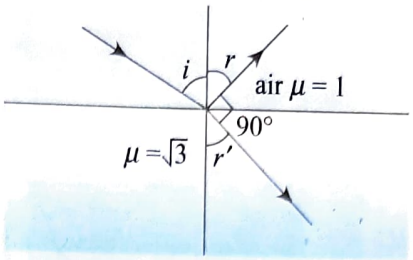
**Sol.** Snell's law:

$$\mu_a \sin\theta_a = \mu_b \sin\theta_b \Rightarrow \frac{c}{v_a}\sin\theta_a = \frac{c}{v_b}\sin\theta_b$$

$$\frac{c}{v_a}\sin 45° = \frac{c}{c/3}\sin 60° \Rightarrow v_a = \frac{c}{3}\sqrt{\frac{2}{3}}$$

## ILLUSTRATION 1.42

A ray of light is incident on a transparent glass slab of refractive index $\sqrt{3}$. If the reflected and refracted rays are mutually perpendicular, what is the angle of incidence?




**Sol.** Let the angle of incidence, angle of reflection, and angle of refraction be $i$, $r$ and $r'$, respectively.

Now, as per the equation $(90° - r) + (90° - r') = 90°$

$r' = (90° - i)$ (because $i = r$, in case of reflection according to Snell's law, $1 \sin i = \mu \sin r'$)

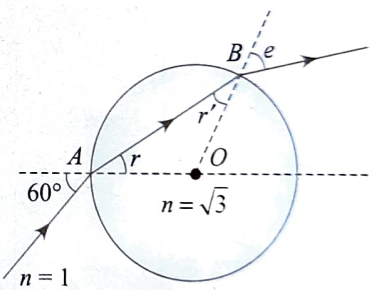
or $\sin i = \mu \sin r'$

or $\sin i = \mu \sin(90° - i) \Rightarrow \tan i = \mu$

or $i = \tan^{-1}\mu = \tan^{-1}\sqrt{3} = 60°$

## ILLUSTRATION 1.43

A light ray is incident on a glass sphere of reflective index $\mu = \sqrt{3}$ at an angle of incidence 60° as shown in figure. Find the angles $r$, $r'$, $e$ and the total deviation after two refractions.



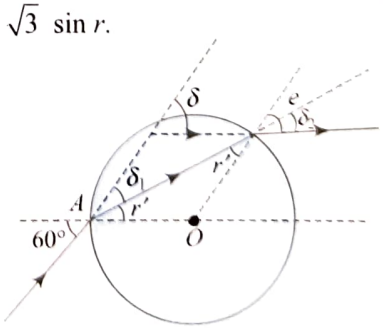

**Sol.** At point '$A$': Applying Snell's law

$1 \sin 60° = \sqrt{3} \sin r \Rightarrow r = 30°$

From symmetry $r' = r = 30°$.

Again applying Snell's law at second surface (at point '$B$')

$1 \sin e = \sqrt{3} \sin r$.




$\Rightarrow \quad e = 60°$

Deviation at the first surface, $\delta_1 = i - r = 60° - 30° = 30°$

Deviation at the second surface, $\delta_2 = e - r' = 60° - 30° = 30°$

Therefore, total deviation = 60°.

## ILLUSTRATION 1.44

A light beam passes from medium 1 to medium 2. Show that the emerging beam is parallel to the incident beam.

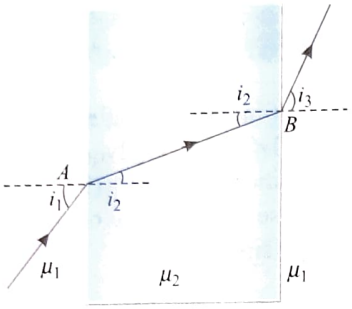
**Sol.** Applying Snell's law at $A$, $\mu_1 \sin i_1 = \mu_2 \sin i_2$

$$\text{or} \quad \frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_1} \qquad \ldots(i)$$

Similarly at $B$, $\mu_2 \sin i_2 = \mu_1 \sin i_3$

$$\therefore \quad \frac{\mu_1}{\mu_2} = \frac{\sin i_2}{\sin i_3} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we have $i_3 = i_1$ i.e., the emergent ray is parallel to incident ray.

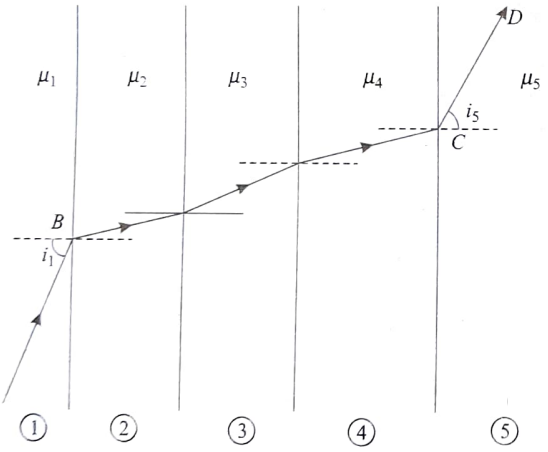



**Important Point:**

If the boundaries of the media are parallel, the emergent ray although laterally displaced, is parallel to the incident ray, if $\mu_1 = \mu_5$. We can also directly apply the Snell's law ($\mu \sin I$ = constant) in medium 1 and 5, i.e.,

$$\mu_1 \sin i_1 = \mu_5 \sin i_5$$

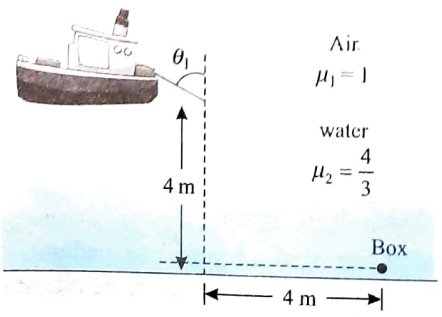
So, $i_1 = i_5$ *if* $\mu_1 = \mu_5$




If any of the boundary is not parallel we cannot use this law directly by jumping the intervening media.

## ILLUSTRATION 1.45

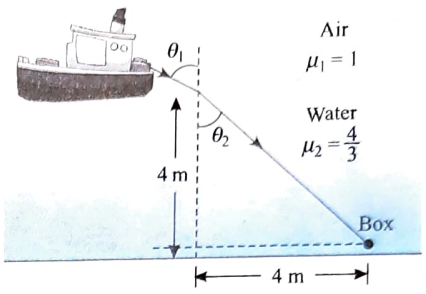
A searchlight on a yacht is being used at night to illuminate a sunken box, as in figure. At what angle of incidence $\theta_1$ should the light be aimed?




**Sol.** $\tan\theta_2 = \frac{4}{4} \Rightarrow \theta_2 = 45°$; Given: $\mu_1 = 1$ for air and $\mu_2 = \frac{4}{3}$ for water; Snell's law gives $\mu_1 . \sin\theta_1 = \mu_2 . \sin\theta_2$

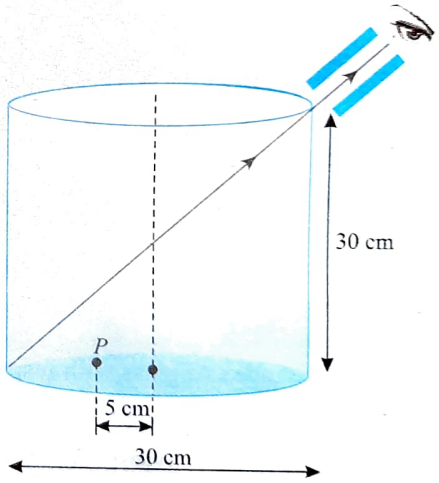
$$1.\sin\theta_1 = \frac{4}{3}.\sin 45° \Rightarrow \sin\theta_1 = \frac{4}{3}.\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin\theta_1 = \frac{2\sqrt{2}}{3} \Rightarrow \theta_1 = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$




## ILLUSTRATION 1.46

A cylindrical vessel, whose diameter and height both are equal to 30 cm, is placed on a horizontal surface and a small particle $P$ is placed in it at a distance of 5.0 cm from the centre. An eye is placed at a position such that the edge of the bottom is just visible. The particle $P$ is in the plane of drawing. Up to what minimum height should water be poured in the vessel to make the particle $P$ visible?




**Sol.** If we pour water in vessel, refraction will take place at air and water interface.

Applying Snell's law at $A$, we get $1 \cdot \sin 45° = \frac{4}{3} \sin\theta$

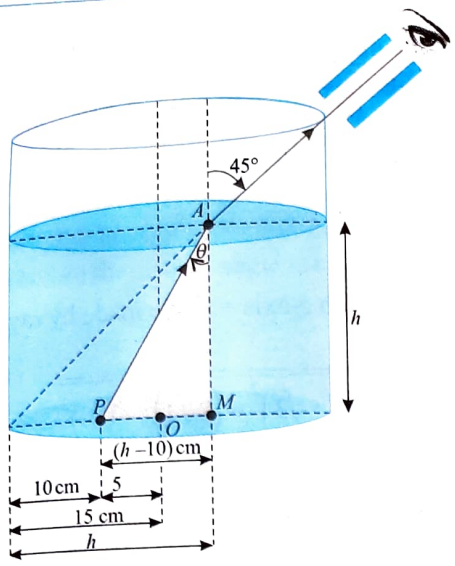
Here, $\theta$ is the angle which the incidence ray of light makes with the normal.

$$\Rightarrow \quad 1\left(\frac{1}{\sqrt{2}}\right) = \frac{4}{3}\sin\theta \quad \Rightarrow \quad \sin\theta = \frac{3}{4\sqrt{2}}$$

$$\tan\theta = \frac{3}{\sqrt{16\times 2 - 9}} = \frac{3}{\sqrt{23}}$$

$\Rightarrow$ In $\Delta APM$,

$$\tan\theta = \frac{3}{\sqrt{23}} = \frac{PM}{AM} = \frac{5+x}{h} = \frac{5+h-15}{h}$$



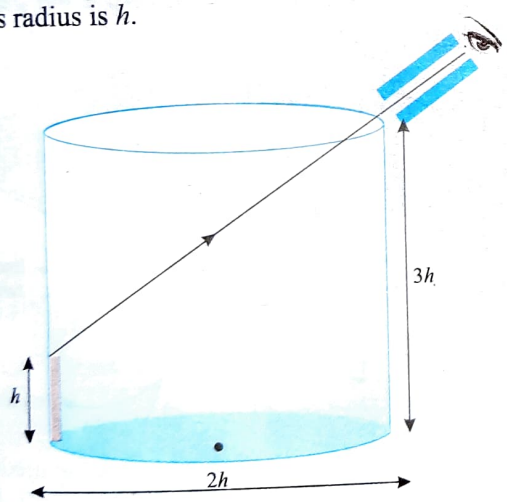

$$\frac{3}{\sqrt{23}}=\frac{h-10}{h}\Rightarrow 3h=h\sqrt{23}-10\sqrt{23}$$

$$h=\left(\frac{10\sqrt{23}}{\sqrt{23}-3}\right)\text{cm}$$

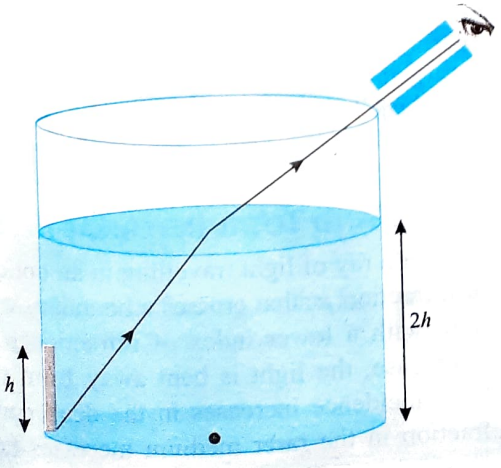
Hence, water should be poured up to height $\frac{10\sqrt{23}}{\sqrt{23}-3}$ cm to make the particle '*P*' visible.

## ILLUSTRATION 1.47

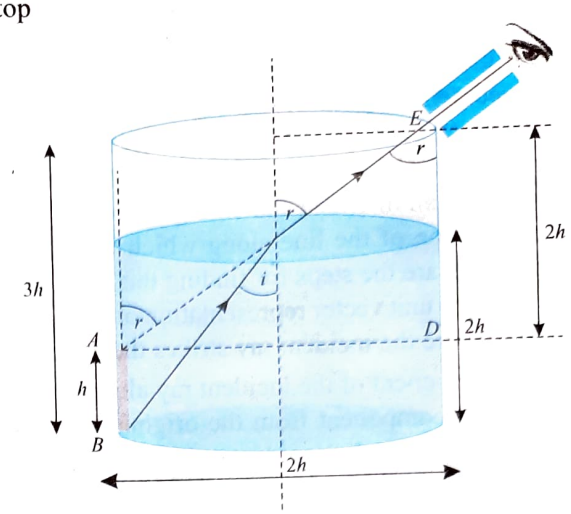
An observer can see through a pin hole, the top of a thin rod of height $h$, placed as shown in figure. The beaker's height is $3h$ and its radius is $h$.




When the beaker is filled with a liquid upto a height $2h$, he can see the lower end of the rod. Find the refractive index of liquid.




**Sol.** Considering refraction from top surface, applying Snell's law at top




$$n.\sin i=1\sin r \qquad \ldots(\text{i})$$

Since $\tan r=\frac{2h}{2h}=1\Rightarrow r=45°$

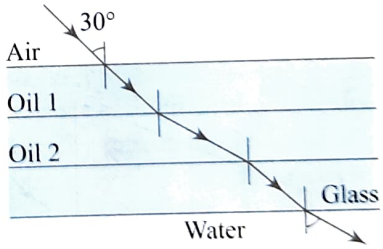
And $\sin i=\frac{h}{h\sqrt{5}}\Rightarrow \sin i=\frac{1}{\sqrt{5}}$

Substituting the values of $i$ and $r$ in Eq. (i)

$$\therefore\quad n\cdot\frac{1}{\sqrt{5}}=1\cdot\frac{1}{\sqrt{2}}\Rightarrow n=\sqrt{\frac{5}{2}}$$

## ILLUSTRATION 1.48

Light is incident from air on an oil layer at an incident angle of 30°. After moving through the oil 1, oil 2, and glass it enters water. If the refraction index of glass and water are 1.5 and 1.3, respectively. Find the angle which the ray makes with the normal in water.




**Sol.** As we know that $\mu \sin i$ = constant

$$\Rightarrow\quad \mu_{\text{air}}\sin i_{(\text{air})}=\mu_{\text{glass}}\sin r_{(\text{glass})}$$

$$\sin i_{(\text{glass})}=\frac{\mu_{\text{air}}}{\mu_{\text{glass}}}\sin i_{(\text{air})} \qquad \ldots(\text{i})$$

Again $\mu_{\text{glass}}\sin i_{\text{glass}}=\mu_{\text{water}}\sin r_{\text{water}}$ ...(ii)

From Eqs. (i) and (ii), $\sin 30°=1.3\sin r$

$$\Rightarrow\quad \sin r=\frac{1}{2\times 1.3}=\frac{1}{2.6};\ r=\sin^{-1}\frac{1}{2.6}$$

**Note:** For parallel layers, we can apply Snell's law directly at initial and final parts.

## VECTOR REPRESENTATION OF A LIGHT RAY

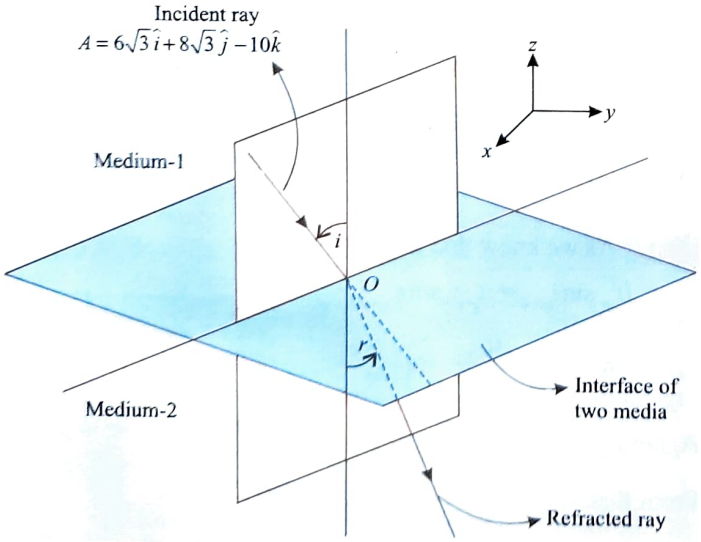
The angle of incidence of a light ray on an interface is usually determined geometrically by simply drawing the ray diagram. However, in some situations, the vector representation of the line along which the light ray travels is given. The ray may then reflect or refract at an interface and emerge. How do we find the vector representation of the line along which the emergent ray travels? Following are the steps for finding this:

- Determine the unit vector representation of the normal to the interface where the incident ray strikes the surface, say $\hat{e}_n$.
- Find the component of the incident ray along the normal.
- Subtract this component from the original ray to compute the plane in which the incident ray travels. Calculate the unit vector that represents the plane of the incident ray and the normal, say $\hat{e}_p$.
- Using the vector dot product, calculate the angle between the incident ray and the normal.
- From the governing equation calculate the angle of reflection/refraction, say $r$.
- We know that the emergent ray will be in the same plane as that of the incident ray and the normal. Thus the unit vector representing the direction of the emergent beam is simply $\cos(r)\hat{e}_n + \sin(r)\hat{e}_p$.

Let us learn to apply these steps through following illustrations.

### ILLUSTRATION 1.49

The $x$-$y$ plane is the boundary between two transparent media. Medium-1 with $z \geq 0$ has refractive index $\sqrt{2}$ and medium-2 with $z \leq 0$ has a refractive index $\sqrt{3}$. A ray of light in medium-1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in the medium 2.




**Sol.** Unit vector representing the normal to the plane $\hat{e}_n = \hat{k}$.

Component of the incident ray along the normal is $-10\hat{k}$. The unit vector that represents the plane of the incident ray and the normal also in the direction of $q$.

Unit vector in the direction of $q$, $\hat{e}_p = \hat{q} = \dfrac{6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2}}$

$$= \frac{6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}}{\sqrt{36\times 3 + 64\times 3}} = \frac{6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j}}{10\sqrt{3}}$$

$$\Rightarrow \quad \hat{q} = \left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)$$

Angle made by ray with $z$-axis = angle made by ray with normal

$$|\cos\gamma| = \frac{A_z}{|A|} = \frac{-10}{\sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2 + (10)^2}}$$

$$= \frac{-10}{\sqrt{400}} = -\frac{1}{2} = \cos\theta_z \Rightarrow \theta_z = 120^\circ$$

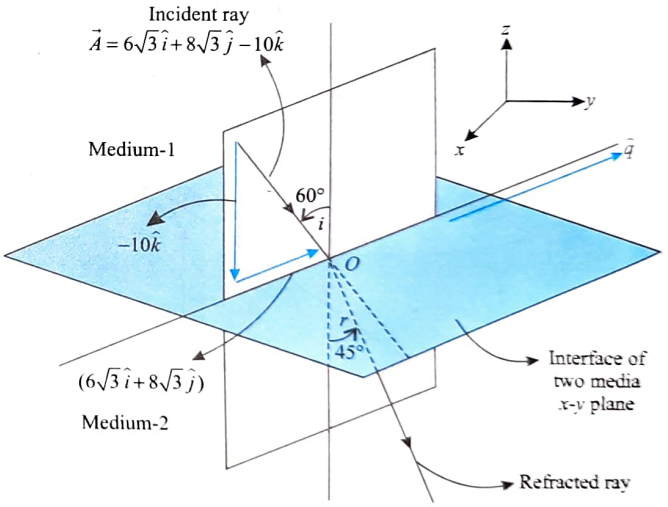
The angle of incidence is $i = 180^\circ - 120^\circ = 60^\circ$

Angle made by refracted ray with $z$-axis = angle made by refracted ray with normal

Applying Snell law $n_1 \sin 60^\circ = n_2 \sin r$

$$\sqrt{2}.\frac{\sqrt{3}}{2} = \sqrt{3}.\sin r$$

$$\Rightarrow \quad \sin r = \frac{1}{\sqrt{2}} \Rightarrow r = 45^\circ$$




Unit vector in the direction of refracted ray in the medium-2,

The equation of the emergent ray is $\cos(r)\hat{e}_n + \sin(r)\hat{e}_p$

$$\hat{r} = \cos 45^\circ(-\hat{k}) + \sin 45^\circ.\hat{q}$$

$$= \frac{1}{\sqrt{2}}.(-\hat{k}) + \frac{1}{\sqrt{2}}.\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)$$

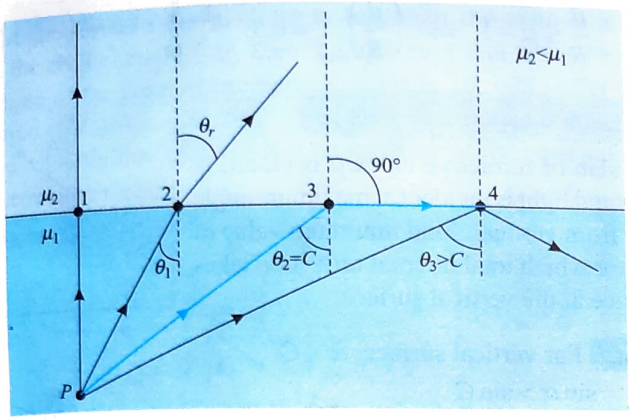
$$\Rightarrow \quad \hat{r} = \frac{1}{\sqrt{2}}.\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right) - \frac{1}{\sqrt{2}}.\hat{k}$$

## CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION

Now let's consider a ray of light travelling in an optical medium with index of refraction $\mu_1$ that crosses a boundary with another optical medium with a lower index of refraction $\mu_2$ such that $\mu_2 < \mu_1$. In this case, the light is bent away from the normal. As the angle of incidence increases in the denser medium the angle of refraction in the rarer medium increases (see figure)

The angle of incidence for which the angle of refraction becomes 90° is called **critical angle** ($\theta_c$ or $C$).

When the angle of incidence of a ray travelling from a denser medium to rarer medium is greater than the critical angle, no refraction occurs. The incident ray is totally reflected back into the same medium, **total internal reflection** takes place instead of refraction. Here, the laws of reflection hold good. Some light is also reflected before the critical angle is achieved, but not totally.




Applying Snell's law at the position of critical angle

$$\frac{\sin C}{\sin 90^\circ} = {}_1\mu_2 = \frac{\mu_1}{\mu_2} \Rightarrow \sin C = \frac{1}{{}_2\mu_1} = \frac{\mu_2}{\mu_1}$$

or $$\sin C = \frac{\text{RI of rarer medium}}{\text{RI of denser medium}}$$

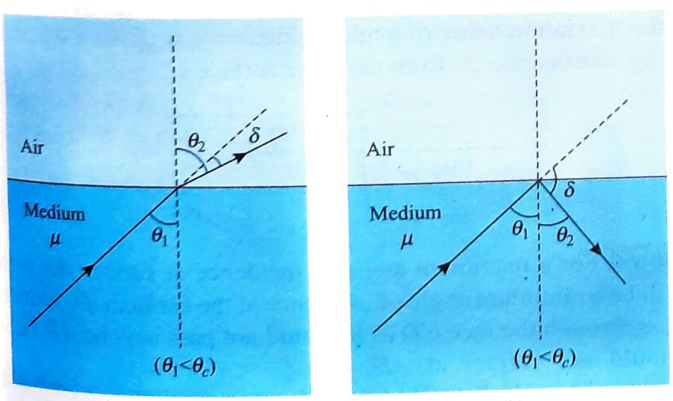
## DEVIATION OF RAY DUE TO REFRACTION

The figure shows a light ray travelling from a denser to rarer medium at an angle $\theta_1$ smaller than the critical angle $C$. The deviation of the light ray is given by $\delta = \theta_2 - \theta_1$. Since $\mu_{denser} \sin\theta_1 = \mu_{rarer} \sin\theta_2$, therefore,

$$\sin\theta_2 = \frac{\mu_D}{\mu_R}\sin\theta_1 = \mu\sin\theta_1$$

$$\theta_2 = \sin^{-1}(\mu\sin\theta_1)$$

$$\delta = \sin^{-1}(\mu\sin\theta_1) - \theta_1$$




(a) (b)

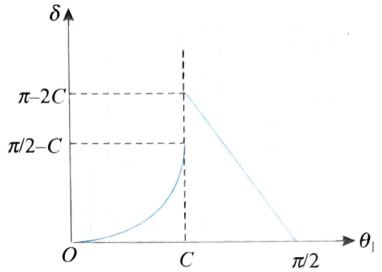
This is a non-linear increasing equation. The maximum value of $\delta$ occurs when $\theta_1 = C$, and is equal to

$$\delta_{max} = \frac{\pi}{2} - C$$

If the light is incident at angle $\theta_1 > C$, then the angle of deviation is given by, $\delta = \theta - 2\theta_1$.

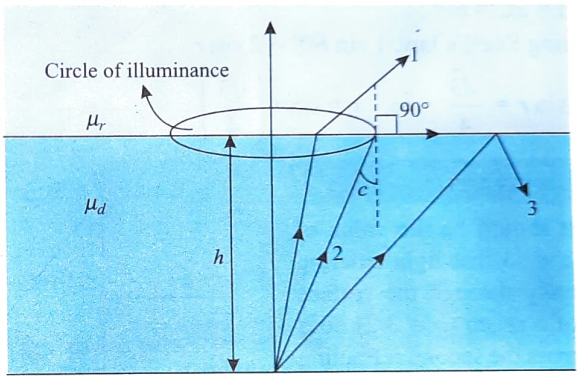
This is a linearly decreasing function. The maximum value of $\delta$ occurs when $\theta_1 = C$ and is equal to $\delta_{max} = \pi - 2C$.

The variation of $\delta$ with the angle of incidence $\theta_1$ is plotted in figure.




## Circle of Illuminance

For total internal reflection, angle of incidence should be greater than the critical angle ($i > C$). Figure shows a luminous object placed in denser medium at a distance $h$ from an interface separating two media of refractive indices $\mu_r$ and $\mu_d$. Subscripts $r$ and $d$ stand for rarer and denser media, respectively.



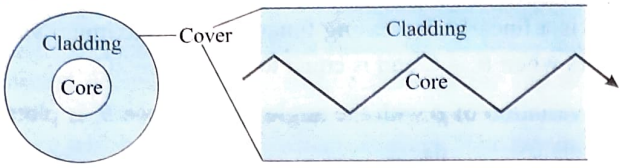

In the figure, ray 1 strikes the surface at an angle less than critical angle $C$ and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle greater than the critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called **circle of illuminance (COI)**.

All light rays striking inside the circle of illuminance get refracted in the rarer medium. If an observer is in the rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in the rarer medium and then the object cannot be seen from the rarer medium. Radius of COI can be easily found.

## Optical Fiber

An important application of total internal reflection is the transmission of light in **optical fibers**. Light is injected into a fiber so that the angle of incidence at the outer surface of the fiber is greater than the critical angle for total internal reflection. The light is then transported the length of the fiber as it bounces repeatedly from the fiber surface. Thus, optical fibers can be used to transport light from a source to a destination. Figure shows a bundle of optical fibers with the end of the fibers open to the

camera. The other end of the fibers is optically connected to a light source.




Note that optical fibers can transport light in directions other than a straight line. The fiber may be bent, as long as the radius of curvature of the bend is not small enough to allow the light travelling in the optical fiber to have angles of incidence $\theta_i$ less than $\theta_c$.

### ILLUSTRATION 1.50

Find the angle of refraction in a medium ($\mu = 2$) if light is incident in vacuum, making an angle equal to twice the critical angle.

**Sol.** Since the incident light is in rarer medium, total internal reflection cannot take place.

$$C = \sin^{-1}\frac{1}{\mu} = 30^\circ$$

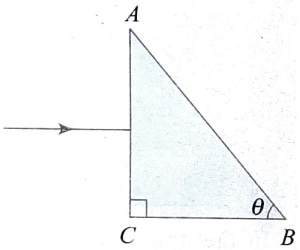
$$\therefore \quad i = 2C = 60^\circ$$

Applying Snell's law, $1 \sin 60^\circ = 2 \sin r$

$$\sin r = \frac{\sqrt{3}}{4} \quad \Rightarrow \quad r = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$$

### ILLUSTRATION 1.51

What should be the value of angle $\theta$ so that light entering normally through the surface $AC$ of a prism ($n = 3/2$) does not cross the second refracting surface $AB$?




**Sol.** Light ray will pass the surface $AC$ without bending since it is incident normally. Suppose it strikes the surface $AB$ at an angle of incidence $i$.

$$i = 90 - \theta$$

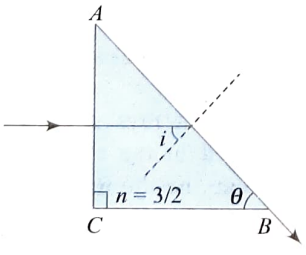
For the required condition: $i > C$

$$\Rightarrow \quad 90^\circ - \theta > C$$

or $\quad \sin(90^\circ - \theta) > \sin C$

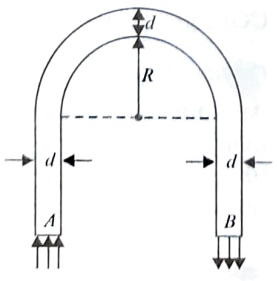
or $\quad \cos\theta > \sin C = \frac{1}{3/2} = \frac{2}{3}$

or $\quad \theta < \cos^{-1}\frac{2}{3}$




### ILLUSTRATION 1.52

A glass rod having square cross-section is bent into the shape as shown in the figure. The radius of the inner semi-circle is $R$ and width of the rod is $d$. Find the minimum value of $d/R$ so that the light that enters at $A$ will emerge at $B$. Refractive index of glass is $\mu = 1.5$



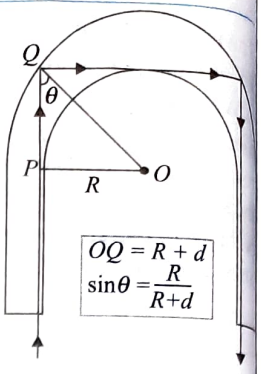

**Sol.** Consider the figure. If smallest angle of incidence $\theta$ is greater than critical angle then all light will emerge out at $B$.

$$\Rightarrow \quad \theta \geq \sin^{-1}\left(\frac{1}{\mu}\right) \quad \Rightarrow \quad \sin\theta \geq \frac{1}{\mu}$$

From figure, $\sin\theta = \dfrac{R}{R+d}$

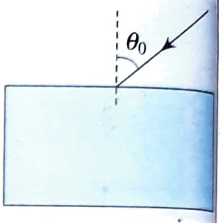
$$\Rightarrow \quad \frac{R}{R+d} \geq \frac{1}{\mu} \quad \Rightarrow \quad \left(1+\frac{d}{R}\right) \leq \mu$$

$$\Rightarrow \quad \frac{d}{R} \leq \mu - 1 \quad \Rightarrow \quad \left(\frac{d}{R}\right)_{max} = 0.5$$




### ILLUSTRATION 1.53

A slab of refractive index $\mu$ is placed in air and light is incident at maximum angle $\theta_0$ from vertical. Find minimum value of $\mu$ for which total internal reflection takes place at the vertical surface.



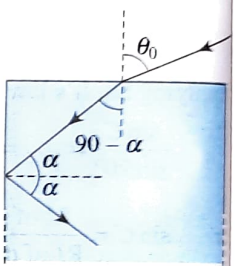

**Sol.** For vertical surface, $\alpha > C$

$$\Rightarrow \quad \sin\alpha > \sin C$$

$$\Rightarrow \quad \sin\alpha > \frac{1}{\mu} \qquad \ldots(i)$$

For horizontal surface $\sin\theta_0 = \mu\cos\alpha$

$$\Rightarrow \quad \sin\alpha = \sqrt{\frac{\mu^2 - \sin^2\theta}{\mu^2}} \qquad \ldots(ii)$$




From Eqs. (i) and (ii), we get

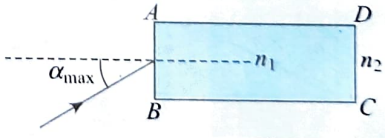
$$\frac{\sqrt{\mu^2 - \sin^2\theta}}{\mu} > \frac{1}{\mu} \quad \Rightarrow \quad \mu^2 - \sin^2\theta > 1$$

$$\Rightarrow \quad \mu > \sqrt{1 + \sin^2\theta}$$

So, minimum value of $\mu = \sqrt{1 + \sin^2\theta}$

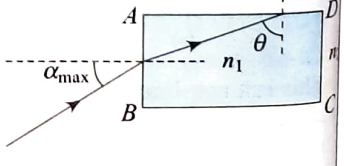
### ILLUSTRATION 1.54

A rectangular slab $ABCD$, of refractive index $n_1$, is immersed in water of refractive index $n_2$ ($n_1 < n_2$). A ray of light is incident at the surface $AB$ of the slab as shown in figure. Find the maximum value of angle of incidence $\alpha_{max}$, such that the ray comes out only from the other surface $CD$.




**Sol.** For a maximum angle of incidence at surface $AB$ there will be a minimum angle of incidence at the surface $AD$. A ray to pass through the face $CD$ as it should not pass beyond $AD$ i.e. should not refract at $AD$.

Hence, the angle $\theta$ should be the critical angle.




By Snell's law, $\sin\theta = \dfrac{n_2}{n_1}$

$$n_2 \sin\alpha_{max} = n_1 \sin(90 - \theta)$$

$$\sin\alpha_{max} = \frac{n_1}{n_2}\cos\theta \Rightarrow \alpha_{max} = \sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\frac{n_2}{n_1}\right)\right]$$

## ILLUSTRATION 1.55

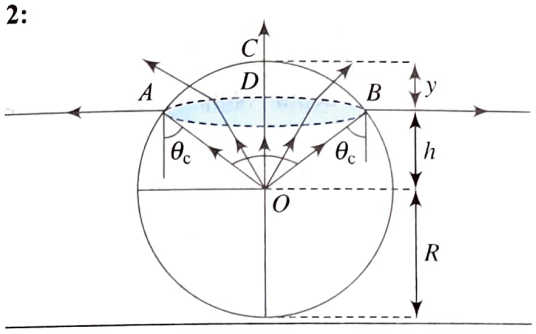
A point source of light is placed at distance $h$ below the surface of a large and deep lake. What fraction of light will escape through the surface of water?

**Sol.** **Method 1:** Due to total internal reflection some of the light rays incident at the interface will return back into water. So, only that portion of light will escape for which the angle of incidence at the interface of the medium is less than the critical angle.

If the critical angle is $\theta_c$, then the light rays that reach beyond the base of the cone whose vertical angle is $2\theta_c$ will suffer total internal reflection.

Hence, only the light incident on the base of the cone refracts and escapes.

**Method 2:**




The fraction of light escaping,

$$f = \frac{\text{Area of the cap}}{\text{Area of sphere}} = \frac{2\pi Ry}{4\pi R^2}, \text{ i.e.,}$$

$$f = \frac{1}{2}\left[\frac{y}{R}\right] = \frac{1}{2}\left[\frac{R-h}{R}\right]$$

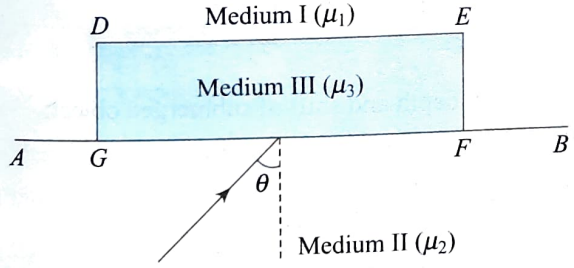
Area of cap $ABCD$ can be calculated by using method of integration,

i.e., $$f = \frac{1}{2}\left[1 - \frac{h}{R}\right] = \frac{1}{2}[1 - \cos\theta_c]$$

$$= \frac{1}{2}\left[1 - \sqrt{1 - \sin^2\theta_c}\right] = \frac{1}{2}\left[1 - \sqrt{1 - \frac{1}{n^2}}\right]$$

## ILLUSTRATION 1.56

A monochromatic light is incident on the plane interface $AB$ between two media of refractive indices $\mu_1$ and $\mu_2$ ($\mu_2 > \mu_1$) at an angle of incidence $\theta$ as shown in figure.




The angle $\theta$ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now, if a transparent slab $DEFG$ of uniform thickness and of refractive index $\mu_3$ is introduced on the interface (as shown in the figure), show that for any value of $\mu_3$ all light will ultimately be reflected back into medium II.

**Sol.** We will use the symbol $\leq$ to mean 'infinitesimally greater than'.

When the slab is not inserted,

$$\theta \leq \theta_c = \sin^{-1}(\mu_1/\mu_2) \quad \text{or} \quad \sin\theta \geq \mu_1/\mu_2$$

When the slab is inserted, we have two cases

$$\mu_3 \leq \mu_1 \quad \text{and} \quad \mu_3 > \mu_1$$

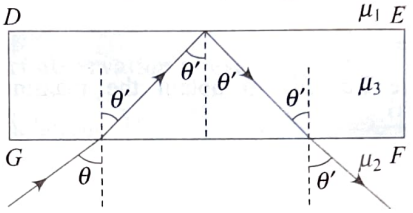
**Case I:** $\mu_3 < \mu_1$. We have $\sin\theta \geq \mu_1/\mu_2 \geq \mu_3/\mu_2$

Thus, the light is incident on $AB$ at an angle greater than the critical angle $\sin^{-1}(\mu_3/\mu_2)$. It suffers total internal reflection and goes back to medium II.

**Case II:** $\mu_3 > \mu_1$

$$\sin\theta \geq \mu_1/\mu_2 < \mu_3/\mu_2$$

Thus, the angle of incidence $\theta$ may be smaller than the critical angle $\sin^{-1}(\mu_3/\mu_2)$ and hence it may enter medium III. The angle of refraction $\theta'$ is given by (see figure).




$$\frac{\sin\theta}{\sin\theta'} = \frac{\mu_3}{\mu_2} \qquad \text{...(i)}$$

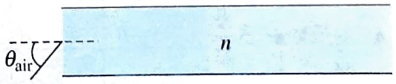
$$\Rightarrow \quad \sin\theta' = \frac{\mu_2}{\mu_3}\sin\theta \leq \frac{\mu_2}{\mu_3}\cdot\frac{\mu_1}{\mu_2}$$

Thus, $\sin\theta' \geq \dfrac{\mu_1}{\mu_3} \quad \Rightarrow \quad \theta' \geq \sin^{-1}\left(\dfrac{\mu_1}{\mu_3}\right)$ ...(ii)

As the slab has parallel faces, the angle of refraction at the face $FG$ is equal to the angle of incidence at the face $DE$. Equation (ii) shows that this angle is infinitesimally greater than the critical angle here. Hence, the light suffers total internal reflection and falls at the surface $FG$ at an angle of incidence $\theta'$. At this face, it will refract into medium II and the angle of refraction will be $\theta$ as shown by Eq. (i). Thus, the total light energy is ultimately reflected back into medium II.

## ILLUSTRATION 1.57

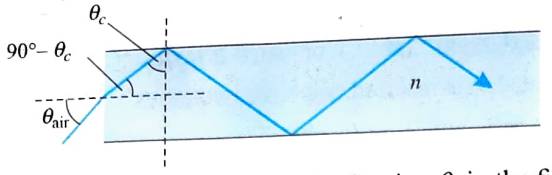
Consider a long optical fiber with index of refraction $n$, that is surrounded by air. (There is no cladding.) The end of the fiber is polished to be flat and is perpendicular to the length of the fiber. A light ray from a laser is incident from air onto the centre of the circular face of the optical fiber at angle $\theta_{air}$.




What is the maximum angle of incidence for this light ray such that it will be confined and transported by the optical fiber? (Neglect any reflection as the light ray enters the fiber.)

**Sol.** The light ray will refract as it enters the optical fiber. Once inside the fiber, if the angle of incidence on the surface of the optical fiber is greater than the critical angle for total internal reflection, then the light is transmitted without loss.

Figure shows a sketch of the light ray entering the optical fiber and reflecting off its inner surface.




The critical angle for total internal reflection $\theta_c$ in the fiber for light entering from air is given by

$$\sin\theta_c = \frac{1}{n} \Rightarrow \theta_c = \sin^{-1}\left(\frac{1}{n}\right) \quad ...(i)$$

where $n$ is the index of refraction of the optical fiber. For the light entering the optical fiber, Snell's law tells us that since $n_{air} = 1$,

$$\sin\theta_{air} = n\sin\theta_{medium} \quad ...(ii)$$

where $\theta_{medium}$ is the angle of the refracted light ray in the fiber. From figure we can see that

$$\theta_{medium} = 90° - \theta_c \quad ...(iii)$$

We can solve Eq. (ii) to obtain the maximum angle of incidence,

$$\theta_{air} = \sin^{-1}\left(n\sin\theta_{medium}\right)$$

Using Eq. (iii) we can write

$$\theta_{air} = \sin^{-1}\left(n\sin\left(90° - \theta_c\right)\right) = \sin^{-1}\left(n\cos\theta_c\right)$$

We can then use Eq. (i) to obtain

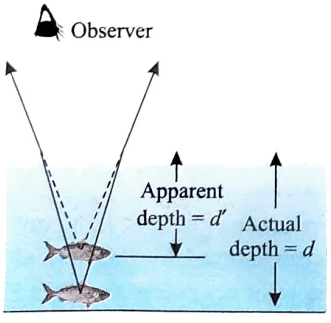
$$\theta_{air} = \sin^{-1}\left(n\cos\left(\sin^{-1}\left(\frac{1}{n}\right)\right)\right)$$

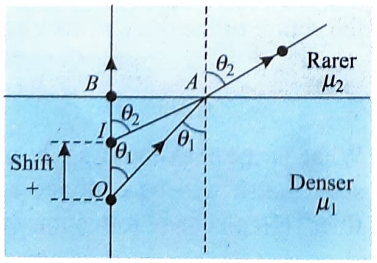
## Apparent Shift of an Object Due to Refraction

Due to bending of light at the interface of two different media, the image formed due to refraction appears at a place other than the object position. This image formation due to refraction creates illusion of shifting of the object position.

Locating the position of image formed becomes much simpler if we restrict ourselves to nearly normal incident rays.

Consider an object $O$ in the medium (RI = $\mu_1$). After refraction, the ray at the interface bend. When the bent ray falls in our eye our eye perceives it along a straight line and it appears at $I$. (see figure).




For nearly normal incident rays, $\theta_1$ and $\theta_2$ will be very small.

$$\tan\theta_1 \simeq \sin\theta_1 = \frac{AB}{\text{object distance from the refracting surface}}$$

$$\tan\theta_2 \simeq \sin\theta_2 = \frac{AB}{\text{image distance from the refracting surface}}$$

$$\Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{\text{Image distance from the refracting surface}}{\text{Object distance from the refracting surface}} = \frac{\tan\theta_1}{\tan\theta_2}$$

Using Snell's law, $\mu_1\sin\theta_1 = \mu_2\sin\theta_2 \Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{\mu_2}{\mu_1} = \frac{\tan\theta_1}{\tan\theta_2}$

$$\Rightarrow \frac{\frac{AB}{OB}}{\frac{BA}{BI}} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{BI}{OB} = \frac{\text{Apprant depth}}{\text{Real depth}} = \frac{\mu_2}{\mu_1}$$

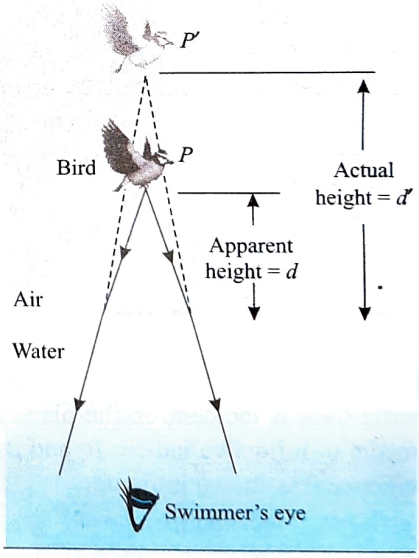
$$\Rightarrow \text{Apprant depth} = \frac{\text{Real depth}}{\mu_1/\mu_2} = \frac{\text{Real depth}}{\mu_{relative}}$$

$$\mu_{relative} = \frac{\mu_i(\text{RI of medium of incidence})}{\mu_r(\text{RI of medium of refraction})}$$

So, Apparent shift = Real depth – Apparent depth

$$\Rightarrow s = \text{Real depth}\left(1 - \frac{1}{\mu_{relative}}\right)$$

**Note:** If $\mu_1 < \mu_2$, shift becomes negative (in the direction opposite to initial, say travelling). In the given figure, the refractive index of medium of incident ($\mu_1$, the medium where the bird is located, *air*) is less than the refractive index of medium of refraction ($\mu_2$, the medium where the observer is located, *water*).In this case, Image distance > object distance, i.e., image is farther from the refracting surface (see figure).

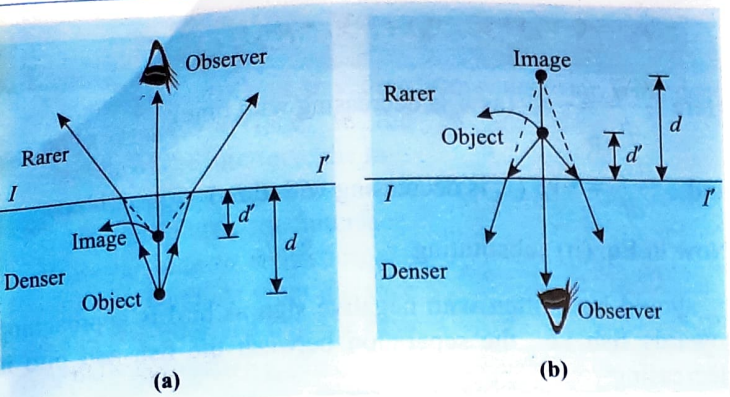



**Note:** Apparent depth and shift of submerged object:
At near normal incidence (small angle of incidence $i$), apparent depth ($d'$) is given by: $d' = \frac{d}{n_{relative}}$

where $n_{relative} = \frac{n_i(\text{RI of medium of incidence})}{n_r(\text{RI of medium of refraction})}$

$d$ = distance of object from the interface = real depth
$d'$ = distance of image from the interface = apparent depth

In general, we can write $\frac{n_i}{d} = \frac{n_r}{d'}$




## ILLUSTRATION 1.58

An object lies 100 cm inside water ($\mu = 4/3$). It is viewed from air nearly normally. Find the apparent depth of the object.

**Sol.** Here object is placed inside water and the observer is situated in air. Hence, apparent depth

$$d' = \frac{d}{(n_i/n_r)} = \frac{d}{\left(\frac{n_{water}}{n_{air}}\right)} = \frac{d}{n_{relative}} = \frac{100}{\frac{4/3}{1}} = 75 \text{ cm}$$

## ILLUSTRATION 1.59

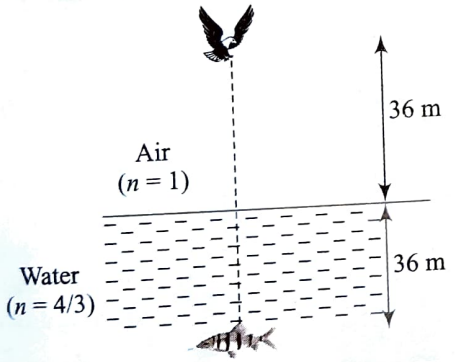
See figure and answer the following questions.

**(a)** Find apparent height of the bird.

**(b)** Find apparent depth of the fish.

**(c)** At what distance will the bird appear to the fish?

**(d)** At what distance will the fish appear to the bird?




**Sol.**

**(a)** Here bird is an object and fish is an observer. Hence, apparent height observed by the fish

$$d'_B = \frac{d}{n_{rel}} = \frac{d}{\left(\frac{n_{air}}{n_{water}}\right)} \Rightarrow d'_B = \frac{36}{\frac{1}{\left(\frac{4}{3}\right)}} = \frac{36}{3/4} = 48 \text{ m}$$

**(b)** Here the fish is an object and the bird is an observer. Hence, apparent height observed by the bird

$$d'_F = \frac{d}{n_{rel}} = \frac{d}{\left(\frac{n_{water}}{n_{air}}\right)} \Rightarrow d'_F = \frac{36}{4/3} = 27 \text{ m}$$

**(c)** For the fish, the bird will be observed at a distance $d'_B$ from the fish: $d_B = 36 + 48 = 84$ m

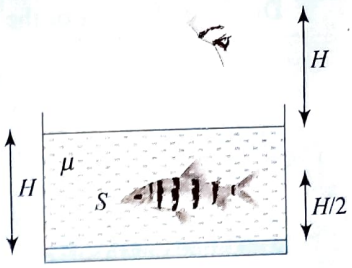
**(d)** For the bird, the fish will be observed at a distance '$d_F$' from the bird: $d_F = 27 + 36 = 63$ m

## ILLUSTRATION 1.60

Consider the situation in figure. The bottom of the pot is a reflecting plane mirror, $S$ is a small fish, and $T$ is a human eye. Refractive index of water is $\mu$.

**(a)** At what distance(s) from itself will the fish see the image(s) of the eye?

**(b)** At what distance(s) from itself will the eye see the image(s) of the fish?




**Sol.**

**(a)** The fish will observe the images of eye one from direct observation and the other reflected image from the plane mirror.

**(i) Direct observation of eye from fish:**

Apparent height, $H' = \frac{H}{n_{rel}} = \frac{H}{\left(\frac{n_{air}}{n_{water}}\right)} = \frac{H}{\left(\frac{1}{\mu}\right)}$

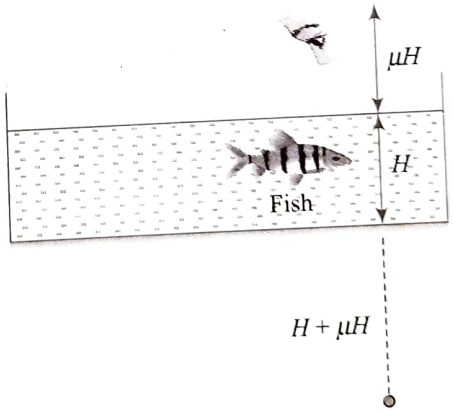
Hence, $H' = \mu H$

Distance of image of eye from fish

$$d = \frac{H}{2} + \mu H = H\left(\frac{1}{2} + \mu\right)$$

**(ii) Observation of reflected image of the eye from the fish:**

For mirror, the distance of eye from it will be $(H + \mu H)$. Hence, the image of eye from mirror will be $(H + \mu H)$ behind the mirror. Hence, distance of image of eye from the fish




$$d' = \frac{H}{2} + H + \mu H = \frac{3}{2}H + \mu H$$

$$\Rightarrow \quad d' = H\left(\frac{3}{2} + \mu\right)$$

**(b)** The eye will also observe two images of the fish, one from direct observation and the other reflected image from the mirror.

**(i) Direct observation of fish from eye:**

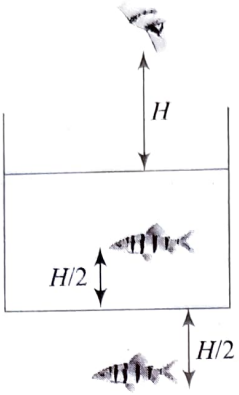
Apparent depth of the fish observed by the eye

$$H' = \frac{H/2}{n_r} = \frac{H/2}{\left(\frac{n_{water}}{n_{air}}\right)} = \frac{H/2}{\mu} = \frac{H}{2\mu}$$

Distance of image of the fish from the eye,

$$d = H + \frac{H}{2\mu} = H\left(1 + \frac{1}{2\mu}\right)$$

**(ii) Eye observing image of fish:**
The eye will observe the image of fish reflected from the mirror.
Apparent depth of image of the fish from air and water interface.




$$H' = \frac{\text{Real depth}}{n_{rel}} = \frac{\frac{3}{2}H}{\left(\frac{n_{water}}{n_{air}}\right)}$$

Here real depth from top surface of water $= H + \frac{H}{2} = \frac{3}{2}H$

$$H' = \frac{3H}{2\mu}$$

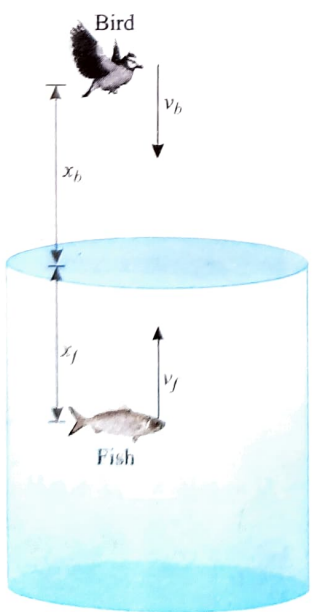
Hence, distance between this image and the eye,

$$d' = H + \frac{3H}{2\mu} = H\left(1 + \frac{3}{2\mu}\right)$$

## ILLUSTRATION 1.61

A fish in an aquarium approaches the top surface of the water at a rate of 3 m $s^{-1}$ observes a bird approaching it at 8 $ms^{-1}$. If the refractive index of water is (4/3), find the actual velocity of the bird.

**Sol.** The fish (in denser medium) observing the bird (in rarer medium).




The distance of the bird as seen from fish,

$$h_{b,f} = x_f + \mu x_b \qquad \ldots(i)$$

For finding speed we need to differential Eq. (i) w.r.t. time

$$\frac{dh_{b,f}}{dt} = \frac{dx_f}{dt} + \mu\frac{dx_b}{dt}$$

$$v_{bf} = (-v_f) + \mu(-v_b) = -(v_f + \mu v_b) \qquad \ldots(ii)$$

Here $\frac{dx_f}{dt} = -v_f$ (as $x_f$ is decreasing with time)

and $\frac{dx_b}{dt} = -v_b$ ($x_b$ is decreasing with time)

Now in Eq. (ii) substituting $v_{bf} = -8$ m/s

$v_{bf}$ should be written with negative sign as bird is approaching towards fish i.e., the separation between the fish and bird is decreasing.

$$v_f = 3\text{ m/s} \text{ and } \mu = \frac{4}{3}$$

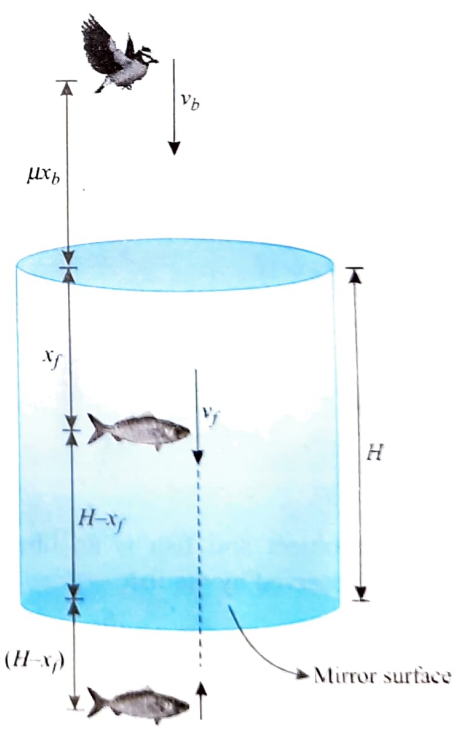
We get $-8 = -\left(3 + \frac{4}{3} \times v_b\right)$

$$8 = 3 + \frac{4}{3} \times v_b \Rightarrow v_b = \frac{15}{4}\text{ m/s}$$

## ILLUSTRATION 1.62

A bird is moving towards an aquarium with speed 4 m/s. A fish in the aquarium is moving down with speed 2 m/s. Suppose the base of the aquarium is silvered. Find the speed of the bird as seen by fish in mirror.

**Sol.** Distance of the bird as seen by fish through mirror.




$$h_{b,f} = \mu x_b + H + H - x_f$$

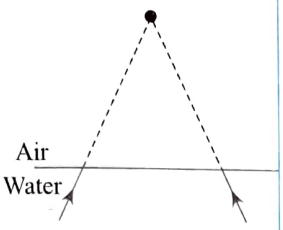
$$\frac{dx_{bf}}{dt} = \mu\frac{dx_b}{dt} - \frac{dx_f}{dt}$$

$$= \mu(-v_b) - v_f = -(\mu v_b + v_f)$$

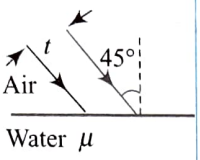
$$= -\left(\frac{4}{3} \times 4 - 2\right) = -\frac{22}{3}\text{ m/s}$$

## CONCEPT APPLICATION EXERCISE 1.4

1. A converging set of rays, travelling from water to air, is incident on a plane interface. In the absence of the interface, the rays would have converged to a point $O$, 60 cm above the interface. However, due to refraction the rays will bend. At what distance above the interface will the rays actually converge?

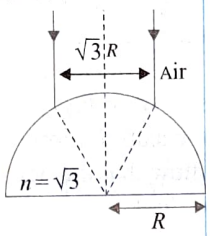



2. A beam of width $t$ is incident at 45° on an air–water boundary. The width of the beam in water is _____.

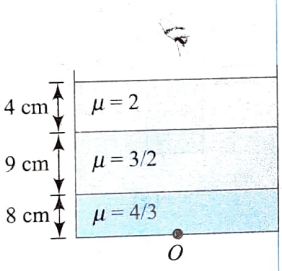



3. A fish is 60 cm under water ($\mu = 4/3$). A bird directly overhead looks at the fish. If the bird is at a distance of 120 cm from the water surface, **(a)** what is the apparent position of the fish as seen by the bird and **(b)** what is the apparent position of the bird as seen by the fish?

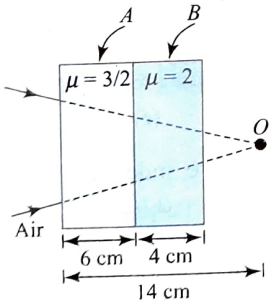
4. A light beam of diameter $\sqrt{3R}$ is incident symmetrically on a glass hemisphere of radius $R$ and of refractive index $n = \sqrt{3}$. Find radius of the beam at the base of hemisphere.



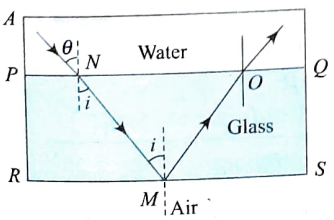

5. A tank contains three layers of immiscible liquids. The first layer is of water with refractive index 4/3 and thickness 8 cm. The second layer is of oil with refractive index 3/2 and thickness 9 cm while the third layer is of glycerine with refractive index 2 and thickness 4 cm. Find the apparent depth of the bottom of the container.



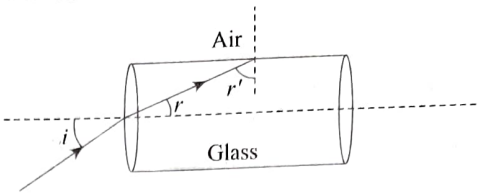

6. A convergent beam is incident on two slabs placed in contact as shown in figure. Where will the rays finally converge?



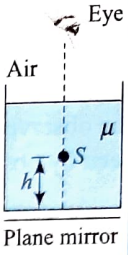

7. A slab of water is on the top of a glass slab of refractive index 2. At what angle to the normal must a ray be incident on the top surface of the glass slab so that it is reflected from the bottom surface as shown in figure?



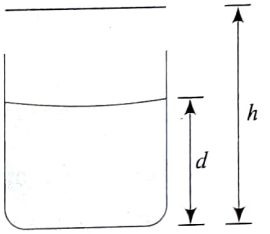

8. What should be the value of refractive index $\mu$ of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod?



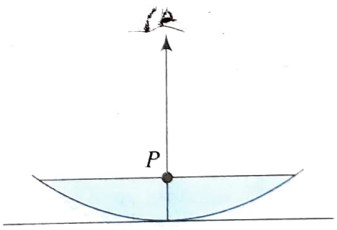

9. In figure, a point source $S$ is placed at a height $h$ above the plane mirror in a medium of refractive index $\mu$.
   **(a)** Find the number of images seen for normal view.
   **(b)** Find the distance between the images.



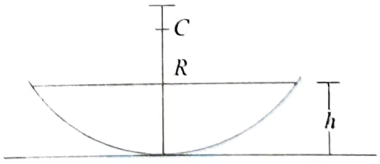

10. Consider the situation shown in figure. A plane mirror is fixed at a height $h$ above the bottom of a beaker containing water (refractive index $\mu$) up to a height $d$. Find the position of the image of the bottom formed by the mirror.




11. A concave mirror of radius 40 cm lies on a horizontal table and water is filled in it up to a height of 5.00 cm. A small dust particles floats on the water surface at a point $P$ vertically above the point of contact of the mirror with the table. Locate the image of the dust particle as seen from a point directly above it. The refractive index of water is 1.33.




12. A concave mirror of radius $R$ is kept on a horizontal table. Water (refractive index = $\mu$) is poured into it up to a height $h$. Where should an object be placed so that its image is formed on itself?



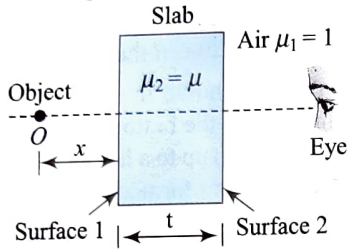

### ANSWERS

1. 45 cm
2. $\frac{t\sqrt{2\mu^2-1}}{\mu}$
3. 45 cm; 160 cm
4. $\frac{R}{\sqrt{3}}$
5. 14 cm
6. 18 cm (left)
7. $\sin^{-1}\left(\frac{3}{4}\right)$
9. (a) Two; (b) $\frac{2h}{\mu}$
10. $h - d + (d/\mu)$
11. 8.75 cm
12. $\frac{R-h}{\mu}$

# REFRACTION THROUGH A PARALLEL SLAB

A slab is formed when a medium is isolated from its surroundings by two plane surfaces parallel to each other. In this section, we will determine the position and nature of the image formed when a slab is placed in front of an object.

Consider an object $O$ placed a distance $x$ in front of a glass slab of thickness "$t$" and refractive index $\mu$. The observer is on the other side of the slab. A ray of light from the object first refracts at surface (1) then refracts at surface (2) before reaching the observer (see figure). Let us analyse the location of image as seen by observer taking one step at a time.

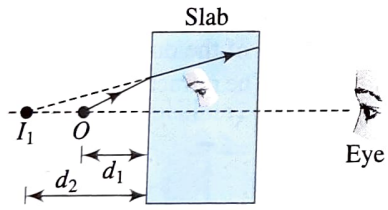



**All objects viewed through a transparent glass slab**

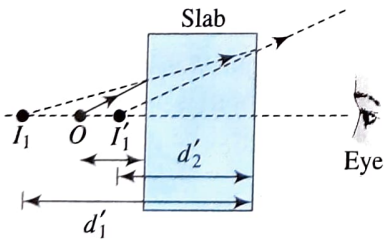
**Refraction at surface 1:**

Here, $\mu_1 = 1$, $\mu_2 = \mu$, $d_1$ = real depth, $d_2$ = apparent depth

$$\text{Apparent depth, } d_2 = \frac{d_{\text{real}}}{n_{\text{relative}}} = \frac{d_1}{\left(\frac{n_{\text{incident}}}{n_{\text{refraction}}}\right)} = \frac{d_1}{\left(\frac{\mu_1}{\mu_2}\right)}$$




**(a) The apparent position of the object after refraction at the first surface**




**(b) Final position of the object after refraction at both surfaces**

$$d_2 = \frac{x}{(1/\mu)}$$

$$d_2 = \mu x \qquad \ldots(i)$$

Thus, the first image is formed behind the object at the point $I_1$. $I_1$ now serves as the object for the second surface [see Fig. (a)].

**Refraction at surface 2:**

Here, $n_{\text{incident}} = \mu_2 = \mu$

$n_{\text{refraction}} = \mu_1 = 1$

$$\text{Apparent depth, } d_2' = \frac{d_{\text{real}}}{n_{\text{real}}}$$

$$d_2' = \frac{d_1'}{\left(\frac{\mu}{1}\right)} \text{ but } d_1' = (d_2 + t)$$

$$d_2' = \frac{(\mu x + t)}{\mu} \qquad \ldots(ii)$$

Thus, the final image $I_1'$ is at a distance $x + (t/\mu)$ behind the second interface. The original object was at a distance $x + t$ behind the second interface. Therefore, the image appears shifted by

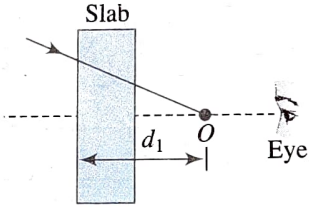
$$s = \left[x + \frac{t}{\mu}\right] - [x + t] = t\left[1 - \frac{1}{\mu}\right] \qquad \ldots(iii)$$

We can say a slab appears to shift the object along the perpendicular to the slab by a distance $t[1 - (1/\mu)]$ in the direction of the travelling ray [see Fig. (b)].

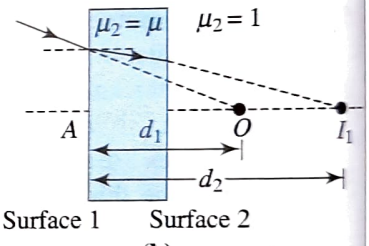
**How about the direction of the shift?**

As discussed in the previous section the object is shifted towards the glass slab. Is it always true?

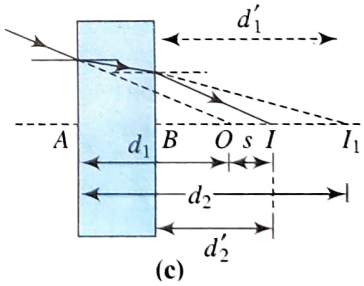
Consider a converging set of rays incident on a glass slab as shown in Fig. (a). Here, we have a virtual object to the right of the slab. Following the same procedure as that in earlier, we can show that the net shift is same as given by Eq. (iii). However, let us look a little closely at the direction of the shift.




**(a)**




**(b)**




**(c)**

Here, $\mu$ is the refractive index of the slab with respect to surrounding medium ($\mu = \mu_2/\mu_1$). If the refractive index of medium is greater than the refractive index of slab, then shifting ($s$) will be negative. In this case, the shifting will be in the direction opposite to the travelling ray.

**Refraction at surface 1:**

The converging rays will be focused at $O$. The point $O$ will act as virtual object for refraction through surface 1.

For surface 1: Real depth for surface 1, $AO = d_1$

Here, $n_{\text{incident}} = \mu_1 = 1$

$n_{\text{refraction}} = \mu_2 = \mu$

$$\text{Apparent depth, } d_2 = \frac{d_{\text{real}}}{n_{\text{relative}}} = \frac{d_1}{\left(\frac{n_{\text{incident}}}{n_{\text{refraction}}}\right)}$$

$$d_2 = \frac{d_1}{\left(\frac{1}{\mu}\right)} = \mu d_1$$

**Refracion at surface 2:**

After refraction from surface 1, the image of $O$ will form at $I_1$. For surface 2, $I_1$ will act as an object.

For surface 2: Real depth for surface $BI_1 = d_1' = (d_2 - t) = (\mu d_1 - t)$

Here, $n_{\text{incident}} = \mu_2 = \mu$

$n_{\text{refraction}} = \mu_1 = 1$

Apparent depth, $d_2' = \dfrac{d_{\text{real}}}{n_{\text{relative}}} = \dfrac{d_1'}{\left(\frac{n_{\text{incident}}}{n_{\text{refraction}}}\right)}$

$$= \frac{d_1'}{\left(\frac{\mu}{1}\right)} = \frac{(\mu d_1 - t)}{\mu}$$

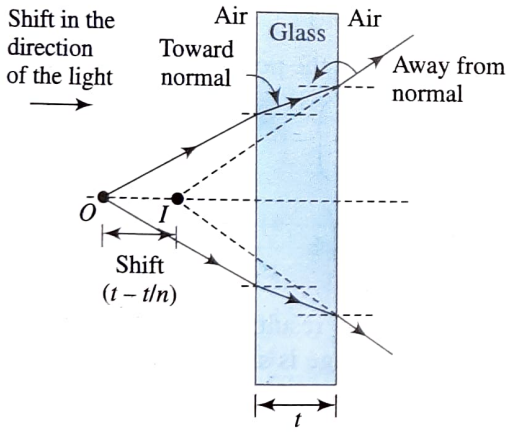
$$d_2' = \frac{(\mu d_1 - t)}{\mu} = \left(d_1 - \frac{t}{\mu}\right)$$

Hence, shifting of object position from slab

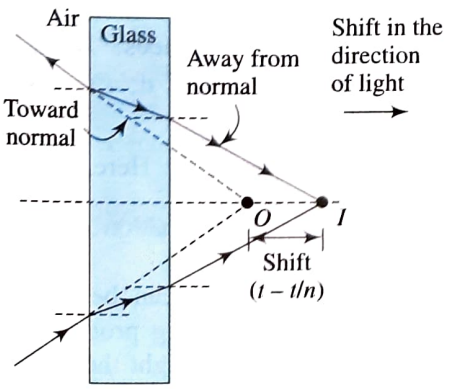
$$OI = s = BI - BO = d_2' - (d_1 - t)$$

$$s = \left(d_1 - \frac{t}{\mu}\right) - (d_1 - t) = \left(t - \frac{t}{\mu}\right) = t\left(1 - \frac{1}{\mu}\right)$$

**Image formation by a slab:**




**(a) Image formed is virtual**

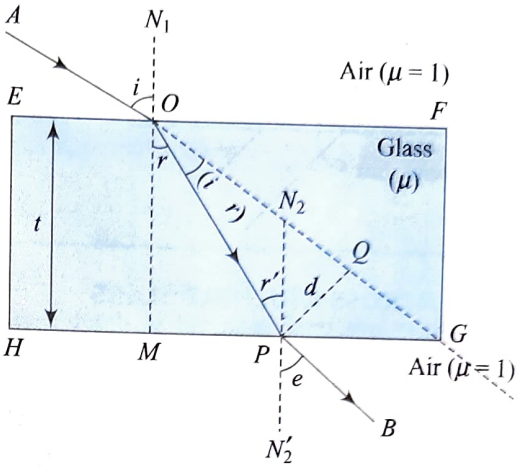



**(b) Image formed is real**

In both cases, shifting is in the direction of the travelling ray.

# LATERAL DISPLACEMENT OF EMERGENT BEAM THROUGH A GLASS SLAB

Figure represents the refraction of a ray $AO$ incident on the slab at an angle of incidence $i$ through the glass slab $EFGH$. At face $EF$, the incident ray $AO$ is refracted along $OP$, the angle of refraction being $r$.




1. If $e$ is angle of emergence, then from Snell's law at faces $EF$ and $HG$,

   $\mu_a \sin i = \mu \sin r$ and $\mu \sin r' = \mu_a \sin e$

   $r' = r$ and $\mu_a = 1$, we have

   $\sin i = \sin e$ or $e = i$

   That is the emergent ray is parallel to the incident ray.
2. Let $PQ$ be the perpendicular dropped from $P$ on incident ray produced.

   Then, the lateral displacement caused by the plate,

$$d = PQ = OP \sin(i - r) = \frac{OM}{\cos r} \sin(i - r)$$

$$= \frac{t \sin(i - r)}{\cos r}$$

**Special Cases**

If $i$ is very small, $r$ is also very small, then

$\sin i \to i$, $\sin r \to r$ and $\cos r \to 1$ so that

$\dfrac{\sin i}{\sin r} = \mu$ takes the form $\dfrac{i}{r} = \mu$.

Therefore, the expression for lateral displacement takes the form

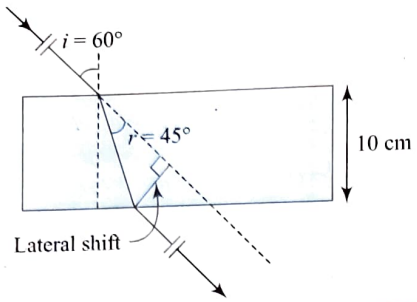
$$d = \frac{t(i - r)}{i} = ti\left(1 - \frac{r}{i}\right) = ti\left(1 - \frac{1}{\mu}\right)$$

**ILLUSTRATION 1.63**

Find the lateral shift of a light ray while it passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60° and the angle of refraction in glass is 45°.

**Sol.** $d = \dfrac{t\sin(i-r)}{\cos r} = \dfrac{10\sin(60°-45°)}{\cos 45°}$

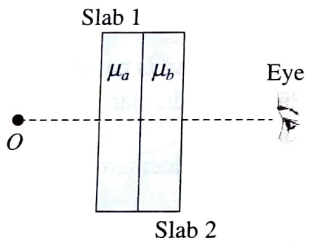
$= \dfrac{10\sin 15°}{\cos 45°} = 10\sqrt{2}\ \sin 15°$




## Refraction Across Multiple Slabs

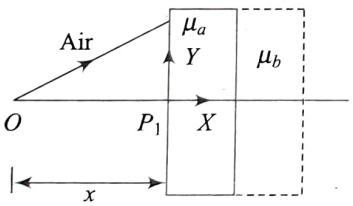
In figure, an object is placed in front of two slabs in contact. The thickness and refractive indices of the slabs are $t_1$, $\mu_a$ and $t_2$, $\mu_b$, respectively. Where will the final image of the object appear to be?

A light ray emerging from $O$ now refracts at three surfaces. The first is between air and $\mu_a$ the second between $\mu_a$ and $\mu_b$ while the third is between $\mu_b$ and air. Let us solve the problem taking one step at a time.




**An object placed in front of two glass slabs in contact**

**First interface** (see figure): Here, $\mu_1 = 1$, $\mu_2 = \mu_a$




$d_1 = x$, $d_2 = ?$

$$d_2 = \frac{d_1}{\mu_{\text{relative}}} = \frac{d_1}{\mu_1/\mu_2}$$

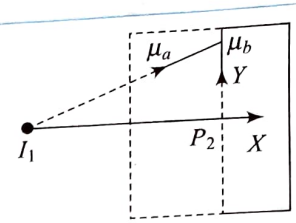
$$\Rightarrow \quad \frac{\mu_1}{d_1} = \frac{\mu_2}{d_2}$$

or $$d_2 = \frac{\mu_2}{\mu_1} d_1 = \mu_a d_1$$

Therefore, the image distance, $d_2 = -\mu_a x$ ...(i)

**Second interface** (see figure):

Here, $\mu_1 = \mu_a$, $\mu_2 = \mu_b$

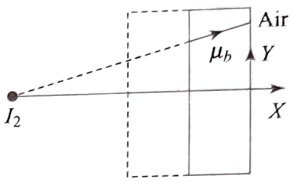



$d_1 = -(\mu_a x + t_1)$, $d_2 = ?$

Since $\dfrac{\mu_1}{d_1} = \dfrac{\mu_2}{d_2}$

The image distance, $d_2 = -\mu_b\left(x + \dfrac{t_1}{\mu_a}\right)$ ...(ii)

**Third interface** (see figure):




Here, $\mu_1 = \mu_b$, $\mu_2 = 1$

$$d_1 = -\mu_b\left(x + \frac{t_1}{\mu_a}\right) + t_2,\ d_2 = ?$$

and the final image distance from the 3rd interface is

$$d_2 = -\left[x + \frac{t_1}{\mu_a} + \frac{t_2}{\mu_b}\right] \quad \text{...(iii)}$$

Therefore, the net shift in the position of the image is

$$s = -\left(x + \frac{t_1}{\mu_a} + \frac{t_2}{\mu_b}\right) - [-(x + t_1 + t_2)]$$

$$= t_1\left(1 - \frac{1}{\mu_a}\right) + t_2\left(1 - \frac{1}{\mu_b}\right) \quad \text{...(iv)}$$

Looking at the above result, we realize that the net shift in the position of the image is simply the sum of the individual shifts at each of the slabs if they were independently placed in air.

Thus, the simple problem of refraction in a glass slab can be tackled in two ways:

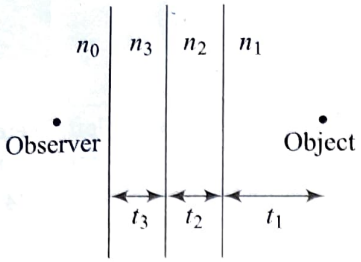
1. By the method of interfaces: Here, the refraction formula, equation $(\mu_1/d_1 = \mu_2/d_2)$, is applied at each interface.
2. By the method of elements: Here, the slab itself is an element with a governing equation $t\left[1 - \dfrac{1}{\mu}\right]$.

Most problems in ray optics can be solved by either of the two methods. In the following problems, we shall solve problems in both ways and highlight the method that gives quicker solution.

**Note:** Refraction through a composite slab (or refraction through a number of parallel media, as seen from a medium of RI $n_0$)

- Apparent depth (distance of final image from final surface)

$$= \frac{t_1}{n_{1rel}} + \frac{t_2}{n_{2rel}} + \frac{t_3}{n_{3rel}} + \cdots + \frac{t_n}{n_{nrel}}$$



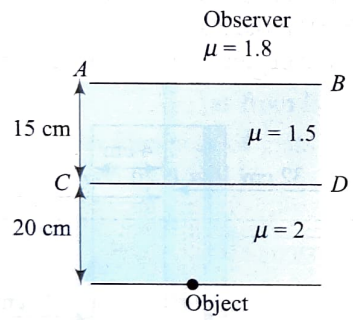

- Apparent shift

$$= t_1\left[1 - \frac{1}{n_{1\text{rel}}}\right] + t_2\left[1 - \frac{1}{n_{2\text{rel}}}\right] + \cdots + t_n\left[1 - \frac{n}{n_{n\text{rel}}}\right]$$

where $t$ represents thickness and $n$ represents the RI of the respective media, relative to the medium of observer (i.e., $n_{1\text{rel}} = n_1/n_0$, $n_{2\text{rel}} = n_2/n_0$, etc.)

## ILLUSTRATION 1.64

In figure, find the apparent depth of the object seen below surface $AB$.




**Sol.** $D_{app} = \sum \frac{d}{\mu} = \frac{20}{\left(\frac{2}{1.8}\right)} + \frac{15}{\left(\frac{1.5}{1.8}\right)} = 18 + 18 = 36$ cm

## ILLUSTRATION 1.65

A layer of oil 3 cm thick is floating on a layer of coloured water 5 cm thick. Refractive index of coloured water is 5/3 and the apparent depth of the two liquids appears to be 36/7 cm. Find the refractive index of oil.

**Sol.** Apparent depth (AI) $= \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2}$

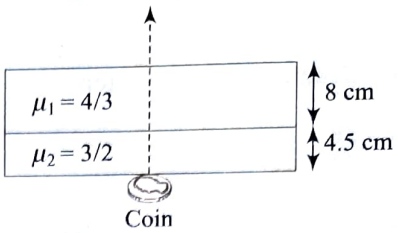
$\therefore \quad \frac{36}{7} = \frac{5}{5/3} + \frac{3}{\mu_2}$

$\Rightarrow \quad \frac{3}{\mu_2} = \frac{36}{7} - 3 = \frac{15}{7}$

$\Rightarrow \quad \mu_2 = \frac{7}{5} = 1.4$

## ILLUSTRATION 1.66

In figure, determine the apparent shift in the position of the coin. Also, find the effective refractive index of the combination of the glass and water slab.




**Sol.** Total apparent shift is

$$s = t_1\left(1 - \frac{1}{\mu_1}\right) + t_2\left(1 - \frac{1}{\mu_2}\right) = 8\left(1 - \frac{1}{\frac{4}{3}}\right) + 4.5\left(1 - \frac{1}{\frac{3}{2}}\right)$$

$= 2 + 1.5 = 3.5$ cm

The apparent depth of the coin from the top is $t = (8 + 4.5) - 3.5 = 9$ cm and, the real depth of the coin is

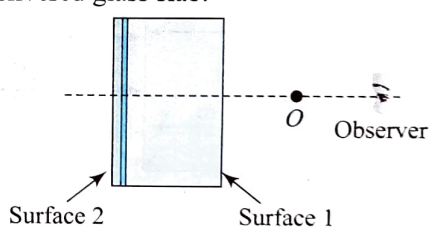
$t_1 + t_2 = 8 + 4.5 = 12.5$

Therefore, the effective refractive index is

$$\mu_{\text{eff}} = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{t_1 + t_2}{t} = \frac{12.5}{9} = 1.39$$

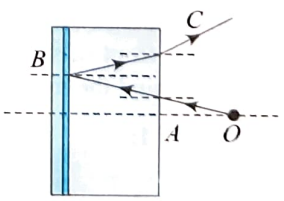
# SLAB AND MIRROR COMBINED

Let us observe what happens if one surface of the slab is silvered. Consider the silvered slab shown in Fig. (a). An object is placed in front of a silvered glass slab.




**(a)**

Here, a ray of light from the object first refracts at surface 1. It is then reflected from surface 2 before refracting again at surface 1 and emerging [see Fig. (b)]. So, we can consider a silvered slab as a combination of




**(b)**

1. A refracting surface, 2. A reflecting surface, and
3. A refracting surface again.

The above situation can be considered as a combination of a slab and a plane mirror placed together. Thus, a silvered slab is a combination of

1. A glass slab, 2 A plane mirror, and
3. A glass slab again.

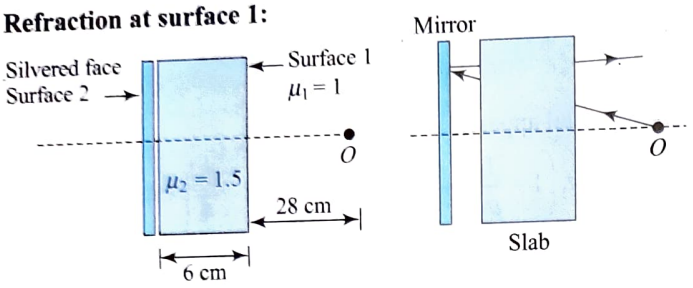
To learn the concept, we will discuss the situation through an illustration.

**ILLUSTRATION 1.67**

An object is placed in front of a slab ($\mu = 1.5$) of thickness 6 cm at a distance 28 cm from it. Other face of the slab is silvered. Find the position of final image.

**Sol.** **Method 1. Method of interface:** A ray of light from the object $O$ undergoes refraction, reflection and then refraction.

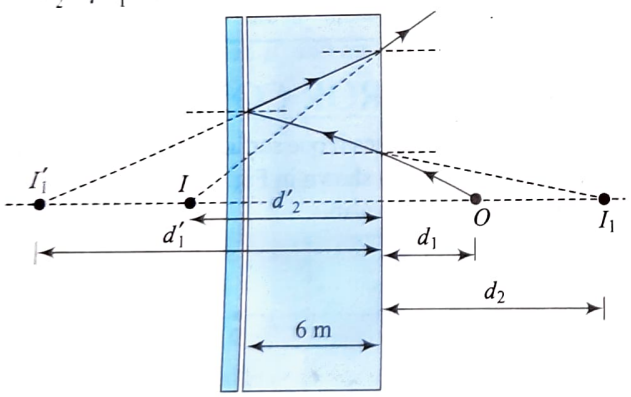
**Refraction at surface 1:**




Here, $\mu_1 = 1$, $\mu_2 = 1.5$, $d_1 = 28$ cm, $d_2 = ?$

Since $d_2 = \dfrac{d_1}{n_{\text{relative}}} = \dfrac{d_1}{(1/\mu)}$; $\left[n_{\text{rel}} = \dfrac{n_{\text{incident}}}{n_{\text{refracted}}} = \dfrac{1}{\mu}\right]$

$\Rightarrow \quad d_2 = \mu d_1 = 1.5 \times 28 = 42$ cm




Therefore, $d_2 = 42$ cm from the first interface. The first image $I_1$ is formed 42 cm in front of the slab.

**Reflection at surface 2:**

The object for reflection at the second surface is the image from refraction at the first.

Therefore, object distance from the mirror is = 42 + 6 = 48 cm.

As a result of reflection, the image will be formed as far behind the mirror as the object is in front of it. Therefore, the second image $I_1'$ is formed 48 cm behind the mirror.

**Second refraction at surface 1:**

Object distance from surface 1,

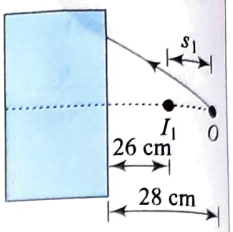
$$d_1 = 48 + 6 = 54 \text{ cm}$$

$$d_2' = \frac{d_1'}{n_{\text{relative}}} = \frac{d_1'}{\left(\dfrac{n_{\text{incident}}}{n_{\text{refraction}}}\right)} = \frac{54}{(1.5/1)} = 36 \text{ cm}$$

So, the final image is at a distance $d_2 = 36$ cm behind the first interface.

Hence, final image is formed 36 cm behind surface 1 or 30 cm behind surface 2.

**Method 2. Shifting of object:** A ray of light from the object first encounters a glass slab, then a mirror, and finally a glass slab again.



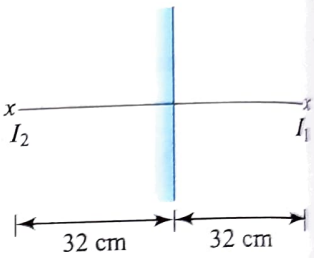

**Glass slab:** A slab simply shifts the object along the axis by a distance

$$s_1 = t\left(1 - \frac{1}{\mu}\right) = 2 \text{ cm}$$

Direction of shift of object is towards left. Therefore, the object appears to be at $I_1$ which is 28 – 2 = 26 cm from the slab.

For mirror, the object for the mirror is the image $I_1$ formed after shift due to the slab.

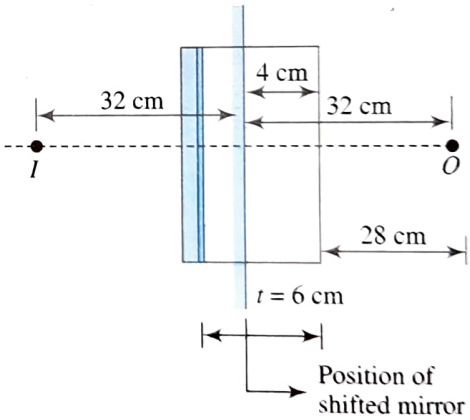
Therefore, object distance from the mirror is 26 + 6 = 32 cm. The image will now be formed 32 cm behind the mirror. Now, reflected rays are travelling from left to right.




The ray now travels through the slab again but this time from left to right. Therefore, it is shifted again by a distance of 2 cm, but towards the right. Thus, final position of the image is 32 – 2 = 30 cm behind the mirror.

**Method 3. Shifting of mirror:** By the principle of reversibility of light, we can say if light rays are coming from the mirror and passing through the slab, the mirror will shift 2 cm towards right for observer in front of the slab.

The position of the object from shifted mirror = 32 cm.



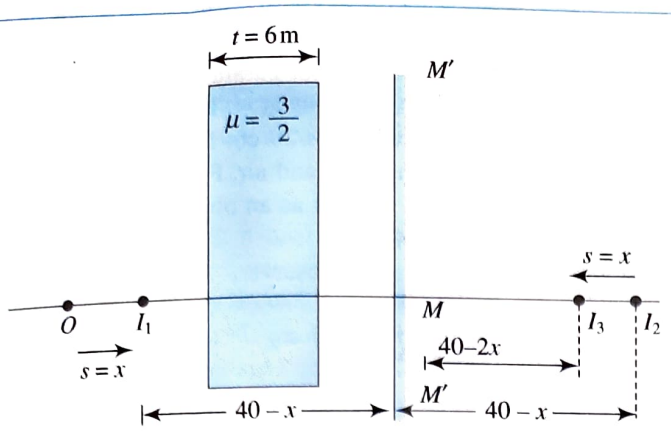

So, the position of the image formed by shifted mirror will be 32 cm behind it. Hence, the position of the image from surface 2 is 30 cm left to it and 36 cm left of surface 1.

Let us learn the combination of the slab and mirror through some more illustrations.

**ILLUSTRATION 1.68**

A 20 cm thick glass slab of refractive index 1.5 is kept in front of a plane mirror. Find the position of the image (relative to mirror) as seen by an observer through the glass slab when a point object is kept in air at a distance of 40 cm from the mirror.

**Sol.** The rays from $O$ will first pass through slab and produce shifting towards right. The glass slab will form an image of $O$ at $I_1$ such that




$$OI_1 = x = d\left[1-\frac{1}{\mu}\right] = 20\left[1-\frac{2}{3}\right] = \frac{20}{3}\text{ cm} \qquad ...(i)$$

Now, the rays after passing through slab will (towards right) So, the distance of $I_1$ from the mirror $MM'$,

$I_1M = (40 - x)$

This image $I_1$ will act as an object for the mirror and the mirror will form an image $I_2$, such that

$MI_2 = MI_1 = 40 - x$ [with $x$ given by Eq. (i)]

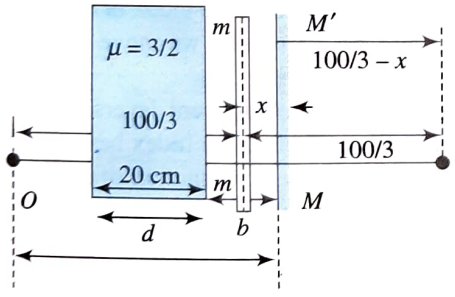
Now, image $I_2$ will act as an object again for the glass slab which by producing a shift of $x$ forms the final image $I_3$ such that $I_2I_3 = x$. Hence, the distance of final image $I_3$ from the mirror will be

$$MI_3 = MI_2 - I_3I_2 = (40 - x) - x = 40 - 2x$$

$$MI_3 = 40 - 2\times\left[\frac{20}{3}\right] = \left[\frac{80}{3}\right]\text{ cm}$$

[as from Eq. (i), $x = 20/3$ cm]

**Alternative solution:** As thickness of glass slab is 20 cm and $\mu = (3/2)$, so shift produced by it will be




$$x = d\left[1-\frac{d}{\mu}\right] = 20\left[1-\frac{2}{3}\right] = \frac{20}{3}\text{ cm}$$

So, the glass slab will shift the mirror from $MM'$ to $mm'$ as shown in.

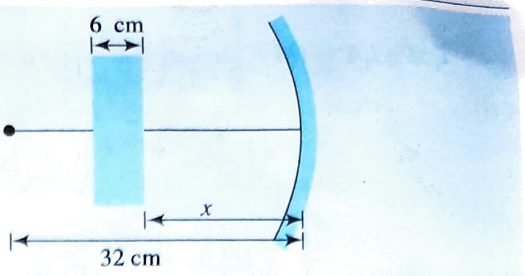
The distance of object from this virtual mirror will be

$40 - x = 40 - (2/3) = (100/3)$ cm

This virtual mirror will form the image of object $O$ at a distance (100/3) behind it and so the distance of image from actual mirror $MM'$ will be $\frac{100}{3}-\frac{20}{3}=\frac{80}{3}$ cm [as $mm'$ is 20/3 cm in front of $MM'$]

### ILLUSTRATION 1.69

A point object $O$ is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index $\mu = 3/2$ and thickness 6 cm is inserted between the object and mirror.




Find the position of final image when the distance $x$ shown in figure is **(a)** 5 cm and **(b)** 20 cm.

**Sol.** The normal shift produced by a glass slab is

$$S = \left(1-\frac{1}{\mu}\right)t = \left(1-\frac{2}{3}\right)(6) = 2\text{ cm}$$

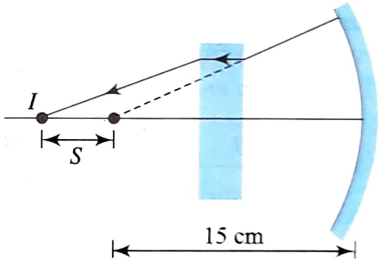
i.e., for the mirror the object is placed at a distanced $(32 - S) = 30$ cm from it.

Applying mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

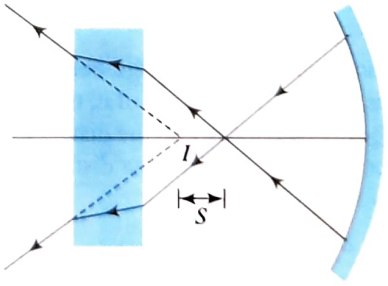
or $\frac{1}{v}+\frac{1}{-30}=-\frac{1}{10}$

or $v = -15$ cm

**(a)** When $x$ = 5 cm: The light falls on the slab on its return journey as shown. But the slab will again shift it by a distance $S$ = 2 cm. Hence, the final real image is formed at a distance (15 + 2) = 17 cm from the mirror.




**(b)** When $x$ = 20 cm: This time also the final image is at a distance 17 cm from the mirror but it is virtual as shown.

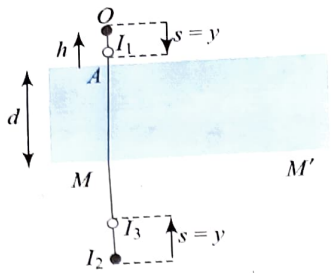



### ILLUSTRATION 1.70

A vessel having perfectly reflecting plane bottom is filled with water ($\mu = 4/3$) to a depth $d$. A point source of light is placed at a height $h$ above the surface of water. Find the distance of final image from water surface.

**Sol.** As shown in figure, water will form the image of object $O$ at $I_1$ such that $OI_1 = y = d[1 - (1/\mu)]$, so that the distance of image $I_1$ from water surface will be $I_1A = h - y = h - d[1 - (1/\mu)]$. Hence, the distance of this image $I_1$ from mirror $MM'$,

$$I_1M = I_1A + AM = \left[h - d\left(1 - \frac{1}{\mu}\right)\right] + d = h + \left[\frac{d}{\mu}\right]$$




Now, image $I_1$ will act as object for mirror $MM'$. As a plane mirror forms image at same distance behind the mirror as the object is in front of it, the image of $I_1$ formed by the mirror $MM'$ will be $I_2$ such that $I_1M = I_2M = h + (d/\mu)$.

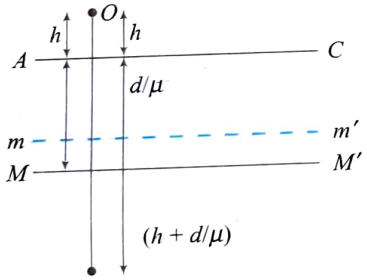
Now, this image $I_2$ will act as object for water again and water will produce image $I_3$ such that $I_2I_3 = y = d\left(1 - \frac{1}{\mu}\right)$. So, the distance of image $I_3$ from the surface of water $AC$ will be

$$AI_3 = AM + MI_2 - I_2I_3 = d + \left[h + \frac{d}{\mu}\right] - d\left[1 - \frac{1}{\mu}\right]$$

$$AI_3 = h + 2\frac{d}{\mu} = h + \frac{3}{2}d$$

**Alternative solution:**

As shown in figure, water will form the image of bottom, i.e., mirror $MM'$ at a depth $(d/\mu)$ from its surface. So, the distance of object $O$ from virtual mirror $mm'$ will be $h + (d/\mu)$.

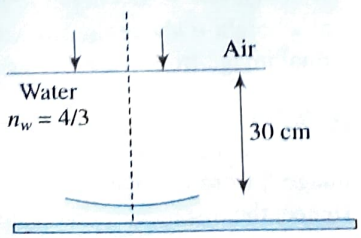



Now, as a plane mirror forms image behind the mirror at the same distance as the object is in front of it, the distance of image $I$ from $mm'$ will be $h + (d/\mu)$. Also, as the distance of virtual mirror from the surface of water is $(d/\mu)$, the distance of image $I$ from the surface of water will be

$$\left[h + \frac{d}{\mu}\right] + \frac{d}{\mu} = h + \frac{2d}{\mu} = h + \frac{3}{2}d \qquad \left[\text{as } \mu = \frac{4}{3}\right]$$

**ILLUSTRATION 1.71**

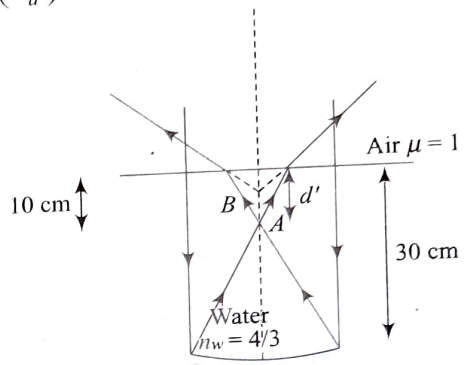
A concave mirror of focal length 20 cm is placed inside water with its shining surface upwards and principal axis vertical as shown in figure. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.




**Sol.** The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore, for the mirror, object is at ∞. Its image $A$ (in figure) will be formed at focus which is 20 cm from the mirror. Now, for the interface between water and air. For observer at air, the image formed by mirror will act as an object for observer with real depth $d = 30 - 20 = 10$ cm.

Apparent depth observed by observer,

$$d' = \frac{d}{\left(\frac{n_w}{n_a}\right)} = \frac{10}{\left(\frac{4/3}{1}\right)} = 7.5 \text{ cm}$$




$R = 40$ cm

## REFRACTION IN A MEDIUM WITH VARIABLE REFRACTIVE INDEX

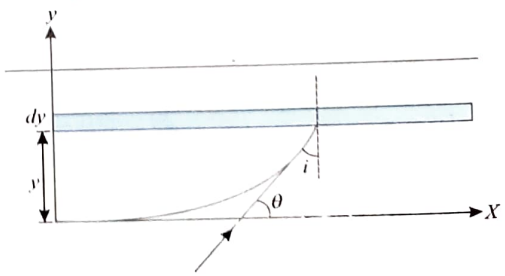
So far, we have assumed that the refractive index of the slab is a constant. This need not be necessarily true. An example of such a situation is the atmosphere. The atmosphere becomes thinner as we go up. Hence, the refractive index of air is highest close to surface of earth and decreases as we move upward. How do we analyze such a problem?

Divide the medium into different layers. This indicates that each layer has different refractive index having value according to a given expression. If $\mu$ is a function of $x$, then a differential layer will be a thin layer of thickness $dx$. If $\mu$ is a function of $y$ then a differential layer will be a thin layer of thickness $dy$.

At a given layer, let the angle of incidence be $i$. Relate this angle of incidence to the initial condition and the refractive index at that point using the relation

$$\mu \sin(i) = \text{constant} \qquad \ldots(i)$$

From the given ray diagram (figure), draw a tangent to the path taken by the ray of light. Geometrically, relate the slope of this tangent to the angle of incidence.




$$\tan\theta = \frac{dy}{dx} = \tan(90 - i) \qquad \ldots(ii)$$

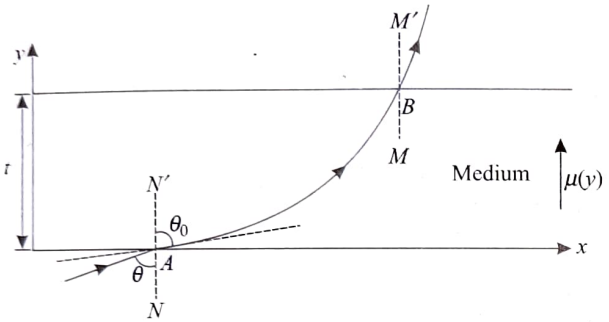
Substitute for $i$ from Eq. (i) and determine $dy/dx$ as a function of $x$ and $y$.

Integrate and obtain an expression for $y$ as a function of $x$.

Let us discuss a case where refractive index is changing in $y$-direction.

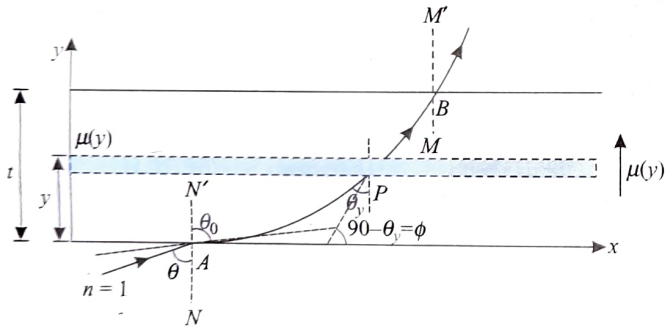
As 1: $\mu = f(y)$

Consider a slab with a medium where the refractive index varies from $\mu_1$ to $\mu_2$ in relation with $y$. Assume that the outside medium is air. Let light be incident on the glass slab at an angle $\theta$. The light ray strikes the glass slab at the points. The normal at this point is $NN'$ as shown in figure. Once the ray enters the medium, the refractive index varies continuously as if there are infinite number of successive glass slabs. Therefore, the angle of incidence also varies continuously and the path of the ray is curved. The ray finally reaches the second surface at the point $B$ before exiting the slab, once again refracting at the exit point.




How can we derive a mathematical expression for the equation of the ray in the medium?

Consider refraction at some height $y$ (figure). Here, the angle of incidence is $\theta_y$ and the refractive index of the medium is $\mu_y$.




The product of the refractive index and the sine of the angle of incidence is constant. Therefore, at the height $y$ we can say that

$$\mu_y \sin\theta_y = \text{constant} \qquad \ldots(i)$$

Furthermore, applying the law of refraction at points $A$ which is the interface between air and the medium, we have

$$1 \times \sin(\theta) = \mu_y \sin\theta_y \qquad \ldots(ii)$$

$$\therefore \quad \sin\theta_y = \frac{\sin\theta}{\mu_y}$$

$$\cos\theta_y = \sqrt{\frac{\mu_y^2 - \sin^2\theta}{\mu_y^2}}$$

Hence, $\cot\theta_y = \dfrac{\sqrt{\mu_y^2 - \sin^2\theta}}{\sin\theta}$

Now, the slope of the curve at point $P$ is given by

$$\tan\phi = \frac{dy}{dx} \qquad \ldots(iii)$$

But $\phi = 90 - \theta_y$

Therefore, $\dfrac{dy}{dx} = \cot(\theta_y) = \dfrac{\sqrt{\mu_y^2 - \sin^2\theta}}{\sin\theta}$ ...(iv)

$$\frac{\sin\theta\, dy}{\sqrt{\mu_y^2 - \sin^2\theta}} = dx$$

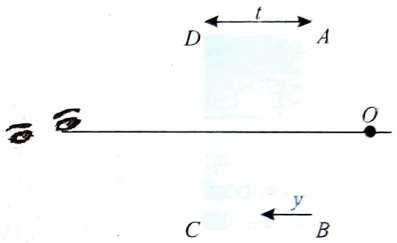
Integrating both side with proper limits.

If the functional dependence of "$\mu_y$" is known, we can easily solve for the equation of the curve.

$$\sin\theta \int_{y_1}^{y_2} \frac{dy}{\sqrt{\mu_y^2 - \sin^2\theta}} = \int_{x_1}^{x_2} dx$$

## ILLUSTRATION 1.72

A glass slab is placed between an object $(O)$ and an observer $(E)$ with its refracting surfaces $AB$ and $CD$ perpendicular to the line $OE$.




The refractive index of the glass slab changes with distance $(y)$ from the face $AB$ as $\mu = \mu_0(1 + y)$. Thickness of the slab is $t$.

Find how much closer (compared to original distance) the object appears to the observer. Consider near normal incidence only.

**Sol.** Shift in position of the object caused due to thickness $dy$ of the slab is

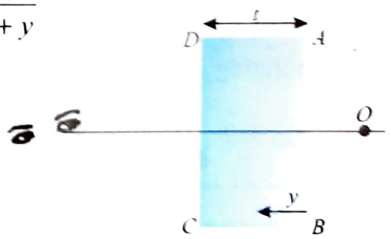
$$ds = dy\left(1 - \frac{1}{\mu}\right) = dy\left(1 - \frac{1}{\mu_0(1+y)}\right)$$

Total shift due to all such layers is

$$s = \int ds = \int_0^t dy - \frac{1}{\mu_0}\int_0^t \frac{dy}{1+y}$$

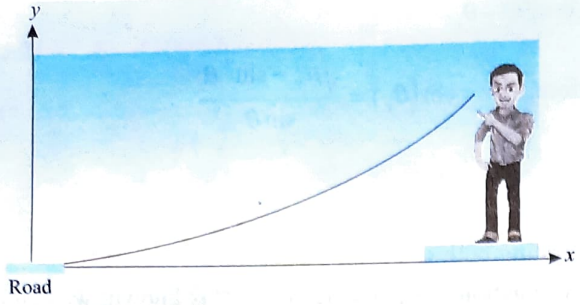
$$= t - \frac{1}{\mu_0}[\ln(1+y)]_0^t$$

$$\Rightarrow \quad s = t - \frac{\ln(1+t)}{\mu_0}$$

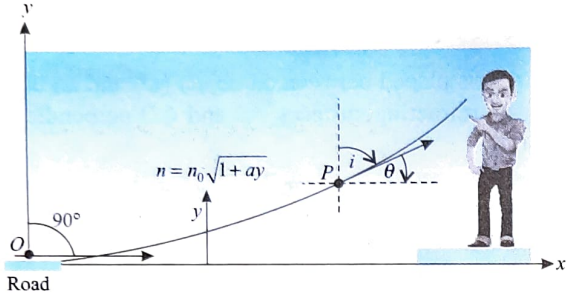



## ILLUSTRATION 1.73

Due to a vertical temperature gradient in the atmosphere the index of refraction varies. Suppose index of refraction varies as $n = n_0\sqrt{1 + ay}$ where $n_0$ is the index of refraction at the surface and $a = 2.0 \times 10^{-6}\ \text{m}^{-1}$. A person of height $h = 2.0$ m stands on a level surface. Beyond what distance he cannot see the runway?




**Sol.** As refractive index is changing along $y$-direction, we can assume a number of thin layers of air placed parallel to $x$-axis. Let $O$ be the distant object just visible to the man. Let $P$ be a point on the trajectory of the ray.



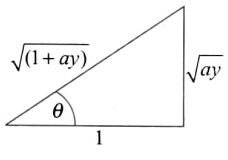

From Snell's law, $n.\sin i = \text{constant}$

At the surface $n = n_0$ and $i = 90°$

$$n_0 \sin 90° = n \sin i = (n_0\sqrt{(1+ay)})\sin(90° - \theta)$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{1+ay)}}$$

$$\Rightarrow \tan\theta = \sqrt{ay}$$




$$\tan\theta = \sqrt{ay} = \frac{dy}{dx} \Rightarrow dx = \frac{dy}{\sqrt{ay}}$$

$$\int_0^x dx = \int_0^y \frac{dy}{\sqrt{ay}} \Rightarrow x = \frac{1}{\sqrt{a}}\left[\frac{y^{\left(-\frac{1}{2}+1\right)}}{\left(-\frac{1}{2}+1\right)}\right]_0^y = 2\frac{y^{\frac{1}{2}}}{\sqrt{a}} = 2\sqrt{\frac{y}{a}}$$

Hence equation of trajectory $x = 2\sqrt{\frac{y}{a}}$ or $y = \frac{ax^2}{4}$

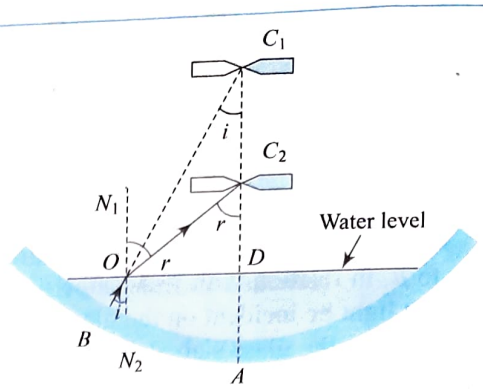
On substituting $y = 2.0$ m and $a_0 = 2\times10^{-6}$ m$^{-1}$,

$$x_{max} = 2\sqrt{\frac{2}{2\times10^{-6}}} = 2000 \text{ m}$$

## Measurement of Refractive Index of a Liquid by a Concave Mirror

A concave mirror of large radius of curvature is placed on a table with its principal axis vertical, as shown in figure. A horizontal pin is placed with its tip on the principal axis of the mirror. The pin is moved till there is no parallax between the tip of the pin and its image, when the pin lies at the centre of curvature of the mirror.

A small quantity of liquid whose refractive index is to be measured is poured into the mirror. The pin is moved down in order to remove the parallax between the tip of the pin and its image.




In the figure, $C_1$ is the position of the pin when its image coincides with itself without liquid, and $C_2$ is the position of the pin when its image coincides with itself after pouring the liquid into the concave mirror.

The ray $BO$ that is normal to the mirror passes through $C_1$ before pouring the liquid. It is refracted away from the normal when the liquid is poured, now it passes through $C_2$.

In figure, $\sin i = \frac{OD}{OC_1}$, $\sin r = \frac{OD}{OC_2}$

From Snell's law, $\mu \sin i = 1 \sin r$

$$\mu = \frac{OD}{OC_2} \times \frac{OC_1}{OD} = \frac{OC_1}{OC_2}$$

Taking paraxial ray assumption, $OC_1 \simeq DC_1$; $OC_2 \simeq DC_2$

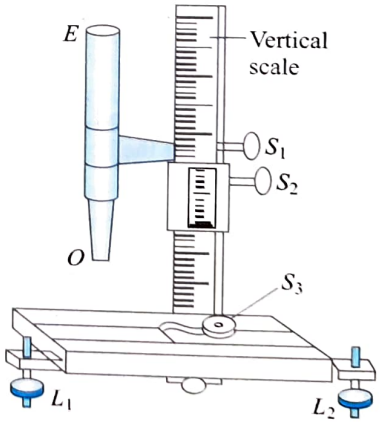
Thus, $\mu = \frac{DC_1}{DC_2}$

If we neglect the depth of the liquid, we have $\mu = \frac{AC_1}{AC_2}$

## Measurement of Refractive Index of a Liquid by a Travelling Microscope

Using the fact that $\mu = \frac{\text{Real depth}}{\text{Apparent depth}}$

We may determine the refractive index of a glass slab (see figure)




The travelling microscope is first focused on a fine mark on a piece of paper and the reading on the scale is noted as $Y_1$. Then the glass slab is placed on the paper.

The microscope is now shifted up and is focused again on the mark so that it is distinctly visible. The reading of the microscope is noted as $Y_2$.

Now, a little lycopodium powder is scattered on the upper surface of the glass block. The microscope is focused on the powder and the third reading of the scale is noted as $Y_3$.

The real thickness of the glass slab is $Y_3 - Y_1$ and apparent thickness of the glass slab is $Y_3 - Y_2$. Thus, $\mu = \dfrac{Y_3 - Y_1}{Y_3 - Y_2}$.

The same method can also be used to find the refractive index of a liquid. Any mark on the bottom of a glass vessel is first focused and then liquid is poured into it and the same mark is again focused. Finally, a little lycopodium powder is sprinkled on the surface of the liquid and is focused as before.

## CONCEPT APPLICATION EXERCISE 1.5

**1.** Identify the True or False statements.

**(a)** A glass slab cannot deviate the light.

**(b)** A glass slab can produce lateral displacement.

**(c)** The shift produced by a slab depends on the converging and diverging nature of beam.

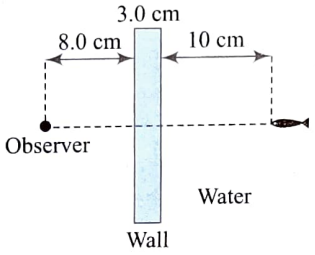
**(d)** Apparent shift in case of a slab always occurs in the direction of light ray travelling.

**(e)** The shift produced by a slab can never exceed its thickness.

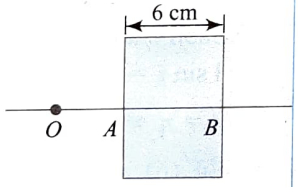
**2.** In figure, a fish watcher watches a fish through a 3.0 cm thick glass wall of a fish tank. The watcher is in level with the fish; the index of refraction of the glass is 8/5 and that of the water is 4/3.

**(a)** To the fish, how far away does the watcher appear to be?

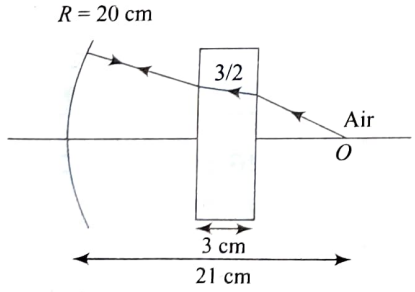
**(b)** To the watcher, how far away does the fish appear to be?




**3.** An object $O$ is placed at 8 cm in front of a glass slab, whose one face is silvered as shown in figure. The thickness of the slab is 6 cm. If the image formed 10 cm behind the silvered face, find the refractive index of glass.




**4.** An object is placed on the principal axis of a concave mirror of focal length 10 cm at a distance of 21 cm from it. A glass slab is placed between the mirror and the object as shown in figure.




Find the distance of final image formed by the mirror.

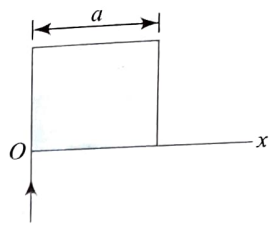
**5.** The image of an object kept at a distance of 30 cm in front of a concave mirror is found to coincide with itself. If a glass slab ($\mu = 1.5$) of thickness 3 cm is introduced between the mirror and the object, then

**(a)** Identify, in which direction the mirror should be displaced so that the final image may again coincide with the object itself.

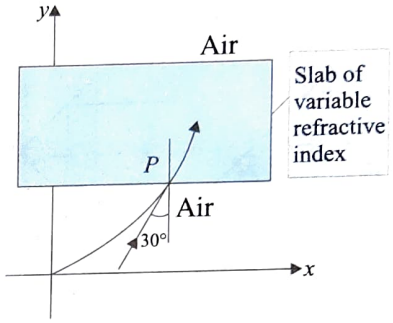
**(b)** Find the magnitude of displacement.




**6.** The refractive index of an anisotropic medium varies as $\mu = \mu_0 \sqrt{(x+1)}$, where $0 \le x \le a$. A ray of light is incident at the origin just along $y$-axis (shown in figure). Find the equation of ray in the medium.

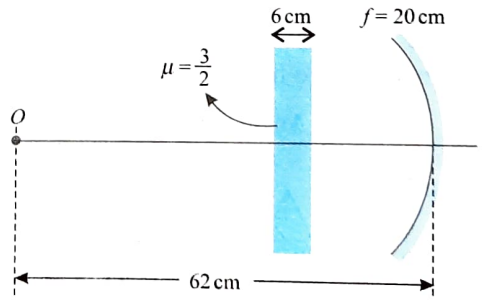



**7.** A ray of light travelling in air is incident at angle of incident 30° on one surface of slab in which refractive index varies with $y$. The light travels along the curve $y = 4x^2$ ($y$ and $x$ are in meter) in the slab. Find out the refractive index of the slab at $y$ = 1/2 m in the slab.




**8.** A ray of light is incident on a glass slab at grazing incidence. The refractive index of the material of the slab is given by $\mu = \sqrt{(1+y)}$. If the thickness of the slab is $d = 2$ m, determine the equation of the trajectory of the ray inside the slab and the coordinates of the point where the ray exits from the slab. Take the origin to be at the point of entry of the ray.

**9.** A point object $O$ is placed at a distance of 62 cm in front of a concave mirror of focal length $f = 20$ cm. A glass slab of refractive index $\mu = \dfrac{3}{2}$ and thickness 6 cm is inserted between the object and the mirror. Let's define final image as image formed after the light ray originating from $O$ passes through the slab, gets reflected from the mirror and then again passes through the slab. Where final image will form?




## ANSWERS

**1.** (a) True; (b) True; (c) False; (d) False; (e) True

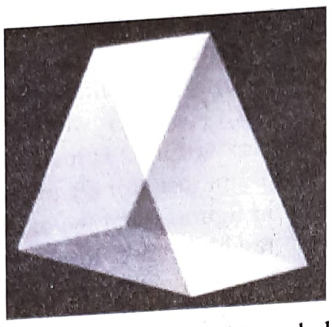
**2.** (a) $\dfrac{139}{6}$ cm; (b) $\dfrac{139}{8}$ cm **3.** 1.5 **4.** 21 cm

**5.** (a) Rightwards; (b) 1 cm **6.** $y = 2\sqrt{x}$ **7.** $\dfrac{3}{2}$

**8.** $y = \dfrac{x^2}{4}$; $2\sqrt{2}$ m, 2 m **9.** 32 cm

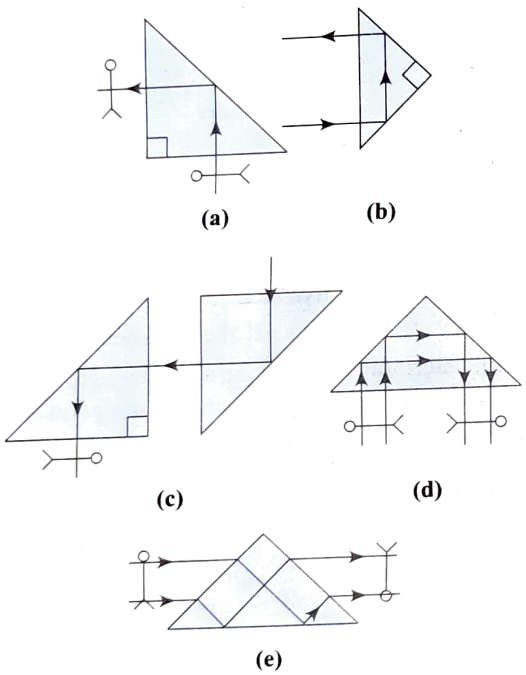
# PRISM

A prism is a transparent medium whose refracting surfaces are not parallel but are inclined to each other (see figure).



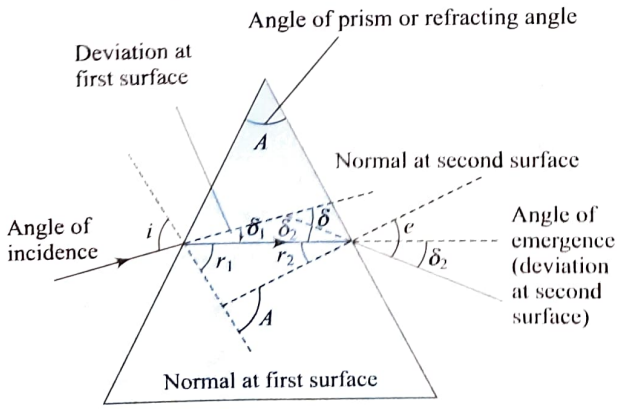
Prisms have the property to change the direction of light. Figures (a) to (e) present some of the useful applications.

The right-angled prism in Fig. (a) turns the light through 90°. A porro prism shown in Fig. (b) is a right-angled prism, it turns light through 180°. Figure (c) shows two right-angled prisms used in binoculars. Figures (d) and (e) illustrate image formation with or without deviation; in both the cases the image is laterally inverted.




*Basic Terms*

1. **Angle of prism or refracting angle ($A$):** The angle between the faces on which light is incident and from which it emerges (see figure).

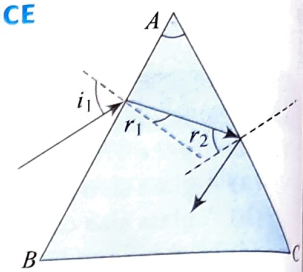



2. **Angle of deviation ($\delta$):** It is the angle between the emergent and the incident ray. It other words, it is the angle through which incident ray turns in passing through a prism.

$$\delta = (i - r_1) + (e - r_2)$$
$$= i + e - (r_1 + r_2)$$
$$= i + e - A$$

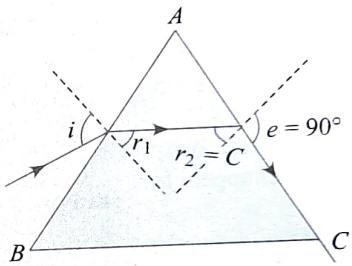
## CONDITION OF NO EMERGENCE

A ray of light incident on a prism of angle $A$ and refractive index $\mu$ will not emerge out of a prism (whatever may be the angle of incidence) if $A > 2\theta_c$, where $\theta_c$ is the critical angle, i.e., $\mu > 1 / [\sin (A/2)]$ (see figure).




### ILLUSTRATION 1.74

What should be the minimum value of refractive index of a prism, refracting angle $A$, so that there is no emergent ray irrespective of the angle of incidence?




**Sol.** If the ray just emerges from face $AC$,

$e = 90°$ and $r_2 = C$ ...(i)

From Snell's law at face $AB$, we have

$1 \sin i = n \sin r_1$ ...(ii)

$A = r_1 + r_2 = r_1 + C$ ...(iii)

From Eq. (ii), $n$ is minimum when $r_1$ is maximum, i.e., $r_2 = C$. In this case, $i = 90°$

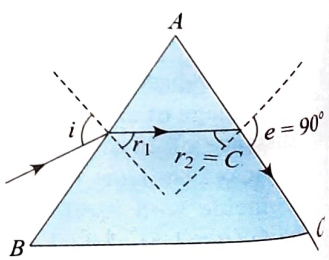
From Eq. (iii), $A = 2C$ or $C = A/2$

As $\sin C = \frac{1}{n} \Rightarrow \sin \frac{A}{2} = \frac{1}{n}$

$n = \text{cosec}\frac{A}{2}$

## CONDITION OF GRAZING EMERGENCE

By the condition of grazing emergence, we mean the angle of incidence $i$ at which the angle of emergence becomes 90° (see figure).




Consider a prism with refracting angle $A$ and refractive index $n$.

The ray grazes face $AC$.

So, $e = 90°, r_2 = C$ ...(i)

and $A = r_1 + r_2 = r_1 + C$ ...(ii)

Also, $\sin C = 1/n$

From Snell's law at face $AB$, $1 \sin i = n \sin r_1$

$$\sin i = n \sin (A - C)$$

$$= n[\sin A \cos C - \cos A \sin C]$$

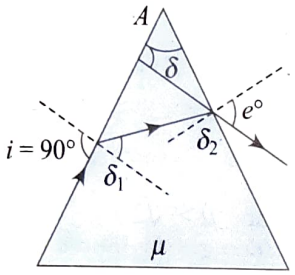
$$= n\left[\sin A\sqrt{1 - \sin\ C} - \cos A \sin C\right]$$

$$= \sqrt{n^2 - 1} \sin A - \cos A$$

$$i = \sin^{-1}\left[\sqrt{n^2 - 1} \sin A - \cos A\right]$$

## CONDITION OF MAXIMUM DEVIATION

Maximum deviation occurs when the angle of incidence is 90° (see figure).

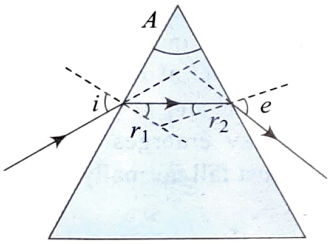



$$\delta_{max} = 90° + e - A$$

where $e = \sin^{-1}[\mu \sin (A - \theta_c)]$

## CONDITION OF MINIMUM DEVIATION

The minimum deviation occurs when the angle of incidence is equal to the angle of emergence (see figure),

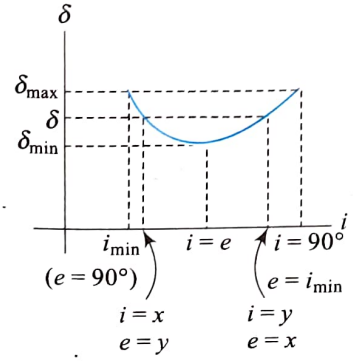



i.e., $i = e$; $\delta_{min} = 2i - A$.

Using Snell's law, we get $\mu = \dfrac{\sin[(\delta_{min} + A)/2]}{\sin[A/2]}$

Note that in the condition of minimum deviation the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.

1. Variation of $\delta$ versus $i$ (shown in figure).

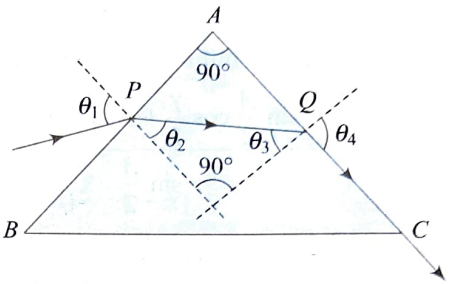



For each $\delta$ (except $\delta_{min}$), there are two values of angle of incidence. If $i$ and $e$ are interchanged, then we get the same value of $\delta$ because of reversibility principle of light.

2. There is one and only one angle of incidence for which the angle of deviation is minimum.

### ILLUSTRATION 1.75

Figure shows a triangular prism of refracting angle 90°. A ray of light incident at face $AB$ at an angle $\theta_1$ refracts at point $Q$ with an angle of refraction 90°.




**(a)** What is the refractive index of the prism in terms of $\theta_1$?

**(b)** What is the maximum value that the refractive index can have?

**(c)** What happens to the light at $Q$ if the incident angle at $Q$ is **(i)** increased slightly, and **(ii)** decreased slightly?

**Sol.**

**(a)** Let the ray be incident at an angle $\theta_1$ at the face $AB$. It refracts at an angle $\theta_2$ and is incident at an angle $\theta_3$ at face $AC$. Finally, the ray comes out at an angle $\theta_4 = 90°$.
From the above figure, the normals at faces $AB$ and $AC$ make an angle of 90° with each other, $\theta_3 = 90° - \theta_2$

$$\sin \theta_3 = \sin(90° - \theta_2) = \cos \theta_2 = \sqrt{1 - \sin^2 \theta_2} \quad ...(i)$$

From Snell's law at face $AC$, $n \sin \theta_3 = 1$

$$n\sqrt{1 - \sin^2 \theta_2} = 1 \quad ...(ii)$$

From Snell's law at face $AB$, $1 \sin \theta_1 = n \sin \theta_2$

$$\sin \theta_2 = \frac{\sin \theta_1}{n} \quad ...(iii)$$

From Eqs. (ii) and (iii), we have

$$n\sqrt{1 - \frac{\sin^2 \theta_1}{n^2}} = 1 \quad ...(iv)$$

On squaring Eq. (iv) and solving for $n$, we get

$$n = \sqrt{1 + \sin^2 \theta_1}$$

**(b)** The greatest possible value of $\sin^2 \theta_1$ is 1; hence, the greatest possible value of $n$ is $n_{max} = \sqrt{2} = 1.41$.

**(c)** **(i)** For a given $n$, if $\theta_1$ is increased the angle of refraction $\theta_2$ increases. As $\theta_3 = 90° - \theta_2$, the angle $\theta$ decreases, i.e., the angle of incidence at face $AC$ is less than the critical angle for total reflection; hence light emerges into air.

**(ii)** If the angle of incidence is decreased, the angle of refraction $\theta_2$ decreases. So, the angle $\theta_3$ increases. The

angle of incidence at the second surface is greater than the critical angle; so light is reflected at $Q$.

**Note:** In the condition of minimum deviation, the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.

## ILLUSTRATION 1.76

A prism has refracting angle equal to $\pi/2$. It is given that $\gamma$ is the angle of minimum deviation and $\beta$ is the deviation of the ray entering at grazing incidence. Prove that $\sin\gamma = \sin^2\beta$.

**Sol.** Applying condition of minimum deviation,

$$\mu = \frac{\sin\frac{(A+\gamma)}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{A}{2}\cos\frac{\gamma}{2} + \cos\frac{A}{2}\sin\frac{\gamma}{2}}{\sin\frac{A}{2}}$$

$$= \cos\frac{\gamma}{2} + \cot\frac{A}{2}\sin\frac{\gamma}{2}$$

Using $A = 90°$, $\mu = \cos\frac{\gamma}{2} + \cot 45° \sin\frac{\gamma}{2}$

$$\Rightarrow \quad \cos\frac{\gamma}{2} + \sin\frac{\gamma}{2} = \mu$$

Squaring, $\cos^2\frac{\gamma}{2} + \sin^2\frac{\gamma}{2} + \sin\gamma = \mu^2$

$$\Rightarrow \quad \sin\gamma = \mu^2 - 1 \qquad ...(i)$$

Deviation at grazing incidence,

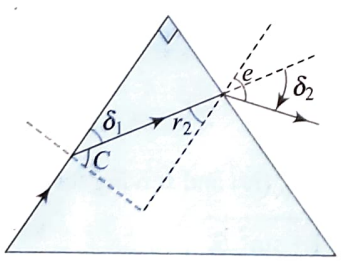
$$\beta = \delta_1 + \delta_2 = \left(\frac{\pi}{2} - C\right) + (e - r_2)$$

$$\Rightarrow \quad \beta = \left(\frac{\pi}{2} - C\right) + \left[e - \left(\frac{\pi}{2} - C\right)\right]$$

$$\Rightarrow \quad \beta = e$$

or $\quad \sin\beta = \sin e = \mu \sin r_2 = \mu \sin\left(\frac{\pi}{2} - C\right)$

$$\Rightarrow \quad \sin\beta = \mu\cos C \qquad ...(ii)$$




Squaring Eq. (ii),

$$\sin^2\beta = \mu^2\cos^2 C$$

$$\Rightarrow \quad \sin^2\beta = \mu^2(1 - \sin^2 C)$$

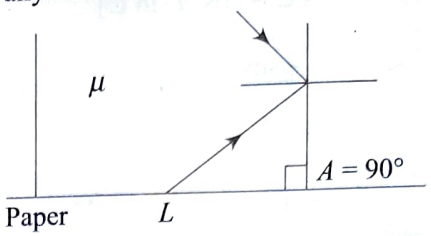
Using $\sin C = \frac{1}{\mu}$, $\sin^2\beta = \mu^2\left(1 - \frac{1}{\mu^2}\right)$

$$\Rightarrow \quad \sin^2\beta = \mu^2 - 1 \qquad ...(iii)$$

From Eqs. (i) and (iii), $\sin\gamma = \sin^2\beta$

## ILLUSTRATION 1.77

A rectangular block of refractive index $\mu$ is placed on a printed page lying on a horizontal surface as shown in figure. Find the minimum value of $\mu$ so that the letter $L$ on the page is not visible from any of the vertical sides.




**Sol.** The letter $L$ will not be visible from the vertical sides if the light ray does not enter through it.

We can apply the condition of no emergence for a prism of angle $A = 90°$.

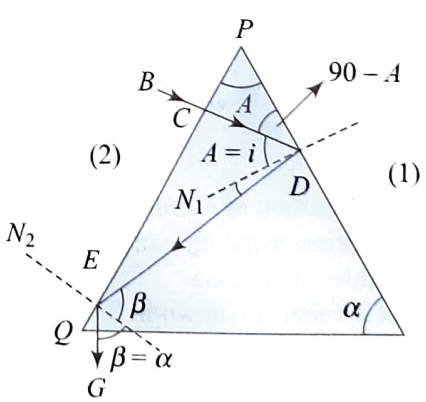
That is, $\mu > \frac{1}{\sin\frac{A}{2}}$

or $\quad \mu > \frac{1}{\sin\frac{90°}{2}}$ or $\mu > \sqrt{2}$

## ILLUSTRATION 1.78

The cross section of a glass prism has the form of an isosceles triangle. One of the refracting faces is silvered. A ray of light falling normally on the other refracting face, being reflected twice, emerges through the base of the prism perpendicular to it. Find the angles of the prism.

**Sol.** The incident ray $BC$ at normal incidence is reflected at silvered face, along $DE$ and at $E$ it again suffers reflection along $EF$. Since the ray emerges normally from the base, therefore the ray $EF$ must fall normally on the base and emerges along $EG$.




We find $i = A$

Also, $\beta = \alpha$

Since $EN_2 \parallel CD$, $\beta = 2i$ (alternate angles)

$\therefore \quad \alpha = 2A \quad (\beta = \alpha, i = A)$ ...(i)

Also, $2\alpha + A = 180°$ ...(ii)

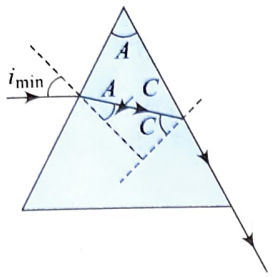
($\therefore$ Sum of angles of a triangle = 180°)

Solving Eqs. (i) and (ii), we get $A = 36°$, $\alpha = 72°$.

**ILLUSTRATION 1.79**

For a prism, $A = 60°$, $n = \sqrt{7/3}$. Find the minimum possible angle of incidence, so that the light ray is refracted from the second surface. Also, find $\delta_{max}$.

**Sol.** In minimum incidence case, the angles will be as shown in figure.




Applying Snell's law, we get

$$1 \times \sin i_{min} = \sqrt{\frac{7}{3}} \sin (A - C)$$

$$= \sqrt{\frac{7}{3}} (\sin A \cos C - \cos A \sin C)$$

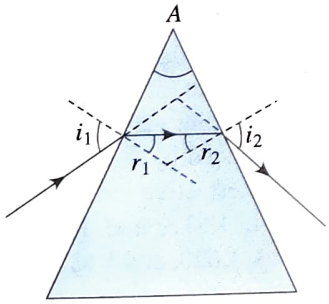
$$= \sqrt{\frac{7}{3}}\left( \sin 60 \sqrt{1 - \frac{3}{7}} - \cos 60 \sqrt{\frac{3}{7}} \right) = \frac{1}{2}$$

$\therefore \quad i_{min} = 30°$

$\therefore \quad \delta_{max} = i_{min} + 90° - A = 30° + 90° - 60° = 60°$

## THIN PRISMS

In thin prisms, the distance between the refracting surfaces is ineligible and the angle of prism ($A$) is very small. Since $A = r_1 + r_2$, therefore if $A$ is small then both $r_1$ and $r_2$ are also small, and the same is true for $i_1$ and $i_2$.




According to Snell's law, $\sin i_1 = \mu \sin r_1$

or $\quad i_1 = \mu r_1$

$\quad \sin i_2 = \mu \sin r_2$

or $\quad i_2 = \mu r_2$

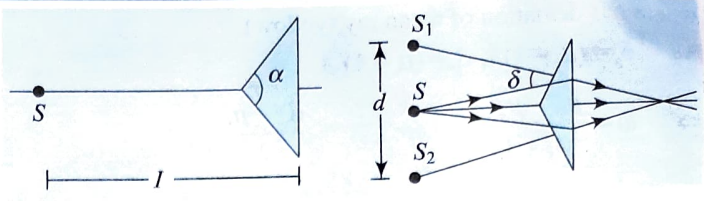
Therefore, deviation,

$$\delta = (i_1 - r_1) + (i_2 - r_2)$$

$$\Rightarrow \quad \delta = (r_1 + r_2)(\mu - 1) \quad \Rightarrow \quad \delta = A(\mu - 1)$$

**ILLUSTRATION 1.80**

A thin biprism (see figure) of obtuse angle $\alpha = 178°$ is placed at a distance $l = 20$ cm from a slit. How many images are formed and what is the separation between them? Refractive index of the material $\mu = 1.6$.




**Sol.** Two images are formed by the two thin prisms—one above the axis and the other below the axis by the same distance. The refracting angle of each thin prism is

$$\frac{\pi - \alpha}{2} = \frac{1}{2}(\pi - \alpha)$$

Then, $\delta$ (deviation of a ray) $= (\mu - 1)\frac{1}{2}(\pi - \alpha)$

$$\frac{d}{2} = l\delta$$

$$d = 2l(\mu - 1)\frac{1}{2}(\pi - \alpha)$$

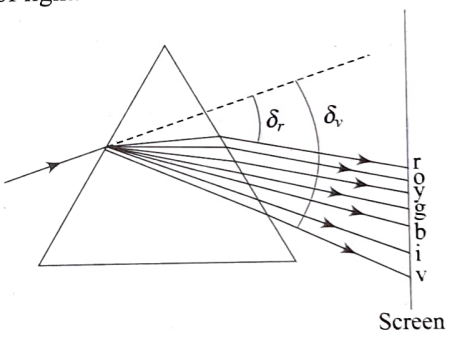
$$d = (\mu - 1)l(\pi - \alpha)$$

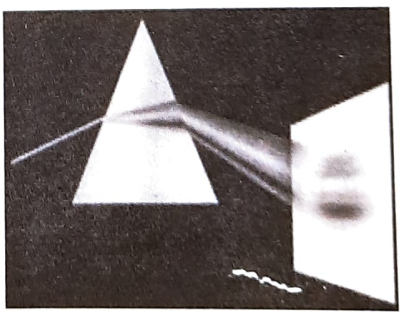
$$d = (1.6 - 1) \times 0.20\left(\pi - 178 \times \frac{\pi}{180}\right)$$

$$= 0.6 \times 0.20 \times \pi \times \frac{1}{90} = 0.004 \text{ m} = 4 \text{ mm}$$

## DISPERSION OF LIGHT

When a ray of light passes through a prism, it splits up into rays of constituent colours or wavelengths. This phenomenon is called dispersion of light.



This phenomenon arises due to the fact that refractive index varies with wavelength. It has been observed for a prism that $\mu$ decreases with the increase of wavelength, i.e., $\mu_{blue} > \mu_{red}$.

Angular dispersion: $\theta = \delta_v - \delta_r$

Dispersive power: Ratio of angular dispersion to mean deviation.

$$\omega = \frac{\delta_v - \delta_r}{\delta}$$

**Sol.** We know that dispersion produced by a thin prism,

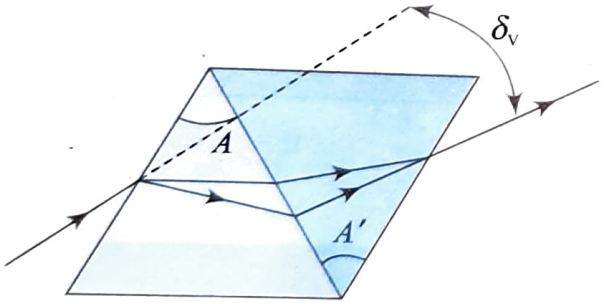
$$\theta = (\mu_v - \mu_r)\, A$$

Here, $\mu_v = 1.68$, $\mu_r = 1.56$ and $A = 18°$

$$\Rightarrow \quad \theta = (1.68 - 1.56) \times 18° = 2.16°$$

## DEVIATION WITHOUT DISPERSION

This means an achromatic combination of two prisms in which net or resultant dispersion is zero and deviation is produced.




For the two prisms,

$$(\mu_v - \mu_1)\, A + (\mu_v - \mu_r)\, A' = 0$$

$$\Rightarrow \quad A' = \frac{(\mu_v - \mu_r)\, A}{\mu_v - \mu_r} \quad \text{and} \quad \omega\delta + \omega'\delta' = 0$$

$$\delta = \delta_1 \left[1 - \frac{\omega}{\omega'}\right]$$

where $\omega$ and $\omega'$ are the dispersive powers of the two prisms and $\delta$ and $\delta'$ their mean deviations.

## DISPERSION WITHOUT DEVIATION

A combination of two prisms in which deviation produced for the mean ray by the first prism is equal and opposite to that produced by the second prism is called a direct vision prism.

**Sol.** Condition for achromatic combination.

$$\theta = \theta'$$

$$(\mu_v - \mu_r)\, A = (\mu'_v - \mu'_r)\, A'$$

$$\Rightarrow \quad A = \frac{(1.42 - 1.35)\, 4°}{1.9 - 1.7} = 1.4°$$

$$\delta_{Net} = \delta - \delta' = (\mu_Y - 1)A - (\mu'_Y - 1)\, A$$

$$= (1.40 - 1)\, 4° - (1.8 - 1)\, 1.4° = 0.48°$$

## ILLUSTRATION 1.84

A crown glass prism of angle 5° is to be combined with a flint prism in such a way that the mean ray passes undeviated. Find **(a)** the angle of the flint glass prism needed and **(b)** the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow, and violet light are 1.514, 1.517, and 1.523, respectively, for crown glass and 1.613, 1.620, and 1.632 for flint glass.

**Sol.** The deviation produced by the crown prism is $\delta = (\mu - 1)A$ and by the flint prism is $\delta' = (\mu' - 1)A'$.

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus the net deviation is

$$\delta_{net} = \delta - \delta' = (\mu - 1)A - (\mu - 1)A' \quad \ldots(i)$$

**(a)** If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A'$$

$$\text{or} \quad A' = \frac{(\mu - 1)}{(\mu' - 1)} A = \frac{1.517 - 1}{1.620 - 1} \times 5° \simeq 4.2°$$

**(b)** The angular dispersion produced by the crown prism is

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$

and that by the flint prism is $\delta_v' - \delta_r' = (\mu_v' - \mu_r')A'$

The net angular dispersion is $(\delta_v - \delta_r)A - (\mu_v' - \mu_r')A'$

$= (1.523 - 1.514) \times 5° - (1.632 - 1.613) \times 4.2°$

$= -0.0348°$

The angular dispersion has magnitude $0.0348°$.

## CONCEPT APPLICATION EXERCISE 1.6

1. The refractive indices of flint glass for red and violet lights are 1.613 and 1.632, respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5°.

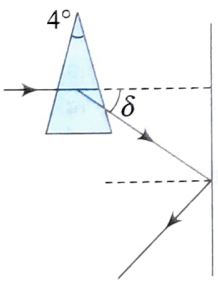
2. Refractive index of glass for red and violet colours are 1.50 and 1.60, respectively. Find:
   (a) The refractive index for yellow colour, approximately.
   (b) Dispersive power of the medium.

3. An equilateral prism is made of glass of refractive index 1.5. Calculate the angles of minimum and maximum deviations. Given:

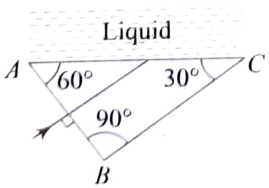
$$A = 60°,\ \mu = 1.5,\ \sin 48.6° = \frac{3}{4};\ \sin 41.8° = \frac{3}{2};\ \sin 27.9° = \frac{3}{2}\sin 18.2$$

4. A prism is made of glass of refractive index 1.5. If the angle of minimum deviation is equal to the refracting angle of the prism, calculate the angle of the prism.

5. A horizontal ray of light passes through a prism whose apex angle is 4° and then strikes a vertical mirror $M$ as shown in figure. For the ray to become horizontal after reflection, find the angle by which the mirror must be rotated.

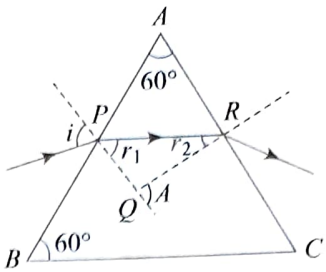



6. Light is incident normally on face $AB$ of a prism as shown in figure. A liquid of refractive index $\mu$ is placed on face $AC$ of the prism. The prism is made of glass of refractive index 3/2. Find the limits of $\mu$ for which total internal reflection takes place on the face $AC$.

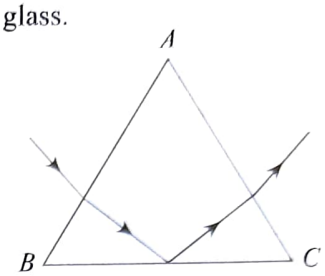



7. A ray of light suffers minimum deviation through a prism of refractive index $\sqrt{2}$. What is the angle of prism if the angle of incidence is double the angle of refraction within the prism?

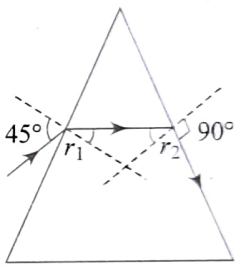
8. A ray of light undergoes a deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$.
   What is the angle subtended by the ray inside the prism with the base of the prism?




9. The path of a ray of light passing through an equilateral glass prism $ABC$ is shown in figure. The ray of light is incident on face $BC$ at the critical angle for just total internal reflection. The total angle of deviation after the refraction at face $AC$ is 108°. Calculate the refractive index of the glass.



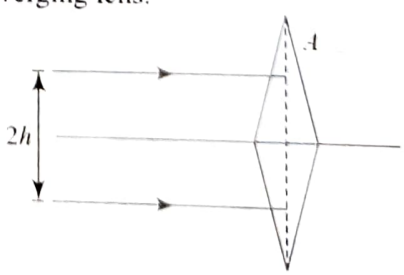

10. In an isosceles prism of angle 45°, it is found that when the angle of incidence is same as the prism angle, the emergent ray grazes the emergent surface.




Find the refractive index of the material of the prism. For what angle of incidence, the angle of deviation will be minimum?

11. Refracting angle of a prism $A = 60°$ and its refractive index is $n = 3/2$. What is the angle of incidence $i$ to get minimum deviation. Also, find the minimum deviation. Assume the surrounding medium to be air ($n = 1$).

12. Two identical thin isosceles prisms of refracting angle $A$ and refractive index $\mu$ are placed with their bases touching each other and this system can collectively act as a crude converging lens. A parallel beam of light is incident on this system as shown in figure. Find the focal length of this so called converging lens.




## ANSWERS

**1.** 0.095° **2.** (a) 1.55; (b) 2/11 **3.** 37.2°; 57.9°
**4.** $2\cos^{-1}\left(\frac{3}{4}\right)$ **5.** 1° **6.** $\mu > \sqrt{3}$ **7.** 90°
**8.** 60° **9.** 1.447 **10.** 58.8° **11.** 48.5° **12.** $h/(\mu-1)A$

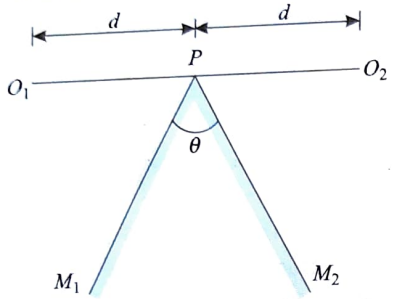
# Solved Examples

## EXAMPLE 1.1

Two plane mirrors are joined together at angle $\theta$ with each other, as shown in figure. Two point objects $O_1$ and $O_2$ are placed symmetrically such that $PO_1 = PO_2 = d$.
$[O_1PO_2$ is a straight line]

**(a)** If the images of $O_1$ and $O_2$ coincide find $\theta$ (call it $\theta_0$).

**(b)** Keeping the position of objects unchanged the angle between the two mirrors is increased to $\theta = \frac{4}{3}\theta_0$. Now find the distance between the images of a $O_1$ and $O_2$.

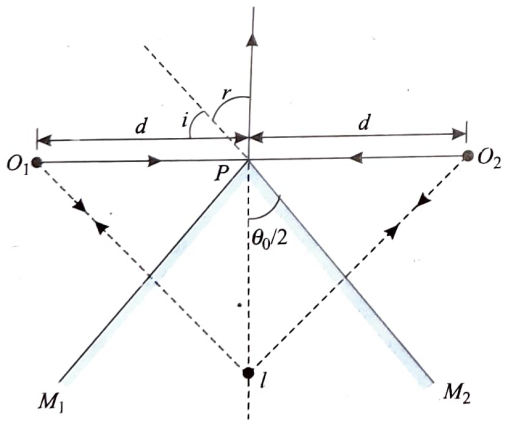



**Sol.**

**(a)** In case of plane mirror, the line joining the object and the image is perpendicular to the mirror, and object and the image are at equal distance from the mirror. Figure shows the image formed by the mirror.

From ray diagram; $\angle i = \angle r = 45°$

But $\frac{\theta_0}{2} = 45° \Rightarrow \theta_0 = 90°$



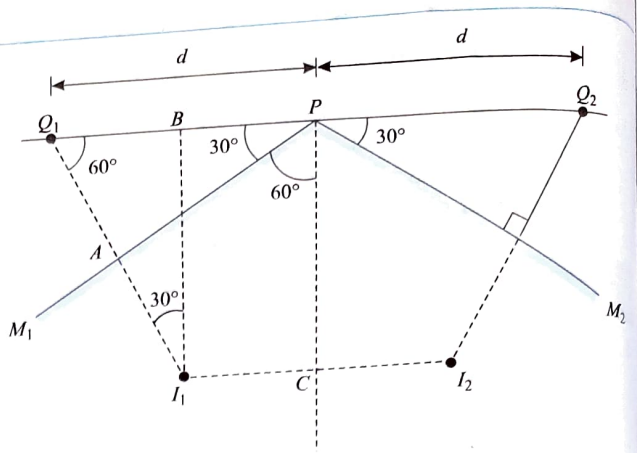

**(b)** In this case $\theta = \frac{4}{3}\theta_0$ and but $\theta_0 = 90°$,

hence $\theta = \frac{4}{3} \times 90° = 120°$

Let us draw the images formed by the mirror,

$O_1A = d\sin 30° = \frac{d}{2} \Rightarrow O_1I_1 = 2O_1A = d$

$\therefore O_1B = (O_1I_1)\cos 60° = \frac{d}{2}$



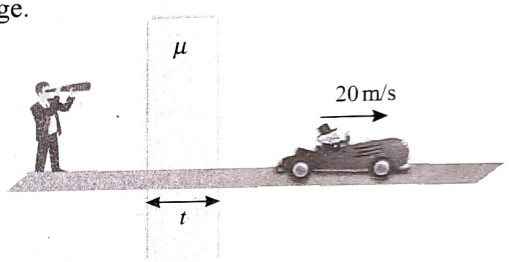

$\Rightarrow BP = \frac{d}{2}(= I_1C)$

$\therefore I_1I_2 = d$

## EXAMPLE 1.2

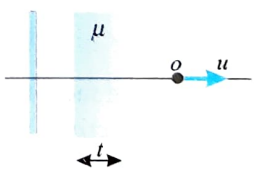
A car is moving with velocity $u$ = 20 m/s. What will be change in velocity of car observed if a glass slab is introduced between car and observer.

**Sol.** Presence of glass slab makes no difference to the velocity of the image.

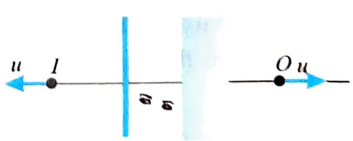



## EXAMPLE 1.3

Object $O$ is placed in front of a plane mirror and glass slab as shown in figure. The object is made to move with a velocity $u$ perpendicular to the mirror surface. Find the speed of the image formed after refraction from the slab followed by reflection from the mirror.

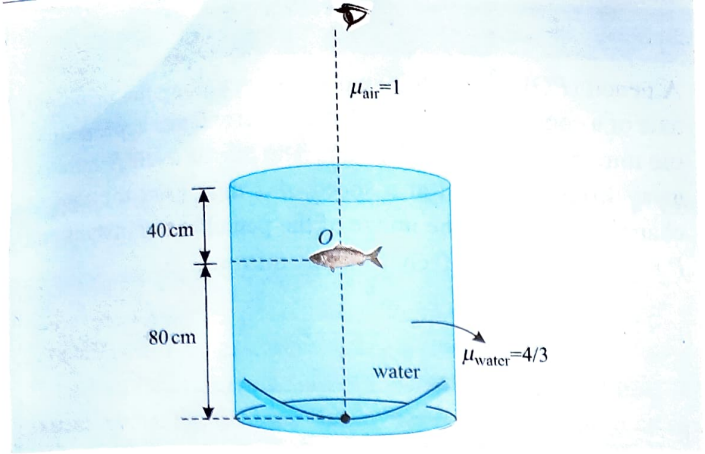



**Sol.** Presence of glass slab makes no difference to the velocity of the image.




## EXAMPLE 1.4

Consider the situation sketched. A fish lies 40 cm under the surface of water ($\mu_{water} = 4/3$), 80 cm above a concave spherical mirror whose radius of curvature is 120 cm. An observer looking down from above in the air ($\mu_{air} = 1$) sees two images of fish. Find distance (in meters) between these two images.

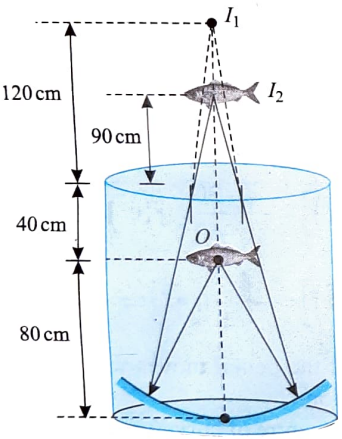



**Sol.** The observer will observe two images. One by direct observation and second will be reflected image forward by mirror.

**Case (a)** Observer directly observes the fish:

$$\text{Apparent depth} = \frac{\text{Real depth}}{\mu_{rel}} = \frac{h_0}{\left(\frac{\mu_{water}}{\mu_{air}}\right)} = \frac{40}{40/3} = 30 \text{ cm}$$

The observer will observe fish 30 cm below the water surface when he see directly.




**Case (b)** Observer observes the reflected image formed by mirror: Considering reflection from mirror,

$$u = -80 \text{ cm}, f = -\frac{R}{2} = \frac{-120}{2} = -60 \text{ cm}$$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v}+\frac{1}{(-80)}=\frac{1}{(-60)} \Rightarrow v = -240 \text{ cm}$$

i.e., 240 cm above mirror or 120 cm above water surface.

Now this image formed by mirror will acts as object for top water surface. Now rays are moving from water to air.

The position of this 'image' as seen by observer observing from top of the water surface

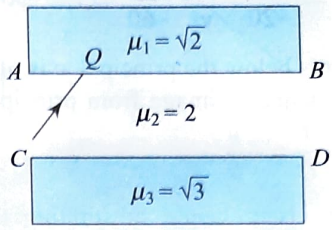
$$\text{Apparent depth } h' = \frac{h_{real}}{\mu_{rel}} = \frac{h_{rel}}{\mu_{water}/\mu_{air}} = \frac{120}{\left(\frac{4/3}{1}\right)} = 90 \text{ cm above water surface}$$

Hence, distance between two images

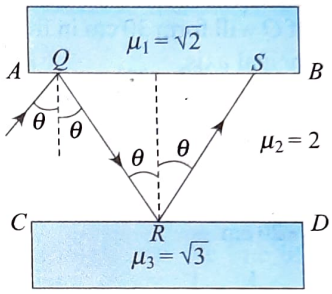
$$d = (30 + 90) = 120 \text{ cm} = 1.2 \text{ m}$$

## EXAMPLE 1.5

*AB* and *CD* are surfaces of two slabs as shown in figure. The medium between the slabs has refractive index 2. Refractive index of the slab above *AB* is $\sqrt{2}$ and below *CD* is $\sqrt{3}$. Find the minimum angle of incidence at *Q*, so that the ray is totally reflected by both the slabs.




**Sol.** Let $\theta$ be the angle of incidence at face *AB*, then for total internal reflection at face *AB*




$$\sin\theta > C_1 = \frac{\mu_1}{\mu_2} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \quad \sin\theta > \frac{1}{\sqrt{2}} \quad \text{or} \quad \sin\theta > \sin 45° \quad \Rightarrow \quad \theta > 45°$$

For total internal reflection at face *CD*,

$$\sin\theta > \sin C_2 = \frac{\mu_3}{\mu_2} = \frac{\sqrt{3}}{2} \Rightarrow \sin\theta > \frac{\sqrt{3}}{2}$$

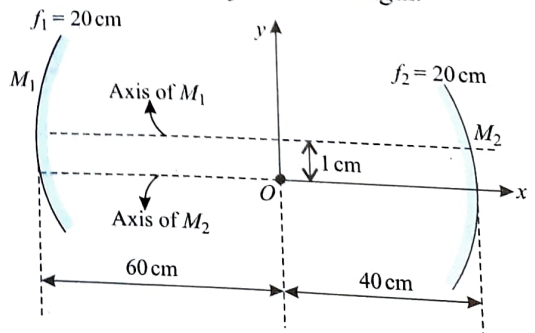
or $\sin\theta > \sin 60°$ or $\theta > 60°$

Therefore, for total internal reflection at both the surfaces,

$$\theta_{min} = 60°$$

## EXAMPLE 1.6

Two concave mirrors each of focal length 20 cm are placed such that their principal axes are parallel to each other and at a distance of 1 cm to each other. Both the mirrors are at a distance of 100 cm to each other. Consider first reflection at $M_1$ and then at $M_2$, find the coordinates of the image thus formed. Take location of object as the origin.



**Sol.** Initial incident ray direction is towards left, hence we will consider leftward direction as positive direction for any of the mirror.

Using mirror formula for first reflection:

$u = -60$ cm and $f = -20$ cm

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-20} = \frac{1}{v} + \frac{1}{-60} \Rightarrow v = -30 \text{ cm}$$

In this case object is below the principal axis of $M_1$, here we can find the lateral distance of image from principal axis of $M_1$ by using the formula $\frac{I}{O} = -\frac{v}{u}$.

In this case $O_1 = -1$ cm, hence substituting the values of $u$, $v$ and $O$ in above equation

$$\frac{I_1}{-1} = -\frac{-30}{-60} \Rightarrow I_1 = 0.5 \text{ cm}$$

It means the image of $O$ will form 30 cm in front of mirror $M_1$ and 0.5 cm above its principal axis.

Let us consider second reflection from mirror $M_2$. The image from mirror $M_1$ will act as object for mirror $M_2$. As we have consider leftward direction as positive direction. For mirror $M_2$

$u = 70$ cm and $f = 20$ cm

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{+20} = \frac{1}{v} + \frac{1}{+70} \Rightarrow v = +28 \text{ cm}$$

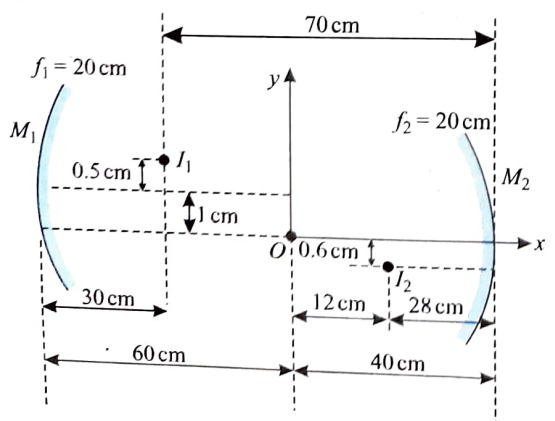
In this case object is above the principal axis of $M_2$ here we can find the lateral distance of image from principal axis of $M_2$ by using the formula $\frac{I}{O} = -\frac{v}{u}$.

In this case $O_2 = +(1+0.5) = 1.5$ cm, hence substituting the values of $u$, $v$ and $O$ in above equation

$$\frac{I_2}{+1.5} = -\frac{+28}{+70} \Rightarrow I_2 = -0.6 \text{ cm}$$

It means the image of $I_2$ will form 28 cm in front of mirror $M_2$ and 0.6 cm below its principal axis.

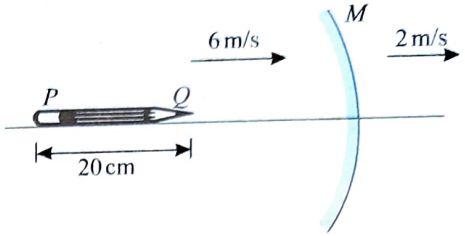
The image formation from mirrors $M_1$ and $M_2$ is given in following figure.




Hence the coordinate of $I_2$ = (12 cm, – 0.6 cm)

## EXAMPLE 1.7

A pencil ($PQ$) of length 20 cm is moving along the principal axis of a concave mirror $M$, with a velocity 6 m/s approaching the mirror of focal length 20 cm. The mirror itself is moving away from the pencil at a speed of 2 m/s. Find the rate of change of length of the image of the pencil at the instant end $P$ is at a distance of 60 cm from the mirror.




**Sol.** The velocity of the image w.r.t. mirror is given by,

$$\vec{V}_{i,m} = -\left(\frac{v}{u}\right)^2 \vec{V}_{o,m} \text{ Or } \left(\vec{V}_i - \vec{V}_m\right) = -\left(\frac{v}{u}\right)^2 \left(\vec{V}_o - \vec{V}_m\right)$$

Let us first calculate the velocity of end. For this we need to find the position of image of $P$

For $P$, $u = -60$ cm and $f = -20$ cm

Using mirror formula: $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v_P} + \frac{1}{(-60)} = \frac{1}{(-20)} \Rightarrow v_P = -30 \text{ cm}$$

$$\left[\vec{V}_i - \left(2\hat{i}\right)\right] = -\left(\frac{-30}{-60}\right)^2 \left[\left(6\hat{i}\right) - \left(2\hat{i}\right)\right]$$

$$\left[\vec{V}_i - \left(2\hat{i}\right)\right] = -\left(\frac{1}{4}\right)\left[4\hat{i}\right] \Rightarrow \left(\vec{V}_i\right)_P = 1\hat{i} \text{ m/s}$$

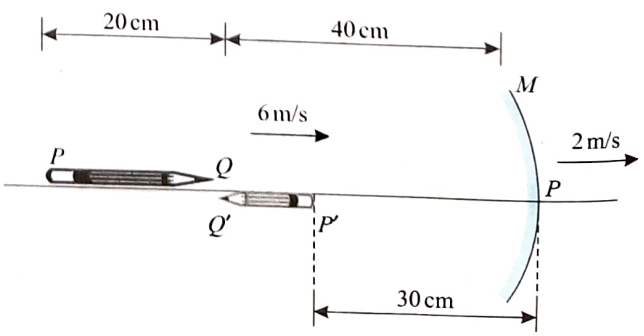
It means end $P$ of the pencil moves with velocity 1 m/s towards rightwards direction.

Point $Q$ is at the centre of curvature of the mirror and its image will be at the same position i.e. 40 cm from the mirror.

Now let us calculate the velocity of end $Q$ of the pencil

$$\left[\vec{V}_i - \left(2\hat{i}\right)\right] = -\left(\frac{-40}{-40}\right)^2 \left[\left(6\hat{i}\right) - \left(2\hat{i}\right)\right]$$

$$\left[\vec{V}_i - \left(2\hat{i}\right)\right] = -\left[4\hat{i}\right] \Rightarrow \left(\vec{V}_i\right)_Q = -2\hat{i} \text{ m/s}$$



It means end $Q$ of the pencil moves with velocity 2 m/s towards leftwards direction.

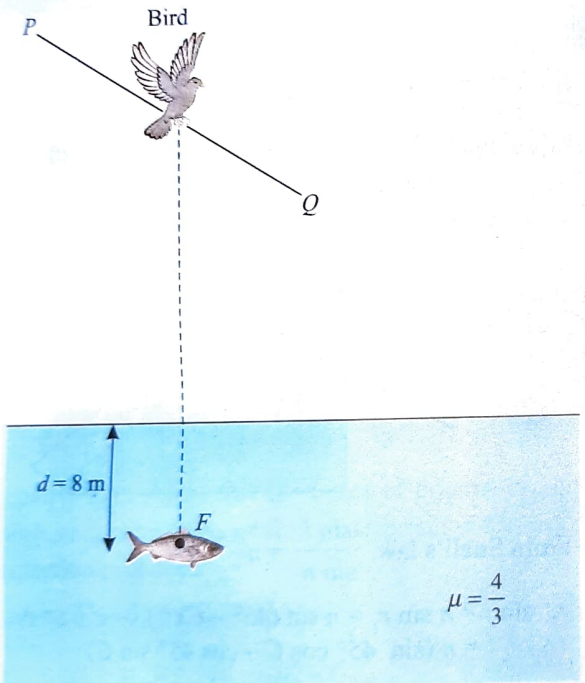
The velocity of end $Q$ w.r.t. $P$,

$$\vec{V}_{Q,P} = \vec{V}_Q - \vec{V}_P = \left(-2\hat{i}\right) - \left(1\hat{i}\right) = 3\hat{i} \text{ m/s}$$

It means the length of the pencil is increasing at a rate of 3 m/s.

## EXAMPLE 1.8

A fish $F$ is still under water at a depth $d = 8$ m. A bird is diving along line $PQ$ at a constant velocity in air. When the bird is exactly above the fish, the fish observes it at a height of 40 m from itself and velocity of the bird appears to be inclined at 45° to the horizontal. At what distance from the fish the bird actually hits the water surface. $\left(\mu_{water} = \frac{4}{3}\right)$




**Sol.** The fish in denser medium observing the bird in rarer medium, the apparent height of the bird from water surface is given by, $h_{app} = \mu h$.

Here, $h$ is the real height of the bird from water surface,

Here, $h_{app} = 40 - 8 = 32\,m$ and $\mu = \frac{4}{3}$

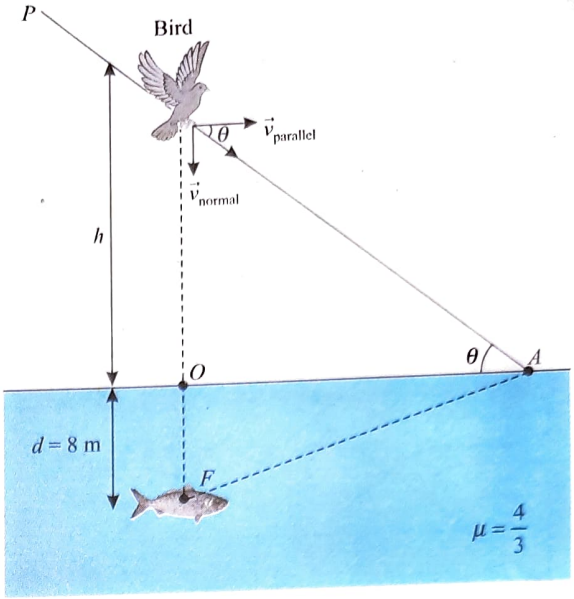
$$32 = \frac{4}{3}h \Rightarrow h = 24\,\text{m}$$

Apparent velocity of the bird normal to water surface

$$\left(V_{app}\right)_{\text{normal}} = \mu V_{\text{normal}}$$

The component of apparent velocity of the bird parallel to water surface should be equal to its velocity parallel to water surface

$$\left(V_{app}\right)_{\text{parallel}} = V_{\text{parallel}}$$




Given $\frac{(V_{app})_{\text{normal}}}{(V_{app})_{\text{palallel}}} = \tan 45° = 1$

$$\therefore \quad \frac{\mu V_{\text{normal}}}{V_{\text{parallel}}} = 1 \Rightarrow \tan\theta = \frac{1}{\mu} = \frac{3}{4} \text{ Or } \theta = 37°$$

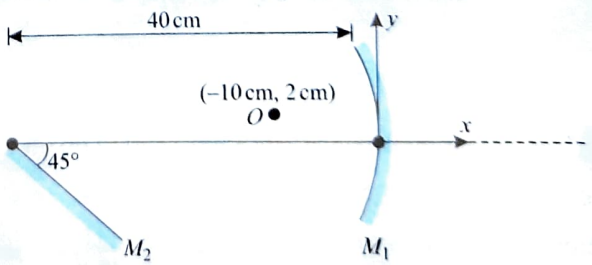
Here $\theta$ is the true angle that the line of motion of the bird makes with horizontal.

$$OA = h\tan\theta = 24 \cdot \frac{3}{4} = 18\,\text{m}$$

$$\therefore \quad FA = \sqrt{18^2 + 8^2} = 2\sqrt{97} \text{ m}$$

## EXAMPLE 1.9

Two mirrors $M_1$ and $M_2$ are arranged as shown in figure. An object $O$ is at the position (–10 cm, 2 cm) with respect to the origin $P$. The concave mirror $M_1$ has radius of curvature 30 cm. A plane mirror $M_2$ is kept at a distance of 40 cm in front of the concave mirror. Considering first reflection on the concave mirror $M_1$ and second on the plane mirror $M_2$. Find the coordinates of the second image w.r.t. the origin $P$.




**Sol.** Initial incident ray direction is towards right, hence we will consider rightward direction as positive direction.

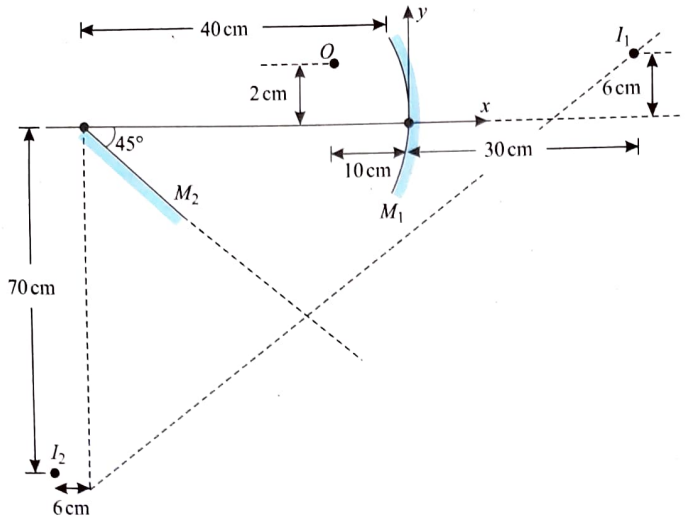
Using mirror formula for first reflection:

For $M_1$, $u = -10$ cm, $f = -15$ cm, $h = 2$ cm.

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{v} + \frac{1}{-10} = \frac{1}{-15} \Rightarrow v = 30 \text{ cm}$$

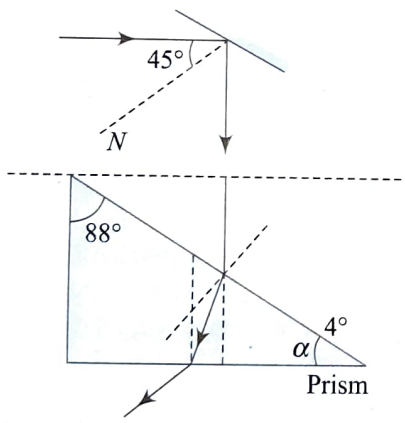
and $\frac{I}{O} = -\frac{v}{u} \Rightarrow \frac{I}{.2} = -\frac{(+30)}{(-10)} \Rightarrow I = 6\,\text{cm}$




The image formed by the plane mirror is 70 cm below the principal axis and 70 + 6 – 30 = 46 cm to the left of the mirror. Therefore, coordinates of $I_2$ w.r.t. $P$ are (–46 cm, –70 cm).

## EXAMPLE 1.10

A ray of light strikes a plane mirror at an angle of incidence 45° as shown in figure. After reflection, the ray passes through a prism of refractive index 1.5 whose apex angle is 4°. Through what angle must the mirror be rotated if the total deviation of the ray be 90° ?




**Sol.** The deviation produced by small angled prism,

$$\delta_1 = (\mu - 1)\alpha = (1.5 - 1)4° = 2° \text{ (always)}$$

Deviation caused by mirror,

$$\delta_2 = 180° - 2i = 180° - 2 \times 45° = 90°$$

Net deviation produced by system = $\delta_1 + \delta_2 = 2° + 90° = 92°$
This is more than 90°.

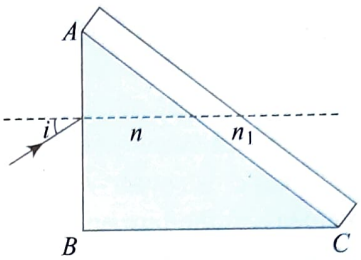
Greater is angle of incidence on the mirror, smaller is the deviation.

If $\beta$ is the angle of rotation of mirror in clockwise direction to increase angle of incidence, then deviation produced by the mirror will be $180° - 2(45° + \beta) = 90° - 2\beta$
Therefore, total deviation produced = $90° - 2\beta + 2° = 92° - 2\beta$
Given, $92° - 2\beta = 90° \Rightarrow \beta = 1°$

## EXAMPLE 1.11

A right angled prism (45°, 90°, 45°) of refractive index $n$ has a plate of refractive index ($n_1 < n$) cemented to its diagonal face. The assembly is in air. A ray is incident on $AB$.

**(a)** Calculate the angle of incidence at $AB$ for which the ray strikes the diagonal face at the critical angle.



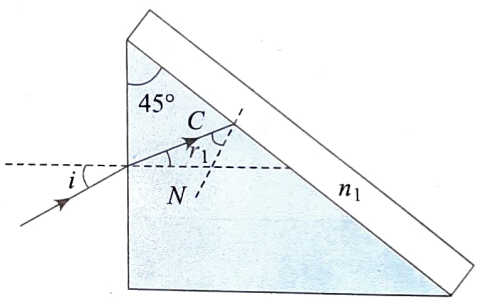

**(b)** Assuming $n = 1.351$, calculate the angle of incidence at $AB$ for which the refracted ray passes through the diagonal face undeviated.

**Sol.**

**(a)** $\sin C = \frac{n_1}{n}$

From figure, $(90° - r_1) + 45° + (90° - C) = 180°$

$\Rightarrow \quad r_1 = 45° - C$




From Snell's law, $\frac{\sin i}{\sin r_1} = n$

$$\therefore \sin i = n \sin r_1 = n \sin (45° - C)$$
$$= n (\sin 45° \cos C - \cos 45° \sin C)$$
$$= \frac{n}{\sqrt{2}}(\cos C - \sin C) = \frac{n}{\sqrt{2}}\left[\sqrt{1 - \sin^2 C} - \sin C\right]$$
$$= \frac{n}{\sqrt{2}}\left[\sqrt{1 - \left(\frac{n_1}{n}\right)^2} - \frac{n_1}{n}\right]$$
$$= \frac{1}{\sqrt{2}}\left[\sqrt{(n^2 - n_1^2)} - n_1\right]$$

$$\Rightarrow i = \sin^{-1}\left\{\frac{1}{\sqrt{2}}\left(\sqrt{n^2 - n_1^2}\right) - n_1\right\}$$

**(b)** When refracted ray passes through the diagonal face undeflected, the incidence at diagonal face is perpendicular.

$r_2 = 0$ so $r_1 + r_2 = 45° \Rightarrow r_1 = 45°$

Again, $\frac{\sin i}{\sin r_1} = n$

$\Rightarrow \quad \sin i = n \sin r_1 = 1.352 \sin 45°$

or $\quad \sin i = 1.3520020 \frac{1}{\sqrt{2}} = 0.956$

or $\quad i = \sin^{-1}(0.956) = 72°58'.$

## EXAMPLE 1.12

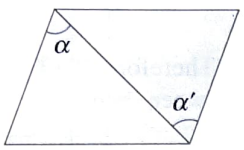
The refractive indices of the crown glass for blue and red lights are 1.51 and 1.49, respectively, and those of the flint glass are 1.77 and 1.73, respectively. An isosceles prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light. Determine the angle of the flint glass prism. Calculate the net dispersion of the combined system.

**Sol.** Deviation produced by a prism $= (\mu_y - 1)\,\alpha$

For no deviation caused by combination of two prisms,

$$(\mu_y - 1)\alpha + (\mu'_y - 1)\,\alpha' = 0$$

$$\Rightarrow \quad \alpha' = -\frac{\mu_y - 1}{\mu'_y - 1}\alpha$$




Here $\alpha = 6°$

$$\mu'_y = \frac{\mu'_b + \mu'_r}{2} = \frac{1.77 + 1.73}{2} = 1.75$$

$$\mu_y = \frac{\mu_b + \mu_r}{2} = \frac{1.51 + 1.49}{2} = 1.50$$

Refracting angle of flint glass prism,

$$\alpha' = \frac{\mu_y - 1}{\mu'_y - 1}\alpha = -\left[\frac{(1.50 - 1)}{(1.75 - 1)}\right] \times 6°$$

$$= -\frac{0.50}{0.75} \times 6° = -4°$$

The negative sign shows that the bases of prisms are opposite. Therefore, refracting angle of flint glass prism = 4°.

Net dispersion produced

$$\Delta\theta = (\delta_b - \delta_r) + (\delta'_b - \delta'_r)$$

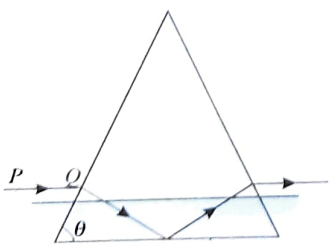
$$= (\mu_b - \mu_r)\alpha + (\mu'_b - \mu'_r)\alpha'$$

$$= (1.51 - 1.49) \times 6° - (1.77 - 1.73) \times 4°$$

$$= 0.12° - 0.16° = -0.04° = 0.04° \text{ (numerically)}$$

## EXAMPLE 1.13

The base of an isosceles glass prism just submerged in water. The base of the prism is kept in horizontal orientation. A horizontal light ray $PQ$ is incident on the prism and takes a path shown in figure and emerge out of the prism. Find the maximum value of base angle $\theta$ of the prism for which total internal reflection can take place at the base.

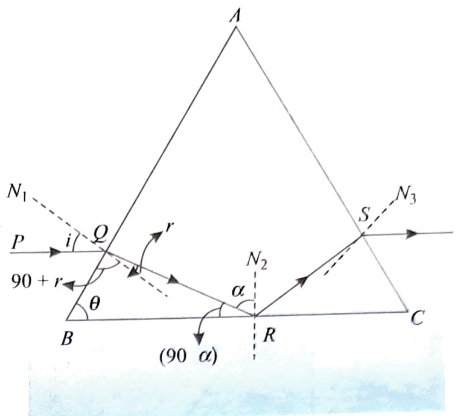



Given: $\mu_{glass} = \frac{3}{2}$ and $\mu_{water} = \frac{4}{3}$

**Sol.** The ray incident on first refracting surface at angle of incidence, $i = \frac{\pi}{2} - \theta$

Snell's law at this surface gives, $\mu_g \sin r = 1.\sin i$

$$\frac{3}{2}\sin r = \cos\theta \qquad \ldots(i)$$




In $\Delta QBR$,

$$\angle Q + \angle B + \angle R = 180°$$

$$90° + r + \theta + 90° - \alpha = 180°$$

$$\Rightarrow \quad \alpha = r + \theta$$

For total internal reflection at the base, $\alpha > C$

$$r + \theta > C$$

$$\Rightarrow \quad \sin(r + \theta) > \sin C = \frac{4/3}{3/2} = \frac{8}{9}$$

$$\Rightarrow \quad \sin(r + \theta) > \frac{8}{9}$$

$$\Rightarrow \quad \cos\theta \cdot \sin r + \sin\theta \cos r > \frac{8}{9}$$

$$\frac{2}{3}\cos^2\theta + \sin\theta\sqrt{1 - \frac{4}{9}\cos^2\theta} > \frac{8}{9} \qquad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad \sin^2\theta\left(1 - \frac{4}{9}\cos^2\theta\right) > \left(\frac{8}{9} - \frac{2}{3}\cos^2\theta\right)^2$$

Let $\cos^2\theta = x$ so $\sin^2\theta = 1 - x$

$$\Rightarrow \quad (1 - x)\left(1 - \frac{4}{9}x\right) > \left(\frac{8}{9} - \frac{2}{3}x\right)^2$$

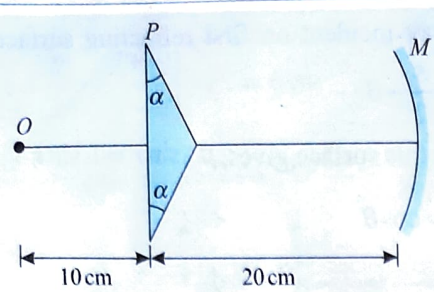
$$\Rightarrow \quad 9(1 - x)(9 - 4x) > (8 - 6x)^2$$

On solving we get, $x < \frac{17}{21}$

$$\Rightarrow \quad \cos \sqrt{\frac{17}{21}} \quad \therefore \quad \theta_{max} = \cos^{-1}\sqrt{\frac{17}{21}}$$

## EXAMPLE 1.14

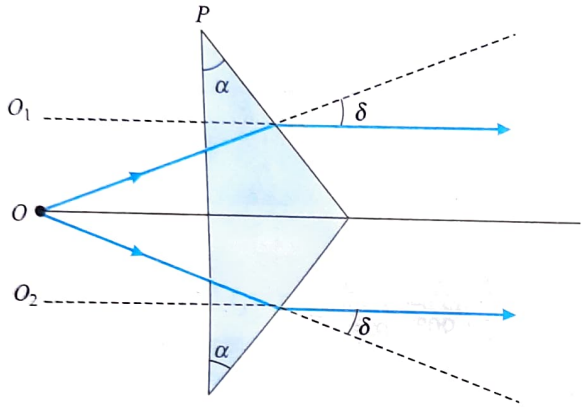
$O$ is a point object kept on the principal axis of a concave mirror $M$ of radius of curvature 20 cm. $P$ is a prism of angle 1.8°. Light falling on the prism (at small angle of incidence) gets refracted through the prism and then falls on the mirror. Refractive index of the prism is 3/2. Find the distance between the images formed by the concave mirror due to this light.




**Sol.** Deviation in case of thin prism, $\delta = (\mu - 1)A$

$$\delta = (1.5-1)1.8^\circ$$

$$= (0.5)1.8 \times \frac{\pi}{180}\text{rad} = \frac{\pi}{200}\text{rad}$$




$$\therefore \text{ Distance } OO_1 = 10 \times \frac{\pi}{200} = \frac{\pi}{20}\text{ cm}$$

Similarly, distance $OO_2 = \frac{\pi}{20}$ cm

$$\therefore \quad O_1O_2 = \frac{\pi}{10}\text{cm}$$

For the mirror $M$, $O_1$ and $O_2$ are two point objects, at a distance of 30 cm.

Now, applying mirror formula

$$\frac{1}{v} + \frac{1}{(-30)} = \frac{1}{(-10)} \Rightarrow v = -15\,\text{cm}$$

$$\text{Lateral magnification} = -\frac{v}{u} = \frac{-(-15)}{-30} = -\frac{1}{2}$$

(–ve sign indicates inverted image)

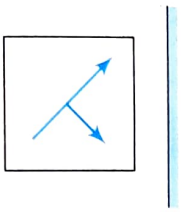
Therefore, if $O_1'$ and $O_2'$ are the images formed, then distance between them,

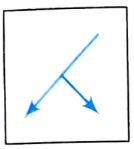
$$O_1'O_2' = \frac{1}{2}O_1O_2 = \frac{1}{2} \times \frac{\pi}{10} = \frac{\pi}{20}\text{cm}$$

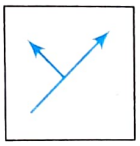
# Exercises

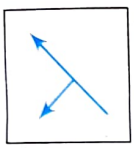
## Single Correct Answer Type

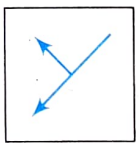
**1.** Choose the correct mirror image of object as shown in the following figure.



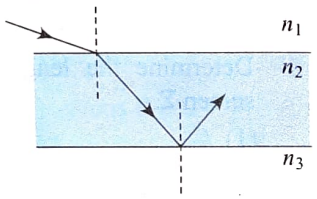
(1) 

(2) 

(3) 

(4) 

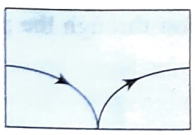
**2.** Light travelling through three transparent substances follows the path shown in figure. Arrange the indices of refraction in order from smallest to largest. Note that total internal reflection does occur on the bottom surface of medium 2.

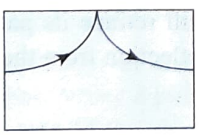


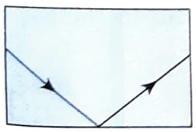

(1) $n_1 < n_2 < n_3$ (2) $n_2 < n_1 < n_3$

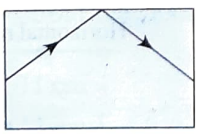
(3) $n_1 < n_3 < n_2$ (4) $n_3 < n_1 < n_2$

**3.** A cubic container is filled with a liquid whose refractive index increases linearly from top to bottom. Which of the following represents the path of a ray of light inside the liquid?

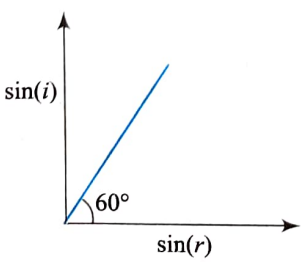
(1) 

(2) 

(3) 

(4) 

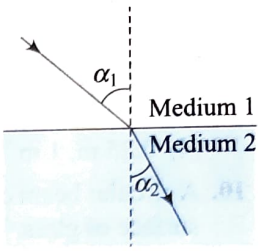
**4.** A ray of light is incident on a medium with angle of incidence $i$ and refracted into a second medium with angle of refraction $r$. The graph of $\sin(i)$ vs. $\sin(r)$ is as shown in figure. Then, the velocity of light in the first medium in $n$ times the velocity of light in the second medium. What should be the value of $n$?




(1) $\sqrt{3}$ (2) $1/\sqrt{3}$

(3) $\sqrt{3}/2$ (4) $2/\sqrt{3}$

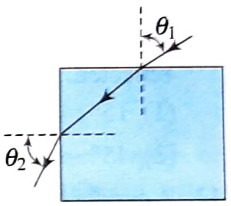
**5.** A beam of light propagates through a medium 1 and falls onto another medium 2, at an angle $\alpha_1$ as shown in figure. After that, it propagates in medium 2 at an angle $\alpha_2$ as shown. The light's wavelength in medium 1 is $\lambda_1$. What is the wavelength of light in medium 2?




(1) $\frac{\sin\alpha_1}{\sin\alpha_2}\lambda_1$ (2) $\frac{\sin\alpha_2}{\sin\alpha_1}\lambda_1$

(3) $\frac{\cos\alpha_1}{\cos\alpha_2}\lambda_1$ (4) $\frac{\cos\alpha_2}{\cos\alpha_1}\lambda_1$

**6.** Light is incident on a glass block as shown in figure. If $\theta_1$ is increased slightly, what happens to $\theta_2$?



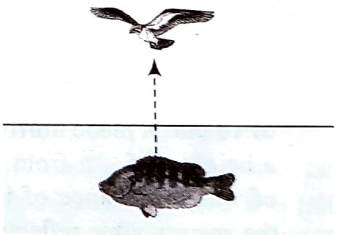

(1) $\theta_2$ also increases slightly

(2) $\theta_2$ is unchanged

(3) $\theta_2$ decreases slightly

(4) $\theta_2$ changes abruptly, since the ray experiences total internal reflection

**7.** A fish is vertically below a flying bird moving vertically down toward water surface. The bird will appear to the fish to be



(1) moving faster than its speed and also away from the real distance

(2) moving faster than its real speed and nearer than its real distance

(3) moving slower than its real speed and also nearer than its real distance

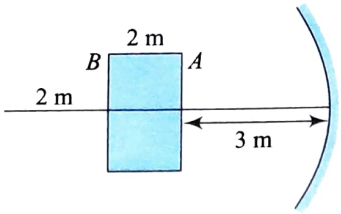
(4) moving slower than its real speed and away from the real distance

**8.** What is the angle of incidence for an equilateral prism of refractive index $\sqrt{3}$ so that the ray is parallel to the base inside the prism?

(1) 30° (2) 45°

(3) 60° (4) Either 30°or 60°

**9.** A cube of side 2 m is placed in front of a concave mirror of focal length 1 m with its face $A$ at a distance of 3 m and face $B$ at a distance of 5 m form the mirror. The distance between the images of faces $A$ and $B$ and heights of images of $A$ and $B$ are, respectively,




(1) 1 m, 0.5 m, 0.25 m
(2) 0.5 m, 1 m, 0.25 m
(3) 0.5 m, 0.25 m, 1 m
(4) 0.25 m, 1 m, 0.5 m

**10.** A circular beam of light of diameter $d = 2$ cm falls on a plane surface of glass. The angle of incidence is 60° and refractive index of glass is $\mu = 3/2$. The diameter of the refracted beam is

(1) 4.00 cm (2) 3.0 cm
(3) 3.26 cm (4) 2.52 cm

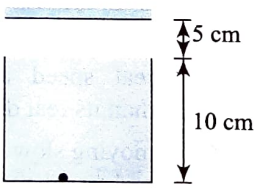
**11.** The velocity of light in a medium is half its velocity in air. If a ray of light emerges from such a medium into air, the angle of incidence, at which it will be totally internally reflected, is

(1) 15° (2) 30°
(3) 45° (4) 60°

**12.** A candle is placed 20 cm from the surface of a convex mirror and a plane mirror is also placed so that the virtual images in the two mirrors coincide. If the plane mirror is 12 cm away from the object, what is the focal length of the convex mirror?

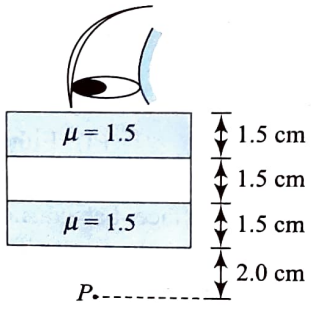
(1) 20 cm (2) 15 cm
(3) 10 cm (4) 5 cm

**13.** Consider the situation shown in figure. Water ($\mu$ = 4/3) is filled in a beaker up to a height of 10 cm. A plane mirror is fixed at a height of 5 cm from the surface of water. Distance of image from the mirror after reflection from it of an object $O$ at the bottom of the beaker is




(1) 15 cm (2) 12.5 cm
(3) 7.5 cm (4) 10 cm

**14.** The image of point $P$ when viewed from top of the slabs will be




(1) 2.0 cm above $P$ (2) 1.5 cm above $P$
(3) 2.0 cm below $P$ (4) 1 cm above $P$

**15.** One of the refracting surfaces of a prism of angle 30° is silvered. A ray of light incident at angle of 60° retraces its path. The refractive index of the material of prism is

(1) $\sqrt{2}$ (2) $\sqrt{3}$
(3) 3/2 (4) 2

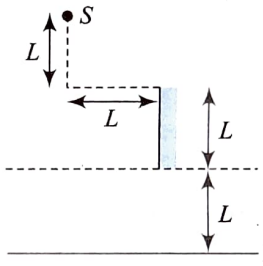
**16.** Angle of minimum deviation is equal to the angle of prism $A$ of an equilateral glass prism. The angle of incidence at which minimum deviation will be obtained is

(1) 60° (2) 30°
(3) 45° (4) $\sin^{-1}(2/3)$

**17.** A spherical mirror forms an image of magnification 3. The object distance, if focal length of mirror is 24 cm, may be

(1) 32 cm, 24 cm (2) 32 cm, 16 cm
(3) 32 cm only (4) 16 cm only

**18.** A point source of light is placed in front of a plane mirror as shown in the figure.



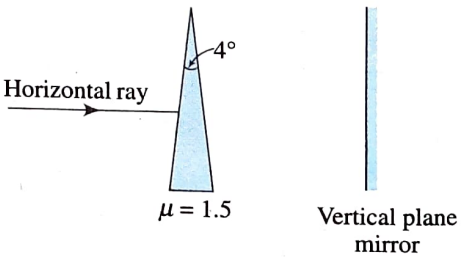

Determine the length of reflected path of light on the screen Σ.

(1) $L$ (2) $2L$
(3) $3L/2$ (4) $L/2$

**19.** A fish looks upward at an unobstructed overcast sky. What total angle does the sky appear to subtend? (Take refractive index of water as $\sqrt{2}$.)

(1) 180° (2) 90°
(3) 75° (4) 60°

**20.** For the situations shown in figure, determine the angle by which the mirror should be rotated, so that the light ray will retrace its path after refraction through the prism and reflection from the mirror?




(1) 1° ACW (2) 1° CW
(3) 2° ACW (4) 2° CW

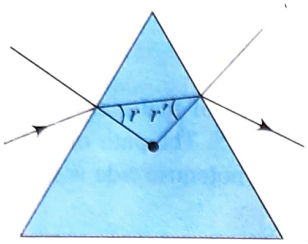
**21.** A real object is placed in front of a convex mirror (fixed). The object is moving toward the mirror. If $v_0$ is the speed of object and $v_i$ is the speed of image, then

(1) $v_i < v_0$ always
(2) $v_i > v_0$ always

(3) $v_i > v_0$ initially and then $v_0 > v_i$

(4) $v_i < v_0$ initially and then $v_i > v_0$

**22.** Let $r$ and $r'$ denote the angles inside an equilateral prism, as usual, in degrees. Consider that during some time interval from $t = 0$ to $t = t$, $r'$ varies with time as $r' = 10 + t^2$. During this time, $r$ will vary as (assume that $r$ and $r'$ are in degree)




(1) $50 - t^2$ (2) $50 + t^2$

(3) $60 - t^2$ (4) $60 + t^2$

**23.** For a prism kept in air, it is found that for an angle of incidence 60°, the angle of refraction $A$, angle of deviation $\delta$ and angle of emergence $e$ become equal. Then, the refractive index of the prism is

(1) 1.73 (2) 1.15

(3) 1.5 (4) 1.33

**24.** The critical angle for light going from medium $X$ into medium $Y$ is $\theta$. The speed of light in medium $X$ is $v$. The speed of light in medium $Y$ is

(1) $v \cos\theta$ (2) $v/\cos\theta$

(3) $v \sin\theta$ (4) $v/\sin\theta$

**25.** A concave mirror is placed on a horizontal table with its axis directed vertically upward. Let $O$ be the pole of the mirror $C$ its centre of curvature and $F$ is the focus. A point object is placed at $C$. It has a real image, also located at $C$. If the mirror is now filled with water, the image will be

(1) real and will remain at $C$

(2) real and located at a point between $C$ and $\infty$

(3) real and located at a point between $C$ and $O$

(4) real and located at a point between $C$ and $F$

**26.** A plane glass mirror of thickness 3 cm of material of $\mu = 3/2$ is silvered on the back surface. When a point object is placed 9 cm from the front surface of the mirror, then the position of the brightest image from the front surface is

(1) 9 cm (2) 11 cm

(3) 12 cm (4) 13 cm

**27.** An object is placed at a distance of 25 cm from the pole of a convex mirror and a plane mirror is set at a distance 5 cm from convex mirror so that the virtual images formed by the two mirrors do not have any parallax. The focal length of the convex mirror is

(1) 37.5 cm (2) –7.5 cm

(3) –37.5 cm (4) +7.5 cm

**28.** When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm. If the object is moved with a speed of 9 cm $s^{-1}$ the speed with which the image moves is

(1) 0.1 m $s^{-1}$ (2) 1 m $s^{-1}$

(3) 3 m $s^{-1}$ (4) 9 m $s^{-1}$

**29.** A convex mirror and a concave mirror of radius 10 cm each are placed 15 cm apart facing each other. An object is placed midway between them. If the reflection first takes place in the concave mirror and then in convex mirror, the position of the final image is

(1) on the pole of the convex mirror

(2) on the pole of the concave mirror

(3) at a distance of 10 cm from the convex mirror

(4) at a distance of 5 cm from the concave mirror

**30.** A small piece of wire bent into an L shape, with upright and horizontal portions of equal lengths, is placed with the horizontal portion along the axis of the concave mirror whose radius of curvature is 10 cm. If the bend is 20 cm from the pole of the mirror, then the ratio of the lengths of the images of the upright and horizontal portions of the wire is

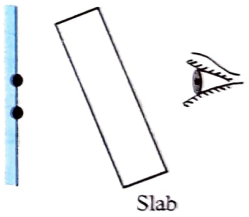
(1) 1 : 2 (2) 3 : 1

(3) 1 : 3 (4) 2 : 1

**31.** The observer at $O$ views two closely spaced spots on a vertical wall through an angled glass slab as shown in figure. As seen by observer, the spots appear

(1) shifted upward, such that distance between them remains same

(2) shifted downward, such that distance between them remains same




(3) spaced farther apart

(4) spaced closer together

**32.** The image of an object placed on the principal axis of a concave mirror of focal length 12 cm is formed at a point which is 10 cm more distance from the mirror than the object. The magnification of the image is

(1) 8/3 (2) 2.5

(3) 2 (4) –1.5

**33.** An object is approaching a fixed plane mirror with velocity 5 m $s^{-1}$ making an angle of 45° with the normal. The speed of image w.r.t. the mirror is

(1) 5 m $s^{-1}$ (2) $5/\sqrt{2}$ m $s^{-1}$

(3) $5\sqrt{2}$ m $s^{-1}$ (4) 10 m $s^{-1}$

**34.** A clear transparent glass sphere ($\mu = 1.5$) of radius $R$ is immersed in a liquid of refractive index 1.25. A parallel beam of light incident on it will converge to a point. The distance of this point from the centre will be

(1) $-3R$ (2) $+3R$

(3) $-R$ (4) $+R$

**35.** A ray of light from a denser medium strikes a rarer medium at an angle of incidence $i$. The reflected and refracted rays

make an angle of $\pi/2$ with each other. If the angles of reflection and refraction are $r$ and $r'$, then the critical angle will be

(1) $\tan^{-1}(\sin i)$ (2) $\sin^{-1}(\sin r)$

(3) $\sin^{-1}(\tan i)$ (4) $\sin^{-1}(\tan r)$

**36.** A ray of light travelling in glass ($\mu = 3/2$) is incident on a horizontal glass–air surface at the critical angle $\theta_c$. If a thin layer of water ($\mu = 4/3$) is now poured on the glass–air surface, the angle at which the ray emerges into air at the water–air surface is

(1) 60° (2) 45°

(3) 90° (4) 180°

**37.** A ray of light enters a rectangular glass slab of refractive index $\sqrt{3}$ at an angle of incidence 60°. It travels a distance of 5 cm inside the slab and emerges out of the slab. The perpendicular distance between the incident and the emergent rays is

(1) $5\sqrt{3}$ cm (2) $\frac{5}{2}$ cm

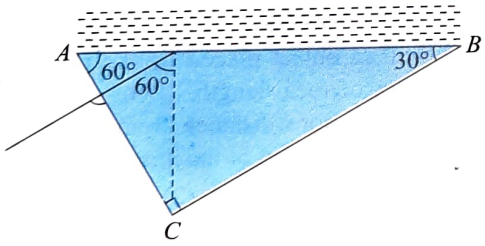
(3) $5\sqrt{3/2}$ cm (4) 5 cm

**38.** A ray of monochromatic light is incident on the refracting face of a prism (angle 75°). It passes through the prism and is incident on the other face at the critical angle. If the refractive index of the prism is $\sqrt{2}$, then the angle of incidence on the first face of the prism is

(1) 15° (2) 30°

(3) 45° (4) 60°

**39.** *ACB* is right-angled prism with other angles as 60° and 30°. Refractive index of the prism is 1.5. *AB* has thin layer of liquid on it as shown. Light falls normally on the face AC. For total internal reflections, maximum refractive index of the liquid is




(1) 1.4 (2) 1.3

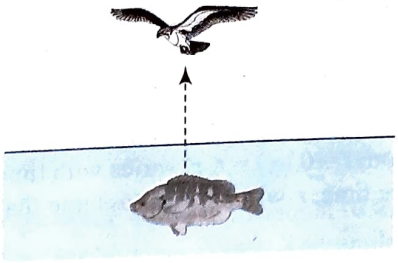
(3) 1.2 (4) 1.6

**40.** A glass prism has refractive index $\sqrt{2}$ and refracting angle 30°. One of the refracting surface of the prism is silvered. A beam of monochromatic light will retrace it path if its angle of incidence on the unsilvered refracting surface of the prism is

(1) 0 (2) $\pi/6$

(3) $\pi/4$ (4) $\pi/3$

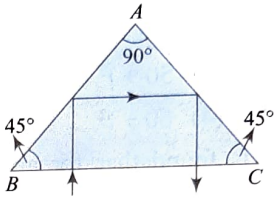
**41.** A fish rising up vertically toward the surface of water with speed 3 $ms^{-1}$ observes a bird diving down vertically towards it with speed 9 $ms^{-1}$. The actual velocity of bird is



(1) 4.5 m $s^{-1}$ (2) 5.4 m $s^{-1}$

(3) 3.0 m $s^{-1}$ (4) 3.4 m $s^{-1}$

**42.** The refractive index of material of a prism of angles 45°, –45°, and –90° is 1.5. The path of the ray of light incident normally on the hypotenuse side is shown in

(1)
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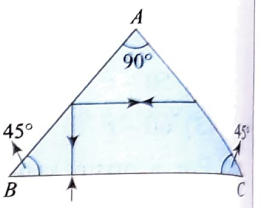
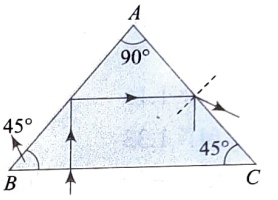
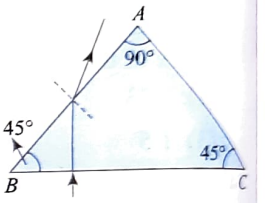



(3)



(4)



**43.** An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, it is found that there is no parallax between the images formed by the two mirrors. What is the radius of curvature of the convex mirror?

(1) 25 cm (2) 7 cm

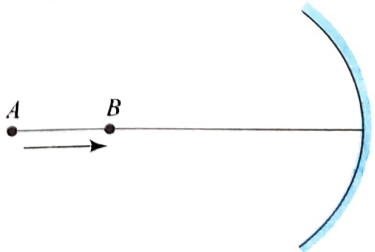
(3) 18 cm (4) 27 cm

**44.** The image produced by a concave mirror is one-quarter the size of object. If the object is moved 5 cm closer to the mirror, the image will only be half the size of the object. The focal length of mirror is

(1) $f = 5.0$ cm (2) $f = 2.5$ cm

(3) $f = 7.5$ cm (4) $f = 10$ cm

**45.** A linear object *AB* is placed along the axis of a concave mirror. The object is moving towards the mirror with speed *U*. The speed of the image of the point *A* is $4U$ and the speed of the image of *B* is also $4U$ but in opposite direction. If the centre of the line *AB* is at a distance *L* from the mirror then find out the length of the object.




(1) $3L/2$ (2) $5L/3$

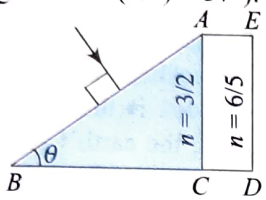
(3) $L$ (4) None of these

**46.** A mango tree is at the bank of a river and one of the branch of tree extends over the river. A tortoise lives in the river. A mango falls just on the tortoise. The acceleration of the mango falling from tree as it appears to the tortoise is (refractive index of water is 4/3 and the tortoise is stationary)

(1) $g$ (2) $3g/4$

(3) $4g/3$ (d) none of these

**47.** In figure, $ABC$ is the cross section of a right-angled prism and $ACDE$ is the cross section of a glass slab. The value of $\theta$ so that light incident normally on the face $AB$ does not cross the face $AC$ is (given $\sin^{-1}(3/5) = 37°$).




(1) $\theta \leq 37°$ (2) $\theta < 37°$

(3) $\theta \leq 53°$ (4) $\theta < 53°$

**48.** A point object '$O$' is at the centre of curvature of a concave mirror. The mirror starts to move at a speed $u$, in a direction perpendicular to the principal axis. Then, the initial velocity of the image is

(1) $2u$, in the direction opposite to that of mirror's velocity

(2) $2u$, in the direction same as that of mirror's velocity

(3) zero

(4) $u$, in the direction same as that of mirror's velocity

**49.** Refractive index of a prism is $\sqrt{7/3}$ and the angle of prism is 60°. The minimum angle of incidence of a ray that will be transmitted through the prism is

(1) 30° (2) 45°

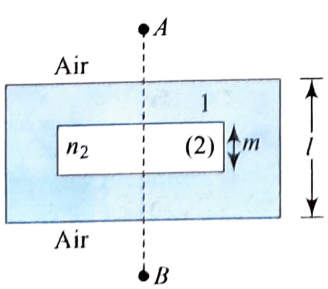
(3) 15° (4) 50°

**50.** For a prism kept in air, it is found that for an angle of incidence 60°, the angle of refraction 'A', angle of deviation '$\delta$', and angle of emergence '$e$' become equal. The minimum angle of incidence of a ray that will be transmitted through the prism is

(1) 1.73 (2) 1.15

(3) 1.5 (4) 1.33

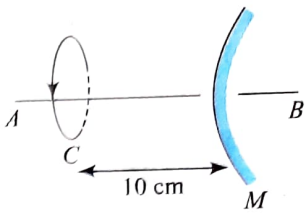
**51.** In a thick glass slab of thickness $l$, and refractive index $n_1$, a cuboidal cavity of thickness '$m$' is carved as shown in figure and is filled with a liquid of RI $n_2$ ($n_1 > n_2$). The ratio $l/m$, so that shift produced by this slab is zero when an observer $A$ observes an object $B$ with paraxial rays is




(1) $\dfrac{n_1 - n_2}{n_2 - 1}$ (2) $\dfrac{n_1 - n_2}{n_2(n_1 - 1)}$

(3) $\dfrac{n_1 - n_2}{n_1 - 1}$ (4) $\dfrac{n_1 - n_2}{n_1(n_2 - 1)}$

**52.** A particle revolves in clockwise direction (as seen from point $A$) in a circle $C$ of radius 1 cm and completes one revolution in 2 sec. The axis of the circle and the principal axis of the mirror $M$ coincide, call it $AB$. The radius of curvature of the mirror is 20 cm. Then, the direction of revolution (as seen from $A$) of the image of the particle and its speed is



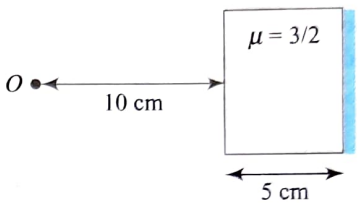

(1) clockwise, 1.57 cm s$^{-1}$

(2) clockwise, 3.14 cm s$^{-1}$

(3) anticlockwise, 1.57 cm s$^{-1}$

(4) anticlockwise, 3.14 cm s$^{-1}$

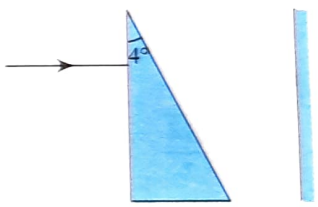
**53.** A point object is placed in front of a thick plane mirror as shown in figure. Find the location of final image w.r.t. object.




(1) 15/2 cm (2) 15 cm

(3) 40/3 cm (4) 80/3 cm

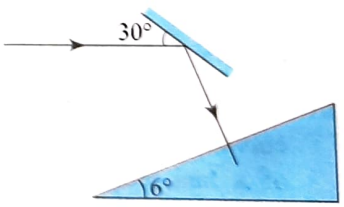
**54.** A right-angled prism of apex angle 4° and refractive index 1.5 is located in front of a vertical plane mirror as shown in figure. A horizontal ray of light is falling on the prism. Find the total deviation produced in the light ray as it emerges 2nd time from the prism.




(1) 8° CW (2) 6° CW

(3) 180° CW (4) 174°CW

**55.** Find the net deviation produced in the incident ray for the optical instrument shown in figure. (Take refractive index of the prism material as 2.)




(1) 66° CW

(2) 66° ACW

(3) 54° ACW

(4) 54° CW

**56.** The refracting angle of a prism is $A$ and refractive index of the material of prism is $\cot(A/2)$. The angle of minimum deviation will be

(1) $180° - 3A$ (2) $180° + 3A$

(3) $90° - 3A$ (4) $180° - 2A$

**57.** An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror. The distance of the near surface of the slab from the mirror is 1 cm. The final image from the mirror will be formed at

(1) 4.67 cm (2) 6.67 cm

(3) 5.67 cm (4) 7.67 cm

**58.** A concave mirror of radius of curvature 40 cm forms an image of an object placed on the principal axis at a distance 45 cm in front of it. Now if the system (including object) is completely immersed in water ($\mu = 1.33$), then

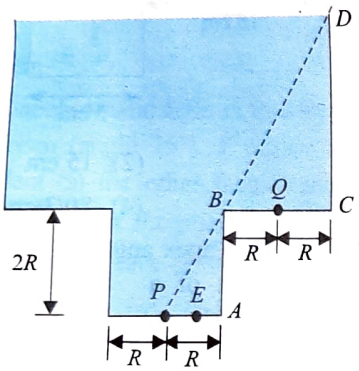
(1) the image will shift towards the mirror

(2) the magnification will reduce

(3) the image will shift away from the mirror and magnification will increase.

(4) the position of the image and magnification will not change.

**59.** In a vessel, as shown in figure, point $P$ is just visible when no liquid is filled in vessel through a telescope in the air. When liquid is filled in the vessel completely, point $Q$ is visible without moving the vessel or telescope. Find the refractive index of the liquid.




(1) $\frac{\sqrt{14}}{3}$ (2) $\frac{\sqrt{85}}{5}$

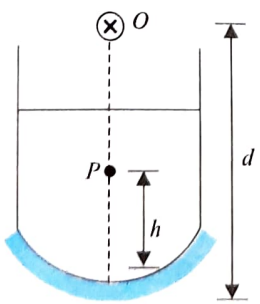
(3) $\sqrt{2}$ (4) $\sqrt{3}$

**60.** In the previous problem, what will be refractive index of the liquid so that point $E$ is visible ($PE = R/3$) through the telescope in air when the vessel is filled with liquid completely, without moving the telescope and vessel

(1) 1.5 (2) $\sqrt{3}$

(3) $\sqrt{2}$ (4) none of these

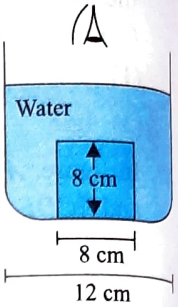
**61.** A concave mirror of radius of curvature $h$ is placed at the bottom of a tank containing a liquid of refractive index $\mu$ up to a depth $d$. An object $P$ is placed at height $h$ above the bottom of the mirror. Outside the liquid, an observer $O$ views the object and its image in the mirror. The apparent distance between these two will be




(1) $h\left(1-\frac{1}{\mu}\right)$ (2) $\frac{2h}{\mu-1}$

(3) zero (4) $\frac{2h}{\mu}$

**62.** A cylindrical vessel of diameter 12 cm contains $800\pi$ cm$^3$ of water. A cylindrical glass piece of diameter 8.0 cm and height 8.0 cm is placed in the vessel. If the bottom of the vessel under the glass piece is seen by the paraxial rays, locate its image. The index of refraction of glass is 1.50 and that of water is 1.33.



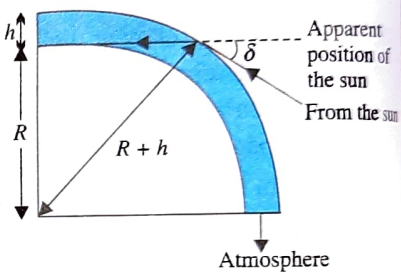

(1) 9.2 cm above the bottom

(2) 12.5 cm above the bottom

(3) 7.1 cm above the bottom

(4) 7.1 cm below the bottom

**63.** When the sun is either rising or setting and appears to be just in the horizon, it is in fact below the horizon. The explanation for this seeming paradox is that light from the sun bends slightly when entering the earth's atmosphere as shown in figure. Assume that the atmosphere has uniform density and hence uniform index of refraction $n$ and extends to a height $h$ above the earth's surface, at which point is abruptly stop. The angle $\delta$ above the Sun's true position is given by




(1) $\sin^{-1}\left(\frac{nR}{R+h}\right)$

(2) $\sin^{-1}\left(\frac{R}{R+h}\right)$

(3) $\sin^{-1}\left(\frac{nR}{R+h}\right)+\sin^{-1}\left(\frac{R}{R+h}\right)$

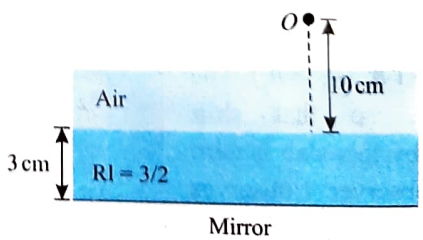
(4) $\sin^{-1}\left(\frac{nR}{R+h}\right)-\sin^{-1}\left(\frac{R}{R+h}\right)$

**64.** A gun of mass $m_1$ fires a bullet of mass $m_2$ with a horizontal speed $v_0$. The gun is fitted with a concave mirror of focal length $f$ facing toward a receding bullet. Find the speed of separation of the bullet and the image just after the gun was fired.

(1) $\left(1+\frac{m_2}{m_1}\right)v_0$ (2) $2\left(1-\frac{m_2}{m_1}\right)v_0$

(3) $2\left(1+\frac{2m_2}{m_1}\right)v_0$ (4) $2\left(1+\frac{m_2}{m_1}\right)v_0$

**65.** The position of final image of an object $O$ as shown in figure is

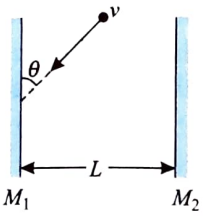



(1) 11 cm behind mirror (2) 12.33 cm behind mirror

(3) 18 cm behind mirror (4) 15 cm behind mirror

**66.** A bird is flying with a velocity $v$ between two long vertical plane mirrors making an angle $\theta$ with mirror $M_1$ as shown. Then what will be the relative velocity of approach between

the images formed by the mirrors due to the 1st reflection in each of them




(1) $v\sin\theta$ (2) $2v\sin\theta$

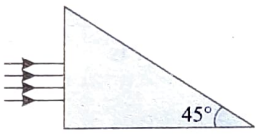
(3) $2(v\sin\theta + L)$ (4) zero

**67.** The $x$–$z$ plane separates two media $A$ and $B$ of refractive indices $\mu_1 = 1.5$ and $\mu_2 = 2$. A ray of light travels from $A$ to $B$. Its directions in the two media are given by unit vectors $\vec{u}_1 = a\hat{i} + b\hat{j}$ and $\vec{u}_2 = c\hat{i} + d\hat{j}$. Then

(1) $\frac{a}{c} = \frac{4}{3}$ (2) $\frac{a}{c} = \frac{3}{4}$

(3) $\frac{b}{d} = \frac{4}{3}$ (4) $\frac{b}{d} = \frac{3}{4}$

**68.** A beam of light consisting of red, green and blue and is incident on a right angled prism. The refractive index of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will



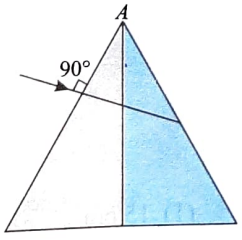

(1) separate part of the red color from the green and blue colors

(2) separate part of the blue color from the red and green colors

(3) separate all the three colors from the other two colors

(4) not separate even partially, any colors from the other two colors

**69.** A ray of light is incident normally on the first refracting face of the prism of refracting angle $A$. The ray of light comes out at grazing emergence. If one half of the prism (shaded position) is knocked off, the same ray will



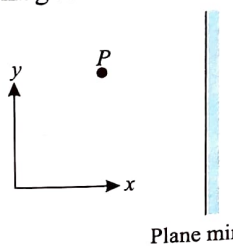

(1) Emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2}\sec A/2\right)$

(2) Not emerge out of the prism

(3) Emerge at an angle of emergence $\sin^{-1}\left(\frac{1}{2}\sec A/4\right)$

(4) None of these

**70.** Shown plane mirror is lying in $y$–$z$ plane. A particle $P$ has velocity $\vec{u} = 5\hat{i} + 6\hat{j}$ at $t = 0$ and acceleration $\vec{a} = 2t\hat{j}$. Velocity of its image at $t = 2$ s relative to it is




(1) $10\hat{i}$ (2) $-10\hat{i}$

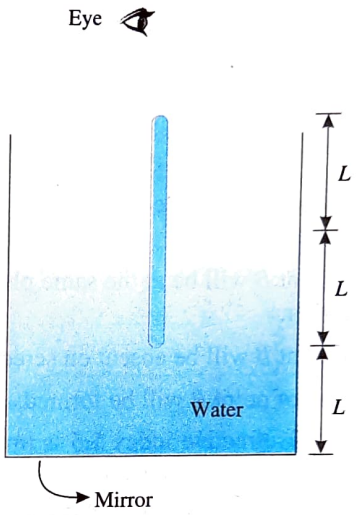
(3) $20\hat{j}$ (4) $-20\hat{j}$

**71.** An infinitely long rod lies along the axis of a concave mirror of focal length $f$. The near end of the rod is at distance $u > f$ from the mirror. Its image will have a length

(1) $\frac{uf}{u-f}$ (2) $\frac{uf}{u+f}$

(3) $\frac{f^2}{u+f}$ (4) $\frac{f^2}{u-f}$

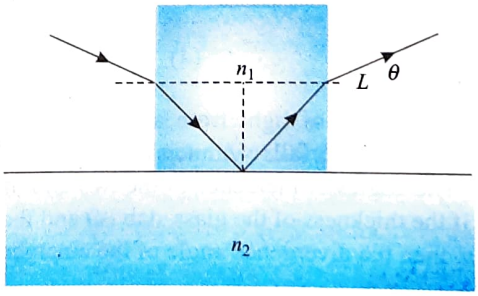
**72.** What is the length of the image of the rod in mirror, according to the observer in air? ( refractive index of the liquid is $m$)




(1) $\mu L + L$ (2) $L + \frac{L}{\mu}$

(3) $L\mu + \frac{L}{\mu}$ (4) None of these

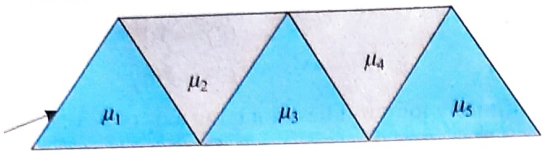
**73.** A cubical block of glass of refractive index $n_1$ is in contact with the surface of water of refractive index $n_2$. A beam of light is incident on vertical face of the block (see figure). After refraction, a total internal reflection at the base and refraction at the opposite vertical face, the ray emerges out at an angle $\theta$. The value of $\theta$ is given by




(1) $\sin\theta < \sqrt{n_1^2 - n_2^2}$ (2) $\tan\theta < \sqrt{n_1^2 - n_2^2}$

(3) $\sin\theta < \frac{1}{\sqrt{n_1^2 - n_2^2}}$ (4) $\tan\theta < \frac{1}{\sqrt{n_1^2 - n_2^2}}$

**74.** The diagram shows five isosceles right angled prisms. A light ray incident at 90° at the first face emerges at same angle with the normal from the last face. Which of the following relations will hold regarding the refractive indices?



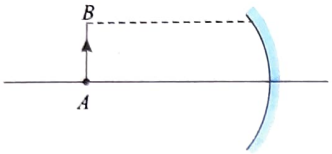

(1) $\mu_1^2+\mu_3^2+\mu_5^2=\mu_2^2+\mu_4^2$

(2) $\mu_1^2+\mu_3^2+\mu_5^2=1+\mu_2^2+\mu_4^2$

(3) $\mu_1^2+\mu_3^2+\mu_5^2=2+\mu_2^2+\mu_4^2$

(4) none of these

**75.** A converging mirror forms real image of object $AB$ on screen. Now a hole is made on mirror just in front of point $B$. Select the correct option.



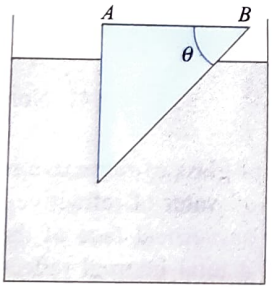

(1) Image of point $B$ will be slightly below the previous position in screen.

(2) Image of point $B$ will be at the same place where it was formed earlier.

(3) Image of point $B$ will be absent on screen.

(4) Two images of point $B$ will be formed.

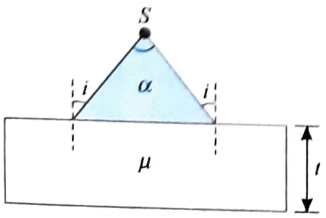
**76.** A glass prism of refractive index 1.5 is immersed in water (refractive index 4/3). A light beam normally on the face $AB$ is totally reflected to reach on the face $BC$ if




(1) $\sin\theta \geq \frac{8}{9}$ (2) $\frac{2}{3} < \sin\theta < \frac{8}{9}$

(3) $\sin\theta \leq \frac{2}{3}$ (4) $\sin\theta \leq \frac{8}{9}$

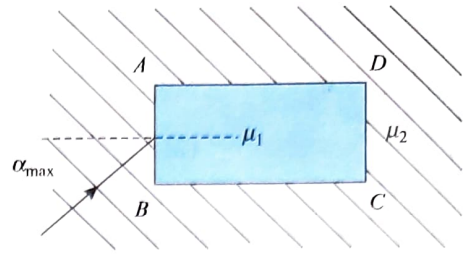
**77.** A diverging beam of light from a point source $S$ having divergence angle $\alpha$, falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is $t$ and the refractive index $n$, then the divergence angle of the emergent beam is




(1) zero (2) $\alpha$

(3) $\sin^{-1}\left(\frac{1}{n}\right)$ (4) $2\sin^{-1}\left(\frac{1}{n}\right)$

**78.** A rectangular glass slab $ABCD$ of refractive index $n_1$ is immersed in water of refractive index $n_2(n_1 < n_2)$. A ray of light is incident at the surface $AB$ of the slab as shown. The maximum value of the angle of incidence $\alpha_{max}$ such that the ray comes out from the other surface $CD$ is given by

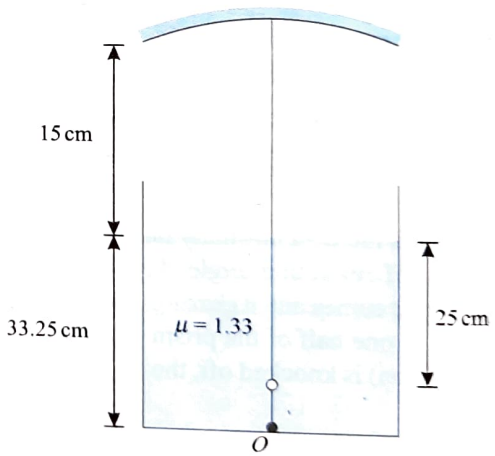



(1) $\sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\left(\frac{n_2}{n_1}\right)\right)\right]$

(2) $\sin^{-1}\left[n_1\cos\left(\sin^{-1}\left(\frac{1}{n_2}\right)\right)\right]$

(3) $\sin^{-1}\left(\frac{n_1}{n_2}\right)$ (4) $\sin^{-1}\left(\frac{n_2}{n_1}\right)$

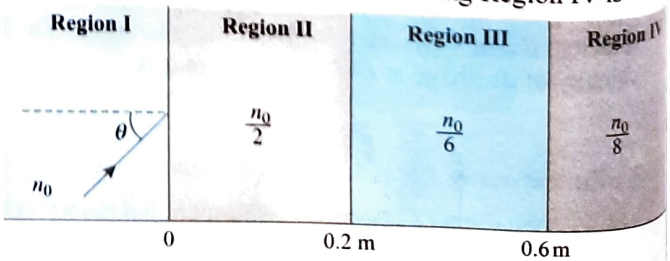
**79.** A container is filled with water ($\mu = 1.33$) up to a height of 33.25 cm. A convex mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. Focal length of the mirror is




(1) 15 cm (2) 20 cm

(3) −18.31 cm (4) 10 cm

**80.** A light beam is travelling from Region I to Region IV (refer figure). The refractive indices in Regions I, II, III, and IV are $n_0$, $n_0/2$, $n_0/6$, and $n_0/8$, respectively. The angle of incidence for which the beam just misses entering Region IV is

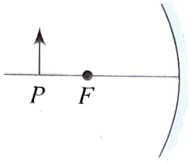



(1) $\sin^{-1}(3/4)$ (2) $\sin^{-1}(1/8)$

(3) $\sin^{-1}(1/4)$ (4) $\sin^{-1}(1/3)$

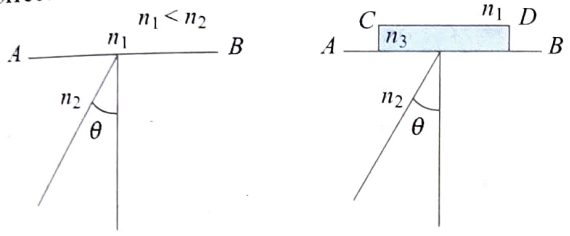
## Multiple Correct Answers Type

1. Figure shows an object located at point $P$, 0.25 meter from concave spherical mirror with principal focus $F$. The focal length of the mirror is 0.10 m. How does the image change if the object is moved from point $P$ toward point $F$?



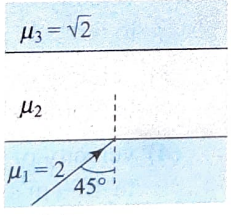

(1) Its distance from the mirror decreases
(2) The size of image decreases
(3) Its distance from the mirror increases
(4) The size of image increases

2. In figure, light is incident at an angle $\theta$ which is slightly greater than the critical angle. Now, keeping the incident angle fixed a parallel slab of refractive index $n_3$ is placed on surface $AB$. Which of the following statements are correct?



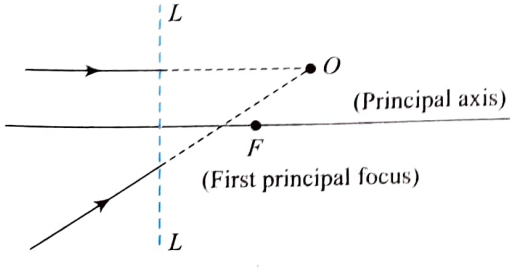

(1) Total internal reflection occurs at $AB$ for $n_3 < n_1$
(2) Total internal reflection occurs at $AB$ for $n_3 > n_1$
(3) The ray will return back to the same medium for all values of $n_3$
(4) Total internal reflection occurs at $CD$ for $n_3 < n_1$

3. In figure, a light ray is incident on the lower medium boundary at an angle if 45° with the normal. Which of the following statements is/are true?



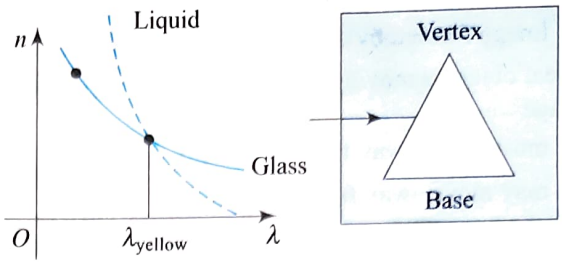

(1) If $\mu_2 > \sqrt{2}$, then angle of deviation is 45°.
(2) If $\mu_2 < \sqrt{2}$, then angle of deviation is 90°.
(3) If $\mu_2 < \sqrt{2}$, then angle of deviation is 135°.
(1) If $\mu_2 > \sqrt{2}$, then angle of deviation is 0°.

4. Consider the rays shown in figure as paraxial. The image of the virtual point object $O$ formed by the lens $LL$ is



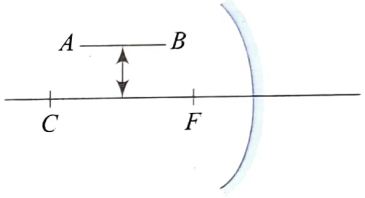

(1) virtual (2) real
(3) located below the principal axis
(4) located to the left of the lens

5. A glass prism is immersed in a hypothetical liquid. The curves showing the refractive index $n$ as a function of wavelength $\lambda$ for glass and liquid are as shown in Figs. (a) and (b). When a ray of white light is incident on the prism parallel to the base




(1) yellow ray travels without deviation
(2) blue ray is deviated toward the vertex
(3) red ray is deviated toward the base
(4) there is no dispersion

6. An object $AB$ is placed parallel and close to the optical axis between focus $F$ and centre of curvature $C$ of a converging mirror of focal length $f$ as shown in figure. Then,




(1) Image of $A$ will be closer than that of $B$ from the mirror.
(2) Image of $AB$ will be parallel to the optical axis.
(3) Image of $AB$ will be a straight line inclined to the optical axis.
(4) Image of $AB$ will not be a straight line.

7. Which of the following statements is/are correct about the refraction of light from a plane surface when light ray is incident in denser medium. [$C$ is critical angle]
(1) The maximum angle of deviation during refraction is $(\pi/2) - C$, it will be at angle of incidence $C$
(2) The maximum angle of deviation for all angles of incidence is $\pi - 2C$, when angle of incidence is slightly greater than $C$
(3) If angle of incidence is less than $C$, then deviation increases if angle of incidence is also increased
(4) If angle of incidence is greater than $C$, then angle of deviation decreases if angle of incidence is increased

8. Mark the correct statement(s) w.r.t. a concave spherical mirror.
(1) For real extended object, it can form a diminished virtual image
(2) For real extended object, it can form a magnified virtual image
(3) For virtual extended object, it can form a diminished real image
(4) For virtual extended object, it can form a magnified real image

**9.** Mark the correct statement(s) from the following:

(1) Image formed by a convex mirror can be real

(2) Image formed by a convex mirror can be virtual

(3) Image formed by a convex mirror can be magnified

(4) Image formed by a convex mirror can be inverted

**10.** A real object is moving toward a fixed spherical mirror. The image

(1) must move away from the mirror

(2) may move away from the mirror

(3) may move toward the mirror if the mirror is concave

(4) must move toward the mirror if the mirror is convex

**11.** A real point source is 5 cm away from a plane mirror whose reflecting ability is 50%, while the eye of an observer (pupil diameter 5 mm) is 10 cm away from the mirror. Assume that both source and eye are on the same line perpendicular to the surface and refracted rays have no effect on intensity. Then,

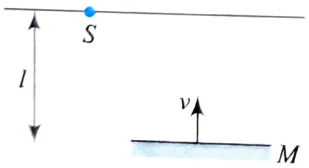
(1) the area of the mirror used in observing the image of source is $(25\pi/36)$ mm$^2$

(2) the area of the mirror used in observing the image of source is $25\pi$ mm$^2$

(3) the ratio of the intensities of light as received by the observer in the presence to that in the absence of mirror is (10/9)

(4) the ratio of the intensities of light as received in the presence to that in the absence of mirror is 19/18

**12.** A plane mirror $M$ is arranged parallel to a wall $W$ at a distance $l$ from it. The light produced by a point source $S$ kept on the wall is reflected by the mirror and produces a patch of light on the wall. The mirror moves with velocity $v$ towards the wall.




Which of the following statement(s) is/are correct?

(1) The patch of light will move with speed $v$ on the wall

(2) The patch of light will not move on the wall

(3) As the mirror comes closer, the patch of light will become larger and shift away from the wall with speed larger than $v$

(4) The size of the patch of light on the wall remains the same

**13.** Which of the following statements are correct?

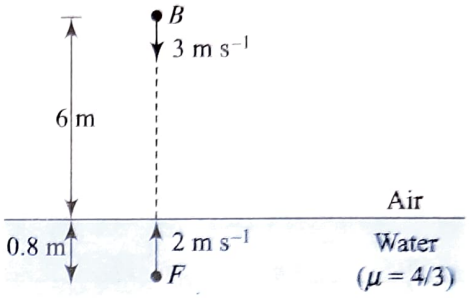
(1) A ray of light is incident on a plane mirror and gets reflected. If the mirror is rotated through an angle $\theta$, then the reflected ray gets deviated through angle $2\theta$.

(2) A ray of light gets reflected successively from two mirrors which are mutually inclined. Angular deviation suffered by the ray does not depend upon angle of incidence on first mirror.

(3) A plane mirror cannot form real image of a real object.

(4) If an object approaches toward a plane mirror with velocity $v$, then the image approaches the object with velocity $2v$.

**14.** A fish $F$, in the pond is at a depth of 0.8 m from the water surface and is moving vertically upward with velocity 2 ms$^{-1}$. At the same instant, a bird $B$ is at a height of 6 m from the water surface and is moving downward with velocity 3 ms$^{-1}$. At this instant, both are on the same vertical line as shown in figure. Which of the following statements are correct?




(1) Height of $B$, observed by $F$ (from itself), is equal to 5.30 m

(2) Depth of $F$, observed by $B$ (from itself), is equal to 6.60 m

(3) Height of $B$, observed by $F$ (from itself), is equal to 8.80 m

(4) None of the above

**15.** In the previous question,

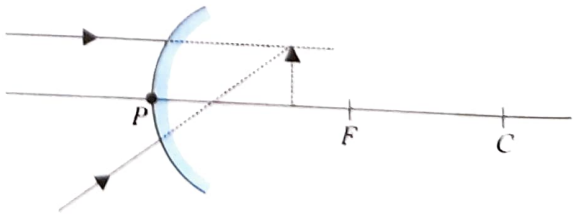
(1) velocity of $B$, observed by $F$ (relative to itself), is equal to 4.25 m s$^{-1}$

(2) velocity of $B$, observed by $F$ (relative to itself), is equal to 6 m s$^{-1}$

(3) velocity of $B$, observed by $F$ (relative to itself), is equal to 5.50 m s$^{-1}$

(4) velocity of $F$, observed by $B$ (relative to itself), is equal to 4.50 m s$^{-1}$

**16.** Converging rays strike a spherical convex mirror such that they can form the image (in the absence of mirror) between pole and focus. Now what can you say about final image formed by mirror?




(1) real

(2) virtual

(3) erect

(4) inverted

**17.** In the above question, if the rays were to converge between $F$ and $C$ of mirror, then find the nature of final image formed

(1) real

(2) virtual

(3) erect

(4) inverted

**18.** In the above question, if the rays were to converge beyond $C$, then find the nature of final image formed.

(1) real (2) virtual

(3) erect (4) inverted

**19.** A concave mirror forms an image of the sun at a distance of 12 cm from it.

(1) The radius of curvature of this mirror is 6 cm.

(2) To use it as a shaving mirror, it can be held at a distance of 8–10 cm from the face.

(3) If an object is kept at a distance of 24 cm from it, the image formed will be of the same size as the object.

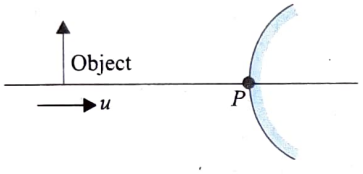
(4) All the above alternatives are correct.

**20.** A real object is placed in front of a convex mirror (focal length $f$). It moves towards the mirror, the image also moves. If $V_i$ = speed of image and $V_0$ = speed of the object and $u$ is the distance of object from mirror along principal axis, then

(1) $V_i < V_0$ if $|u| < |f|$ (2) $V_i > V_0$ if $|u| > |f|$

(3) $V_i < V_0$ if $|u| > |f|$ (4) $V_i = V_0$ if $|u| = |f|$

**21.** An object is moving towards a convex mirror with a constant velocity. $P$ is the pole of mirror.



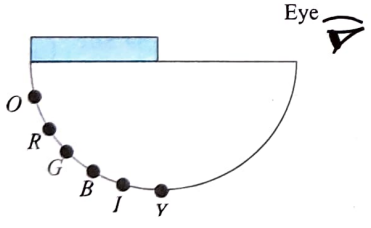

(1) Magnification of the image increases with time.

(2) Magnification of the image decreases with time.

(3) Velocity of the image increases with time.

(4) Velocity of the image decreases with time.

**22.** A person is looking at the flat surface of a transparent hemisphere of refractive index $\mu = \sqrt{2}$. Half of the flat surface is coloured black and half of curved surface is coloured in six equally spaced strips as shown in figure. Then




(1) the person can see green, orange and red

(2) the person can see only yellow, indigo and blue

(3) the ray coming from red and orange strips will be totally reflected

(1) the ray coming from yellow, indigo and blue stips will be totally reflected

**23.** Refractive index of an equilateral prism is $\sqrt{2}$. Then

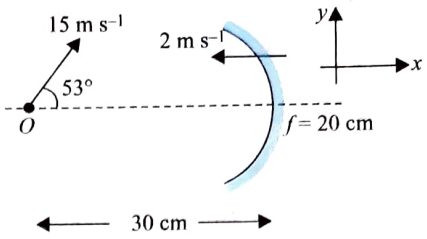
(1) minimum deviation from this prism can be 30°

(2) minimum deviation from this prism can be 45°

(3) at angle of incidence 45°, deviation is minimum

(4) at angle of incidence 60°, deviation is minimum

**24.** In the situation as shown in figure $\left(\cos 53° = \frac{3}{5}\right)$




(1) velocity of image w.r.t. mirror is $-22\hat{i} - 24\hat{j}$

(2) velocity of image w.r.t. mirror is $-44\hat{i} - 24\hat{j}$

(3) velocity of image w.r.t. ground is $-46\hat{i} - 24\hat{j}$

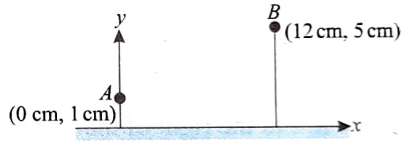
(4) velocity of image w.r.t. ground is $-24\hat{i} - 24\hat{j}$

**25.** For a concave mirror of focal length $f$, image is 2 times larger. Then the object distance from the mirror can be

(1) $\frac{f}{2}$ (2) $\frac{3f}{2}$

(3) $\frac{f}{4}$ (4) $\frac{4f}{3}$

**26.** Points $A$ (0 cm, 1 cm) and $B$ (12 cm, 5 cm) are object–image pair (one of the point acts as object and the other point as image), $x$-axis is the principal axis of the mirror. This object image pair may be



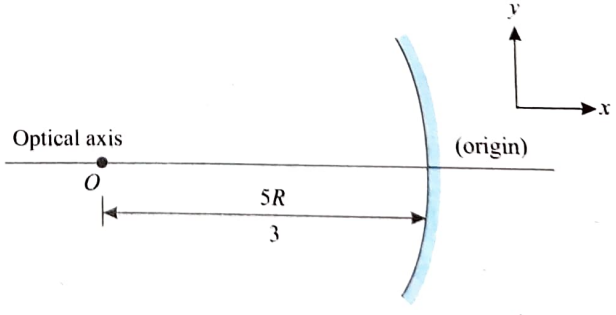

(1) due to a convex mirror of focal length 2.5 cm

(2) due to a concave mirror having its pole at (2, 0)

(3) real virtual pair

(4) due to a concave mirror of focal length 5 cm

**27.** A point object is placed at a distance of $5R/3$ from the pole of concave mirror of small aperture and radius of curvature $R$. Point object oscillates with amplitude 1 mm perpendicular to the optical axis. Then



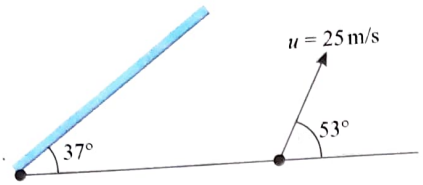

(1) position of image of object from pole is $\left(-\frac{5R}{7}, 0\right)$

(2) image of object is real

(3) amplitude of image is $\frac{3}{7}$ mm

(4) phase difference between motion of object and its image when object crosses optical axis is $\pi$

**28.** A particle is projected with speed 25 m/s at angle 53° from horizontal in front of an inclined plane mirror as shown in figure. Choose the correct options: $(g = 10\ \text{m/s}^2)$



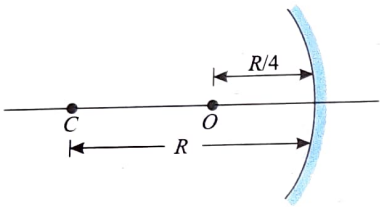

(1) Speed of image w.r.t. object will be maximum after time 4 sec of projection

(2) Minimum speed of image w.r.t. object is zero

(3) Speed of image w.r.t. object will be minimum after time 7/8 sec of projection

(4) Maximum speed of image w.r.t. object is 50 m/s

**29.** The figure below shows an object $O$ placed at a distance $R/4$ to the left of a concave spherical mirror that has radius of curvature $R$. Point $C$ is the centre of curvature of the mirror. Size of the object is much smaller than the radius of curvature of the mirror. The image formed by the mirror is




(1) larger in size and inverted

(2) larger in size and upright

(3) at a distance $R/2$ to the right of the mirror

(4) at a distance $R/2$ to the left of the mirror

**30.** A ray of light moving along vector $(3\sqrt{2}\hat{i} - 3\hat{j} - 3\hat{k})$ under goes refraction at an interface of two media, which is $y$–$z$ plane. The refractive index for $x \le 0$ is one while for $x \ge 0$ it is $\sqrt{2}$. Then,

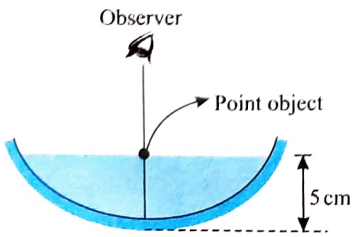
(1) refracted ray bend towards $y$-axis

(2) refracted ray bend towards $x$-axis

(3) the unit vector along the refracted ray is $\dfrac{\sqrt{6}\hat{i} - \hat{j} - \hat{k}}{2}$

(d) the unit vector along the refracted ray is $\dfrac{\sqrt{6}\hat{i} - \hat{j} - \hat{k}}{\sqrt{8}}$

**31.** A concave mirror of radius 40 cm is filled with water (refractive index, $\mu = 4/3$) up to height 5 cm. A point object $P$ is floating on the surface of water as shown in the figure.



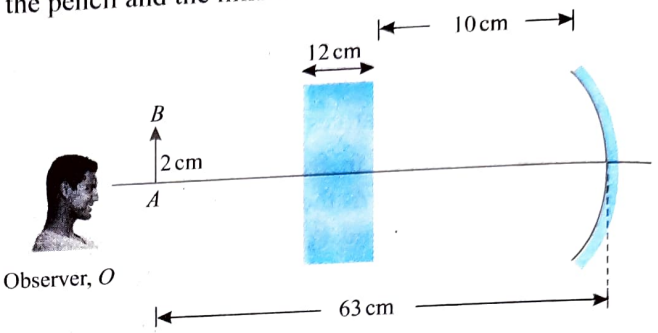

(1) The image of point object $P$ observed by observer is virtual

(2) The image of point object $P$ observed by observer is real

(3) The image of point object $P$ is 8.75 cm below the surface of water

(4) The image of point object is 10 cm above the surface of water

**32.** A pencil $AB$ of height 2 cm is placed at distance 63 cm from a concave mirror of radius of curvature 40 cm. A slab of refractive index 4/3 and thickness 12 cm is placed between the pencil and the mirror.



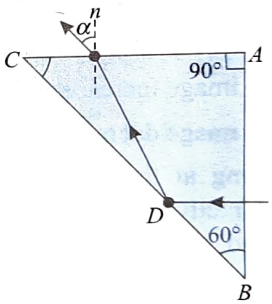

(1) Final image is real, inverted and 1 cm high

(2) Final image is virtual, inverted and 1 cm high

(3) Final image is 33 cm from the pole of mirror

(4) Final image is 27 cm from the pole of mirror

**33.** A ray of light is incident normally on face $AB$ of a prism of refractive index 5/3 immersed in water of refractive index 4/3 as shown in the figure.




(1) The exit angle $\alpha$, of the ray is $\sin^{-1}\left(\dfrac{5}{8}\right)$

(2) The exit angle $\alpha$, of the ray is $\sin^{-1}\left(\dfrac{5}{4\sqrt{3}}\right)$

(3) Total internal reflection does not take place at point $D$, if water is replaced by a medium of refractive index $\dfrac{5}{2\sqrt{3}}$

(4) Total internal reflection does not take place at point $D$, if water is replaced by a medium whose refractive index $\dfrac{6}{5}$

**34.** A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total internal reflection. If $n$ is the refractive index of the medium with respect to air, select the possible value(s) of $n$ from the following:

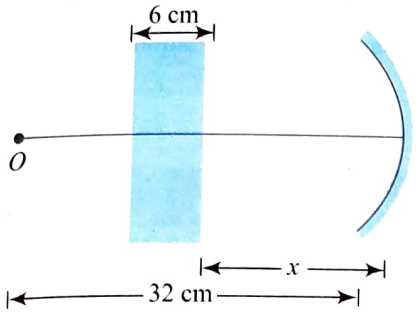
(1) 1.3 (2) 1.4

(3) 1.5 (4) 1.6

## Linked Comprehension Type

**For Problems 1–2**

A point object $O$ is placed in front of a concave mirror of focal length 10 cm. A glass slab of refractive index $\mu = 3/2$ and thickness 6 cm is inserted between the object and mirror.




1. Find the position and nature of the final image when the distance $x$ shown in figure is 5 cm.

(1) 11 cm, virtual (2) 17 cm, real

(3) 14 cm, real (4) 20 cm, virtual

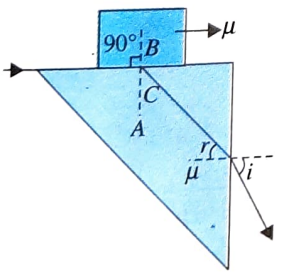
2. Find the position and nature of the final image when the distance $x$ shown in figure is 20 cm.

(1) 17 cm, virtual (2) 17 cm, real

(3) 12 cm, virtual (4) 15 cm, virtual

**For Problems 3–5**

Pulfrich refractometer is used to measure the refractive index of solids and liquid. It consist of right angled prism A having its two faces perfectly plane. One of the face is horizontal and the other is vertical as shown in figure. The solid $B$ whose refractive index is to be determined is taken having two faces cut perpendicular to one another. Light is incident in a direction parallel to the horizontal surface so that the light entering the prism $A$ is at critical angle $C$. Finally, it emerges from the prism at an angle $i$. Let the refractive index of the solid be $\mu$ and that of the prism $A$ be $\mu_0$ (which is known). Here $\mu_0 > \mu$ and by measuring $i$, $\mu$ can be determined.




3. Refractive index of the solid ($\mu$) in terms of $\mu_0$ and $i$ is

(1) $\sqrt{\mu_0^2 + \sin^2 i}$ (2) $\mu_0 + \sin^2 i$

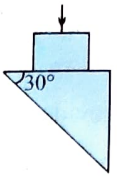
(3) $\sqrt{\mu_0^2 - 2\sin^2 i}$ (4) $\sqrt{\mu_0^2 - \sin^2 i}$

4. If $\mu_0 = \sqrt{2}$ and the ray just fail to emerge from the prism, refractive index $\mu$ of the solid will be

(1) 1.21 (2) $\sqrt{2}$

(3) 1 (4) $\sqrt{3}/2$

5. A ray of light incident normally on the horizontal face of the slab and just fail to emerge from the diagonal face of the prism. If prism angle 30° refractive index of the prism can be



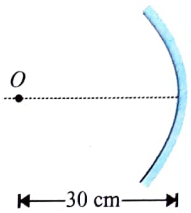

(1) $\sqrt{2}$ (2) $\sqrt{3}$

(3) slightly greater than 2 (4) slightly less than 2

**For Problems 6–8**

An object is present on the principal axis of a concave mirror at a distance 30 cm from it. Focal length of mirror is 20 cm.

6. Image formed by mirror is




(1) at a distance 60 cm in front of mirror

(2) at a distance 60 cm behind the mirror

(3) at a distance 12 cm in front of mirror

(4) at a distance 12 cm behind the mirror.

7. If object starts moving with 2 cm $s^{-1}$ along principal axis towards the mirror then,

(1) image starts moving with 4 cm $s^{-1}$ away from the mirror

(2) image starts moving with 4 cm $s^{-1}$ towards the mirror

(3) image starts moving with 8 cm $s^{-1}$ towards the mirror

(4) image starts moving with 8 cm $s^{-1}$ away from the mirror

8. If object starts moving with 2 cm $s^{-1}$ perpendicular to principal axis above the principal axis then,

(1) image moves with velocity 4 cm $s^{-1}$ below the principal axis

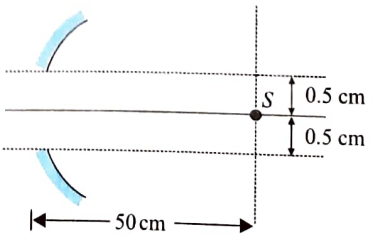
(2) image moves with velocity 4 cm $s^{-1}$ above the principal axis

(3) image moves with velocity 8 cm $s^{-1}$ below the principal axis

(4) image moves with velocity 8 cm $s^{-1}$ above the principal axis

**For Problems 9–11**

A concave mirror forms real image of a point source lying on the optical axis at a distance of 50 cm from the mirror. The focal length of the mirror is 25 cm. The mirror is cut in two, and its halves are drawn at a distance of 1 cm apart in a direction perpendicular to the optical axis.



**9.** The distance between the two images formed by the two halves of the mirror is

(1) 0.5 cm (2) 1 cm

(3) 1.5 cm (4) 2 cm

**10.** Two statements are provided. Choose which is true (T) and which is false (F).

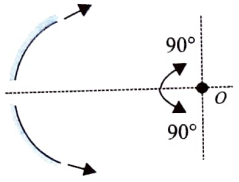
**Statement I:** Image formed by half cut mirror has just half the intensity as that of the intensity of image formed by complete mirror.

**Statement II:** More the lateral separation between the two halves of the mirror, less will be the separation between the two images, as show in the given situation.

(1) T, T (2) T, F

(3) F, T (4) F, F

**11.** In the given situation, if the two halves are each rotated (before being separated) about $S$ by 90°, with reference to original position, then the distance between the two images will be

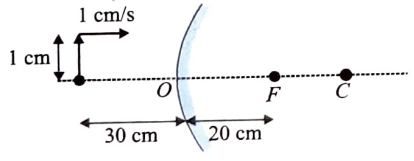



(1) 0.5 cm (2) 2 cm

(3) 1 cm (4) zero

**For Problems 12–13**

An object $AB$ of height 1 cm lying on the axis of convex mirror of focal length 20 cm, at a distance of 30 cm from its pole as shown in figure. Now the object starts moving with a constant speed 1 cm/sec towards the pole. Calculate




**12.** Magnitude of velocity of image at the given instant.

(1) 0.08 cm $s^{-1}$ (2) 0.64 cm $s^{-1}$

(3) 0.16 cm $s^{-1}$ (4) 0.32 cm $s^{-1}$

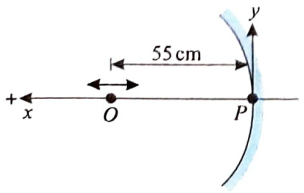
**13.** Rate of change of height of image.

(1) 0.16 cm $s^{-1}$ (2) 0.004 cm $s^{-1}$

(3) 0.016 cm $s^{-1}$ (4) 0.008 cm $s^{-1}$

**For Problems 14–16**

A particle performs simple harmonic motion, about $x$ =55 cm (mean position). The pole of the concave mirror is at origin of the coordinate system given in figure. The focal length of the mirror is 20 cm and the position of oscillating particle from its mean position is $s = 5\cos 10\pi t$.




**14.** Find the range of vibration of image.

(1) 10 cm (2) 5 cm

(3) 3.33 cm (4) 20 cm

**15.** Find the maximum distance of image from pole of mirror,

(1) 10 cm (2) 30 cm

(3) 25 cm (4) 33.33 cm

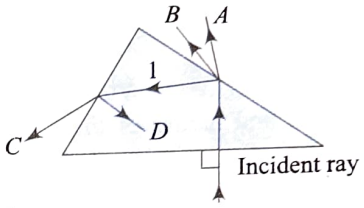
**16.** Find the time period of oscillation of image.

(1) 0.1 s (2) 0.2 s

(3) 0.4 s (4) 0.6 s

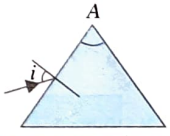
## Matrix Match Type

**1.** A white light ray is incident on a glass prism, and it creates four refracted rays $A$, $B$, $C$, and $D$. Match the refracted rays with the colours given (1 and $D$ are rays due to total internal reflection):




| Column I (Ray) | Column II (Colour) |
|---|---|
| i. A | a. red |
| ii. B | b. green |
| iii. C | c. yellow |
| iv. D | d. blue |

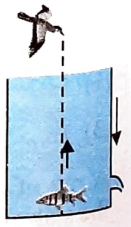
**2.** For a prism of refracting angle $A$ and refractive index Assume rays are incident at all angles of incidence $0° \le i \le$ 90°. Ignore partial reflection.




| Column I | Column II |
|---|---|
| i. $A = 15°$ | a. All rays are reflected back from the second surface |
| ii. $A = 45°$ | b. All rays are refracted into air from the second face |
| iii. $A = 70°$ | c. Some rays are reflected back from second surface |
| iv. $A = 50°$ | d. Some rays are refracted into air from the second surface |

**3.** A bird in air is diving vertically over a tank with speed 5 cm $s^{-1}$. Base of tank is silvered. A fish in the tank is rising upward along the same line with speed 2 cm $s^{-1}$. Water level is falling at rate of 2 cm $s^{-1}$.

[Take: $\mu_{water} = 4/3$]



| Column I | Column II (in cm $s^{-1}$) |
|---|---|
| i. Speed of the fish as seen by the bird directly | a. 8 |
| ii. Speed of the image of fish formed after reflection in the mirror as seen by the bird | b. 6 |
| iii. Speed of bird relative to the fish looking upwards | c. 3 |
| iv. Speed of image of bird relative to the fish looking downwards in the mirror | d. 4 |

4. In column I, types of the mirrors are given and in column II, types of the image for virtual objects are listed, match the following two columns.

| Column I | Column II |
|---|---|
| i. Concave mirror | a. only real image |
| ii. Convex mirror | b. only virtual image |
| iii. Plane mirror | c. may be real or virtual image |

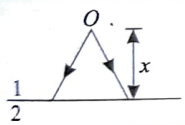
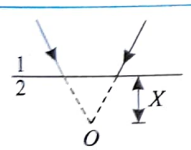
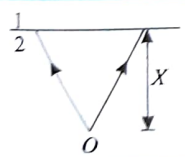
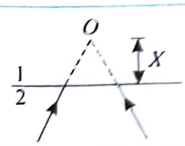
(1) i. → a.; ii. → c.; iii. → a.

(2) i. → b.; ii. → c.; iii. → a.

(3) i. → c.; ii. → b.; iii. → a.

(4) i. → a.; ii. → c.; iii. → b.

5. Match the following two columns corresponding to single refraction from plane surface. In cases shown in Column I, $\mu_1 > \mu_2$.

| Column I | Column II |
|---|---|
| i.   | a. Image distance greater than $x$ from plane surface |
| ii.   | b. Image distance less than $x$ from plane surface |
| iii.   | c. Real image |
| iv.   | d. Virtual image |

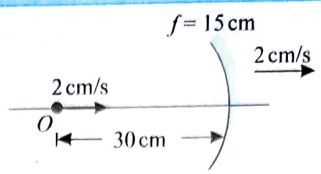
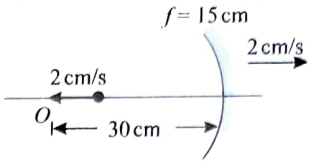
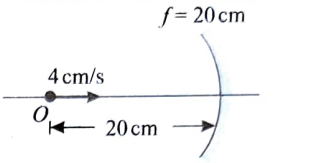
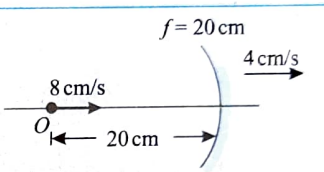
(1) i. → b., d.; ii. → b., c.; iii. → a., d.; iv. → a., c.

(2) i. → b., c.; ii. → b., c.; iii. → a., d.; iv. → a.

(3) i. → b.; ii. → b., c.; iii. → a,. d.; iv. → a., c.

(4) i. → a., d.; ii. → b., c.; iii. → a., c.; iv. → a., c.

6. Column I shows an optical system, in which velocity of object and mirror is shown. Column II has value of magnitude of velocity of image formed by mirror w.r.t. ground. Match the proper entries from column II to column I. Object is moving on principal axis of mirror.

| Column I | Column II |
|---|---|
| i.   | a. 2 m/s |
| ii.   | b. 1 m/s |
| iii.   | c. 6 m/s |
| iv.   | d. 3 m/s |

(1) i. → d.; ii. → b.; iii. → a.; iv. → c.

(2) i. → a.; ii. → c.; iii. → d.; iv. → d.

(3) i. → b.; ii. → a.; iii. → d.; iv. → c.

(4) i. → d.; ii. → d.; iii. → a.; iv. → c.

7. In column I different object positions w.r.t. a concave or convex mirror are given. In column II corresponding magnifications are given. Match the situations given in column I with the situations given in column II.

| Column I | Column II |
|---|---|
| i. An object is placed at a distance equal to focal length from pole before convex mirror | a. $\infty$ |
| ii. An object is placed at focus before a concave mirror | b. 0.5 |
| iii. An object is placed at the centre of curvature before a concave mirror | c. $\frac{1}{3}$ |
| iv. An object is placed at a distance equal to radius of curvature before a convex mirror | d. $-1$ |

(1) i. → a.; ii. → b.; iii. → d.; iv. → c.

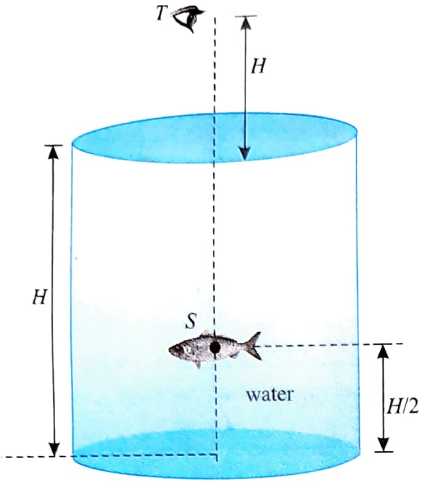
(2) i. → b.; ii. → a.; iii. → d.; iv. → c.

(3) i. → c.; ii. → a.; iii. → d.; iv. → b.

(4) i. → d.; ii. → b.; iii. → a.; iv. → c.

8. The bottom of the container is reflecting plane mirror, $S$ is a small fish and $T$ is a human eye as shown in figure. Refractive index of water is $\mu$. Fish can see two images of human eye, first due to refraction from top surface only and other due to refraction from top surface and than reflection

at bottom. Distance of these images from fish are $d_1$ and $d_2$ respectively. Similarly, human eye can also see the two images of fish, first due to refraction only and other due to, reflection and then refraction. Distance of these images from human eye are $d_3$ and $d_4$ respectively. Match the quantities of column I with their values in column II.




| Column I | Column II |
|---|---|
| i. $d_1$ | a. $H\left[1+\frac{1}{2\mu}\right]$ |
| ii. $d_2$ | b. $H\left[\mu+\frac{1}{2}\right]$ |
| iii. $d_3$ | c. $H\left[1+\frac{3}{2\mu}\right]$ |
| iv. $d_4$ | d. $H\left[\mu+\frac{3}{2}\right]$ |

(1) i. → a.; ii. → d.; iii. → b.; iv. → c.

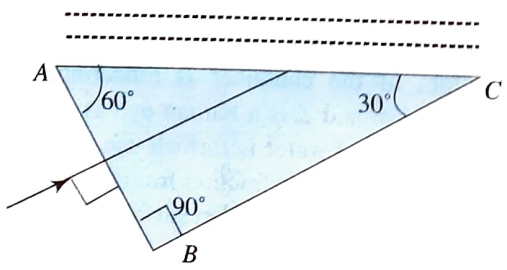
(2) i. → d.; ii. → b.; iii. → a.; iv. → c.

(3) i. → c.; ii. → a.; iii. → d.; iv. → b.

(4) i. → b.; ii. → d.; iii. → a.; iv. → c.

## Numerical Value Type

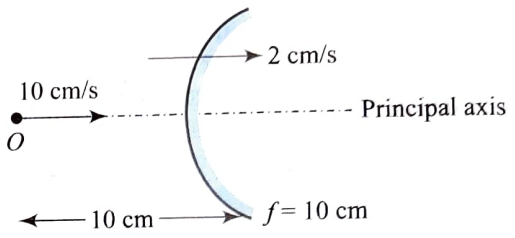
1. As shown in figure, light is incident normally on one face of the prism. A liquid of refractive index $\mu$ is placed on the horizontal face $AC$. The refractive index of prism is 3/2. If total internal reflection taken place on face $AC$, $m$ should be less than $\frac{I\sqrt{3}}{4}$, where $I$ is an integer. Find the value of $I$.



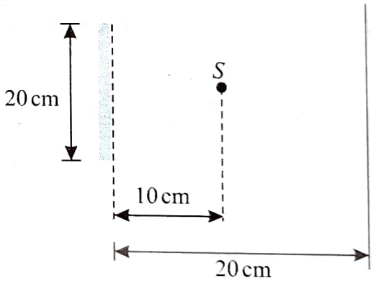

2. A ray of light is incident at an angle of 60° on one face of a prism which has refracting angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. If the refractive index of the material of the prism is $\mu = \sqrt{a}$, find the value of $a$.

3. A container of uniform cross-section has a height of 14 m. Up to what height (in metre) water of refractive index 4/3 should be filled inside the container so that container seems to be half filled for normal viewing.

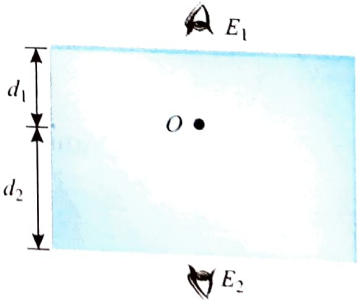
4. What is the velocity (in cm $s^{-1}$) of image in situation shown in figure ($O$ = object, $f$ = focal length)? Object moves with velocity 10 cm/s and mirror moves with velocity 2 cm $s^{-1}$ as shown.



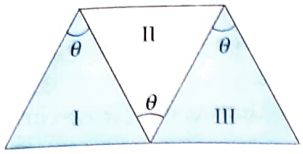

5. A point source of light $S$ is placed at a distance 10 cm in front of the centre of a mirror of width 20 cm suspended vertically on a wall. An insect walks with a speed 10 cm/s in front of the mirror along a line parallel to the mirror at a distance 20 cm from it as shown in figure. Find the maximum time (in s) during which the insect can see the image of the source $S$ in the mirror.




6. Refractive index of the glass slab is 1.5. There is a point object $O$ inside the slab as shown. To eye $E_1$ object appears at a distance of 6 cm (from the top surface) and to eye $E_2$ it appears at a distance of 8 cm (from the bottom surface). Find thickness of the glass slab (in cm).

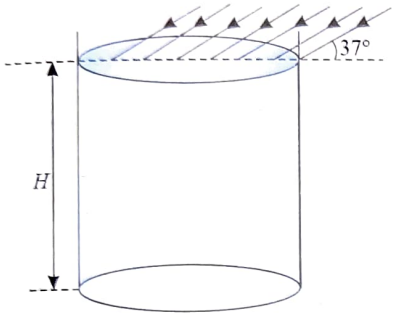



7. A direct-vision prism is made out of three prisms, each with a refracting angle of $\theta = 60°$, attached to each other as shown in figure. Light of a certain wavelength is incident on the first prism. The angle of incidence is 30° and the ray leaves the third prism parallel to the direction of incidence. The refractive index of the glass of the first and third prisms is 1.5. Find the refractive index of the material of the middle prism. $\left(\sqrt{6} = 2.45\right)$



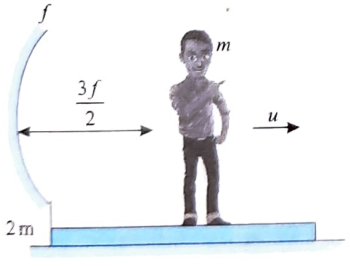

8. A point object is placed 33 cm from a convex mirror which curvature radius 40 cm. A glass plate of thickness 6 cm and index 2.0 is placed between the object and the mirror, closer to the mirror. Find the distance of final image from the object (in cm).

9. An opaque cylindrical tank with an open top has a diameter of 3.00 m and is completely filled with water. When the setting sun reaches an angle of 37° above the horizon, sunlight ceases to illuminate any part of the bottom of the tank. How deep is the tank (in m)?

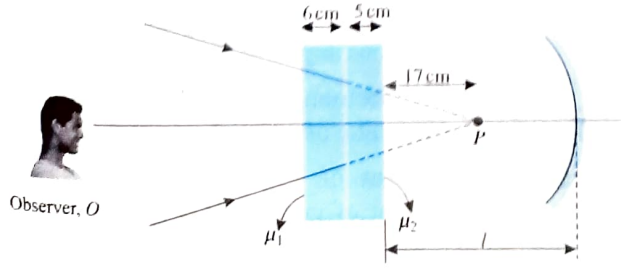



10. A prism of refractive index $\sqrt{2}$ has a refracting angle of 30°. One of the refracting surfaces of the prism is polished. For the beam of monochromatic light to retrace its path, find the angle of incidence(in degree) on the refracting surface.

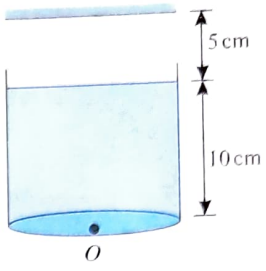
11. A man of mass $m$ starts moving w.r.t. a platform of mass 2 m with a velocity $\mu = 9/13$ m/s as shown in the figure. The platform is fitted with a concave mirror of focal length $f$. The velocity of image ($m$ m/s) at the initial moment is



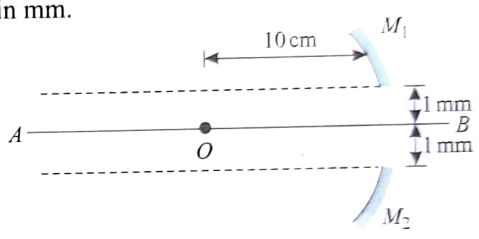

12. A converging beam is incident on a system of two glass slabs with refractive indices $\mu_1 = (3/2)$, $\mu_2 = (5/4)$ and a concave mirror placed at a distance '$l$' from the closest face of the slab 2. The observer $O$ receives the reflected rays from the system and perceives that the image is formed at $P$ itself (see figure) which is also the point of convergence of the incident beam (extrapolated). If the concave mirror's focal length is 20 cm. Find the value of $l$ in cm.




13. Consider the situation shown in figure. Water $\left(\mu_w = \frac{4}{3}\right)$ is filled in a beaker up to a height of 10 cm. A plane mirror is fixed at a height of 5 cm from the surface of water. Find the distance of image from the mirror after reflection from it of an object $O$ at the bottom of the beaker (in cm)

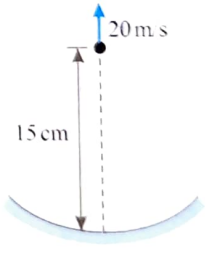



14. A concave mirror of focal length 20 cm is cut into two parts from the middle and the two parts are moved perpendicularly by a distance 1 mm from the previous principal axis $AB$. Find the distance between the images formed by the two parts in mm.

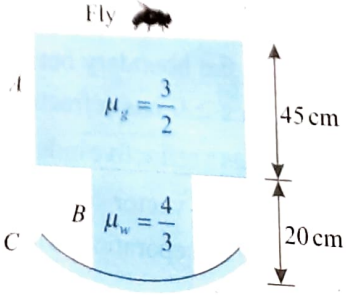



15. A balloon is rising up along the axis of a concave mirror of radius of curvature 20 m. A ball is dropped from the balloon at a height 15 m from the mirror when the balloon has velocity 20 m/s. Find the speed of the image of the ball (in m/s) formed by concave mirror after 4 second?

[Take: $g = 10$ m/s$^2$]

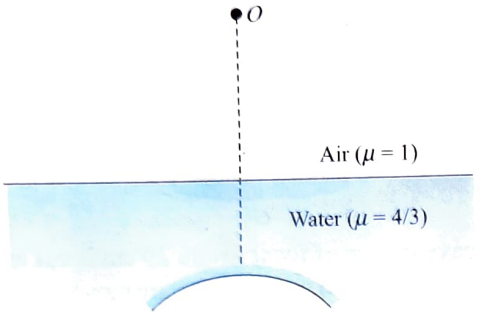



16. A fly $F$ is sitting on a glass slab $A$, 45 cm thick and of refractive index 3/2. The slab covers the top of a container $B$ containing water (Refractive index 4/3) up to a height of 20 cm. The bottom of container is closed by a concave mirror $C$ of radius of curvature 40 cm. Find the distance of the final image from the top surface of slab $A$ in cm, formed by all refractions and reflection assuming paraxial rays.



17. A slab of glass of thickness 6 cm and index 1.5 is placed somewhere in between a concave mirror and a point object, perpendicular to the mirror's optical axis. The radius of curvature of the mirror is 40 cm. If the reflected final image coincides with the object, then find the distance of the object from the mirror (in cm).

18. A point object lies 30 cm above water on the axis of a convex mirror of focal length 40 cm lying 20 cm below water surface. Consider two images:

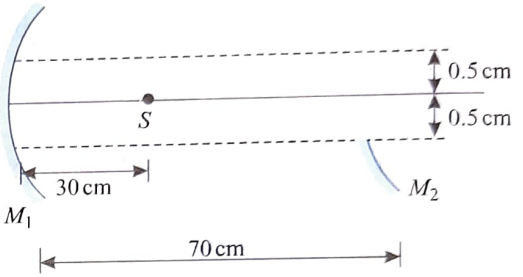



(i) Image formed by partial reflection from water surface

(ii) Image formed by refraction from water surface followed by reflection from mirror and again refraction out of the water surface.

Find the distance between these two images in cm.

19. Two spherical mirrors $M_1$ and $M_2$ are arranged as shown in the figure. The mirror $M_2$ is cut into two halves and its halves are drawn symmetrically at a distance 1 cm apart in a direction perpendicular to optical axis. A point source $S$ is placed at 30 cm from mirror $M_1$, whose focal length is 20 cm. The focal length of mirror $M_2$ is 10 cm. If we consider the image formation due to successive reflection through $M_1$ and $M_2$. Find the separation between image (in mm).

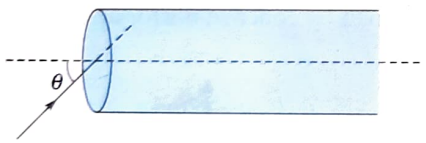



# Archives

## JEE Main

### Single Correct Answer Type

1. A transparent solid cylindrical rod has a refractive index of $2/\sqrt{3}$. It is surrounded by air. A light ray is incident at the midpoint of one end of the rod as shown in the figure.




The incident angle $\theta$ for which the light ray grazes along the wall of the rod is

(1) $\sin^{-1}\left(\frac{1}{2}\right)$ (2) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(3) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

**(AIEEE 2009)**

2. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is

(1) $\frac{1}{10}$ m/s (2) $\frac{1}{15}$ m/s

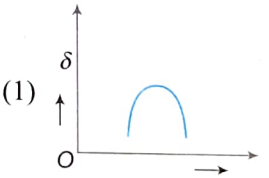
(3) 10 m/s (4) 15 m/s **(AIEEE 2011)**

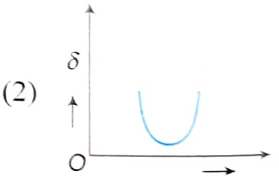
3. Let the $x$–$y$ plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is

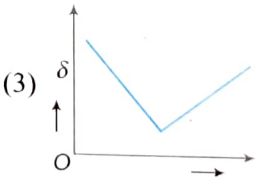
(1) 30° (2) 45°

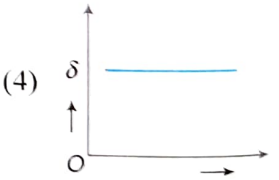
(3) 60° (4) 75° **(AIEEE 2011)**

4. The graph between angle of deviation ($\delta$) and angle of incidence ($i$) for a triangular prism is represented by

(1) 


(2) 


(3) 


(4) 


**(JEE Main 201**

5. A green light is incident from the water to the air–water interface at the critical angle ($\theta$). Select the correct statement.

(1) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium.

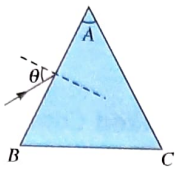
(2) The entire spectrum of visible light will come out of the water at various angles to the normal.

(3) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal.

(4) The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

**(JEE Main 201**

6. Monochromatic light is incident on a glass prism of angle $A$. If the refractive index of the material of the prism is $\mu$, a ray, incident at an angle $\theta$, on the face $AB$ would get transmitted through the face $AC$ of the prism provided:




(1) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(2) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(3) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin\left(\frac{1}{\mu}\right)\right)\right]$

(4) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin\left(\frac{1}{\mu}\right)\right)\right]$

7. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index?

(1) 1.5 (2) 1.6

(3) 1.7 (4) 1.8

**(JEE Main 2016)**

## JEE ADVANCED

### Single Correct Answer Type

1. Two beams of red and violet colors are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be

(1) 30° for both the colors

(2) greater for the violet color

(3) greater for the red color

(4) equal but not 30° for both the colors **(IIT-JEE 2008)**

2. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant when the ball is 12.8 m above the water surface, the fish sees the speed of ball as

(1) 9 m $s^{-1}$ (2) 12 m $s^{-1}$

(3) 16 m $s^{-1}$ (4) 21.33 m $s^{-1}$

**(IIT-JEE 2009)**

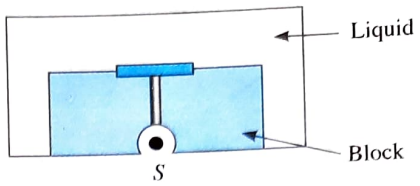
3. A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is

(1) 30° (2) 45°

(3) 60° (4) 75°

**(JEE Advanced 2013)**

4. A point source $S$ is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is

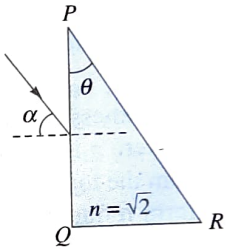



(1) 1.21 (2) 1.30

(3) 1.36 (4) 1.42

**(JEE Advanced 2014)**

5. A parallel beam of light is incident from air at an angle $\alpha$ on the side $PQ$ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face $PR$ when has a minimum value of 45°. The angle $\theta$ of the prism is

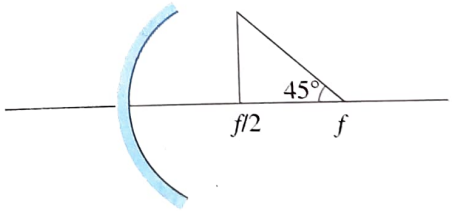


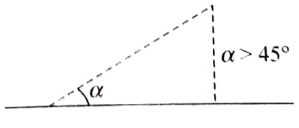

(1) 15° (2) 22.5°

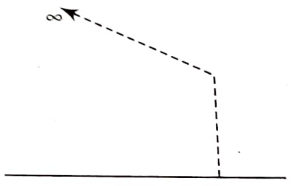
(3) 30° (4) 45°

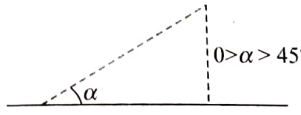
**(JEE Advanced 2016)**

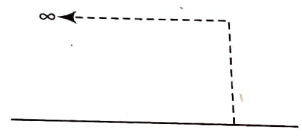
6. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length $f$, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.)?




(1) 


(2) 

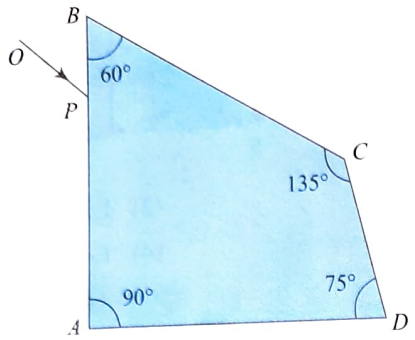
(3) 


(4) 

**(JEE Advanced 2018)**

## Multiple Correct Answers Type

1. A ray $OP$ of monochromatic light is incident on the face $AB$ of prism $ABCD$ near vertex $B$ at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct?



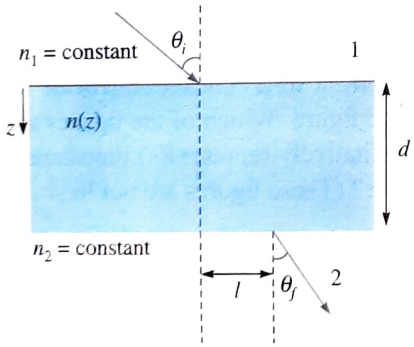

(1) The ray gets totally internally reflected at face $CD$

(2) The ray comes out through face $AD$

(3) The angle between the incident ray and the emergent ray is 90°

(4) The angle between the incident ray and the emergent ray is 120° **(IIT-JEE 2010)**

2. A transparent slab of thickness $d$ has a refractive index $n(z)$ that increases with $z$. Here $z$ is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices $n_1$ and $n_2$ ($> n_1$), as shown in the figure. A ray of light is incident with angle $\theta_i$ from medium 1 and emerges in medium 2 with refraction angle $\theta_f$ with a lateral displacement $l$. Which of the following statement(s) is (are) true?




(1) $l$ is independent of $n_2$ (2) $l$ is dependent on $n(z)$

(3) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$ (4) $n_1 \sin\theta_i = n_2 \sin\theta_f$

**(JEE Advanced 2016)**

3. For an isosceles prism of angle $A$ and refractive index $\mu$, it is found that the angle of minimum deviation $\delta_m = A$. Which of the following options is/are correct?

(1) At minimum deviation, the incident angle $i_1$ and the refracting angle $r_1$ at the first refracting surface are related by $r_1 = (i_1/2)$

(2) For this prism, the refractive index $\mu$ and the angle of prism $A$ are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$

(3) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism

(4) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is $i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$

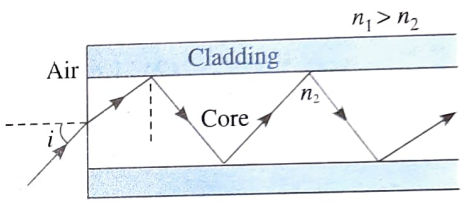
**(JEE Advanced 2017)**

## Linked Comprehension Type

### For Problems 1–2

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index $n_1$ surrounded by a medium of lower refractive index $n_2$. The light guidance in the structure takes place due to successive total internal reflections at the interface of the medium and $n_2$ as shown in the figure. All rays with the angle of incidence $i$ less than a particular value $i_m$ are confined in the medium of refractive index $n_1$. The numerical aperture (NA) of the structure is defined as $\sin i_m$.

**(JEE Advanced 2015)**




1. For two structures namely $S_1$ with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and $S_2$ with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is (are)

(1) $NA$ of $S_1$ immersed in water is the same as that of $S_2$ immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(2) $NA$ of $S_1$ immersed in liquid of refractive index $\frac{16}{\sqrt{15}}$ is that as that of $S_2$ immersed in water

(3) $NA$ of $S_1$ placed in air is the same as that of $S_2$ immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

(4) $NA$ of $S_1$ placed in air is the same as that of $S_2$ placed in water

2. If two structures of same cross-sectional area, but different numerical apertures $NA_1$ and $NA_2$ ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

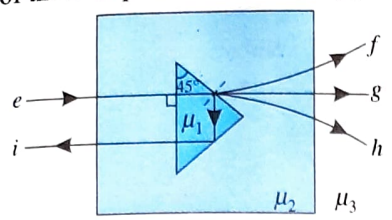
(1) $\frac{NA_1 NA_2}{NA_1 + NA_2}$ (2) $NA_1 + NA_2$

(3) $NA_1$ (4) $NA_2$

## Matrix Match Type

1. A right angled prism of refractive index $\mu_1$ is placed in a rectangular block of refractive index $\mu_2$, which is surrounded by a medium of refractive index $\mu_3$, as shown in the figure. A ray of light $e$ enters the rectangular block at normal incidence.

Depending upon the relationships between $\mu_1, \mu_2$ and $\mu_3$, it takes one of the four possible paths *ef*, *eg*, *eh*, or *ei*.




Match the paths in Column I with conditions of refractive indices in Column II and select the correct answer using the codes given below the lists:

| Column I | Column II |
|---|---|
| i. $e \to f$ | a. $\mu_1 > \sqrt{2}\,\mu_2$ |
| ii. $e \to g$ | b. $\mu_2 > \mu_1$ and $\mu_2 > \mu_3$ |
| iii. $e \to h$ | c. $\mu_1 = \mu_2$ |
| iv. $e \to i$ | d. $\mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$ |

**Codes:**

| | i. | ii. | iii. | iv. |
|---|---|---|---|---|
| (1) | b. | c. | a. | d. |
| (2) | a. | b. | d. | c. |
| (3) | d. | a. | b. | c. |
| (4) | b. | c. | d. | a. |

**(JEE Advanced 2013)**

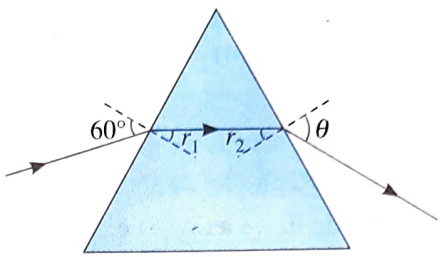
## Numerical Value Type

1. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius $R$ cm. What is the value of $R$?

**(IIT-JEE 2010)**

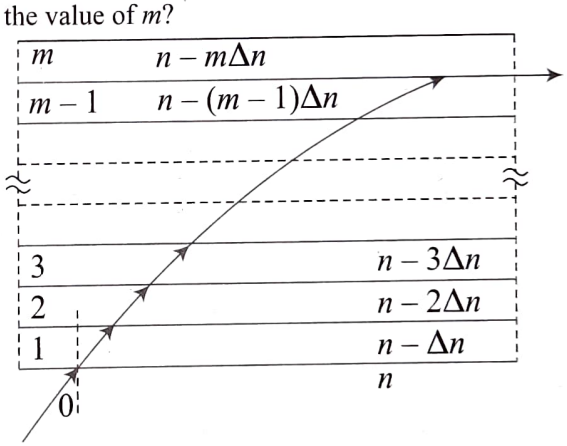
2. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from 25/3 m to 50/7 m in 30 s. What is the speed of the object in km per hour? **(IIT-JEE 2010)**

3. A monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index $n$ and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of $\theta$ is 60° and $\frac{d\theta}{dn} = m$. The value of $m$ is




**(JEE Advanced 2015)**

4. A monochromatic light is travelling in a medium of refractive index $n = 1.6$. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where $n_m$ is the refractive index of the $m^{th}$ slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and $m^{th}$ slabs from the right side of the stack. What is the value of $m$?




**(JEE Advanced 2017)**

# Answers Key

## EXERCISES

### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| 1. (3) | 2. (4) | 3. (1) | 4. (1) | 5. (2) |
| 6. (3) | 7. (1) | 8. (3) | 9. (4) | 10. (3) |
| 11. (2) | 12. (4) | 13. (2) | 14. (4) | 15. (2) |
| 16. (1) | 17. (2) | 18. (3) | 19. (2) | 20. (4) |
| 21. (1) | 22. (1) | 23. (1) | 24. (4) | 25. (4) |
| 26. (4) | 27. (1) | 28. (2) | 29. (1) | 30. (2) |
| 31. (1) | 32. (4) | 33. (1) | 34. (2) | 35. (3) |
| 36. (3) | 37. (2) | 38. (3) | 39. (2) | 40. (3) |
| 41. (1) | 42. (1) | 43. (1) | 44. (2) | 45. (3) |
| 46. (3) | 47. (2) | 48. (2) | 49. (1) | 50. (1) |
| 51. (2) | 52. (1) | 53. (4) | 54. (4) | 55. (4) |
| 56. (4) | 57. (4) | 58. (4) | 59. (2) | 60. (4) |
| 61. (3) | 62. (3) | 63. (4) | 64. (4) | 65. (1) |
| 66. (4) | 67. (1) | 68. (1) | 69. (1) | 70. (2) |
| 71. (4) | 72. (2) | 73. (1) | 74. (2) | 75. (2) |
| 76. (1) | 77. (2) | 78. (1) | 79. (3) | 80. (2) |

### Multiple Correct Answers Type

| | | |
|---|---|---|
| 1. (3),(4) | 2. (1),(3) | 3. (1),(2) |
| 4. (1),(3),(4) | 5. (1),(2),(3) | 6. (1),(3) |
| 7. (1),(2),(3),(4) | 8. (2),(3) | 9. (1),(2),(3),(4) |
| 10. (2),(3),(4) | 11. (1),(4) | 12. (2),(4) |
| 13. (1),(2),(3),(4) | 14. (2),(3) | 15. (2),(4) |
| 16. (1),(3) | 17. (2),(4) | 18. (2),(4) |
| 19. (2),(3) | 20. (1),(3) | 21. (1),(3) |

**22.** (2),(3) **23.** (1),(3) **24.** (2),(3)
**25.** (1),(2) **26.** (1),(2),(3) **27.** (1),(2),(3),(4)
**28.** (1),(2),(3),(4) **29.** (2),(3) **30.** (2),(4)
**31.** (1),(3) **32.** (1),(3) **33.** (1),(3)
**34.** (3),(4)

### Linked Comprehension Type

**1.** (2) **2.** (1) **3.** (4) **4.** (3) **5.** (3)
**6.** (1) **7.** (4) **8.** (1) **9.** (4) **10.** (2)
**11.** (4) **12.** (3) **13.** (4) **14.** (3) **15.** (4)
**16.** (2)

### Matrix Match Type

**1.** i. → a.; ii. → c.; iii. → b.; iv. → d.
**2.** i. → b.; ii. → c., d.; iii. → a.; iv. → c., d.
**3.** i. → b.; ii. → c.; iii. → a.; iv. → d.
**4.** (1) **5.** (1) **6.** (2) **7.** (2) **8.** (4)

### Numerical Value Type

**1.** (3) **2.** (3) **3.** (8) **4.** (0) **5.** (6)
**6.** (21) **7.** (1.95) **8.** (42) **9.** (4) **10.** (45)
**11.** (3) **12.** (60) **13.** (12.5) **14.** (2) **15.** (80)
**16.** (22.50) **17.** (42) **18.** (3) **19.** (5)

## ARCHIVES

### JEE Main

#### Single Correct Answer Type

**1.** (4) **2.** (2) **3.** (2) **4.** (2) **5.** (4)
**6.** (1) **7.** (1)

### JEE Advanced

#### Single Correct Answer Type

**1.** (1) **2.** (3) **3.** (1) **4.** (3) **5.** (1)
**6.** (4)

#### Multiple Correct Answers Type

**1.** (1),(2),(3) **2.** (1),(2),(4) **3.** (1),(3),(4)

#### Linked Comprehension Type

**1.** (1),(3) **2.** (4)

#### Matrix Match Type

**1.** (4)

#### Numerical Value Type

**1.** (6) **2.** (3) **3.** (2) **4.** (8)

# 2 Geometrical Optics: Part 2

## INTRODUCTION

In previous chapter we learned reflection through mirrors and refraction at plane surfaces. As we know all optical image-forming instruments work from a combination of lenses or mirrors. To understand the image formation by lens, we need to learn refraction at spherical surfaces. In this chapter we are simply going to do the applications of the principles discussed in previous chapter.

## REFRACTION AT A SPHERICAL SURFACE

A refracting surface which forms a part of a sphere of a transparent medium is called a spherical refracting surface.

Following terms are related to the study of refraction from spherical refracting surfaces:

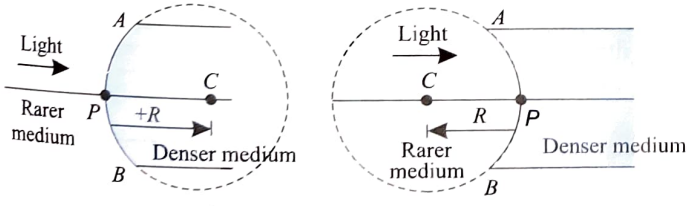
**Pole:** The centre of the spherical surface is called its pole. It is denoted by $P$.

**Centre of curvature:** The centre of the sphere, of which this refracting surface is a part, is called the centre of curvature of the refracting surface. It is denoted by $C$.

**Radius of curvature:** The radius of the sphere of which the curved surface forms a part is called its radius of curvature. It is denoted by $R$.

**Aperture:** The diameter of the spherical refracting surface is called its aperture. The line joining the points $A$ and $B$ represents the aperture of the spherical refracting surface.

**Principal axis:** The line $PC$ produced on either side of the refracting surface is called its principal axis.



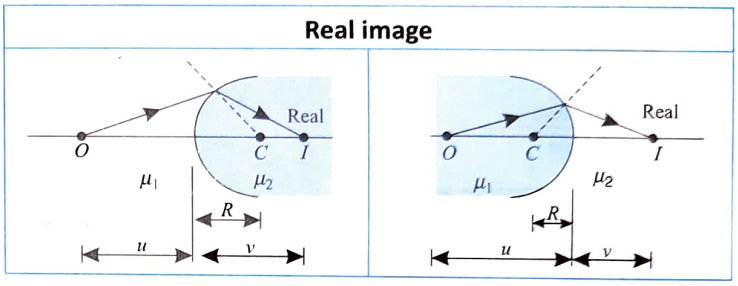

(a) Convex surface (b) Concave surface

## TYPES OF IMAGES

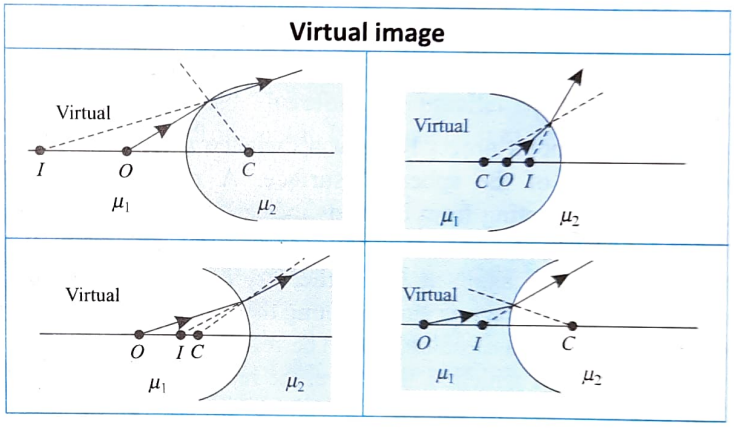
Our concern is whether the light rays, after refracting through the surface, form a real image (no observer necessary) or a virtual image (assuming that an observer intercepts the rays). The answer depends on the relative values of $\mu_1$ and $\mu_2$ and on the geometry of the situation.

Six possible results are shown in figure. In each part of the figure, the medium with the greater index of refraction is shaded, and object $O$ is always in the medium with index of refraction $\mu_1$, to the left of the refracting surface. In each part, a representative ray is shown refracting through the surface.

**Real image**



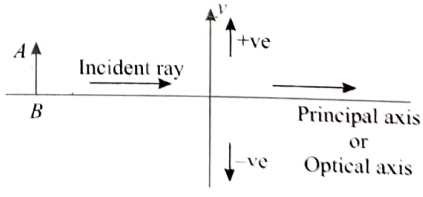

**Virtual image**




**Note:** Real images form on the side of a refracting surface that is opposite the object, and virtual images form on the same side as the object.

## SIGN CONVENTION TO BE USED

According to New Cartesian sign convention:




(1) The distances measured in the direction of incident ray are taken as positive and those in the direction opposite to incident ray are taken as negative.

(2) Distances are taken as positive when above the principal/optical axis and negative when below the axis.

(3) The distances of the objects and images are measured from the optical centre or pole which is regarded as origin of Cartesian coordinate system.

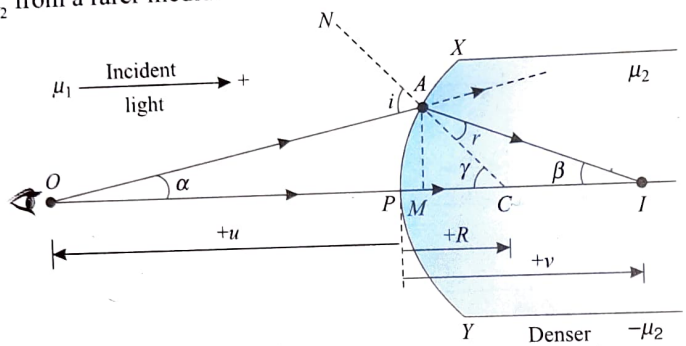
### Assumptions

To study the phenomenon of refraction at spherical surfaces, we make the following assumptions.

1. The aperture of the refracting surface is very small so that distance of any point from the surface is almost equal to the distance of that point from the pole of the surface.
2. The angles of incidence and consequently the angles of refraction are small.

## Refraction at a Spherical Surface Convex towards the Rarer Medium

Let $P$ be the pole and $C$ be the centre of curvature of a spherical refracting surface (figure). The surface is convex towards the rarer medium and separates a denser medium of refractive index $\mu_2$ from a rarer medium of refractive index $\mu_1$.




Let $O$ be a point object lying at a distance $u$ from $P$ on the principal axis of the spherical surface. A ray of light $OA$ (incident ray) starting from $O$ meets the refracting surface at $A$. After refraction, it bends towards the normal $CAN$ and meets the principal axis again at $I$. Another ray of light $OP$, falling normally on the refracting surface along the principal axis suffers no deviation and passes along $PC$. The refracted rays $PC$ and $AI$ actually meet at $I$ which is, therefore, the real image of the point object $O$.

Let $\angle OAN = i$ and $\angle CAI = r$

Applying Snell law at $A$ : $\mu_1 . \sin i = \mu_2 \sin r$

As angles $i$ and $r$ are small, $\sin i \approx i$ and $\sin r \approx r$ and as such

or $\mu_1 i = \mu_2 r$ ...(i)

In $\Delta OAC$, $i = \gamma + \alpha$ and in $\Delta IAC$, $r = \gamma - \beta$

From Eq. (i), $\mu_1(\gamma + \alpha) = \mu_2(\gamma - \beta)$

or $\mu_1\alpha + \mu_2\beta = (\mu_2 - \mu_1)\gamma$ ...(ii)

As the aperture is small, the angles $\alpha$, $\beta$ and $\gamma$ are also small and as such these can be replaced by their tangents.

Thus, $\alpha \approx \tan\alpha = \dfrac{AM}{MO} \approx \dfrac{AM}{PO}$

$\beta \approx \tan\beta = \dfrac{AM}{MI} \approx \dfrac{AM}{PI}$

and $\gamma \approx \tan\gamma = \dfrac{AM}{MC} \approx \dfrac{AM}{PC}$

From Eq. (ii), $\mu_1 \dfrac{AM}{PO} + \mu_2 \dfrac{AM}{PI} = (\mu_2 - \mu_1)\dfrac{AM}{PC}$

or $\dfrac{\mu_1}{PO} + \dfrac{\mu_2}{PI} = \dfrac{(\mu_2 - \mu_1)}{PC}$ ...(iii)

According to New Cartesian sign convention,

$PO = -u, PI = +v, PC = +R$

From Eq. (iii), $\dfrac{\mu_1}{-u} + \dfrac{\mu_2}{v} = \dfrac{(\mu_2 - \mu_1)}{R}$

or $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{(\mu_2 - \mu_1)}{R}$ ...(iv)

**Important Points:**

- Though Eq. (iv) is derived for refraction through convex surface, but above equation can be applied for refraction at concave surface .
- Above equation is applicable for real as well as virtual object.
- In above equation $\mu_1$ is the refractive index of incident medium and $\mu_2$ is the refractive index of refractive medium.
- If a refracting surface is flat, then $R$ is infinite and Eq. (iv) reduces to

$$\frac{\mu_2}{v} = \frac{\mu_1}{u} \Rightarrow \frac{v}{\mu_2} = \frac{u}{\mu_1}$$

$$v = \frac{u}{\mu_1/\mu_2}$$

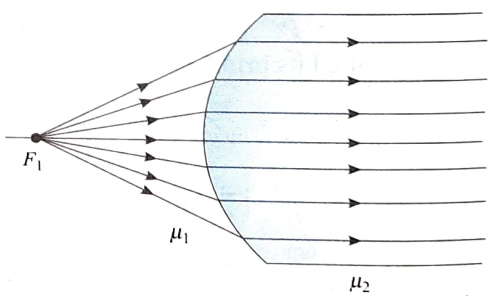
or Apparent depth $= \dfrac{\text{Real depth}}{\mu_r}$

Here $\mu_r$ = relative refractive index of incident medium with respect to refractive medium.

## Focal Length of a Single Spherical Surface

A single spherical surface has two principal focus points which are as follows:

(i) **First focus:** The first principal focus is the point on the axis where when an object is placed, the image is formed at infinity. That is when



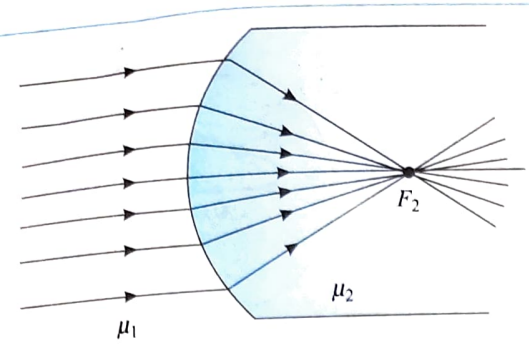

$u = f_1$, $v = \infty$, then from $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{(\mu_2 - \mu_1)}{R}$

We get $-\dfrac{\mu_1}{f_1} = \dfrac{\mu_2 - \mu_1}{R} \Rightarrow f_1 = \dfrac{-\mu_1 R}{(\mu_2 - \mu_1)}$

The first focal length ($f_1$) is also called object focal length ($f_o$).

(ii) **Second focus:** Similarly, the second principal focus is the point where parallel rays focus. That is $u_1 = -\infty$, $v_1 = f_2$

then $\dfrac{\mu_2}{f_2} = \dfrac{\mu_2 - \mu_1}{R}$ ; $f_2 = \dfrac{\mu_2 R}{(\mu_2 - \mu_1)}$




The second focal length ($f_2$) is also called image focal length ($f_1$).

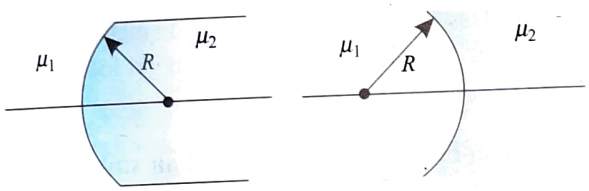
(iii) **Ratio of focal length:** $\frac{f_1}{f_2} = -\frac{\mu_1}{\mu_2}$

## Optical Power of a Curve Surface

The optical power of a refracting surface is given as

$$P = \frac{\mu_2 - \mu_1}{R}$$

The optical power of a refracting surface is the measure of the degree to which the refracting surface can be converge or diverge the rays of light passing through it.




(a) (b)

**Convex surface:**
If $\mu_2 > \mu_1$ (and $R$ is +ve ), $P$ is +ve.
If $\mu_2 < \mu_1$ (and $R$ is +ve), $P$ is –ve.

**Concave surface:**
If $\mu_2 > \mu_1$ (and $R$ is –ve), $P$ is –ve.
If $\mu_2 < \mu_1$ (and $R$ is –ve), $P$ is +ve.

The relation between optical power and object focal length $f_o$ (or $f_1$) is given as

$$P = \frac{\mu_1}{f_1} \quad \text{where } P = \frac{\mu_2 - \mu_1}{R}$$

The relation between optical power and image focal length $f_i$ (or $f_2$) is given as

$$P = \frac{\mu_2}{f_2} \quad \text{where } P = \frac{\mu_2 - \mu_1}{R}$$

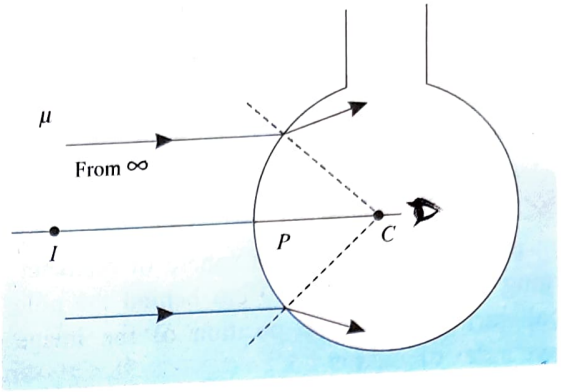
**Important Point:**

The optical power of a refracting surface depends on the refractive index of media at both of its sides and the radius of curvature of the interface. If $P$ is +ve, the surface is said to be converging and if $P$ is –ve, the surface is diverging.

### Illustration 2.1

An empty spherical flask of diameter 15 cm is placed in water of refractive index 4/3. A parallel ray of light strikes the flask. Where does it get focused, if observed from within the flask?

**Sol.** The spherical flask placed inside water and a parallel beam of light striking on flask. The observer is observing from inside the flask.




Using, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$

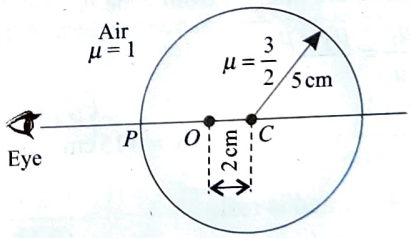
Here, $u = -\infty$, $R = +\frac{15}{2}$ cm, $\mu_1 = 1$ and $\mu_2 = \frac{4}{3}$

$$\frac{4/3}{v} - \frac{1}{(-\infty)} = \frac{(4/3 - 1)}{(+15/2)} \Rightarrow v = -\frac{45}{2}\text{cm} = -22.5\,\text{cm}$$

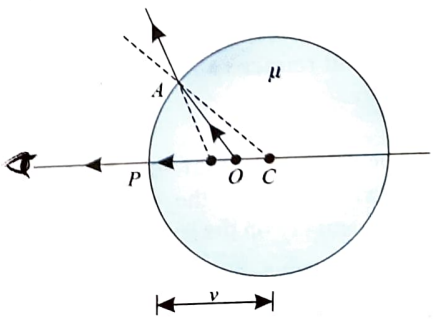
As $v$ is negative, the image will be formed on the same side as that of the object i.e. the side from which the parallel of light is incident on the flask. Therefore, the parallel beam of light will appear to come from point I (virtual image) as shown in the figure.

### Illustration 2.2

Figure shows a solid glass sphere of radius 5 cm that has a small air bubble $O$ trapped at a distance 2 cm from the centre $C$. The refractive index of the material of glass is 1.5. Find the apparent position of the bubble where it will appear, when seen through the surface of the sphere from an outside point $E$.




**Sol.** Here rays are striking on concave side of the sphere




Using, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$

Here, $u = -PO = -(PC - OC) = -(5 - 2)\text{cm} = -3\,\text{cm}$

$R = -5\,\text{cm}$; and $\mu_1 = \frac{3}{2}$ and $\mu_2 = 1$

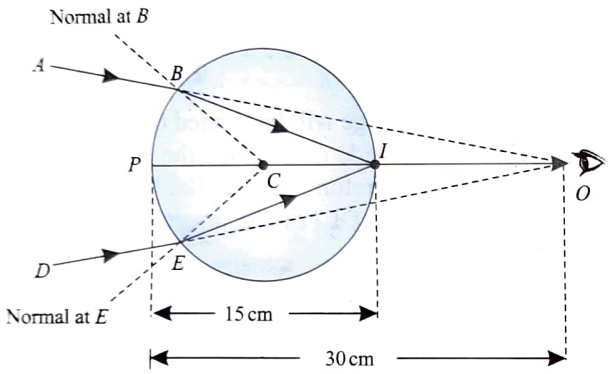
$$\frac{1}{v} - \frac{3/2}{(-3)} = \frac{(1-3/2)}{(-5)} \Rightarrow v = -\frac{5}{2}\,\text{cm} = -2.5\,\text{cm}$$

The course of rays giving the apparent position $I$ of the air bubble $O$ trapped inside the glass sphere will be as shown in the figure.

## ILLUSTRATION 2.3

A beam of light strikes a glass sphere of diameter 15 cm converging towards a point 30 cm behind the pole of the spherical surface. Find the position of the image if the refractive index of glass is 1.5.

**Sol.** Let the incident rays on the sphere from left. In the absence of the spherical surface, they converge at $O$, 30 cm from the pole $P$. Thus, $O$ serves as the virtual object for first refraction at left surface.




Here, $u = PO = +30$ cm, $R = +\frac{15}{2}$ cm, $\mu_1 = 1$ and $\mu_2 = \frac{3}{2}$

Let the distance of the image $I$ from $P$ be $v$.

Using, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

or $$\frac{3/2}{v} - \frac{1}{(+30)} = \frac{(3/2-1)}{(+15/2)} \Rightarrow v = +15\,\text{cm}$$

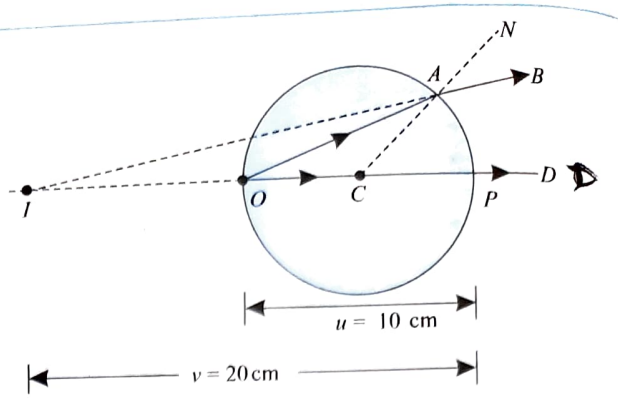
Thus, the image formed is real.

## ILLUSTRATION 2.4

A mark placed on the surface of a sphere is viewed through glass from a position directly opposite. If the diameter of the sphere is 10 cm and refractive index of glass is 1.5, find the position of the image.

**Sol.** Let $O$ be the position of the mark on the sphere on left side. It is viewed from the right side opposite to $O$. Let $OA$ be the incident ray on the surface $AP$. It is refracted along $AB$.

Similarly, another ray incident along $OP$, passes straight along $PD$. The two refracted rays $AB$ and $PD$ when produced backwards appear to come from $I$ ($AB$ and $AI$ are in the same straight line).Thus, $I$ is the virtual image of the mark $O$ (see figure).




Here, $u = PO = -10$ cm, $\mu_1 = 1{,}5$

$R = PC = -10/2 = -5$ cm, $\mu_2 = 1.0$

Let the distance of the image $I$ from $P$ be $v$.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

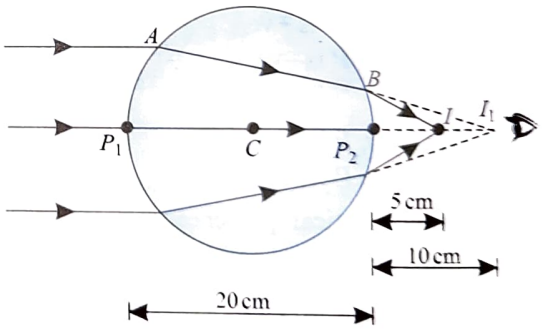
or $$\frac{1}{v} - \frac{3/2}{(-10)} = \frac{1-3/2}{-5} \Rightarrow v = -20\,\text{cm}$$

The negative sign shows that the image is virtual.

## ILLUSTRATION 2.5

A sphere of glass ($\mu = 1.5$) is of 20 cm diameter. A parallel beam enters it from one side. Where will it get focused on the other side?

**Sol.** A parallel beam of light coming in air strikes on the left surface of the sphere and it is being observed from right side of the sphere. Let $P_1$ and $P_2$ be the poles of two surfaces of glass sphere (figure).




**Refraction at surface left surface :** A parallel beam of light coming in air falls on the left surface of the sphere.

Here, $\mu_1 = 1$, $u = -\infty$, $\mu_2 = 1.5$, $R = +20/2 = +10$ cm,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu - \mu_1}{R}$$

or $$\frac{3/2}{v_1} - \frac{1}{-\infty} = \frac{3/2-1}{10} \Rightarrow v = +30\,\text{cm}$$

After refraction, it would converge to a point $I_1$. It means would be the real image of the object at infinity when refraction takes place at the surface $AP_1$.

**Refraction at right surface:** After refraction at surface $AP_1$ the rays before actually converging to a point $I_1$, meet the right surface of the sphere

Here, $u = P_2I_1 = (P_1I_1 - P_1P_2) = (30 - 20) = +10$ cm

$R = -10$ cm

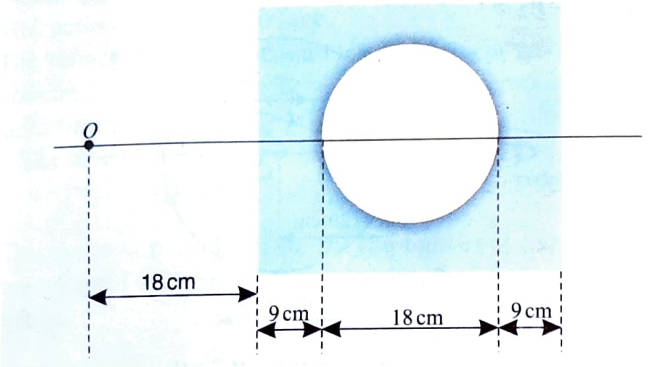
$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

or $$\frac{1}{v}-\frac{3/2}{(+10)}=\frac{1-3/2}{(-10)} \Rightarrow v=+5\,\text{cm}$$

It means finally the rays converge to the point I and I is its real image.

**ILLUSTRATION 2.6**

A cubical block of glass, refractive index 1.5, has a spherical cavity of radius $r = 9$ cm inside it as shown in figure. A luminous point object $O$ is at a distance of 18 cm from the cube (see figure). What is the apparent position of $O$ as seen from other side of cube?



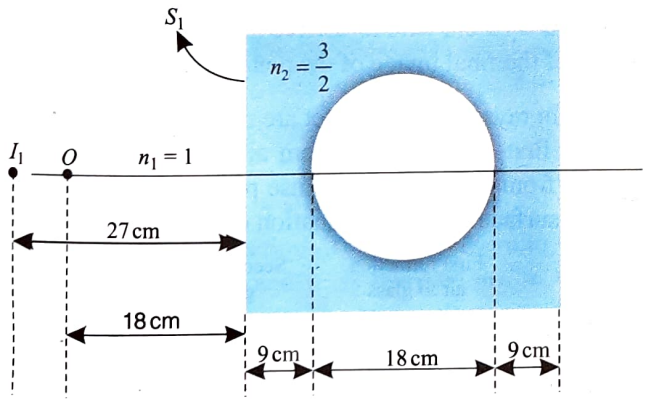

**Sol.** For refraction at first surface $S_1$:

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$$

Here $u=-18\,\text{cm}$, $\mu_2=\frac{3}{2}$ and $\mu_1=1$

$$\frac{3/2}{v_1}-\frac{1}{(-18)}=\frac{(3/2-1)}{\infty} \Rightarrow v_1=-27\,\text{cm}$$

First image lies to the left of $S_1$.



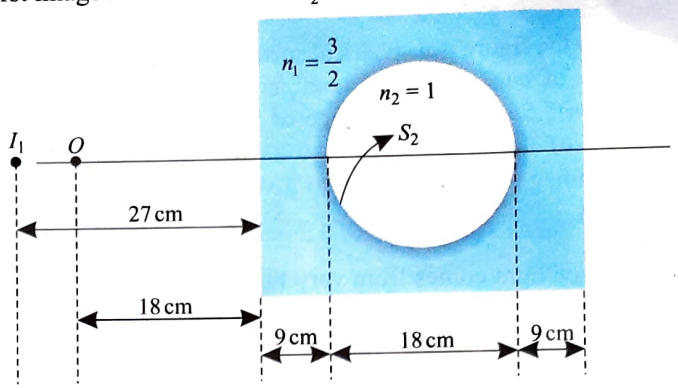

For refraction at second surface $S_2$:

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$$

Here $u_2=-(27+9)=-36\,\text{cm}$, $\mu_2=1$ and $\mu_1=\frac{3}{2}$

$$\frac{1}{v_2}-\frac{3/2}{(-36)}=\frac{(1-3/2)}{+9} \Rightarrow v_2=-\frac{72}{7}\,\text{cm}$$

First image lies to the left of $S_2$.

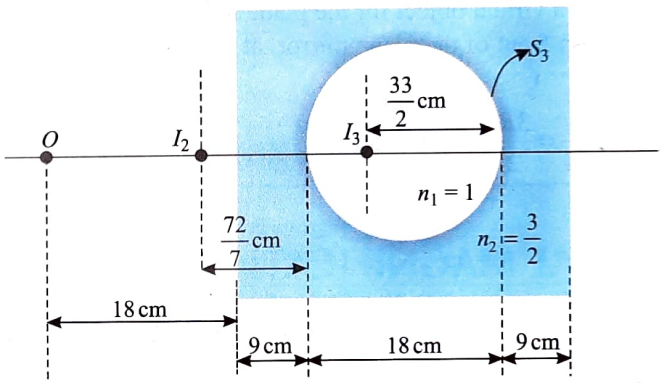



For refraction at third surface $S_3$: $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

Here $u_3=-\left(\frac{72}{7}+18\right)=-\frac{198}{7}\,\text{cm}$, $\mu_2=\frac{3}{2}$ and $\mu_1=1$

$$\frac{3/2}{v_3}-\frac{1}{(-198/7)}=\frac{(3/2-1)}{(-9)} \Rightarrow v_3=-\frac{33}{2}\,\text{cm}$$

Third image lies to the left of $S_3$.

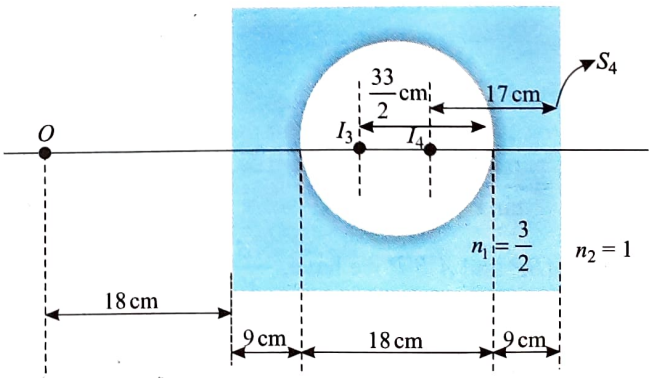



For refraction at third surface $S_4$: $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

Here $u_4=-\left(\frac{33}{2}+9\right)=-\frac{51}{2}\,\text{cm}$, $\mu_2=1$ and $\mu_1=\frac{3}{2}$

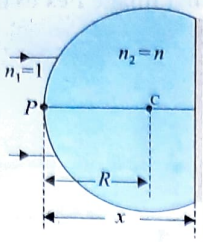
$$\frac{1}{v_4}-\frac{3/2}{(-51/2)}=\frac{(1-3/2)}{\infty}=0 \Rightarrow v_4=-17\,\text{cm}$$

Third image lies to the left of $S_4$.

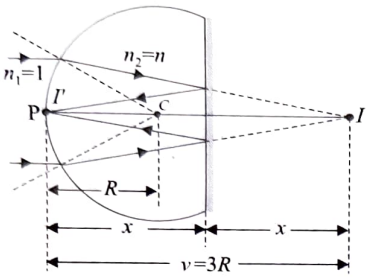


### ILLUSTRATION 2.7

Shown in the figure is a spherical surface of radius of curvature $R$ and refractive index $(=n)1.5$. Find the distance of the silvering of the plane surface so as to form an image at the pole due to a very far object.




**Sol.** Since light comes from very far object




$$u=\infty \Rightarrow \frac{n}{v}-\frac{1}{\infty}=\frac{n-1}{R} \Rightarrow v=\frac{nR}{n-1}=\frac{1.5R}{1.5-1}=3R$$

Due to presence of the plane mirror, the image found at $I$ behaves as a virtual object for the plane mirror & a real image $I'$ is formed in front of the plane mirror, at the pole $P$.

$\Rightarrow \quad x+x=v$

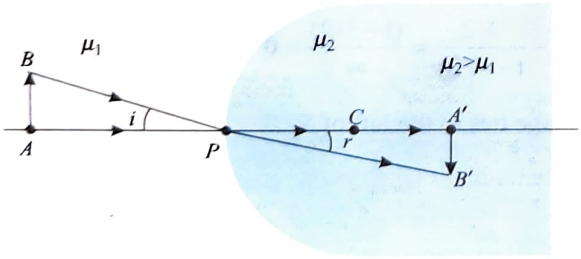
$$\Rightarrow \quad x=\frac{v}{2}=\frac{3R}{2} \Rightarrow x=1.5R$$

## LATERAL MAGNIFICATION FOR REFRACTING SPHERICAL SURFACE

Lateral magnification, $m=\dfrac{\text{Image height}}{\text{Object height}}=\dfrac{-(A'B')}{AB}$.

Now, $\mu_1 \sin i=\mu_2 \sin r \Rightarrow \dfrac{\sin i}{\sin r}=\dfrac{\mu_2}{\mu_1}$;

For small angles of incidence, $\sin i=\tan i$ and $\sin r=\tan r$




$$\frac{\sin i}{\sin r}=\frac{\tan i}{\tan r}=\frac{\mu_2}{\mu_1};$$

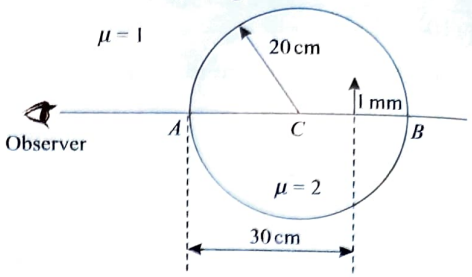
In triangle $ABP$ and $A'B'P$ we have, $\dfrac{AB/PA}{A'B'/PA'}=\dfrac{\mu_2}{\mu_1}$

$$\frac{A'B'}{AB}=\frac{-PA'/\mu_2}{PA/\mu_1}$$

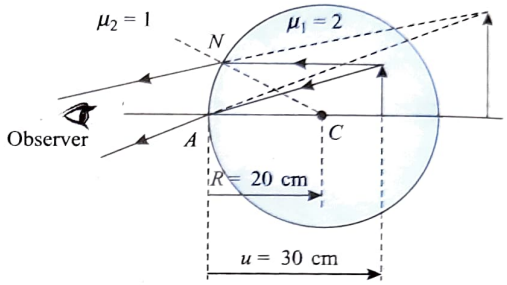
Hence, $m=\dfrac{v/\mu_2}{u/\mu_1}$

### ILLUSTRATION 2.8

Find the position, size and nature of image, for the situation shown in figure. Draw ray diagram.




**Sol.** For refraction near point $A$, $u=-30$ cm; $R=-20$ cm; $\mu_1=2$, $\mu_2=1$




Applying refraction formula $\dfrac{\mu_2}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{R}$

$$\frac{1}{v}-\frac{2}{-30}=\frac{1-2}{-20}$$

$$v=-60\,\text{cm}$$

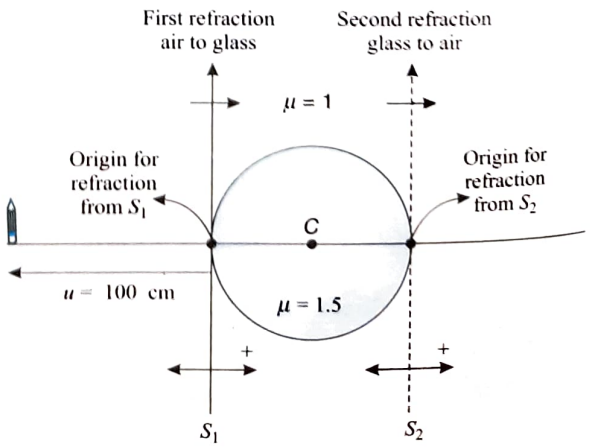
$$m=\frac{h_2}{h_1}=\frac{\mu_1 v}{\mu_2 u}=\frac{2(-60)}{1(-30)}=4$$

$\Rightarrow \quad h_2=4\,\text{mm}$

### ILLUSTRATION 2.9

A pencil of radius 1 cm is placed 1 m in front of a spherical glass ball of radius 25 cm with refractive index 1.50. Determine the position of the final image of the pencil and its magnification.

**Sol.** Light rays from the object are refracted through the glass ball twice; first at surface $S_1$, from air to glass, and second, at surface $S_2$, from glass to air. We use paraxial approximation, so that single surface refraction equation can be used.



**Refraction at first surface**

$\mu_1 = 1, \mu_2 = 3/2, u = -100$ cm, $R = +25$ cm

(The radius of curvature is positive because center of curvature is to the right.)

Substituting these values in single surface refraction equation,

$$\frac{3/2}{v} - \frac{1}{(-100)} = \frac{(3/2-1)}{(+25)}$$

On solving for $v$, we get $v = +150$ cm

The image is located 150 cm to the right of the first refracting surface. The magnification due to refraction at first surface,

$$m_1 = \frac{\mu_1 v}{\mu_2 u} = \frac{1(150)}{3/2(-100)} = -1$$

**Refraction at second surface**

For refraction at second surface, the origin of the cartesian coordinate system has to be shifted to the vertex of the second refracting surface.

The object distance for refraction at $S_2$ is

$$u' = +(150 - 50) = 100 \text{ cm}$$

This is the virtual object for $S_2$; the light rays converging to $I_1$ are refracted at $S_2$ before they can actually converge to form the image.

$$\mu_1 = \frac{3}{2}, \mu_2 = 1, u = 100 \text{ cm}, R = -25 \text{ cm}$$

$$\frac{1}{v} - \frac{3/2}{(+100)} = \frac{(1-3/2)}{(-25)} \Rightarrow v = +\frac{200}{7} \text{ cm}$$

$$m_2 = \frac{\mu_1 v}{\mu_2 u} = \frac{\left(\frac{3}{2} \times \frac{200}{7}\right)}{1 \times 100} = \frac{3}{7}$$

$$m = m_1 \times m_2 = -1 \times \frac{3}{7} = -\frac{3}{7}$$

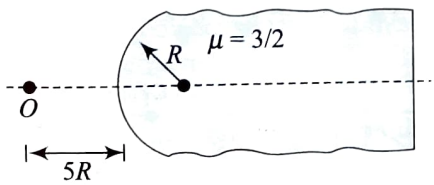
## CONCEPT APPLICATION EXERCISE 2.1

**1.** Identify whether the statements are True or False.

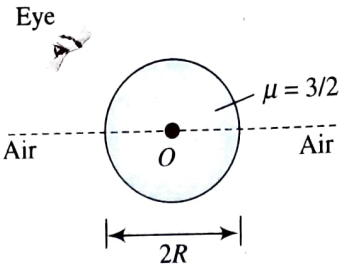
**(a)** The equation $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$ is applicable to a plane surface for $R = \infty$.

**(b)** In the above equation, $\mu_1$ is the medium in which the object is placed and $\mu_2$ is the medium in which the image is formed.

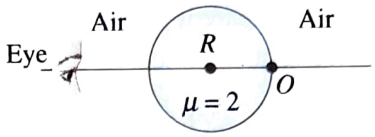
**(c)** In the given figure, the real image of object $O$ is formed at a distance $5R$ in the medium.



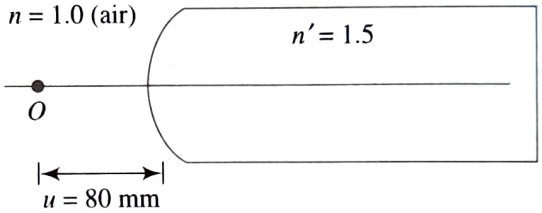

**(d)** In figure, the image of an object $O$ placed on the center face of the sphere is formed at infinity.




**(e)** In figure, the image of an object $O$ placed on the opposite face of the sphere is formed at infinity.




**2.** One end of a cylindrical glass rod shown in figure is ground to a hemispherical surface of radius $R = 20$ mm.

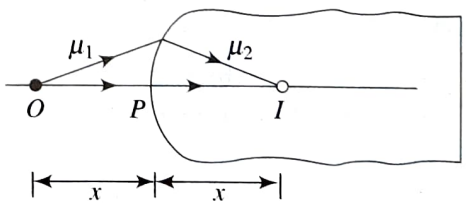



**(a)** Find the image distance of a point object on the axis of the rod, 80 mm to the left of the vertex. The rod is in air.

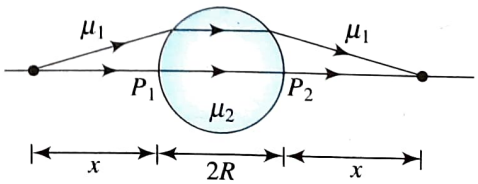
**(b)** Let the same rod be immersed in water of index 4/3, the other quantities having the same values as before. Find the image distance.

**3.** A lens is 5 cm thick and the radii of curvature of its surface are 10 cm and 25 cm, respectively. A point object is placed at a distance of 12 cm from the surface whose radius of curvature is 10 cm. How far beyond the other surface is the image formed?

**4.** A spherical surface of radius $R$ separates two media of refractive indices $\mu_1$ and $\mu_2$, as shown in figure. Where should an object be placed in medium 1 so that a real image is formed in medium 2 at the same distance?



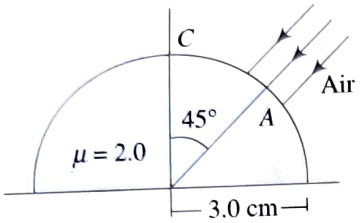

**5.** A sphere of radius $R$ made of material of refractive index $\mu_2$ is placed in a medium of refractive index $\mu_1$. Where would an object be placed so that a real image is equidistant from the sphere?



6. One end of a horizontal cylindrical glass rod ($\mu = 1.5$) of radius 5.0 cm is rounded in the shape of a hemisphere. An object 0.5 mm high is placed perpendicular to the axis of the rod at a distance of 20.0 cm from the rounded edge. Locate the image of the object.

7. There is a small air bubble inside a glass sphere ($\mu = 1.5$) of radius 10 cm. The bubble is 4.0 cm below the surface and is viewed normally from the outside. Find the apparent depth of the bubble.

8. Figure shows a transparent hemisphere of radius 3.0 cm made of a material of refractive index 2.0.



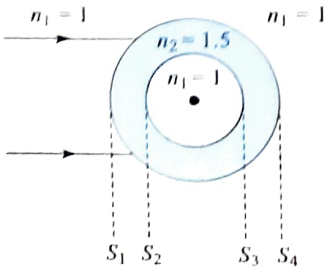

   **(a)** A narrow beam of parallel rays is incident on the hemisphere as shown in the figure. Are the rays totally reflected at the plane surface?
   **(b)** Find the image formed by refraction at the first surface.
   **(c)** Find the image formed by refraction or by reflection at the plane surface.
   **(d)** Trace qualitatively the final rays as they come out of the hemisphere.

9. A small object is embedded in a glass sphere ($\mu = 1.5$) of radius 5.0 cm at a distance 1.5 cm left to the center. Locate the image of the object as seen by an observer standing **(a)** to the left of the sphere and **(b)** to the right of the sphere.

10. A paperweight in the form of a hemisphere of radius 3 cm is used to hold down a printed page (plain surface down). An observer looks at the page vertically through the paperweight. At what height above the page will the printed letters near the center appear to the observer?

11. Solve the previous problem if the paperweight is inverted at its place so that the spherical surface touches the paper.

12. A glass sphere, refractive index 1.5 and radius 10 cm, has a spherical cavity of radius 5 cm concentric with it. A narrow beam of parallel light is directed into the sphere. Locate the final image.




## ANSWERS

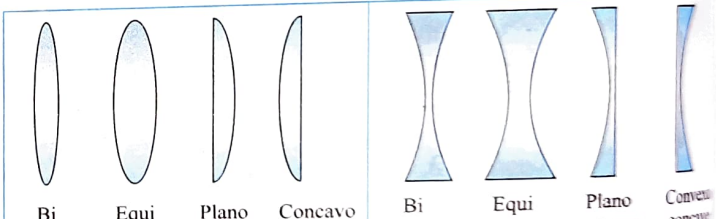
**1.** (a) True; (b) False; (c) True; (d) False; (e) True
**2.** (a) 40 mm, left of the vertex; (b) 60 mm, left of the vertex
**3.** 95 cm **4.** $\left(\frac{\mu_2+\mu_1}{\mu_2-\mu_1}\right)R$ **5.** $\left(\frac{\mu_1}{\mu_2-\mu_1}\right)R$
**6.** 30 cm **7.** $\frac{40}{13}$ cm below the surface **9.** (a) 3 cm; (b) 7.65 cm
**10.** Zero **11.** 1 cm **12.** 25 cm from $S_4$

## THIN LENS

A **lens** is a transparent medium with two refracting surfaces such that at least one is curved and refractive index of its material is different from that of the surroundings.

A thin spherical lens with refractive index greater than that of surroundings behaves as a convergent or convex lens, i.e., converges parallel rays if its central (i.e., paraxial) portion is thicker than marginal one. However, if the central portion of a lens is thinner than marginal, it diverges parallel rays and behaves as divergent or concave lens. This is how we classify and identify convergent and divergent lenses. A thin lens is called convex if it is thicker at the middle and it is called concave if it is thicker at the ends.

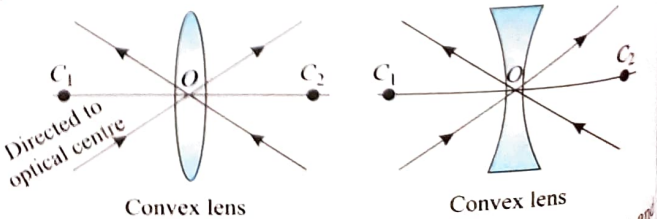
Converging and diverging lenses come in a variety of shapes as figure illustrates. Observe that converging lenses are thicker at the center than at the edges, whereas diverging lenses are thinner at the center.




**Important Points:**

- biconvex or convexo-convex, if the other surface is also convex;
- Plano-convex if the other surface is plane; and
- Concavo-convex if the other surface is concave. Similarly, a concave lens is categorized as concavo-concave or biconcave, plano-concave and convexo-concave.

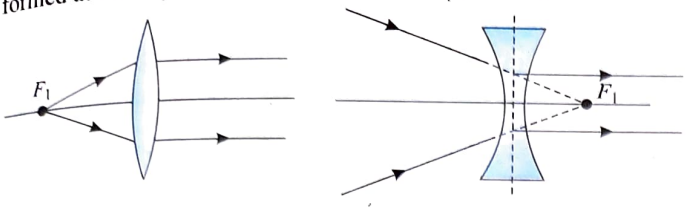
**Optical centre:** $O$ is a point for a given lens through which any ray passes undeviated.



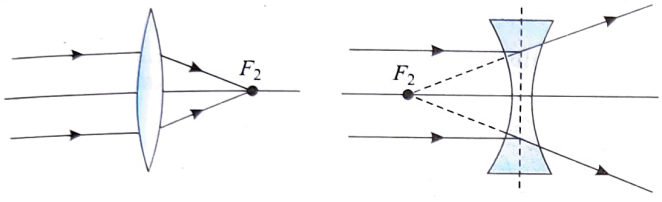

**Principal axis:** $C_1 C_2$ is a line passing through optical centre and perpendicular to the lens.

**Principal focus:** A lens has two surfaces and hence two focal points.

**First principal focus:** First principal focus is the position of the object on the principal axis of the lens, for which the image is formed at infinity. It is denoted by point $F_1$ (figure).




**Second principal focus:** Second principal focus is the position of the image on the principal axis of the lens, when the object lies at infinity. It is denoted by point $F_2$ as shown in figure.




**Note:**

- Second principal focus is the principal focus of the lens.
- When medium on two sides of lens is same then $|F_1| = |F_2|$.

**Focal length ($f$):** It is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appears to converge.

**Aperture:** In reference to a lens, aperture means the effective diameter. Intensity of image formed by a lens which depends on the light passing through the lens will depend on the square of aperture, i.e.,

Intensity $\propto$ (Aperture)$^2$

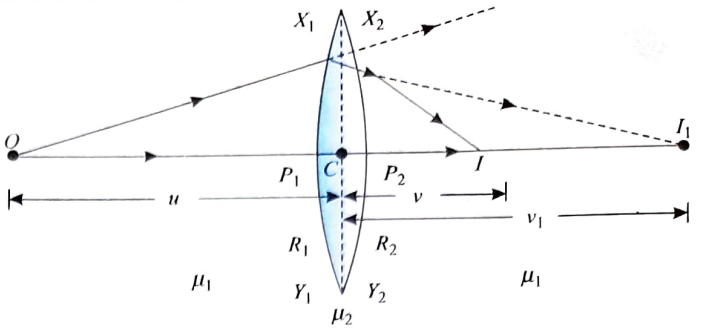
## LENS MAKER'S FORMULA

The lens maker's formula makes a relation of the focal length of a lens to the refractive index of its material and the radii of curvature of its two surfaces. This formula is used to manufacture a lens of a particular focal length from the glass of given refractive index. For this reason, it is called the **lens maker's formula.**

In derivation of the lens maker's formula, we make the following assumptions:

1. The lens is thin, so that the distances measured from the poles of two surfaces of the lens can be taken as to be the same as the distances measured from the optical centre of the lens.
2. The aperture of the lens is small.
3. The object is a point object situated on the principal axis.
4. The incident and refracted rays make small angles with the principal axis.

Suppose that $O$ is a point object placed on the principal axis of the lens made of material of absolute refractive index $\mu_2$ and placed in an optically rarer medium of absolute refractive index $\mu_1$. The radii of curvature of the two surfaces $XP_1Y$ and $XP_2Y$ of the lens are $R_1$ and $R_2$ respectively.




**Refraction at the left surface**: For this surface, $O$ is the object and $I_1$ its image.

Now using, $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R_1}$

$$\Rightarrow \quad \frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \quad (\text{as } v = v_1) \qquad \ldots(\text{i})$$

**Refraction at the right surface:** For this surface, $I_1$ is regarded as the object and $I$ is its image. Since the refraction takes place from a denser medium ($\mu_2$) to rarer medium ($\mu_1$), hence,

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2} = -\frac{\mu_2 - \mu_1}{R_2} \quad (\text{as } u = v_1) \qquad \ldots(\text{ii})$$

Adding Eqs. (i) and (ii), we get

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Or $$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Or $$\frac{1}{v} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(\text{iii})$$

where, $\dfrac{\mu_2}{\mu_1} = \mu$ (refractive index of the lens material w.r.t. the surrounding medium).

If the object is at infinity ($u = \infty$), the image is formed at the focus, i.e., $v = f$. Thus, from Eq. (iii),

$$\frac{1}{f} - \frac{1}{\infty} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Or $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(\text{iv})$$

Equation (iv) is known as lens maker's formula

Also, from Eqs. (iii) and (iv),

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \qquad \ldots(\text{v})$$

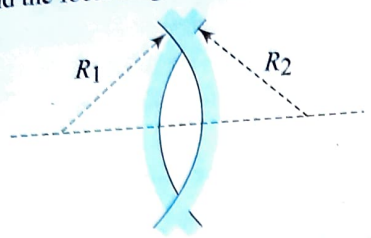
Above equation is known as lens formula.

**Important Points:**

- The lens maker's formula deduced for a convex lens is applicable to both the convex and the concave lenses.
- The focal length of a convex lens is taken as positive and that of a concave lens as negative.

**ILLUSTRATION 2.10**

There are two spherical surfaces of radii $R_1 = 30$ cm and $R_2 = 60$ cm. In how many ways these surfaces may be arranged to get different lenses. If all the lenses are made of glass ($\mu = 1.5$), find the focal length of each lens.




**Sol.** Two surfaces may be arranged in four different ways. Using lens maker's formula, we can obtain the focal length of each lens as

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(a) $\mu = 1.5$, $R_1 = +30$ cm; $R_2 = -60$ cm

$$\therefore \quad \frac{1}{f_1} = (1.5-1)\left(\frac{1}{30} - \frac{1}{-60}\right) = \frac{1}{40}$$

$$\Rightarrow \quad f_1 = 40 \text{ cm (converging)}$$

(b) $\mu = 1.5$, $R_1 = +30$ cm; $R_2 = +60$ cm

$$\therefore \quad \frac{1}{f_2} = (1.5-1)\left(\frac{1}{30} - \frac{1}{60}\right) = \frac{1}{120}$$

$$\Rightarrow \quad f_2 = +120 \text{ cm (converging)}$$

(c) $\mu = 1.5$, $R_1 = -30$ cm; $R_2 = +60$ cm

$$\therefore \quad \frac{1}{f_3} = (1.5-1)\left(\frac{1}{-30} - \frac{1}{60}\right) = \frac{-1}{40}$$

$$\Rightarrow \quad f_3 = -40 \text{ cm (diverging)}$$

(d) $\mu = 1.5$, $R_1 = -30$ cm; $R_2 = -60$ cm

$$\therefore \quad \frac{1}{f_4} = (1.5-1)\left(\frac{1}{-30} - \frac{1}{-60}\right) = \frac{-1}{120}$$

$$\Rightarrow \quad f_4 = -120 \text{ cm (diverging)}$$

**ILLUSTRATION 2.11**

A glass or glycerin convex lens of refractive index 3/2 has got a focal length equal to 50 cm. Find the focal length of the lens, if it is immersed in ethyl alcohol of refractive index 1.36.

**Sol.** According to lens maker's formula,

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = (1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots(i)$$

In ethyl alcohol, $\frac{1}{f_{\text{liquid}}} = \left(\frac{1.5}{1.36} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(ii)

Dividing (i) and (ii), we get

$$\frac{f_{\text{liquid}}}{f} = \frac{(1.5-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{\left(\frac{1.5}{1.36} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$$\Rightarrow \quad f_{\text{liquid}} = 242.75 \text{ cm}$$

## POWER OF A LENS

The **power** of a lens is the measure of its ability of producing convergence of a parallel beam of light. A convex lens of larger focal length produces a smaller converging effect and a convex lens of smaller focal length produces a larger converging effect. The power of a lens is the reciprocal of its focal length.

Mathematically, $\text{Power} = \dfrac{1}{\text{Focal length in metres}}$

That is, $P = \dfrac{1}{f}$ ...(i)

The power of a lens is measured in **dioptre (D).**

From Eq. (i), when $f = 1$ metre, $P = 1$ dioptre i.e.,

$1\text{ D} = 1/\text{m} = 1\text{ m}^{-1}$.

One dioptre is the power of a lens of focal length one metre.

**ILLUSTRATION 2.12**

A convex lens is made of glass of refractive index 1.5. If radius of curvature of each of its two surfaces is 20 cm, find the ratio of the power of the lens, when placed in air to its power, when immersed inside a liquid of refractive index 1.25.

**Sol.** Here, $R_1 = 20\text{ cm} = 0.20\text{ m}$ ; $R_2 = -20\text{ cm} = -0.2\text{ m}$

**When lens is placed in air:** If $P_1$ is power of the lens when placed in air, then

$$P_1 = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.5-1)\left(\frac{1}{0.2} - \frac{1}{-0.2}\right)$$

$$= 0.5(5+5) = 5\text{D}$$

**When lens is placed inside the liquid:** Let $\mu'$ be the refractive index of the material of the lens w.r.t. the liquid. Then, power of the lens, when placed inside the liquid is given by

$$P_2 = (\mu' - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $\mu' = \dfrac{\mu_{\text{glass}}}{\mu_{\text{liquid}}} = \dfrac{1.5}{1.25} = 1.2$

$$\therefore \quad P_2 = (1.2-1)\left(\frac{1}{0.2} - \frac{1}{-0.2}\right) = 0.2(5+5) = 2\text{D}$$

Hence, $\dfrac{P_1}{P_2} = \dfrac{5}{2} = 2.5$

## RULES FOR IMAGE FORMATION

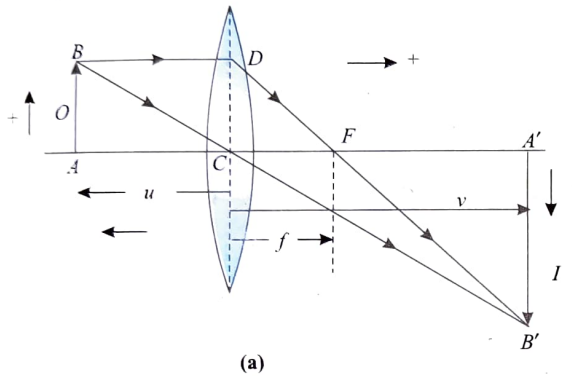
- A ray passing through optical centre proceeds undeviated through the lens.
- A ray passing through first focus or directed towards it, after refraction from the lens, becomes parallel to the principal axis.
- A ray passing parallel to the principal axis after refraction through the lens passes or appears to pass through $F_2$.

## LINEAR MAGNIFICATION PRODUCED BY A LENS

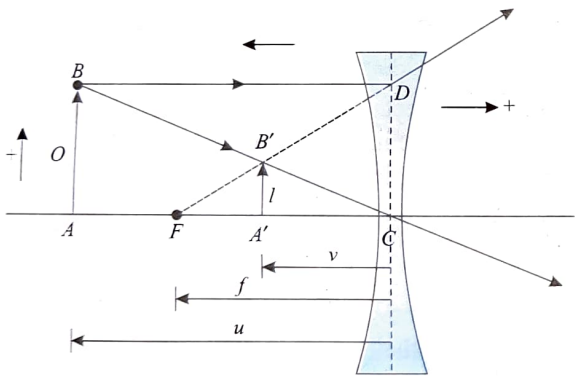
**Linear magnification** ($m$) produced by a lens is defined as the ratio of the size of the image to the size of the object.

That is, $m = \dfrac{I}{O}$ ...(i)

Refer to Figs. (a) and (b). From similar $\Delta ABC$ and $\Delta A'B'C$




(a)




(b)

$$\frac{A'B'}{AB} = \frac{CA'}{CA}$$

In Fig. (a), applying New Cartesian sign convention,

$$\frac{-I}{O} = \frac{v}{-u} \quad \text{or} \quad \frac{I}{O} = \frac{v}{u} \quad \text{or} \quad m = \frac{v}{u}$$

$$\Rightarrow \quad m = \frac{\text{height of image}}{\text{height of object}} = \frac{I}{O} = \frac{v}{u}$$

Also, from Fig. (b), $\dfrac{I}{O} = \dfrac{-v}{-u} = \dfrac{v}{u}$

It can be easily shown that $m = \dfrac{f - v}{f} = \dfrac{f}{f + u}$

Thus, $m = \dfrac{v}{u} = \dfrac{I}{O} = \dfrac{f - v}{f} = \dfrac{f}{f + u}$

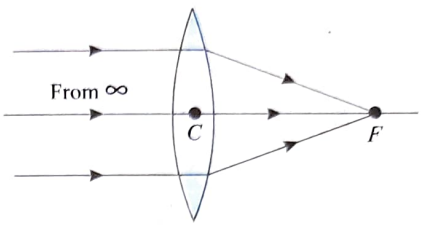
**Important Point:**

For real image, magnification is negative and for virtual image, magnification is positive.

## IMAGE FORMED BY A CONVEX LENS

(i) **When object lies at infinity:**

If an object lies at infinity (figure). Then, $u = \infty$.




From the lens formula, we have

$$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}$$

or $v = f$ (at focus)

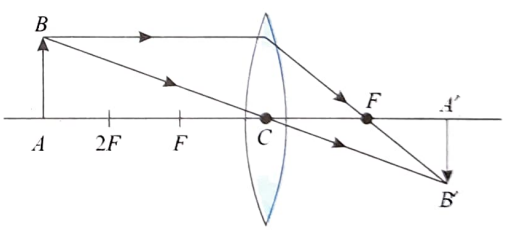
The focal length of the convex lens is positive, $v$ is also positive. Therefore, real image will be formed at focus on the other side of the lens.

Now, magnification, $m = \dfrac{v}{u} = \dfrac{v}{\infty} = 0$ (point in size)

Therefore, when object lies at infinity, image will be formed at $F$ and it will be real, inverted and very small.

(ii) **When object lies beyond $2F$($|u| > 2f$):**

If an object $AB$ lies beyond $2F$ as shown in figure. Then distance of the object from the optical centre of the lens is negative.




Now, $\dfrac{1}{v} - \dfrac{1}{-u} = \dfrac{1}{f}$ or $v = \dfrac{uf}{u + f}$

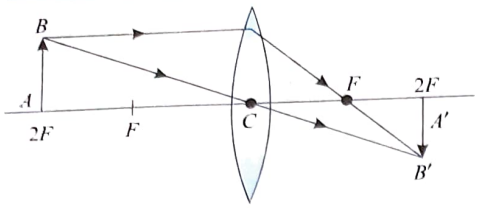
Since $|u| > 2f$, it follows that $v$ will be positive and will have a value between $f$ and $2f$ (image between $F$ and $2F$ on the other side of lens). Since $m = v/u$, the value of $m$ will be negative and $|m| < 1$ (real image and smaller in size). Therefore, when object lies beyond $2F$, the image will be formed between $F$ and $2F$ and will be real, inverted and smaller in size.

(iii) **When object lies at $2F$:**

In this case, $u = -2f$. From the lens equation, we have

$$\frac{1}{v} - \frac{1}{-2f} = \frac{1}{f} \Rightarrow v = 2f$$

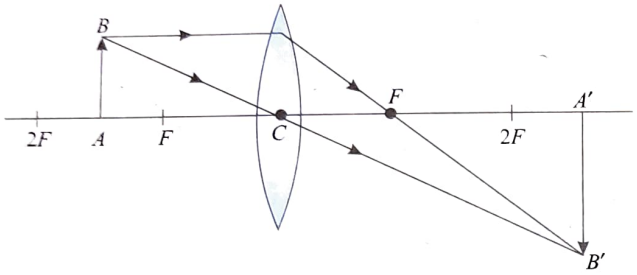
We obtain $v = 2f$ (image at $2F$)




Since $m = v/u$, the value of $m$ will be equal to –1 (real image and equal in size). Therefore, when object lies at $2F$, the image will be formed at $2F$ and will be real, inverted and equal in size (figure).

**(iv) When object lies between $F$ and $2F$**

When object lies between $F$ and $2F$ (figure), $f < |u| < 2f$. From the lens equation, we have $v = \dfrac{uf}{u+f}$

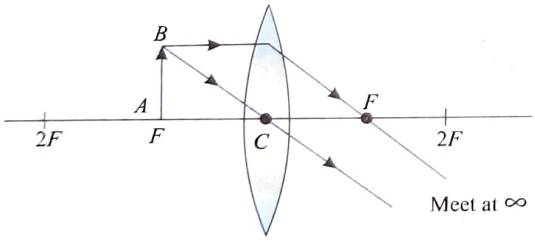



As $f < |u| < 2f$ it means $v$ will be positive and will have a value greater then $2f$ (image beyond $2F$ on the other side of the lens). Since $m = v/u$, $m$ will be negative and $|m| > 1$ (real image and larger in size). Therefore, when object lies between $F$ and $2F$, the image will be formed beyond $2F$ and will be real, inverted and larger in size.

**(v) When object lies at $F$:**

When object lies at $F$, $u = -f$. From the lens equation, we have

$$\frac{1}{v} - \frac{1}{-f} = \frac{1}{f} \text{ or } \frac{1}{v} = 0 \text{ or } v = \infty \qquad \text{(image at infinity)}$$



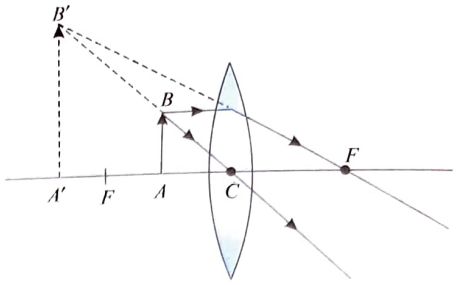

Now, $m = \dfrac{v}{u} = \dfrac{\infty}{-f} = -\infty$ (real image and highly magnified)

Therefore, when object lies at $F$, the image will be formed at infinity and will be real, inverted and highly magnified (figure).

**(vi) When object lies between $F$ and $C$:**

For object lying between $F$ and $C$ (figure), $|u| < f$. From the lens equation, we have $v = \dfrac{uf}{u+f}$

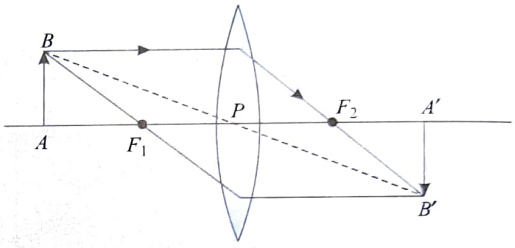
Since $|u| < f$, $v$ will be negative and $|v| > f$. From $m = v/u$, it follows that $m$ will be positive and greater than 1 (virtual and magnified image).




Therefore, when object lies between $F$ and $C$, image will be formed beyond $F$ on the same side of the lens. Further, the image will be virtual, erect and magnified.

**Important Point:**

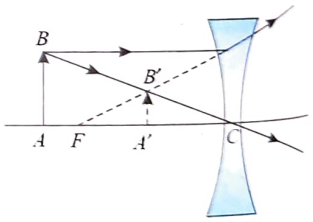
Point $B$, its image $B'$, and the pole $P$ of the lens are collinear. It is due to parallel slab nature of the lens at the middle. This ray goes straight.




## IMAGE FORMED BY A CONCAVE LENS

Consider that an object $AB$ is placed on the principal axis of a concave lens as shown in figure. For a concave lens, both $u$ and $f$ are negative. From the lens equation, we have

$$v = \frac{uf}{u+f}$$

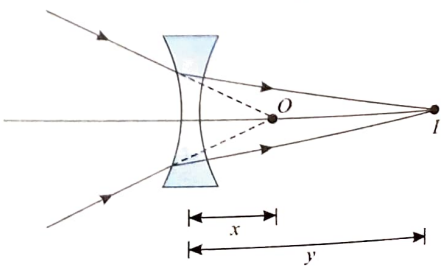



Since both $u$ and $f$ are negative, it follows that $v$ will be negative and $|v| < |f|$ (image on same side of the lens and between $F$ and $C$). From $m = v/u$, it follows that $m$ will be positive and less than 1 (virtual image and smaller in size).

It may be pointed out that irrespective of the position of the object, a concave lens always produces a virtual and erect image on the same side of the lens, which is smaller in size.

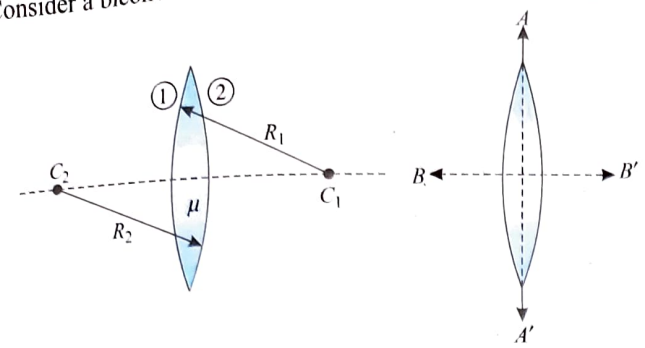
The following conclusions may be obtained for a concave lens:

1. For a real object placed in front of a concave lens, a virtual, erect, and diminished image is formed within focus.
2. A concave lens can form a real image if the object is virtual



## CUTTING THE LENS

Consider a biconvex lens of focal length $f$ as shown in figure.




We can relate its focal length with its radius of curvature as

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

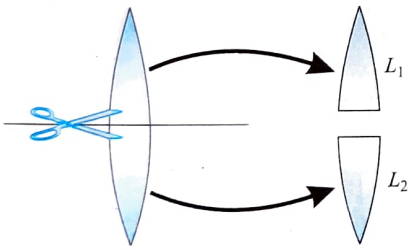
In this case $R_1=+R$ and $R_2=-R$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{+R}-\frac{1}{(-R)}\right)=\frac{2(\mu-1)}{R}\Rightarrow f=\frac{R}{2(\mu-1)}$$

Now let us discuss, how will the lens behave if it is cut in half along axis $AA'$ or along axis $BB'$? Will it behave as a lens? If yes, what is its new focal length?

**Case 1: Lens cut along the principal axis(along $BB'$)**

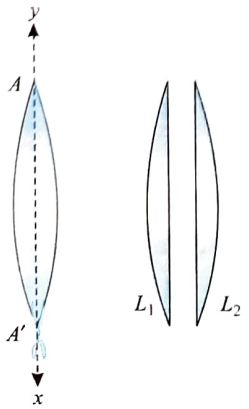
Consider a lens as shown in figure. We cut the lens along the axis $BB'$. Now, we have two lenses, both of which are shown in figure. Will the focal length of these two lenses change?




The answer is no as the radii of curvature of the two surfaces of the lens still remain the same. The only difference between this and a normal lens will be the intensity of the image. If the lens is cut at the center, we can say that the image intensity will reduce by 50%.

**Case 2: Lens cut perpendicular to the principal axis (along $AA'$)**

Now let us cut the lens along the perpendicular bisector $AA'$. We once again have two lenses as shown in figure.




What will be the focal length of the half lenses?

For part $L_1$: $R_1=+R$ and $R_2=\infty$

$$\frac{1}{f_{L_1}}=(\mu-1)\left(\frac{1}{+R}-\frac{1}{(\infty)}\right)=\frac{(\mu-1)}{R}\Rightarrow f_{L_1}=\frac{R}{(\mu-1)}=2f_0$$

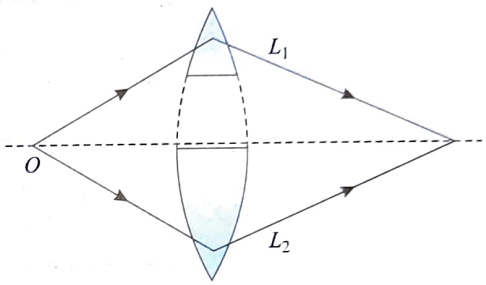
For part $L_2$: $R_1=\infty$ and $R_2=-R$

$$\frac{1}{f_{L_2}}=(\mu-1)\left(\frac{1}{(\infty)}-\frac{1}{(-R)}\right)=\frac{(\mu-1)}{R}\Rightarrow f_{L_2}=\frac{R}{(\mu-1)}=2f_0$$

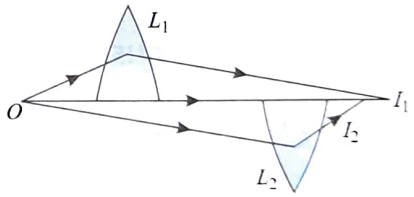
Therefore, the focal length is doubled.

**Important Points:**

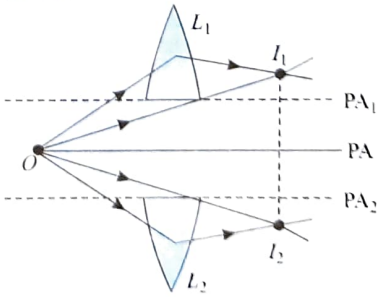
- If we cut and remove a portion of a lens, the principal axis (PA) is not disturbed. Hence the remaining two lenses behave as the original lens and form same image of the object for paraxial beam.




- If we cut the lens along (or parallel to) PA and shift them along the PA, their PA do not change. Then, generally two image will be formed on the PA for a single point object $O$ placed on the PA.




- If we shift the cut lenses $L_1$ and $L_2$ perpendicular to the original PA, their principal axes will be shifted parallel to the original PA. Then $I_1$ and $I_2$ will be formed but not on the original PA.




## GRAPHICAL METHOD FOR DETERMINING FOCAL LENGTH OF A CONVEX LENS

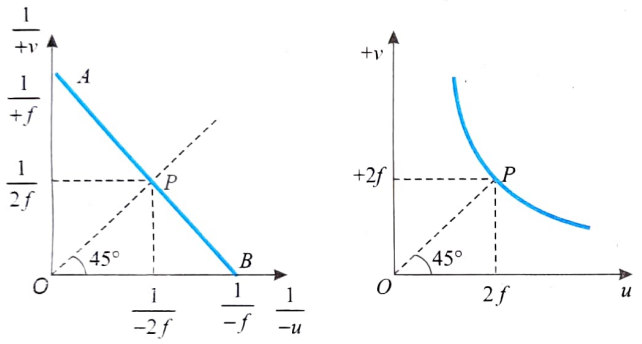
A convex lens forms real and inverted images of an object placed between infinity and focus. From lens equation, using Cartesian sign convention, we have

$u = -x,\ v = +y,\ f = +f$

$$\frac{1}{+y} - \frac{1}{-x} = \frac{1}{+f} \quad \Rightarrow \quad y - x = \text{constant}$$

If a graph is plotted between $\frac{1}{+v}$ and $\frac{1}{-u}$, it represents a straight line with slope equal to –1, as shown in Fig. (a). The intercept with the horizontal axis is $\frac{1}{-f}$ and that with the vertical axis is $\frac{1}{+f}$. If a line $OP$ is drawn at angle 45°, it intersects line $AB$ at point $P$. The coordinates of point $P$ are $\left(\frac{1}{-2f}, \frac{1}{2f}\right)$.

Alternatively, a graph between $v$ and $-u$ can be plotted as shown in Fig. (b). A line drawn at angle 45° from the origin intersects the curve at point $P$ whose coordinates are $(-2f, +2f)$. By measuring the coordinates of this point, the focal length of the lens can also be measured.

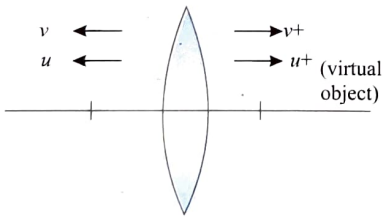



## u-v DIAGRAM FOR CONVEX LENS

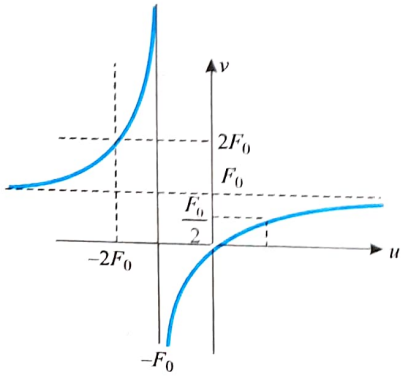
Convex Lens: $v = \frac{uf}{u+f}$,

$f$ is positive let $f = F_0$

$$v = \frac{uF_0}{u+F_0}$$




| $u$ | $-\infty$ | $-2F_0$ | $-F_0$ | 0 | $F_0$ | $\infty$ |
|---|---|---|---|---|---|---|
| $v$ | $F_0$ | $2F_0$ | $\pm\infty$ | 0 | $\frac{F_0}{2}$ | $F_0$ |

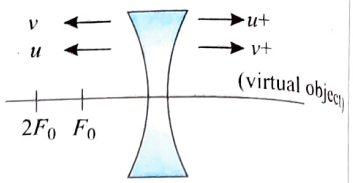



## u-v DIAGRAM FOR CONCAVE LENS

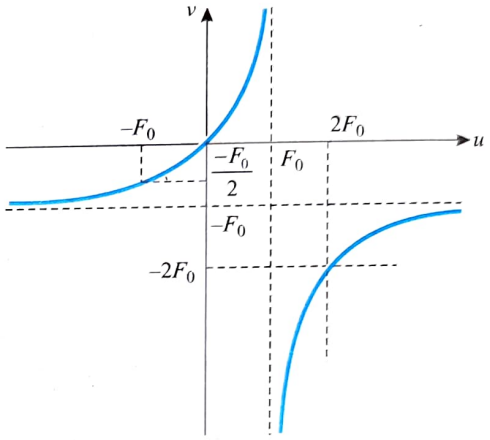
Concave lens: $v = \frac{uf}{u+f}$

$f$ is negative, let $f = -F_0$

$$v = \frac{-uF_0}{u-F_0}$$

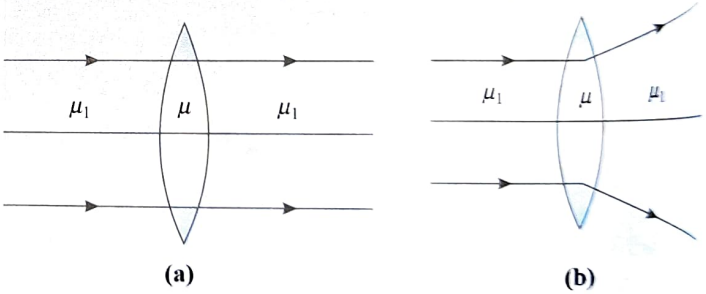



| $u$ | $-\infty$ | $-F_0$ | 0 | $F_0$ | $2F_0$ |
|---|---|---|---|---|---|
| $v$ | $F_0$ | $\frac{-F_0}{2}$ | 0 | $\pm\infty$ | $-2F_0$ |




### ILLUSTRATION 2.13

What is the relation between refractive indices $\mu, \mu_1, \mu_2$, if the behavior of light rays is as shown in Figs. (a) and (b)?




(a) (b)

**Sol.** In Fig. (a): The focal length of the lens is given by

$$\frac{1}{f} = \left(\frac{\mu}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

As the rays pass through the lens undeviated, $f = \infty$

$$\therefore \quad \left(\frac{\mu}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{\infty} = 0$$

Since for a double convex lens, $\frac{1}{R_1} - \frac{1}{R_2}$ is positive, it follows that

$$\frac{\mu}{\mu_1} - 1 = 0 \quad \text{or} \quad \frac{\mu}{\mu_1} = 1$$

In Fig. (b): The focal length of the lens is given by

$$\frac{1}{f} = \left(\frac{\mu}{\mu_2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

As the lens behaves as a diverging lens, $f$ is negative.

Since the factor $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ is positive, it follows that

$\frac{\mu}{\mu_2}-1$ is negative i.e., $\frac{\mu}{\mu_2}-1<0$

$\frac{\mu}{\mu_2}<1$ or $\mu<\mu_2$

## ILLUSTRATION 2.14

Double convex lenses are to be manufactured from a glass of refractive index 1.5, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length of the lens is to be 20 cm?

**Sol.** Let $R$ be the numerical value of the radius of curvature of each surface. For the first surface, it has a positive sign and for the second surface, a negative sign, i.e.,

$R_1=+R$ and $R_2=-R$

Substituting these values in the relation

$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$, we get

$\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{-R}\right)=(\mu-1)\left(\frac{1}{R}+\frac{1}{R}\right)$

or $\frac{1}{f}=(\mu-1)\frac{2}{R}$

Here, $f=+20$ cm (convex lens), $\mu=1.5$

Thus, $\frac{1}{20}=(1.5-1)\times\frac{2}{R}=\frac{1}{R}$

or $R=20$ cm

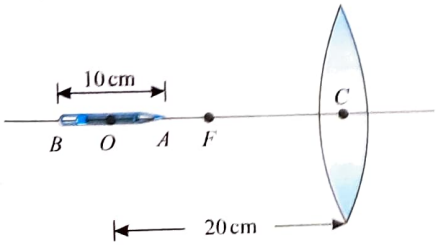
So, the radius of curvature should be 20 cm for each face of lens.

## ILLUSTRATION 2.15

A pencil 10 cm long is placed along the axis of a convex lens of focal length 10 cm, such that the middle point of the pencil is at a distance of 20 cm from the lens. Find the length of the image of the pencil.

**Sol.** Here, the focal length of the lens, $f=+10$ cm

Suppose that the pencil $AB$ of length 10 cm is placed along the axis of the lens, so that the distance of the centre of the pencil from the lens i.e., $CO=20$ cm.




For the end of the pencil $A$:

$u=-CA=-(CO-OA)=-(20-5)=-15$ cm

Let $v_1$ be the distance, at which the image of the end $A$ is formed. Then,

$\frac{1}{v_1}=\frac{1}{f}+\frac{1}{u}=\frac{1}{+10}+\frac{1}{-15}$

or $v_1=30$ cm

For the end of the pencil $B$:

$u=-CB=-(CO+OB)=-(20+5)=-25$ cm

Let $v_2$ be the distance at which the image of the end B is formed. Then,

$\frac{1}{v_2}=\frac{1}{f}+\frac{1}{u}=\frac{1}{+10}+\frac{1}{-25}$

or $v_2=16.67$ cm

Therefore, length of the image of the pencil,

$v_1-v_2=30-16.67=13.33$ cm

## ILLUSTRATION 2.16

The image obtained with a convex lens is erect arid. Its length is four times the length of the object. If the focal length of the lens is 20 cm, calculate the object and image distance.

**Sol.** As the image formed is erect and hence virtual, the magnification produced by the lens is positive i.e., $m=+4$

Also, $f=+20$ cm

Now, $m=\frac{f}{u+f}$

$\therefore \quad 4=\frac{20}{u+20}$ or $u=-15$ cm

Again, $m=\frac{f-v}{f}$

$\therefore \quad 4=\frac{20-v}{20}$

or $v=20-80=-60$ cm

## ILLUSTRATION 2.17

An object is placed at a distance of 1.5 m from a screen and a convex lens is interposed between them. The magnification produced is 4. What is the focal length of the lens?

**Sol.** As the image of the object is formed by the convex lens on the screen, the image formed is real and inverted i.e., $m=-4$.

Suppose that the object is placed at a distance $x$ from the convex lens. As the distance between the object and the screen is 1.5 m, the distance of the image from the lens is $(1.5-x)$. According to the new cartesian sign conventions, $u=-x$; $v=+(1.5-x)$

Now, $m=\frac{v}{u}$

$\therefore \quad -4=\frac{(1.5-x)}{-x}$

or $4x=1.5-x$ or $x=0.3$ m

$\therefore \quad u=-0.3$ m and $v=1.5-0.3=1.2$ m

If $f$ is the focal length of the lens, then

$\frac{1}{f}=-\frac{1}{u}+\frac{1}{v}=-\frac{1}{-0.3}+\frac{1}{1.2}=\frac{5}{1.2}$ or $f=0.24$ m

## ILLUSTRATION 2.18

A beam of light converges to a point $P$. A lens is placed in the path of the convergent beam 12 cm from $P$. At what point does the beam converge if the lens is **(a)** a convex lens of focal length 20 cm **(b)** a concave lens of focal length 16 cm?

**Sol.** The point $P$ acts as a virtual object for the lens.

**(a)** Here, $u = +12$ cm, $f = +20$ cm (convex lens)

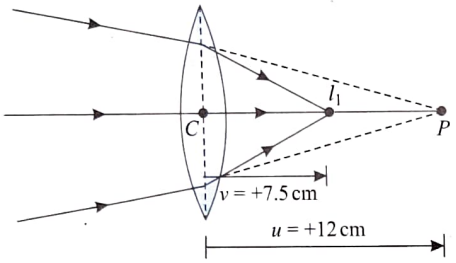
From $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

or $\frac{1}{+20} = \frac{1}{v} - \frac{1}{12}$

or $\frac{1}{v} = \frac{1}{20} + \frac{1}{12} = \frac{2}{15}$

or $v = 7.5$ cm

Since $v$ is positive, the image $I_1$ is real and is at a distance of 7.5 cm from the lens and is on the same side as the point $P$ as shown in Fig. (a).




**(a)**

**(b)** Here, $u = +12$ cm, $f = -6$ cm (concave lens)

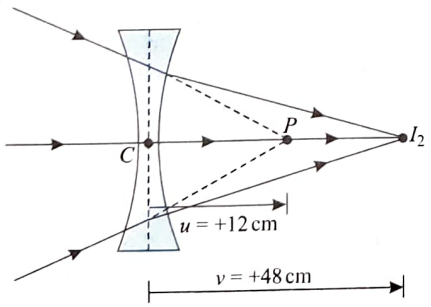
From $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{-16} = \frac{1}{v} - \frac{1}{12}$

or $\frac{1}{v} = -\frac{1}{16} + \frac{1}{12} = \frac{1}{48}$

or $v = 48$ cm

Since $v$ is positive, the image $I_2$ is real and is at a distance of 48 cm from the lens and is on the same side as the point $P$ as shown in Fig. (b).




**(b)**

## ILLUSTRATION 2.19

An object of size 3.0 cm is placed 14 cm in front of a concave lens of focal length 21 cm. Describe the image produced by the lens. What happens if the object is moved farther from the lens?

**Sol.** With usual notation and applying New Cartesian sign convention,

$O = 3.0$ cm, $u = -14$ cm, $f = -21$ cm (concave lens)

As $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{-21} = \frac{1}{v} - \frac{1}{-14} = \frac{1}{v} + \frac{1}{14}$

or $\frac{1}{v} = -\frac{1}{21} - \frac{1}{14} = -\frac{5}{42}$

or $v = -\frac{42}{5} = -8.4\,\text{cm}$

Since $v$ is negative, the image is virtual.

Further, as $\frac{I}{O} = \frac{v}{u}$, $I = \frac{v}{u} \times O$

or $I = \frac{-8.4}{-14} \times 3.0\,\text{cm} = 1.8\,\text{cm}$

The image is erect.

As the object is moved farther from the lens, the image (virtual) moves towards the focus of the lens and its size goes on decreasing. It should however be borne in mind that this image would never cross the focus of the lens.

## ILLUSTRATION 2.20

The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose?

**Sol.** Let the object be placed x in the front of lens of distance of image from the lens is $(3 - x)$ m.

We are given that $u + v = 3$ m

or $v = 3 - u$

Applying lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{+f} = \frac{1}{3-u} - \frac{1}{-u}$

[as $f = +f$ (convex lens), $v = (3 - u)$ (real image), $u = -u$ (real image)]

or $\frac{1}{f} = \frac{1}{3-u} + \frac{1}{u} = \frac{u+3-u}{(3-u)u} = \frac{3}{3u-u^2}$

or $f = \frac{3u-u^2}{3} = u - \frac{u^2}{3}$ ...(i)

or $\frac{df}{du} = 1 - \frac{2u}{3}$

For $f$ to be maximum, $\frac{df}{du} = 0$

or $1 - \frac{2u}{3} = 0$

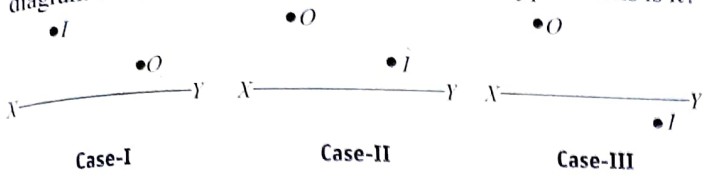
or $u = 1.5$ m ...(ii)

From Eqs. (i) and (ii),

$$f_{max} = 1.5 - \frac{(1.5)^2}{3} = 0.75\,\text{m}$$

## ILLUSTRATION 2.21

An image $I$ is formed of a point object $O$ by a lens whose optic axis is $XY$ as shown in cases (a), (b) and (c). Draw a ray diagram to locate the lens and its focus. What type of lens is it?

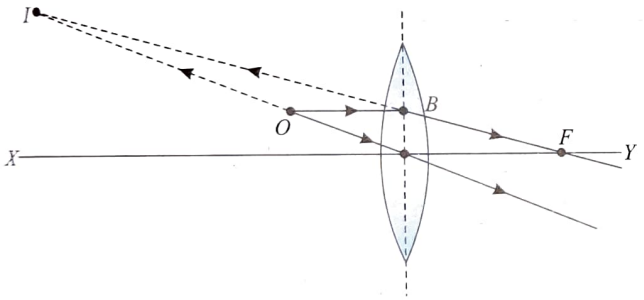



**Sol.** For finding position of lens and focus we follow two steps.

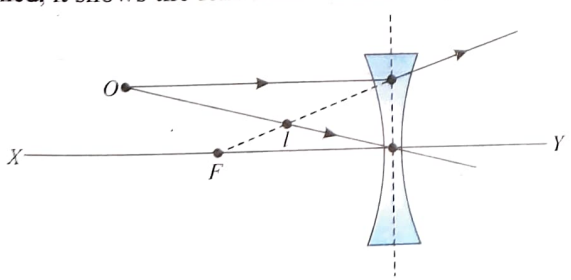
**Step-1:** The ray passing through optical centre of the lens goes undeviated. It means, to obtain the position of optical centre join $O$ and $I$ by a straight line. The point of intersection of this line and optical axis should be the position of the optical centre of the lens.

**Step-2:** The ray travelling parallel to optical axis after refraction from lens passes through focus of the lens. For getting the position of focus draw a line passing through $O$ and parallel to optical axis. It will cut lens at position $B$. Join $I$ and $B$ and extend this line it will intersect optical axis at focus.

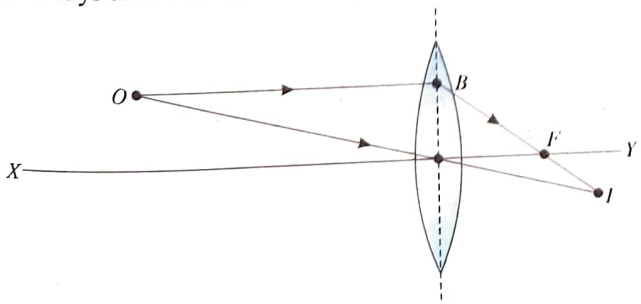
In case-I: Here rays are conversing after refraction, it means the lens should be convex. Also the virtual image is enlarged, hence the lens should be convex.




Case-II: Here the rays are diverging after refraction, it means lens should be concave. Also we can observe the virtual image is diminished, it shows the lens must be concave.




Case-III: In this case the image is real and on other side of the lens, it shows the lens should be convex. Also the converging nature of rays after refraction certify the lens to be convex.




## CONCEPT APPLICATION EXERCISE 2.2

**1.** Choose the following statements as True or False.

**(a)** The real image formed by a lens is always inverted.

**(b)** The image formed by a lens is always inverted.

**(c)** The image formed by a concave lens is always erect and diminished.

**(d)** An air bubble inside water acts like a concave lens.

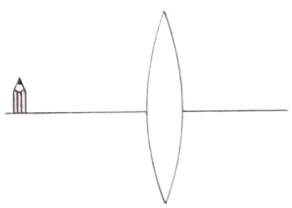
**(e)** The distance between a real object and its real image formed by a single lens cannot be more than $4f$.

**(f)** If an object is moved at a constant speed toward a convex lens from infinity to focus, then its image moves slower in the beginning and faster later on, away from the lens.

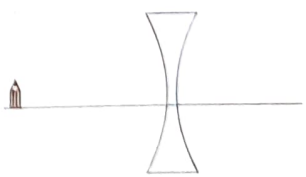
**(g)** The focal length of a glass ($\mu = 1.5$) lens is 10 cm in air. When it is completely immersed in water its focal length will become 40 cm.

**2.** The layered lens shown in figure is made of two kinds of glasses. How many and what kind of images will be produced by this lens with a point source placed on the optical axis? Neglect the reflection of light at the boundaries between layers.

**3.** A pencil of height 1 cm is placed 30 cm from an equiconvex lens, refractive index $n = 3/2$, radius of curvature for both the surfaces, $R_1 = R_2 = R = 10$ cm. Find the location of the image and describe its characteristics.



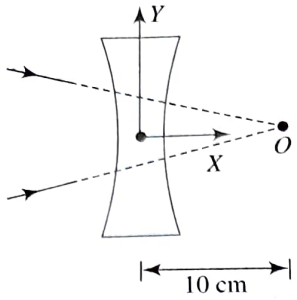
**4.** If in the previous example, we use a diverging lens with a focal length 10.0 cm to form an image of the pencil kept 15 cm in front of the lens, locate and characterize the image.



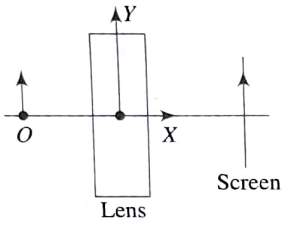
**5.** A biconvex lens has radii of curvature 20 cm and 40 cm. The refractive index of the material of the lens is 1.5. An object is placed 40 cm in front of the lens. Calculate the position of the image.

**6.** A converging bundle of rays is intercepted by a biconcave lens. The radii of curvature of both surfaces are 20 cm and the refractive index of the material of the lens is 1.5. If the

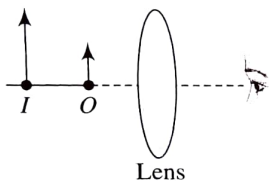
rays originally converged to a point 10 cm in front of the lens, where will they now converge after passing through the lens?




**7.** A candle is placed 15 cm in front of a lens. If the image of the candle captured on a screen is magnified two times, calculate the focal length and nature of the lens.




**8.** A man wishes to view an object through a convex lens of focal length 10 cm. The final image is to be erect and magnified 2 times. How far from the object must he hold the lens?




**9.** An object is placed 12 cm in front of a lens to form an image on a screen. The size of the image is twice that of the object. What is the distance between the object and the image?

**10.** A point source is placed on the axis of a symmetrical convex lens of focal length 20 cm at a distance of 40 cm. If the lens is raised by 1 cm, by how much will the image be lifted relative to the previous axis?

**11.** The source is placed 30 cm from a convex lens which has a focal length of 20 cm. The source is initially located on the axis of the lens. The lens is then cut into two halves in a plane along the principal axis. The two halves are separated by a distance of 4 mm. What will be the locations of the image of the source?

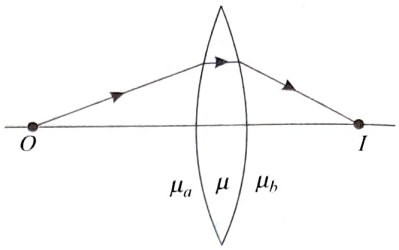
**ANSWERS**

**1.** (a) False; (b) False; (c) True; (d) True; (e) True (f) True; (g) True
**2.** Two, real **5.** 80 cm **6.** 20 cm **7.** 10 cm, convex
**8.** 5 cm **9.** 36 cm **10.** 10 cm

## LENSES WITH DIFFERENT MEDIA ON EITHER SIDE

The lens equation is applicable when medium on both sides of a thin lens is same. If the refractive index of medium on both sides are different, we cannot use lens formula directly, we need to analyse the situation considering refraction through two curved surface one by one.

Consider a lens made of a material with refractive index $\mu$ with a liquid $\mu_a$ on the left and a liquid $\mu_b$ on the right (see figure).




Let an object be placed in medium $\mu_a$. A ray of light from the object undergoes refraction at surface 1 and then refraction at surface 2 before emerging in medium $\mu_b$. Let the radius of curvature of the first surface be $R_1$ and that of the second surface be $R_2$.

At surface 1: $\mu_1 = \mu_a$, $\mu_2 = \mu$, and $R = R_1$

Therefore, $\dfrac{\mu}{v_1} - \dfrac{\mu_a}{u} = \dfrac{\mu - \mu_a}{R_1}$

At surface 2: $\mu_1 = \mu$, $\mu_2 = \mu_b$, and $R = R_2$

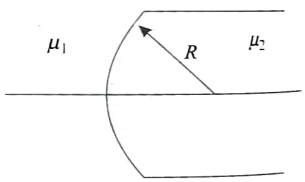
Therefore, $\dfrac{\mu_b}{v} - \dfrac{\mu}{v_1} = \dfrac{\mu_b - \mu}{R_2}$

Adding the above two equations, we get

$$\frac{\mu_b}{v} - \frac{\mu_a}{u} = \frac{\mu - \mu_a}{R_1} + \frac{\mu_b - \mu}{R_2}$$

### POWER OF THIN LENS IN CASE THE MEDIUM ARE DIFFERENT ON BOTH SIDES

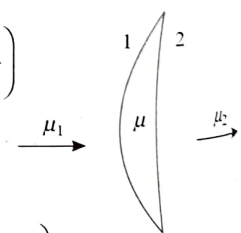
The optical power of a refracting surface is given as $P = \dfrac{\mu_2 - \mu_1}{R}$




A thin lens has two refracting surface 1 and 2. The power of the thin lens is then scalar addition of powers of each refracting surface given as

$$P = P_1 + P_2 = \left(\frac{\mu - \mu_1}{+R_1}\right) + \left(\frac{\mu_2 - \mu}{+R_2}\right)$$

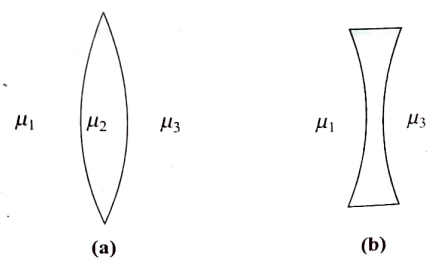
or $$P = \mu\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{\mu_2}{R_2} - \frac{\mu_1}{R_1}$$




Similarly,

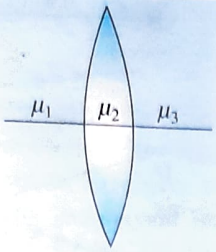
Power for Fig. (a), $P = \left(\dfrac{\mu_2 - \mu_1}{R_1} + \dfrac{\mu_3 - \mu_2}{-R_2}\right)$

Power for Fig. (b), $P = \left(\dfrac{\mu_2 - \mu_1}{R_1} + \dfrac{\mu_3 - \mu_2}{R_2}\right)$



## ILLUSTRATION 2.22

The diagram shows an equiconvex lens. What should be the condition on the refractive indices so that the lens becomes diverging?



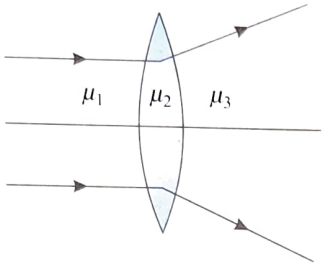

**Sol.** Here the refractive index of the medium on both sides of the lens is different, hence lens maker's formula is not applicable.

Using the equation $\frac{\mu_b}{v}-\frac{\mu_a}{u}=\frac{\mu_a-\mu}{R_1}+\frac{\mu_b-\mu}{R_2}$

Here, $u=\infty, R_1=+R, R_2=-R, \mu_a=\mu_1, \mu=\mu_2$ and $\mu_b=\mu_3$

$$\frac{\mu_3}{v}-\frac{\mu_1}{\infty}=\frac{\mu_2-\mu_1}{R}+\frac{\mu_3-\mu_2}{(-R)}$$

$$\frac{\mu_3}{v}=\frac{\mu_2-\mu_1}{R}-\frac{(\mu_3-\mu_2)}{R}$$



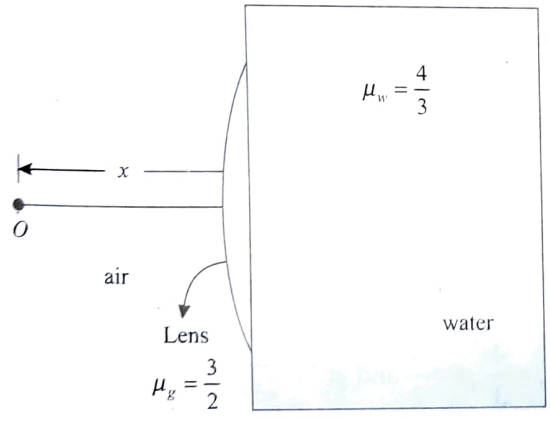

For diverging nature $v<0$,

$$\frac{\mu_2-\mu_1}{R}-\frac{(\mu_3-\mu_2)}{R}<0$$

Or. $\frac{\mu_2-\mu_1}{R}<\frac{\mu_3-\mu_2}{R}\Rightarrow 2\mu_2<\mu_1+\mu_3$

## ILLUSTRATION 2.23

Object $O$ is kept in air in front of a thin plano convex lens of radius of curvature 10 cm. Its refractive index is 3/2 and the medium toward right of plane surface is water of refractive index 4/3. What should be the distance $x$ of the object so that the rays become parallel finally?




**Sol.** The refractive index of the medium on both sides of the lens is different, hence lens maker's formula is not applicable.

Using the equation $\frac{\mu_b}{v}-\frac{\mu_a}{u}=\frac{\mu_a-\mu}{R_1}+\frac{\mu_b-\mu}{R_2}$

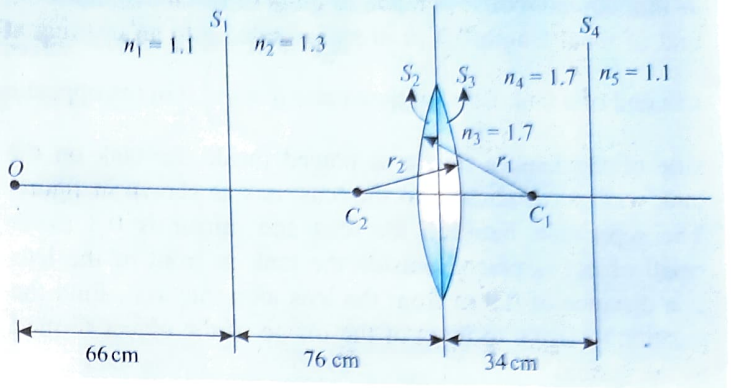
Here, $v=\infty, R_1=+10\,\text{cm}, R_2=\infty, \mu_a=1, \mu=3/2$ and $\mu_b=4/3$

$$\frac{\mu_3}{\infty}-\frac{1}{x}=\frac{3/2-1}{(+10)}+\frac{4/3-3/2}{\infty}$$

$$\frac{-1}{x}=\frac{0.5}{10}\quad\Rightarrow\quad x=-20\ \text{cm}$$

## ILLUSTRATION 2.24

A biconvex lens separates two media of refractive indices 1.3 and 1.7. The refractive index of the lens is 1.5 and the radii of curvature of the two sides of the lens are $r_1=10$ cm and $r_2=60$ cm. The medium of refractive index 1.3 extends to 78 cm from the lens and that of refractive index 1.7 extends to 34 cm from the lens. A luminous object $O$ is at a distance of 144 cm from the lens. Find the position of the final image from the lens.




**Sol.** **For refraction at surface $S_1$:**

Using, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

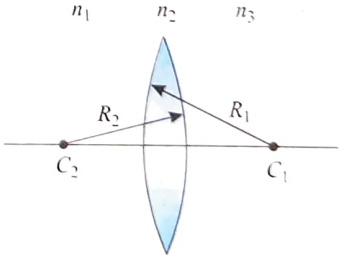
$u=-66$ cm, $R=\infty, \mu_1=1.1$ and $\mu_2=1.3$

$$\frac{1.3}{v}-\frac{1.1}{(-66)}=\frac{(1.3-1.1)}{\infty}=0\quad\Rightarrow v=-78\ \text{cm}.$$

Image lies to the left of the surface $S_1$. This acts as an object for the lens.

**For refraction through the lens:** We apply the general thin lens equation.

$$\frac{\mu_b}{v}-\frac{\mu_a}{u}=\frac{\mu_a-\mu}{R_1}+\frac{\mu_b-\mu}{R_2}$$




Object distance for lens, $u'=-(78+78)$ cm $=-156$ cm

$R_1=+10\,\text{cm}, R_2=-60\,\text{cm}, \mu_a=1.3, \mu=1.5$ and $\mu_b=1.7$

$$\frac{1.7}{v'}-\frac{1.3}{(-156)}=\frac{1.5-1.3}{+10}+\frac{1.7-1.5}{(-60)}$$

$\Rightarrow\quad v'=340$ cm

**For refraction at surface $S_4$:** Object distance for refraction $S_4$ is $u=(340-34)$ cm $=306$ cm

Using, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

$R=\infty, \mu_1=1.1$ and $\mu_2=1.7$

$$\frac{1.7}{v''}-\frac{1.1}{(+306)}=\frac{(1.7-1.1)}{\infty}$$

$$\frac{1.1}{v''}-\frac{1.7}{(+306)}=0 \Rightarrow v''=\frac{1.1\times 3.6}{1.7}=198\text{ cm}$$

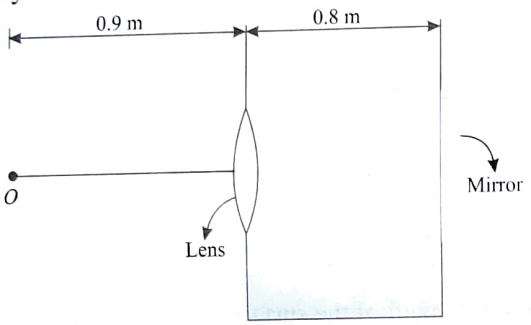
or 198 cm to the right of $S_4$.

Hence the final image will form at a distance,

$(198-34)\text{cm}=154\text{cm}$ (right-side) from the lens.

## ILLUSTRATION 2.25

A thin equiconvex lens made of glass of refractive index 3/2 and of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water $\left(\mu=\frac{3}{2}\right)$. On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis as shown in figure. The separation between the lens and mirror is 0.8 m. A small object is placed outside the tank in front of the lens at a distance of 0.9 m from the lens along its axis. Find the position (relative to lens) of the image of the object formed by the system.




**Sol.** If $R$ is radius of curvature of each lens surface, then for air on either side of lens, we have

$$\frac{1}{f}=\left({}_a\mu_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{3}{2}-1\right)\left(\frac{1}{R}+\frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{f}=\frac{1}{R}\Rightarrow f=R\Rightarrow R=f=0.3\text{ m}$$

If $\mu_2$ is refractive index of lens material and $\mu_1, \mu_3$ are refractive indices on either side of the lens, then the formula is

$$\frac{\mu_3}{v}-\frac{\mu_1}{\mu}=\frac{\mu_2-\mu_1}{R_1}+\frac{\mu_3-\mu_2}{R_2} \qquad ...(i)$$

Here $R_1=0.3$ m, $R_2=-0.3$ m, $\mu_1=\mu_{air}=1$,

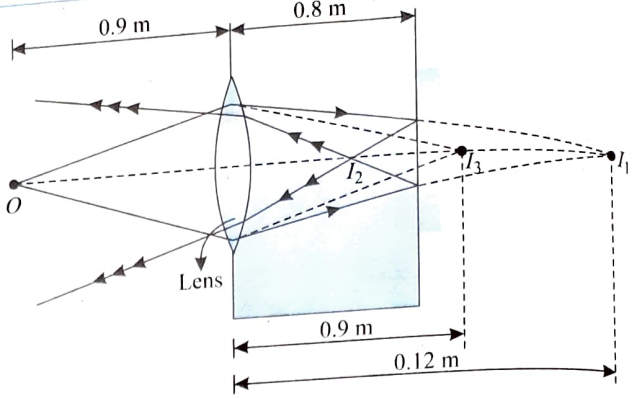
$$\mu_2=\mu_{glass}=\frac{3}{2},\ \mu_3=\mu_{water}=\frac{4}{3},\ u=-0.9\text{ m},\ v=?$$

$$\therefore \quad \frac{4/3}{v}-\frac{1}{(-0.9)}=\frac{\left(\frac{3}{2}-1\right)}{0.3}+\frac{\left(\frac{4}{3}-\frac{3}{2}\right)}{-0.3}$$

or $$\frac{4}{3v}+\frac{1}{0.9}=\frac{1}{0.6}+\frac{1}{1.8}$$

or $$\frac{4}{3v}=\frac{1}{0.6}+\frac{1}{1.8}-\frac{1}{0.9}=\frac{1}{0.9}$$

or $$v=\frac{0.9\times 4}{3}=1.2\text{ m}$$




That is image $I_1$ is at a distance 1.2 m from lens $L$ to the right or at a distance $(1.2-0.8)=0.4$ m to the right of mirror $M$. This image $I_1$ acts as a virtual source for the plane mirror and forms real image $I_2$ at a distance 0.4 m on to the left of mirror and hence at a distance $(0.8-0.4)=0.4$ m to the right of convex lens. This image $I_2$ acts as an object for the lens.

Again, the formula is $\frac{\mu_2'}{v''}-\frac{\mu_1'}{u''}=\frac{\mu_2'-\mu_1'}{R_1}+\frac{\mu_3'-\mu_2'}{R_2}$

But now $\mu_1'=\mu_{water}=\frac{4}{3},\ \mu_2'=\mu_{glass}=\frac{3}{2},\ \mu_3'=\mu_{air}=1$

$R_1=-0.3$ m, $R_2=+0.3$ m, $u''=0.4$ m, $v''=?$

$$\therefore \quad \frac{1}{v''}-\frac{1}{0.3}=-\frac{1}{1.8}-\frac{1}{0.6}$$

or $$\frac{1}{v''}=\frac{1}{0.3}-\frac{1}{1.8}-\frac{1}{0.6}=\frac{1}{0.9}\Rightarrow v''=0.9\text{ m}$$

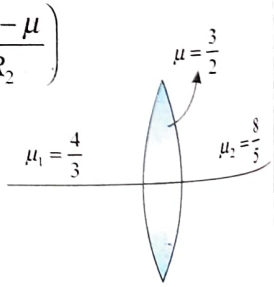
That is image $I_3$ is formed to the right at a distance 0.9 m from the lens and is virtual. That is position of final image will be 0.9 m to the right of lens.

## ILLUSTRATION 2.26

Find the optical power and focal lengths of a bi-convex lens of radius of curvature $R_1=20$ cm, $R_2=30$ cm. Assume that the refractive index of the lens is $\mu=3/2$ and the refractive index of the media at both sides are $\mu_1=4/3$ and $\mu_2=8/5$.

**Sol.** In this case, the power of the lens is

$$P=P_1+P_2=\left(\frac{\mu-\mu_1}{R_1}\right)+\left(\frac{\mu_2-\mu}{R_2}\right)$$




Here, $\mu_1=\frac{4}{3},\mu=\frac{3}{2}$ and $\mu_2=\frac{8}{5}$

Also, $R_1=+\frac{1}{5}\text{m}$, and $R_2=-\frac{3}{10}\text{m}$

$$\Rightarrow \quad P=\left(\frac{3/2-4/3}{+1/5}\right)+\left(\frac{8/5-3/2}{-3/10}\right)=0.5\text{D}$$

The first principal focal length, $f_1=\frac{n_1}{P}=\frac{4/3}{1/2}=\frac{8}{3}\text{m}$

and second principal focal length, $f_2=\frac{n_2}{P}=\frac{1.6}{1/2}=3.2\text{m}$

## COMBINATION OF LENSES AND MIRRORS

When several lenses or mirrors are used coaxially, the image formation is considered one after another in steps. The image formed by the lens facing the object serves as an object for the next lens or mirror, the image formed by the second lens (or mirror) acts as an object for the third, and so on. The total magnification in such situations will be given by

$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times \cdots, \text{ i.e., } m = m_1 \times m_2 \times \cdots$$

We will learn this concept through following illustrations.

### ILLUSTRATION 2.27

An object is placed at a distance of 10 cm to the left on the axis of convex lens $A$ of focal length 20 cm. A second convex lens of focal length 10 cm is placed coaxially to the right of the lens $A$ at a distance of 5 cm from $A$. Find the position of the final image and its magnification. Trace the path of the rays.

**Sol.** Here, for the first lens, $u_1 = -10$ cm, $f_1 = 20$ cm

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1} \quad \Rightarrow \quad \frac{1}{v_1} = \frac{1}{20} - \frac{1}{10}$$

$\Rightarrow \quad v_1 = -20$ cm

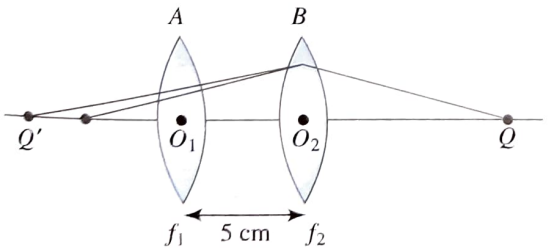
The image is virtual and hence lies on the same side of the object. This will behave as an object for the second lens.

For the second lens:

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

Here, $u_2 = -(20 + 5)$ cm, $f_2 = 10$ cm

$$\frac{1}{v_2} + \frac{1}{25} = \frac{1}{10} \quad \Rightarrow \quad v_2 = \frac{50}{3} = 16\frac{2}{3} \text{ cm}$$



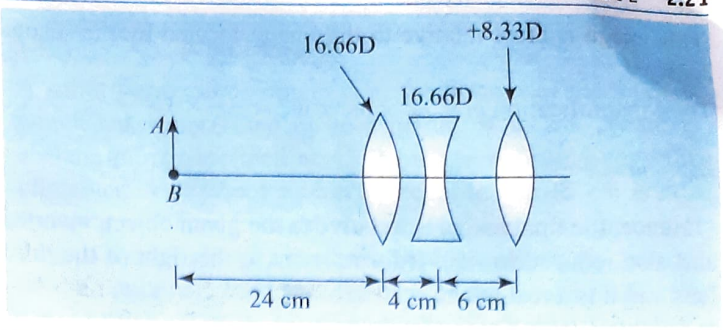

The final image is at a distance of $16\frac{2}{3}$ cm to the right of the second lens.

The magnification of the image is given by,

$$m = \frac{v_1}{u_1} \cdot \frac{v_2}{u_2} = \frac{(-20)}{(-10)} \cdot \frac{50}{3 \times (-25)} = \frac{-4}{3} = -1.33$$

### ILLUSTRATION 2.28

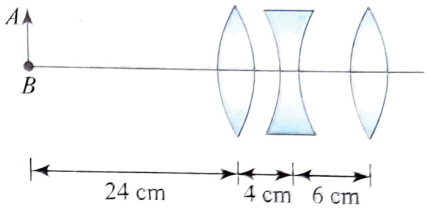
In figure, the length of object $AB$ is 9 cm. Find the nature and position of the final image and also its length. Assume that each lens is a thin lens.




**Sol.** **Action of the first lens:**

Focal length $= \frac{1}{16.66} = 0.06$ m $= 6$ cm

$u = -24$ cm




Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} + \frac{1}{24} = \frac{1}{6}$$

$$\frac{1}{v} = \frac{1}{6} - \frac{1}{24} \quad \text{or} \quad v = 8 \text{ cm}$$

This image would be real and inverted (with respect to object $AB$).

Its magnification, $m_1 = \frac{v}{u} = \frac{8}{-24} = -\frac{1}{3}$

**Action of the second lens:**

Image formed by the first lens, being 8 cm to the right of the first lens, would act as virtual object for the second lens.

Focal length $= \frac{1}{-16.66} = -0.06$ m $= -6$ cm

$u = +4$ cm [distance of image formed by the first lens from the second lens]

$$\frac{1}{v} - \frac{1}{4} = -\frac{1}{6}$$

$$\frac{1}{v} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12} \quad \text{or} \quad v = 12 \text{ cm}$$

Thus, the second lens forms an image 12 cm to its right. This image is real and its magnification,

$$m_2 = \frac{v}{u} = \frac{12}{4} = 3$$

Thus, it is also erect relative to the image formed by the first lens.

**Action of the third lens:**

Image formed by the second lens, being 12 cm to its right, will act as a virtual object for the third lens.

Focal length $= \frac{1}{8.33} = 0.12$ m $= 12$ cm

$u = +6$ cm [distance of image formed by the second lens from the third lens]

$$\frac{1}{v} - \frac{1}{6} = \frac{1}{12}$$

$$\frac{1}{v} = \frac{1}{12} + \frac{1}{6} \quad \text{or} \quad v = 4 \text{ cm}$$

Thus, the final image forms 4 cm to the right of the third lens. The image will be real and its magnification,

$$m_3 = \frac{v}{u} = \frac{4}{6} = \frac{2}{3}$$

This image is erect relative to the image formed by the second lens.

Total magnification $m = m_1 \times m_2 \times m_3 = -\frac{1}{3} \times 3 \times \frac{2}{3}$

$$\Rightarrow \quad m = -\frac{2}{3}$$

Hence, the final image is, relative to the given object, inverted and also reduced in size. It forms 4 cm to the right of the third lens and it is a real image.

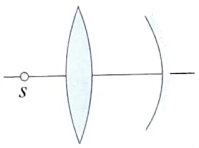
To obtain length of the final image,

$$m = \frac{I}{O} = -\frac{2}{3}$$

$$I = \frac{2}{3} \times 9 = -6 \text{ cm}$$

## ILLUSTRATION 2.29

A converging lens and a converging mirror are placed with their principal axis coinciding. Their separation equals 40 cm. A point source $S$ is placed on the principal axis at a distance of 12 cm from the lens as shown in figure. It is found that the final beam comes out parallel to the principal axis. Focal length of the lens equals 15 cm. Find the focal length of the mirror.




**Sol.** Refraction from lens $\frac{1}{f_L} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{15} = \frac{1}{v} + \frac{1}{12} \qquad [f_L = 15 \text{ cm}, u = -12 \text{ cm}]$$

$$\frac{1}{15} - \frac{1}{12} = \frac{1}{v} \Rightarrow v_1 = -60 \text{ cm}$$

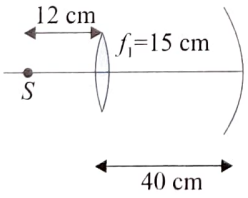
Reflection from mirror: $u_1$ for mirror $u_2 = -100$ cm

If finally rays come parallel to principal axis, then the object should be at focus between mirror and lens. Then image created by mirror will be $v_2 = -25$ cm from the pole of mirror for mirror.

For mirror, $\frac{1}{f_m} = \frac{1}{u} + \frac{1}{v}$

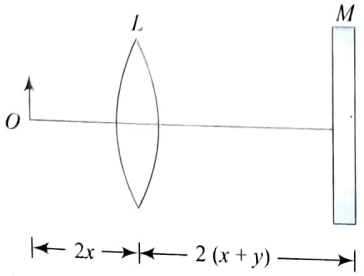
$$\frac{1}{f_m} = -\frac{1}{100} - \frac{1}{25} = -\left(\frac{1+4}{100}\right)$$

$$f_m = -20 \text{ cm}$$




## ILLUSTRATION 2.30

$L$ is a lens such that a parallel beam of light incident on it, after refraction, converges to a point at a distance $x$ from it and $M$ is a mirror such that a parallel beam of light incident on it, after reflection, converges to a point at a distance $y$ from it. Now the lens $L$ and mirror $M$ are placed at a distance $2(x + y)$ from each other. An object of size 2 cm is kept infront of lens at a distance $2x$ from it. Find the nature, position, and size of the image that will be seen by an observer looking toward the mirror through the lens.

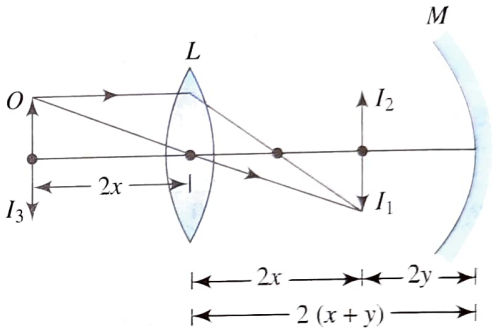



**Sol.** It is given that a parallel beam incident on the lens, after refraction, converges to a point at a distance $x$ from the lens. This implies that the focal length of the lens is $x$.

It is also given that a parallel beam incident on the mirror, after reflection, converges to a point at a distance $y$ from it. This implies that the mirror is a concave mirror of focal length $y$.

Obviously, object $O$ is kept at $2F$ of lens $L$. Hence, its image $I_1$ forms on the other side at $2F$, i.e., at distance $2x$ to the right of the lens. The image will be real, inverted, and magnification of this image $m_1 = \frac{v}{u} = \frac{2x}{-2x} = -1$.

Image $I_1$ of $O$ formed by lens $L$ acts as object for the mirror. Its distance from the pole of mirror will be $2(x + y) - 2x = 2y$. This implies that it will be at the center of curvature of the concave mirror ($y$ being the focal length of the the mirror). Therefore, the concave mirror forms a real, inverted (relative to $I_1$) image $I_2$ at the same position.



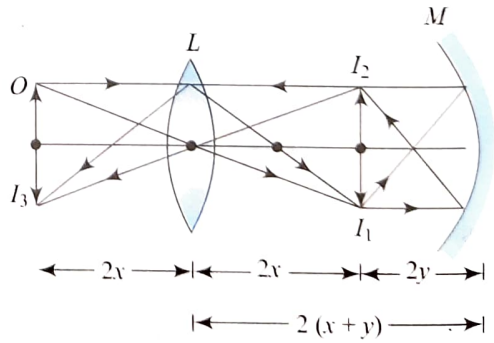

Magnification of $I_2$ (relative to $I_1$), $m_2 = -1$. Finally, $I_2$ acts as object for the lens. Its distance from the lens is $2x$. Hence, it is at $2F$ of the lens ($x$ being the focal length of the lens). Therefore, the lens will form a real and inverted image of $I_2$ at $2F$ on the other side, i.e., to its left. Obviously, $I_3$ will be at the same position as of the object.

Magnification of $I_3$ (relative to $I_2$) $= -1$

Total magnification $= m_1 \times m_2 \times m_3 = -1 \times -1 \times -1 = -1$

Thus, the final image is real and inverted and of the same size (2 cm).

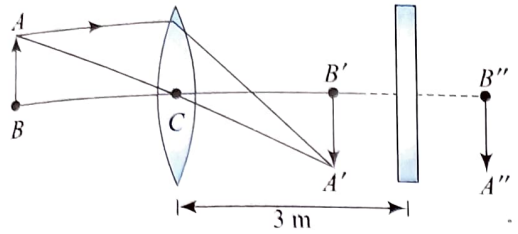
Complete ray diagram is shown in the figure.




## ILLUSTRATION 2.31

An object of height 2 cm is kept 2 m in front of a convex lens of focal length 1 m. A plane mirror is placed at 3 m from the lens on its other side. Find the nature, position, and magnification of the final image that will be seen by an observer looking toward the mirror through the lens.

**Sol.** The lens forms image $A'B'$ of object $AB$, as shown in figure.




Here $u = -2$ cm, $f = 1$ cm

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{v} + \frac{1}{2} = 1 \quad \Rightarrow \quad v = 2 \text{ m}$$

Thus, $A'B'$ is formed at distance 2 m from the lens. Image $A'B'$ is real and inverted and has a magnification

$$m_1 = \frac{v}{u} = \frac{2}{-2} = -1$$

$A'B'$ acts as an object for the plane mirror. Its distance from the plane mirror = 3 – 2 = 1 m. The plane mirror forms its virtual image $A''B''$ which will be 1 m behind the plane mirror and with magnification $m_2 = 1$ (relative to its object $A'B'$).

$A''B''$ now acts as a virtual object for the lens at a distance 4 m to its right.

Applying $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$u = -4 \text{ cm} \quad \Rightarrow \quad \frac{1}{v} + \frac{1}{4} = 1$$

$$\frac{1}{v} = \frac{3}{4} \quad \text{or} \quad v = \frac{4}{3} \text{ m}$$

Since light from $A''B''$ travels from right to left, its image formed by the lens is 4/3 m to the left of lens ($v$ is positive). This image will be real and inverted (relative to $A''B''$). Magnification in this case, $m_3 = \frac{(4/3)}{(-4)} = \frac{-1}{3}$

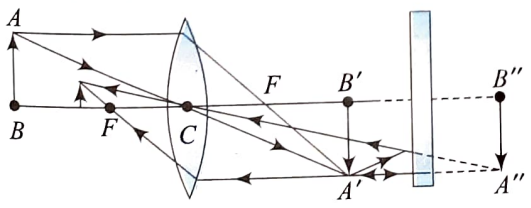
Total magnification,

$$m = m_1 \times m_2 \times m_3 = (-1) \times (1) \times \left(\frac{-1}{3}\right)$$

$$\Rightarrow \quad m = \frac{1}{3}$$

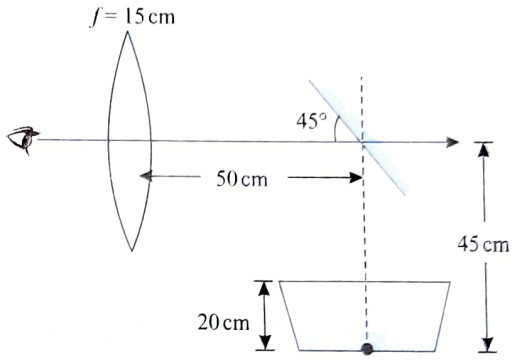
Thus, the final image is real, reduced three times in size and erect (relative to the original object $AB$).

Complete ray diagram is shown in the figure.




## ILLUSTRATION 2.32

A small light source object is at the bottom of a container which has water filled up to a height of 20 cm. There is a plane mirror inclined at 45° to the horizontal above the container. A convex lens having focal length 15 cm is at a distance of 50 cm from the mirror. The horizontal principal axis of the lens is at a distance of 45 cm from the bottom of the container. Find the distance of the image (from the lens) of the object as seen by an observer to the left of the lens.




Light rays from the object hit the lens only after they are reflected from the mirror, (use $\mu_{water} = 4/3$)

**Sol.** The object is situated in denser medium. The apparent depth of the object from top of the water surface

$$d_{app} = \frac{d_{real}}{\mu} = \frac{20}{4/3} = 15 \text{ cm}$$

Now this image formed by refraction through water surface will acts as object for plane mirror. The mirror forms a virtual image at a distance of 40 cm that lies on the principal axis of the lens. This acts as an object for the lens at a distance of 90 cm.

Applying lens formula, $\frac{1}{v} - \frac{1}{-90} = \frac{1}{15}$

$\therefore \quad v = 18$ cm

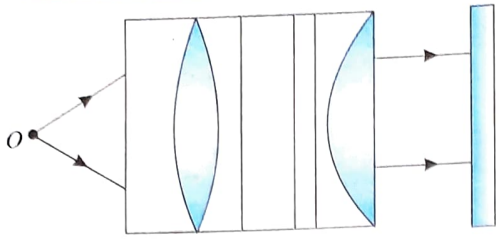
Thus final image is to the left of the lens at a distance of 18 cm.

# CONCEPT OF IMAGE FORMING AT OBJECT ITSELF

In problems in optics, we will usually have a train of optical elements with the stipulation that the image is formed on the object itself. In such cases, there will have to be a mirror at the end of the optical train and the rays have to be incident normally on the mirror in order to retrace their paths. Three kinds of mirrors are possible.

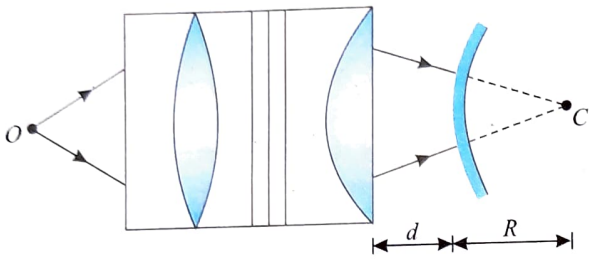
### Case 1: A plane mirror at the end of the optical train

In this situation, the ray of light emerging from the system just before it impinges on the mirror has to be parallel so as to strike the mirror normally. Thus, the image after the last lens must be formed at infinity (see figure).




**Case 2: A concave mirror of the radius of curvature *R***

Here, the bundle of rays must converge on to the center of curvature which is in front of the concave mirror. If the distance of the lens from the mirror is $d$, we can say that the image formed from the last lens must be at a distance $(d + R)$ from the lens (see figure).




**Case 3: A convex mirror of radius of curvature *R***

Here the bundle of rays must converge on to the center of curvature of the convex mirror. If the distance of the last lens from the mirror is $d$, we can say that the image formed from the last lens must be at a distance $(d - R)$ from the lens (see figure).

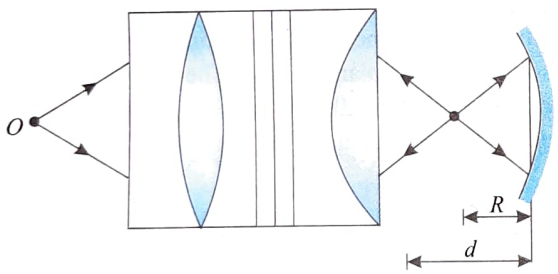
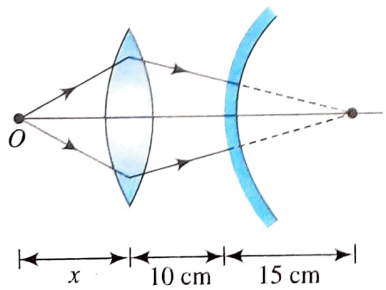



**ILLUSTRATION 2.33**

A convex lens of focal length 20 cm is placed 10 cm in front of a convex mirror of radius of curvature 15 cm. Where should a point object be placed in front of the lens so that it images on to itself?

**Sol.** The convex lens and the convex mirror are shown in figure.




The combination behaves like a concave mirror. Let the distance of the object from the lens be $x$.

For the ray to retrace its path, it should be incident normally on the convex mirror, or in other words the rays should pass through the center of curvature of the mirror.

From the diagram, we see that for the lens

$$u = -x, f = +20 \text{ cm},$$

$$v = +10 + 15 = +25 \text{ cm}$$

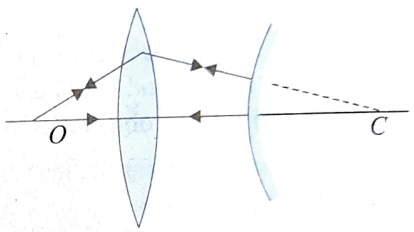
From the lens equation, we get $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

or $\frac{1}{25} - \frac{1}{-x} = \frac{1}{20}$

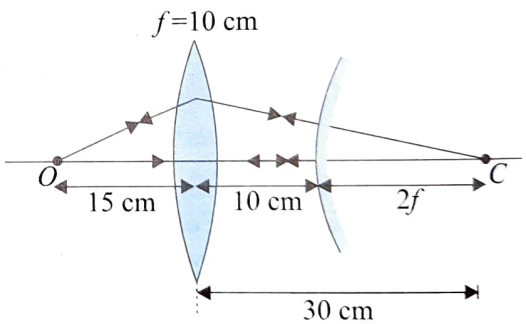
or $x = 100$ cm

**ILLUSTRATION 2.34**

An object is placed at a distance of 15 cm from a convex lens of focal length 10 cm on the other side of the lens, a convex mirror is placed at its focus such that the image formed by combination coincides with the object itself. Find the focal length of concave mirror.




**Sol.** For retracing of ray; ray must fall normally on mirror i.e. toward the center of curvature.




Here, $u = -15$ cm, $f = 10$ cm

For lens: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$;

$$\frac{1}{v} + \frac{1}{15} = \frac{1}{10};$$

$$v = 30 \text{ cm}$$

If image formed by combination coincides with the object, then the rays after interacting with lens should converge at the centre of curvature of the mirror.

Hence, from the ray diagram,

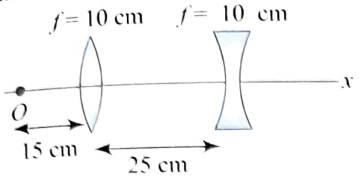
$$2f = 30 - 10$$

$$f = 10 \text{ cm}$$

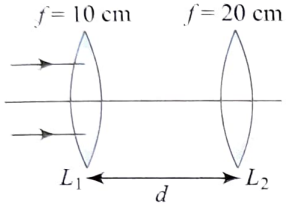
## CONCEPT APPLICATION EXERCISE 2.3

1. A biconvex lens of focal length 20 cm, made of glass of refractive index 1.5, has water ($\mu = 1.33$) on one side and air on the other. An object is placed 15 cm from the lens on the side with water. Where is the image formed?

2. In figure, find the position of final image formed.

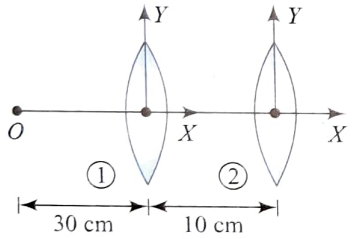



3. Figure shows two converging lenses. Incident rays are parallel to the principal axis. What should be the value of $d$ so that final rays are also parallel?

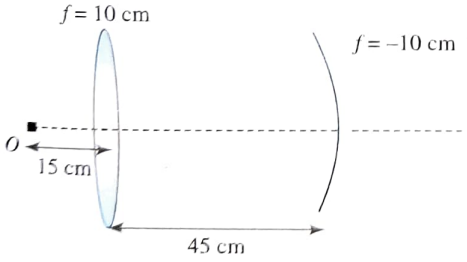



Here, the diameter of ray beam becomes wider.

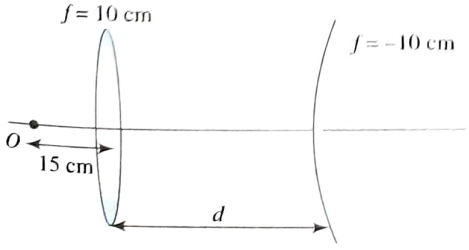
4. A convex lens of focal length 20 cm is placed 10 cm away from a second convex lens of focal length 25 cm. What will be the location of the image of an object at 30 cm in front of the first lens?




5. In figure, find the position of final image formed.

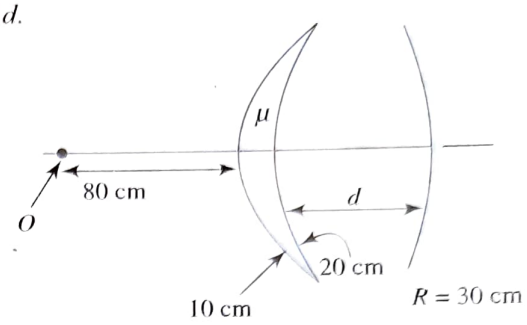



6. In figure, what should be the value of $d$ so that image is formed on the object itself.

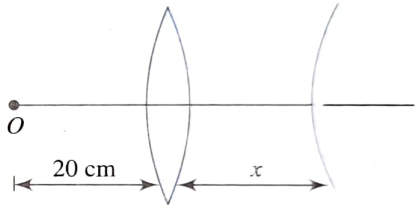



7. A convex lens of focal length 10 cm is placed 30 cm in front of a second convex lens also of the same focal length. A plane mirror is placed after the two lenses. Where should a point object be placed in front of the first lens so that its images on to itself?

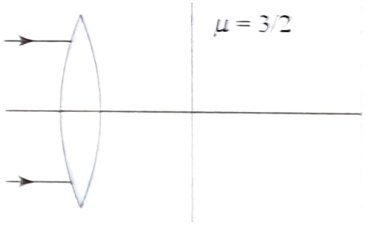
8. If final image after two refractions through the lens and one reflection from the mirror forms at the same point $O$. Refractive index of the material of the lens $\mu = 3/2$. Then find $d$.



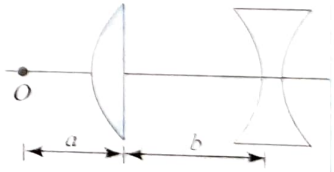

9. Point object $O$ is placed at a distance of 20 cm from a convex lens of focal length 10 cm as shown in figure. At what distance $x$ from the lens should a concave mirror of focal length 60 cm, be placed so that the final image coincides with the object?




10. The focal length of a thin convex lens is 30 cm. At a distance of 10 cm from the lens there is a plane refracting surface of refractive index 3/2. Where will parallel rays incident on lens converge?




11. A plano-convex glass lens ($\mu_g = 3/2$) of radius of curvature $R = 10$ cm is placed at a distance of $b$ from a concave lens of focal length 20 cm. What should be the distance $a$ of point object $O$ from the plano-convex lens so that position of final image is independent of $b$?



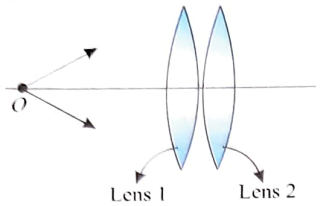

12. When an object is placed at the proper distance to the left of a converging lens, the image is focused in a screen 30 cm to the right of lens. A diverging lens is now placed 15 cm to the right of the converging lens and it is found that the screen must be moved 19.2 cm farther to the right to obtain a sharp image. Find the focal length of the diverging lens.

### ANSWERS

| | | | | |
|---|---|---|---|---|
| 1. 10.3 cm | 2. 10 cm | 3. 30 cm | 4. $\frac{50}{3}$ cm | 5. 30 cm |
| 6. 10 cm | 7. 20 cm | 8. 110 cm | 9. 20 cm | 10. 40 cm |
| 11. 20 cm | 12. 26.7 cm | | | |

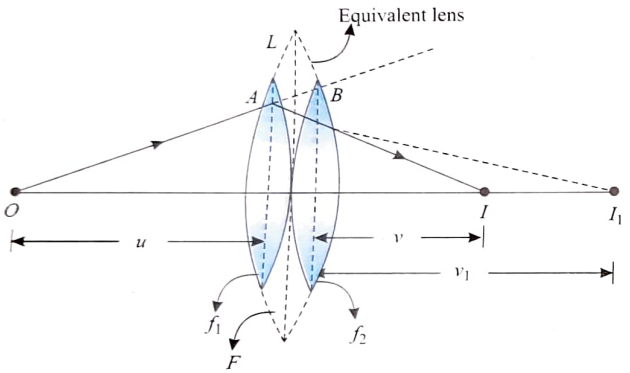
# FOCAL LENGTH OF COMBINATION OF TWO THIN LENSES IN CONTACT

Consider the arrangement shown in figure. We are now interested in finding out the net focal length of the system and position of image for various positions of the object.




Let $A$ and $B$ be two thin lenses of focal lengths $f_1$ and $f_2$ placed coaxially in contact with each other (figure). Note that the lenses themselves may be convex or concave and this is captured in the signs of $f_1$ and $f_2$. We are now interested in finding out the net focal length of the system and position of image for various positions of the object.

Let $O$ be an object lying on the principal axis, $I_1$ is the real image of $O$ formed by the lens $A$ alone.




As $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ and as for the lens $A$, $f=f_1$, $u=u$

$$\frac{1}{f_1}=\frac{1}{v_1}-\frac{1}{u} \quad \text{...(i)}$$

Here $v_1$ be the position of the image formed by the first lens.

The image of the first lens now serves as the object for the second lens. If the image after the second lens if formed at $v$, then for the second lens, we have

$$\frac{1}{f_2}=\frac{1}{v}-\frac{1}{v_1} \quad \text{...(ii)}$$

(as in this case, $f=f_2$, $u=v_1$ and $v=v$)

Adding Eqs. (i) and (ii), we get

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{v}-\frac{1}{u} \quad \text{...(iii)}$$

We can, therefore, see that the system behaves as a single lens with equivalent focal length "$f_e$" given by

$$\frac{1}{f_e}=\frac{1}{f_1}+\frac{1}{f_2} \quad \text{...(iv)}$$

Now we can imagine a single lens $L$ of focal length $F$ placed at the same place where $A$ and $B$ are placed. If this single lens produces the image of the object $O$ at the same point $I$, then such a lens is called the **equivalent** lens and its focal length is **called the equivalent focal length**.

If $P$ is the power of the combination and $P_1$ and $P_2$ are those of the two lenses in contact, then

$$P=P_1+P_2$$

Hence, the power of a combination of lenses in contact with one another is equal to the algebraic sum of the powers of the component lenses.

## MAGNIFICATION DUE TO EQUIVALENT LENS

Let $m_1$ be the magnification of the first lens and $m_2$ be the magnification of the second lens. Let the height of the object be $h_0$, the height of the first image be $h_1$, and the height of the final image be $h_2$. Therefore,

$$m_1=\frac{h_1}{h_0} \quad \text{...(i)}$$

$$m_2=\frac{h_2}{h_1} \quad \text{...(ii)}$$

And the net magnification

$$m=\frac{h_2}{h_0}=\frac{h_2}{h_1}\times\frac{h_1}{h_0}=m_1m_2 \quad \text{...(iii)}$$

**Important Point:**

- We can say that if a set of thin lenses are placed next to each other such that the distance between the lenses is zero, then the net effective focal length of the system $f_{\text{eff}}$ is given by

  $$\frac{1}{f_{\text{eff}}}=\sum_i\frac{1}{f_i}$$

  where $f_i$ is the focal length of every lens.
- When several lenses or mirrors are used coaxially, the image formation is considered one after another in steps. The image formed by the lens facing the object serves as an object for the next lens or mirror, the image formed by the second lens (or mirror) acts as an object for the third and so on. The total magnification in such situations will be given by

  $$m_{\text{eff}}=\frac{I}{O}=\frac{I_1}{O}\times\frac{I_2}{I_1}\times\cdots,$$

  i.e., $m_{\text{eff}}=m_1\times m_2\times m_3\times\cdots$
- If two thin lenses of the same nature are put in contact, then as

  $$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

  $$\frac{1}{F}>\frac{1}{f_1} \quad \text{and} \quad \frac{1}{F}>\frac{1}{f_2}$$

  i.e., $F<f_1$ and $F<f_2$

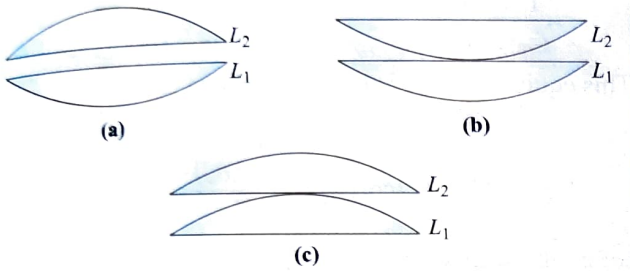
  i.e., the resultant focal length will be lesser than the smallest individual.
- If two thin lenses of opposite nature with different focal lengths are put in contact, the resultant focal length will be of the same nature as that of the lens of the shorter focal length but its magnitude will be more than that of the shorter focal length.

- If a lens of focal length $f$ is divided into two equal parts as shown in Fig. (a) and each part has focal length $f'$ then as

$$\frac{1}{f}=\frac{1}{f'}+\frac{1}{f'}, \quad \text{i.e.,} \quad f'=2f$$

i.e., each part will have focal length $2f$.
Now, if these parts are put in contact as in Fig. (b) or (c), the resultant focal length of the combination will be

$$\frac{1}{F}=\frac{1}{2f}+\frac{1}{2f}, \quad \text{i.e.,} \quad F=f(=\text{initial value})$$



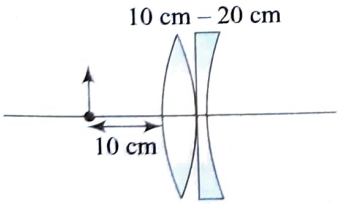

- When surrounding medium of lens is air then, $P=\frac{1}{f_{air}}$
- When surrounding medium, of lens is water then, $P'=\frac{\mu_w}{f_w}$, where $\mu_w$ = refractive index of water, $f_w$ = focal length of lens in water
- If two thin lenses of equal focal length but of opposite nature (i.e., one convergent and other divergent) are put in contact, the resultant focal length of the combination will be

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{-f}=0, \quad \text{i.e.,} \quad F=\infty \text{ and } P=0$$

i.e., the system will behave as a plane glass plate.

## ILLUSTRATION 2.35

Find the lateral magnification produced by the combination of lenses as shown in figure.




**Sol.** If lenses are placed in contact, the focal length of single equivalent length can be given as

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$$

$\Rightarrow$ $f=+20$ cm

Now using lens formula considering system of lenses as a single lens, we get

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\therefore \quad \frac{1}{v}-\frac{1}{-10}=\frac{1}{20}$$

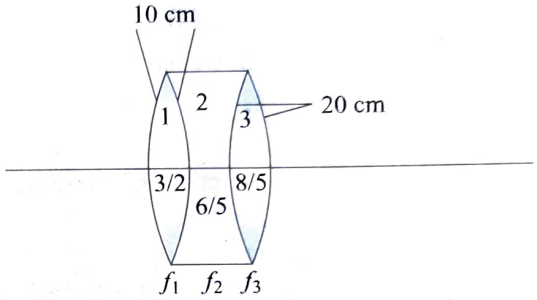
$$\frac{1}{v}=\frac{1}{20}-\frac{1}{10}=\frac{-1}{20} \Rightarrow v=-20 \text{ cm}$$

Now using magnification formula, we get

$$m=\frac{-20}{-10}=2$$

## ILLUSTRATION 2.36

Find the focal length of equivalent system shown in figure.




**Sol.** Here radius of curvatures of different surfaces and refractive index are given, we can calculate focal lengths of lenses by using

$$\frac{1}{f_1}=\left(\frac{3}{2}-1\right)\left(\frac{1}{10}+\frac{1}{10}\right)=\frac{1}{2}\times\frac{2}{10}=\frac{1}{10}$$

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_2}=\left(\frac{6}{5}-1\right)\left(\frac{-1}{10}-\frac{1}{20}\right)=\frac{1}{5}\times\left(\frac{-30}{10\times 20}\right)=\frac{-3}{100}$$

$$\frac{1}{f_3}=\left(\frac{8}{5}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right)=\frac{3}{50}$$

Hence, equivalent focal length

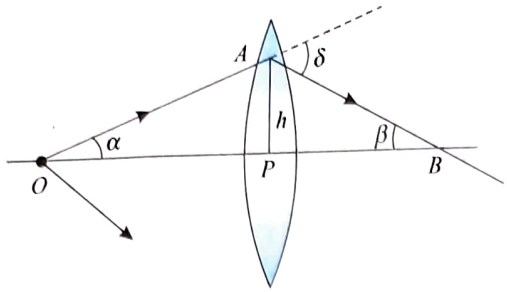
$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}=\frac{1}{10}+\frac{-3}{100}+\frac{3}{50}$$

$\Rightarrow$ $f=100/13$ cm

## THE EQUIVALENT FOCAL LENGTH OF THIN LENSES SEPARATED BY A DISTANCE FROM EACH OTHER

Consider two lenses placed a distance $d$ apart on the same optical axis. Here the object is placed at infinity, the combination may be replaced by an equivalent single thin lens. This lens should converge the bundle of rays from infinity to the same point as the system of lenses and also do so with the same angle of convergence.

Let us first determine the angle of deviation and the corresponding angle of convergence produced by an equivalent single lens (see figure).




**Deviation produced by an equivalent single lens**

Let $O$ be a point object on the axis of the lens. A ray from the object is incident on the lens at points, is refracted, and meets the principal axis at $B$. The angle of deviation produced by the lens is $\angle BAA'$.

From the figure, we see that

$$\angle BAA' = \angle AOB + \angle ABO = \alpha + \beta \qquad ...(i)$$

Also from triangles $OAP$ and $APB$, we have

$$\tan(\alpha) = \frac{AP}{PO},\ \tan(\beta) = \frac{AP}{PB}$$

If the rays are very close to the principal axis, we make the paraxial approximation that

$$\tan(\alpha) \approx \alpha,\ \tan(\beta) \approx \beta \qquad ...(ii)$$

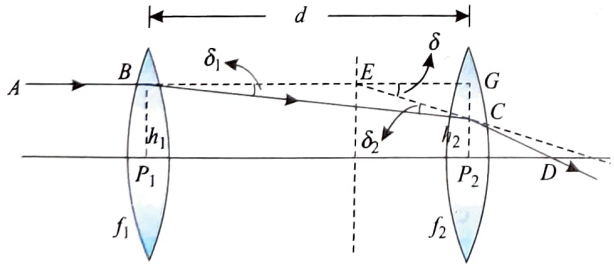
Substituting in Eq. (i), we get the angle of deviation

$$\delta = \frac{AP}{PO} + \frac{AP}{PB}$$

If we now apply the sign convention, we get $u = -PO$, $v = +PB$, $h = +PA$

$$\Rightarrow \quad \delta = h\left[\frac{1}{-u} - \frac{1}{v}\right] = \frac{h}{f} \qquad ...(iii)$$

Now, consider the combination of two lenses placed at distance $d$ apart as shown in figure. The incident ray $AB$ strikes the first lens at $B$, is deflected by $\delta_1$, meets the second lens at $C$, is deflected a further angle $\delta_2$, and the emergent ray $CD$, finally intersects the principal axis at $D$.




**Deviation produced by two lenses at a distance apart**

When the system is replaced by a single equivalent lens, the incident ray will be along $AB$ while the emergent ray will be along $CD$ and the lens can refract the beam only once. Therefore, the equivalent lens must be placed at the point which is the point of intersection of the incident and the emergent rays. $AB$ and $CD$ (extended) intersect at $E$. Therefore, the lens is to be placed at $E$ and the effective focal length of the lens *is* $ED$.

The total deviation of the beam $\delta = \delta_1 + \delta_2$ ...(iv)

But from Eq. (iii), $\delta_1 = \frac{h_1}{f_1}$, $\delta_2 = \frac{h_2}{f_2}$, $\delta = \frac{h_1}{f_{\text{eff}}}$

$$\frac{h_1}{f_{\text{eff}}} = \frac{h_1}{f_1} + \frac{h_2}{f_2}$$

From triangle $BGC$, we have,

$$h_1 - h_2 = GC = d\tan(\delta_1) = d\,\frac{h_1}{f_1}$$

$$h_2 = h_1\left(1 - \frac{d}{f_1}\right)$$

Substituting Eq. (vi) in Eq. (v), we get

$$\frac{h_1}{f_{\text{eff}}} = \frac{h_1}{f_1} + \frac{1}{f_2}h_1\left(1 - \frac{d}{f_1}\right)$$

$$\frac{1}{f_{\text{eff}}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \qquad ...(vii)$$

This equivalent lens is to be placed at a distance

$$EG = (h_1 - h_2)\cot(\delta) = \frac{(h_1 - h_2)}{\delta} = \frac{\left[\frac{h_1 d}{f_1}\right]}{f_{\text{eff}}} = \frac{d \cdot f_{\text{eff}}}{f_1}$$

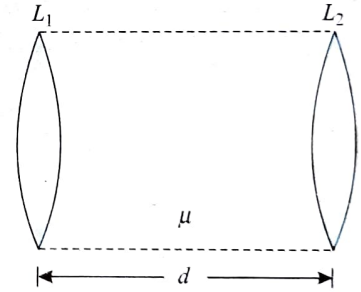
before the second lens.

**Important Points:**

- The Eq. (vii) can be used as the power of the combination of two thin lenses separated by a distance $d$ as

  $$P_{eff} = P_1 + P_2 - dP_1P_2$$

  If we fill a medium of refractive of refractive index $\mu$ between the lenses, the power of the combination is given by




$$P_{eff} = P_1 + P_2 - \frac{dP_1P_2}{\mu}$$

$P_1$ = Power of the lens 1, in the given media.

$P_2$ = Power of the lens 2, in the given media.

$d$ = Distance of separation between the lenses,

$\mu$ = Refractive index between the intervening space.

## ILLUSTRATION 2.37

Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. Find the image of a distant object formed by the combination by:

**(a)** Using thin lens formula separately for the two lenses, and

**(b)** Using the equivalent lens. Note that although the combination forms a real image of a distant object on the other side, it is equivalent to a diverging lens as far as the location of the final image is concerned.

**Sol.**

(a) The first image is formed at the focus of the first lens. This is at 20 cm from the first lens and hence at $u = -40$ cm from the second. Using the lens formula for the second lens,

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = -\frac{1}{40} + \frac{1}{20} \quad \text{or} \quad v = 40 \text{ cm}$$

The final image is formed 40 cm to the right of the second lens.

(b) The equivalent focal length is

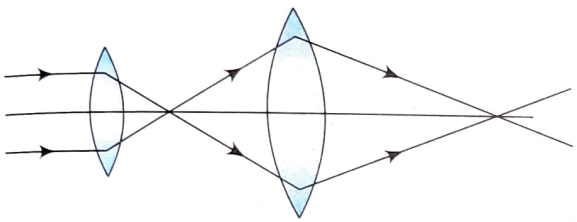
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{20} + \frac{1}{20} - \frac{60}{(20)^2}$$

or $F = -20$ cm

It is a divergent lens. It should be kept at a distance

$$D = \frac{dF}{f_1} \text{ behind the second lens.}$$

Here, $D = \dfrac{60 \times (-20)}{20} = 60$ cm



Thus, the equivalent divergent lens should be placed at a distance of 60 cm to the right of the second lens. The final image is formed at the focus of this divergent lens, i.e., 20 cm of the left of it. It is, therefore, 40 cm to the right of the second lens.

## ILLUSTRATION 2.38

Consider a coaxial system of two thin convex lenses of focal length $f$ each separated by a distance $d$. Draw ray diagrams for image formation corresponding to an object at infinity placed on the principal axis in the following cases: **(a)** $d < f$, **(b)** $d = f$, **(c)** $f < d < 2f$, **(d)** $d = 2f$, and **(e)** $d > 2f$. Indicate the nature of the combination (concave, convex or plane) in each case.

**Sol.** In each of two thin lenses separated by a distance $d$, we have

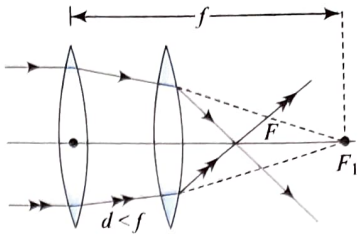
$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

For $f_1 = f, f_2 = f$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} - \frac{d}{f^2}$$

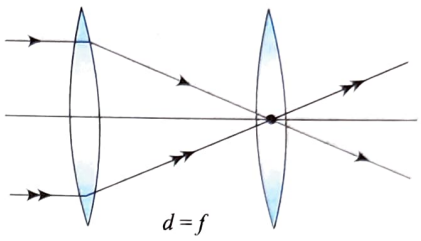
$$F = \frac{f^2}{2f - d} \quad \ldots(i)$$

**(a)** If $d < f$: $F$ will be positive and $> f$. So, the system will behave as a convex lens of focal length $< f$ as shown in Fig. (a).



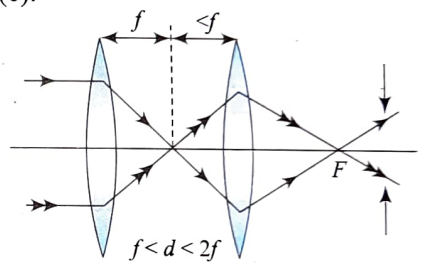

**(a)**

**(b)** If $d = f$: $F = f$, i.e., the system will behave as a convex lens of focal length $f$ as shown in Fig. (b)



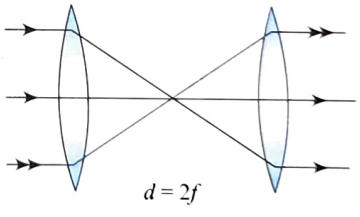

**(b)**

**(c)** If $f < d < 2f$: $F$ will be positive and $< f$, so the system will behave as a convex lens of focal length $> f$ as shown in Fig. (c).



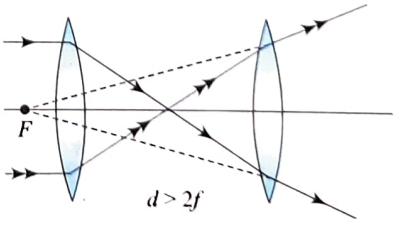

**(c)**

**(d)** If $d = 2f$: $F$ will be infinite, i.e, the system will behave as a plane glass plate of infinite focal length (see Fig. (d)).




**(d)**

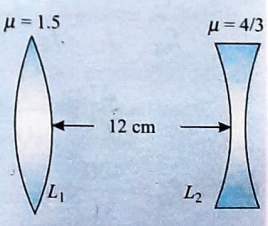
**(e)** If $d > 2f$: $F$ will become negative, i.e., the system will behave as concave lens as shown in Fig. (e).




**(e)**

## ILLUSTRATION 2.39

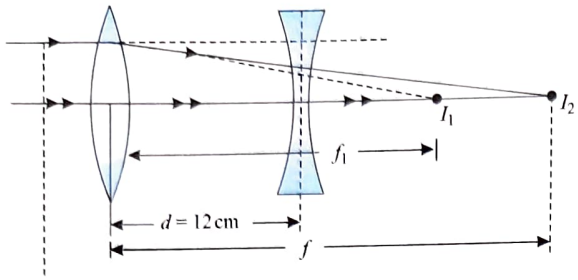
A bi-convex lens $L_1$ of radii of curvature $R_1 = 20$ cm and $R_2 = 30$ cm and RI $= \mu = 1.5$ is kept at a separation of 12 cm from a bi-concave lens $L_2$ whose radii of curvature $R'_1 = 12$ cm, $R'_2 = 24$ cm and RI $= \mu' = 4/3$. Find the focal length of the effective lens.



**Sol.** The focal length of the convex lens is given as

$$\frac{1}{f_1} = (1.5-1)\left(\frac{1}{20}+\frac{1}{30}\right)$$

or $f_1 = 24$ cm




The focal length of the concave lens is given as

$$\frac{1}{f_2} = -\left(\frac{4}{3}-1\right)\left(\frac{1}{12}+\frac{1}{24}\right)$$

$f_2 = -24$ cm

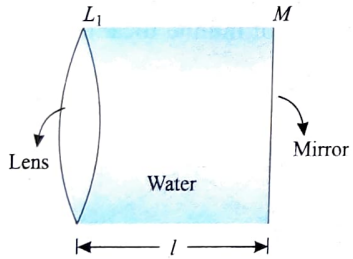
The effective focal length is given as

$$\frac{1}{f} = \frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{\mu f_1 f_2} = \frac{1}{24}+\frac{1}{-24}-\frac{12}{(1)(24)(-24)}$$

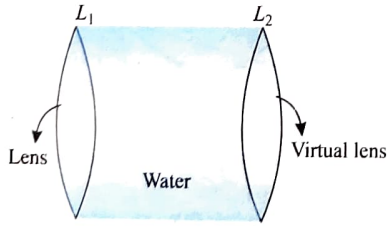
or $f = 48$ cm

### ILLUSTRATION 2.40

A bi-convex lens of focal length 20 cm is placed at a distance $l = 6$ cm from a plane mirror. If the medium between mirror and lens is water, find the equivalent focal length of the system. (The refractive index of the water $\mu_w = \frac{4}{3}$)




**Sol.** The plane mirror will form virtual image of the lens behind the plane mirror. We can consider now two lenses separate by distance $d = 2l$ and the medium between the lenses is water. Hence, the equivalent focal length of the system is given as




$$\frac{1}{f_{eq}} = \frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{\mu f_1 f_2} = \frac{1}{f}+\frac{1}{f}-\frac{2l}{\mu f.f}$$

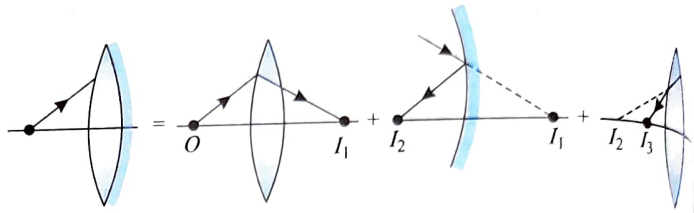
$$= \frac{2}{f}\left(1-\frac{l}{\mu f}\right) = \frac{2}{20}\left(1-\frac{6}{4/3\times 20}\right)$$

Or $f_{eq} = \frac{400}{31}$ cm

## SILVERED LENS

Silvering a surface has the effect of converting the lens into a mirror. In a lens, a ray of light undergoes refraction and emerges on the side opposite to the side of the object. In the case of a silvered lens, after refraction, a ray of light is reflected on the silvered surface and the ray emerges on the same side as the object.

Using the method of superposition, we can treat the system as a combination of lenses and a mirror as shown in figure.




1. First, light will pass through the lens and it will form the image $I_1$.
2. Image $I_1$ will act as an object for silvered surface which acts as curved mirror and forms image $I_2$ of object $I_1$.
3. The light after reflection from silvered surface will again pass through the lens and lens will form final image $I_3$ of object $I_2$.

This is shown in figure. In such a situation, power of the silvered lens will be $P = P_L + P_M + P_L$

With $P_L = \frac{1}{f_L}$, where $\frac{1}{f_L} = (\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

And $P_M = -\frac{1}{f_M}$, where $f_M = \frac{R_2}{2}$

So, the system will behave as a curved mirror of focal length $F$ given by $F = -1/P$.

### ILLUSTRATION 2.41

The radius of curvature of the convex face of a plano-convex lens is 12 cm and its refractive index is 1.5.

**(a)** Find the focal length of this lens. The plane surface of the lens is now silvered.

**(b)** At what distance from the lens will parallel rays incident on the convex face converge?

**(c)** Sketch the ray diagram to locate the image, when a point object is placed on the axis 20 cm from the lens.

**(d)** Calculate the image distance when the object is placed as in (c).

**Sol.**

**(a)** As for a lens, by lensmaker's formula,

$$\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

Here $\mu = 1.5$; $R_1 = 12$ cm; and $R_2 = \infty$

So, $\frac{1}{f} = (1.5-1)\left[\frac{1}{12}-\frac{1}{-\infty}\right]$ i.e., $f = 24$ cm

i.e., the lens is convergent with focal length 24 cm.

**(b)** As light after passing through the lens will be incident on the mirror which will reflect it back through the lens again, so $P = P_L + P_M + P_L = 2P_L + P_M$

But $P_L = \frac{1}{f_L} = \frac{1}{0.24}$ and $P_M = -\frac{1}{\infty} = 0$

$\left[\text{as } f_M = \frac{R}{2} = \infty\right]$

So, $P = 2 \times \frac{1}{0.24} + 0 = \frac{1}{0.12}$ D

The system is equivalent to a concave mirror of focal length $F$.

$$P = -\frac{1}{F}$$

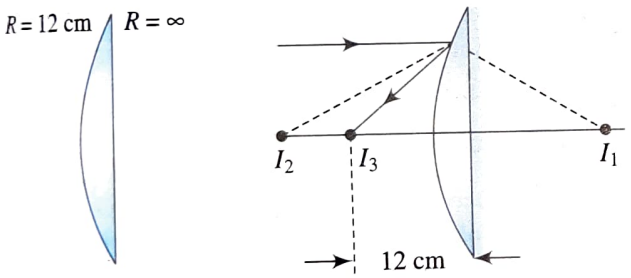
i.e., $F = -\frac{1}{P} = -0.12 \text{ m} = -12 \text{ cm}$

i.e., the system will behave as a concave mirror of focal length 12 cm. So, as for parallel incident rays $u = -\infty$, from mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have

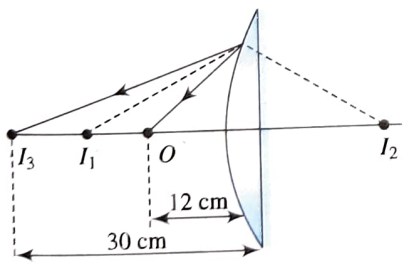
$$\frac{1}{v} + \frac{1}{-\infty} = \frac{1}{-12} \quad \Rightarrow \quad v = -12 \text{ cm}$$

i.e., parallel incident rays will focus at a distance of 12 cm in front of the lens as shown in Figs. (b), (c) and (d). When object is at 20 cm in front of the given silvered lens, which behaves as a concave mirror of focal length 12 cm, from mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we have

$$\frac{1}{v} + \frac{1}{-20} = \frac{1}{-12} \quad \Rightarrow \quad v = -30 \text{ cm}$$




(a) (b)




(c)

i.e., the silvered lens will form an image at a distance of 30 cm in front of it as shown in Fig. (c).

## ILLUSTRATION 2.42

An object in placed 30 cm in front of a concave lens that is made of a glass of refractive index 1.5 and has equal radii of curvature of its two surfaces, each 30 cm. The surface of the lens farther away from the object is silvered. Find the nature and position of the final image.

**Sol.** Focal length of the concave lens, using the lensmaker's formula, is

$$\frac{1}{f_L} = (1.5 - 1)\left(-\frac{1}{30} - \frac{1}{30}\right) \qquad [R_1 = -30 \text{ cm}, R_2 = 30 \text{ cm}]$$

$f_L = -30 \text{ cm} = -0.3 \text{ m}$

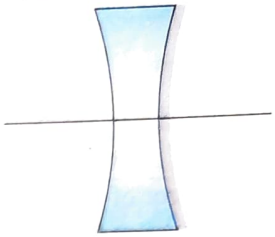
In a silvered lens, light as it enters the lens suffers refraction, then it gets reflected at the silvered surface, and again undergoes refraction as it comes out in air.

In such a situation, power of the silvered lens will be

$$P = P_L + P_M = 2P_L + P_M$$

$$P_L = \frac{1}{f_L} = -\frac{1}{0.3} \text{D}$$

$$P_M = -\frac{1}{f_M} = -\frac{1}{0.15} \text{D}$$



[Silvered surface behaves as a spherical convex mirror of radius of curvature 30 cm so that focal length will be 15 cm = 0.15 m]

$$P = 2P_L + P_M = -\frac{2}{0.3} - \frac{1}{0.15} = -13.33 \text{ D}$$

And focal length of the equivalent mirror,

$$F = -\frac{1}{P} = \frac{-1}{-13.33} = 0.075 \text{ m} = 7.5 \text{ mm}.$$

i.e., the silvered lens behaves as a convex mirror of focal length 7.5 cm.

For an object placed at a distance of 30 cm from the silvered lens, which behaves as a convex mirror of focal length 7.5 cm.

$u = -30$ cm and $f = 7.5$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{30} = \frac{1}{7.5}$$

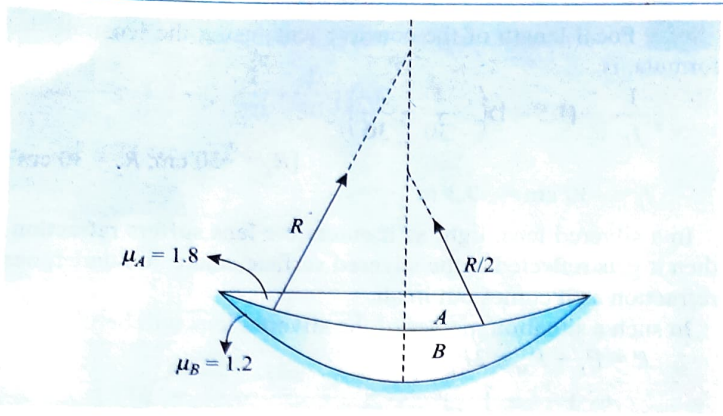
$$\frac{1}{v} = \frac{1}{7.5} + \frac{1}{30}$$

$$\Rightarrow \quad v = 6 \text{ cm}$$

i.e., the image will be 6 cm to the right of the silvered lens and it will be a virtual image [silvered lens behaves as a convex mirror].

## ILLUSTRATION 2.43

A thin plano convex lens $A$ has material of refractive index $\mu_A = 1.8$ and its curved surface has radius of curvature $R$. A thin layer of transparent material $B$ is laid over the curved surface of $A$. The refractive index of $B$ is $\mu_B = 1.2$ and the curved surface of $B$ that is not touching $A$ has radius of curvature $\frac{R}{2}$. This surface of $B$ is silvered. A point object is kept at a distance of 10 cm on the principal axis of the system (above the plane surface) and its image is formed at a distance of 40 cm above the plane surface. Find $R$.




**Sol.** We know the silvering on a surface of the lens has the effect of converting the lens into a mirror. This system also acts as a mirror of focal length $f$.

The system forms the image of an object which is placed 10 cm on the principal axis of the system (above the plane surface) , at a distance of 40 cm above the plane surface

Using mirror formula: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Here $u=-10$ cm, $v=-40$ cm

$$\frac{1}{-40}+\frac{1}{-10}=\frac{1}{f} \Rightarrow f=-8 \text{ cm}$$

The focal length is negative it means the system is like a concave mirror of focal length 8 cm.

Let us calculate the focal lengths and powers of lenses $A$ and $B$ in terms of $R$ (in cm).

For lens $A$, $\frac{1}{f_A}=(\mu_A-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)=\frac{0.8}{R}$

$\therefore \quad f_A=\frac{R}{0.8} \text{ cm}=\frac{R}{80} \text{ m}$

Power of lens $=\frac{1}{f(\text{in m})}$

Hence power of lens $A$, $P_A=\frac{80}{R}\text{D}$

For lens $B$, $\frac{1}{f_B}=(\mu_B-1)\left(\frac{1}{-R}-\frac{1}{-R/2}\right)=\frac{0.2}{R}$

$\therefore \quad f_B=\frac{R}{0.2} \text{ cm}=\frac{R}{20} \text{ m}$

Hence power of lens $B$ , $P_B=\frac{20}{R}\text{D}$

Focal length of mirror surface $f_m=-\frac{R}{4} \text{ cm}=-\frac{R}{400} \text{ m}$

Power of mirror surface. $P_m=-\frac{1}{f_m(\text{in m})}=-\frac{1}{-R/400}=\frac{400}{R}\text{D}$

The power of equivalent mirror formed by the system is given by.

$$P_{eq}=2P_A+2P_B+P_M$$

$$P_{eq}=\frac{2\times 80}{R}+\frac{2\times 20}{R}+\frac{400}{R}=\frac{600}{R}\text{D}$$

Power of mirror $P_{eq}=-\frac{1}{f_{eq}}$

or $\quad f_{eq}=-\frac{1}{P_{eq}}$ or $-\frac{8}{100}=-\frac{1}{600/R} \Rightarrow R=48$ cm

## RELATION BETWEEN OBJECT AND IMAGE VELOCITY

**Case (i). When object is moving along the principal axis:** Let a point object is moving along the principal axis of a lens of focal length $f$. We can relate the object and image positions by the lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Differentiating mirror formula on the both side w.r.t. time we get, $-\frac{1}{v^2}\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0$ (Since focal length of the lens remains constant)

$$\frac{dv}{dt}=+\frac{v^2}{u^2}\frac{du}{dt} \Rightarrow (V_{IL})_{\parallel}=\left(\frac{v^2}{u^2}\right)(V_{OL})_{\parallel}$$

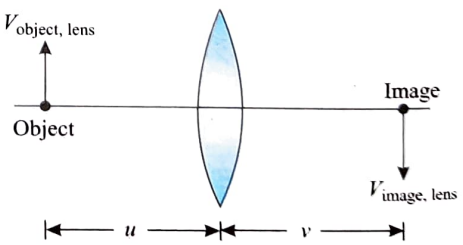
$(V_{IL})_{\parallel}$ = Velocity of image w.r.t. lens along the principal axis.

$(V_{OL})_{\parallel}$ = Velocity of object w.r.t. lens along the principal axis.

**Important Points:**

- Image speed = $m^2$ × Object speed
- In vector notation, Eq. (iii) can be written as $\vec{V}_{\text{image,lens}}=m^2\vec{V}_{\text{object,lens}} \Rightarrow (\vec{V}_{\text{image}}-\vec{V}_{\text{lens}})=m^2(\vec{V}_{\text{object}}-\vec{V}_{\text{lens}})$
- Here it should be noted that the direction of velocity of object and image w.r.t. lens is same to each other.

**Case (ii). When object is moving normal to the principal axis:** In this case a point object is moving normal to the principal axis of a concave mirror as shown in figure. In this situation the object distance will not change, it makes image distance also constant.




For constant values of $v$ and $u$ the value of transverse magnification also constant or $m=\frac{I}{O}=\frac{v}{u}$ is a constant. ...(i)

We can write, $I=mO$

Differentiate Eq. (i) with respect to time, we get

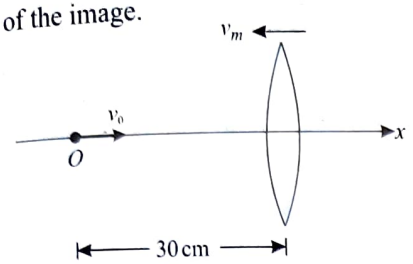
$$\frac{dI}{dt}=m\frac{dO}{dt} \Rightarrow \left(V_{\text{image,lens}}\right)_y=m\left(V_{\text{object,lens}}\right)_y$$

Here $\frac{dI}{dt}=\left(V_{\text{image,lens}}\right)_y$ = velocity of image w.r.t. lens in the direction perpendicular to principal axis,

And, $\frac{dO}{dt} = \left(V_{\text{object,lens}}\right)_y$ = velocity of object w.r.t. lens in the direction perpendicular to principal axis.

**ILLUSTRATION 2.44**

A point object has velocity $v_0 = 2$ cm/s when it is placed at a distance of 30 cm from a lens moving with a speed $v_{\text{lens}} = 2$ cm/s towards left. If the focal length of the lens is $f = 20$ cm, find the velocity of the image.




**Sol.** Velocity of image w.r.t. lens along the principal axis is given by

$$\vec{V}_{\text{image,lens}} = m^2 \vec{V}_{\text{object,lens}}$$

$$\Rightarrow \quad \left(\vec{V}_{\text{image}} - \vec{V}_{\text{lens}}\right) = \left(\frac{v}{u}\right)^2 \left(\vec{V}_{\text{object}} - \vec{V}_{\text{lens}}\right) \qquad \ldots(\text{i})$$

Using: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{(-30)} = \frac{1}{(+20)} \Rightarrow v = +60\,\text{cm}$

Substituting, $u = -30$ cm, $v = +60$ cm, $\vec{V}_{\text{object}} = 2\hat{i}$, $\vec{V}_{\text{lens}} = -2\hat{i}$, is Eq. (i),

$$\vec{V}_{\text{image}} = \left(\frac{+60}{-30}\right)^2 [(2\hat{i}) - (-2\hat{i})] + (-2\hat{i}) = 14\,\hat{i}\ \text{cm/s}$$

Hence image will move with velocity 14 cm/s towards right.

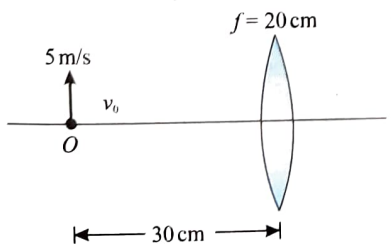
**ILLUSTRATION 2.45**

A point object has velocity $v_0 = 5$ cm/s perpendicular to principal axis of a fixed lens. The object is placed at a distance of 30 cm from the lens of focal length $f = 20$ cm, find the velocity of the image.

**Sol.** The velocity of the image with respect to lens perpendicular to the principal axis :

$$\left(V_{\text{image,lens}}\right)_y = \left(\frac{v}{u}\right)\left(V_{\text{object,lens}}\right)_y \qquad \ldots(\text{i})$$

Apply lens equation: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$




$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{+20} \Rightarrow v = +60\,\text{cm}$$

Substituting, $u = -30$ cm, $v = +60$ cm, $\left(V_{\text{object,lens}}\right)_y = 5$ cm/s, is Eq. (i),

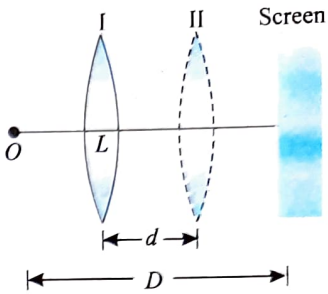
$$\left(V_{\text{image,lens}}\right)_y = \left(\frac{+60}{-30}\right) \times 5 = -10\,\text{cm/s}$$

The image will be moving downwards with velocity 10 cm/s.

# FINDING FOCAL LENGTH OF A CONVEX LENS

## DISPLACEMENT METHOD

Consider a convex lens $L$ placed between an object $O$ and a screen $S$. The distance between the object and the screen is $D$ and the positions of the object and the screen are held fixed. The lens can be moved along the axis of the system and at a position $I$ a sharp image will be formed on the screen. Interestingly, there is another position on the same axis where a sharp image will once again be obtained on the screen. The position is marked as II in figure.




In the figure, let the distance of position $I$ from the object be $x_1$. Then the distance of the screen from the lens is $D - x_1$. Therefore, $u = -x_1$ and $v = +(D - x_1)$.

Substituting in the lens equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{D - x_1} + \frac{1}{x_1} = \frac{1}{f} \qquad \ldots(\text{i})$$

At position II, let the distance of the lens from the screen be $x_2$. Then, the distance of the lens from the object is $D - x_2$. Therefore, $u = -x_2$ and $v = +(D - x_2)$.

Substituting in the lens equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get

$$\frac{1}{D - x_2} + \frac{1}{x_2} = \frac{1}{f} \qquad \ldots(\text{ii})$$

Comparing Eqs. (i) and (ii), we realize that there are only two solutions:

1. $x_1 = x_2$; or
2. $D - x_1 = x_2$ and $D - x_2 = x_1$

The first solution is trivial. Therefore, if the first position of the lens, for a sharp image, is $x_1$ from the object, the second position is at $D - x_1$ from the object.

Let the distance between the two positions I and II be $d$. From the diagram, it is clear that

$$D = x_1 + x_2 \quad \text{and} \quad d = x_2 - x_1 \qquad \ldots(\text{iii})$$

Solving the two equations in (iii), we obtain

$$x_1 = \frac{D - d}{2} \quad \text{and} \quad D - x_1 = \frac{D + d}{2} \qquad \ldots(\text{iv})$$

Substituting Eq. (iv) in Eq. (i), we get $\frac{1}{f} = \frac{2}{D - d} + \frac{2}{D + d}$

or $$f = \frac{D^2 - d^2}{4D} \qquad \ldots(\text{v})$$

Also, $$d = \sqrt{D^2 - 4Df} = \sqrt{D(D - 4f)} \qquad \ldots(\text{vi})$$

We notice from Eq. (vi) that a solution for $d$ is possible only when $D \geq 4f$.

When $D \le 4f$, there is no position for which a sharp image can be formed.

When $D = 4f$, there is only one position where a sharp image is formed.

When $D > 4f$, there are two positions where a sharp image is formed.

The method is suitable for convex lenses only.

What can we say about the size of the object when we know the size of the images?

The magnifications in the first and second positions are

$$m_1 = \frac{h_{i,1}}{h_0} = \frac{D - x_1}{x_1} = \frac{D + d}{D - d}$$

$$m_2 = \frac{h_{i,2}}{h_0} = \frac{D - x_2}{x_2} = \frac{D - d}{D + d}$$

Thus, in the first position the image is magnified while in the second it is reduced. The product of the two magnifications is equal to unity, i.e.,

$$m_1 \times m_2 = \frac{h_{i,1} \times h_{i,2}}{h_0^2} = 1 \quad \text{or} \quad h_0 = \sqrt{h_{i,1} \times h_{i,2}} \qquad ...(vii)$$

The displacement method is not suitable for concave lenses.

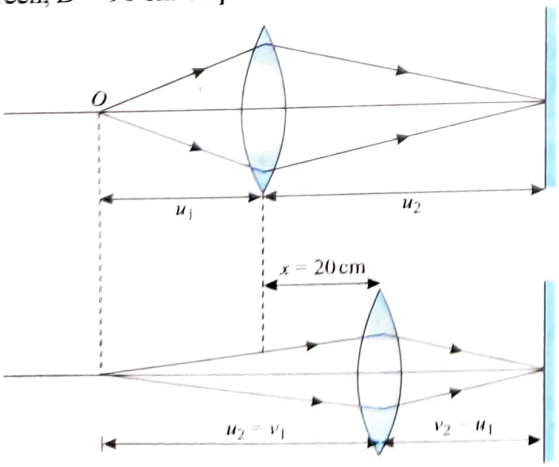
## ILLUSTRATION 2.46

**(a)** A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

**(b)** Suppose the object in (a) above is an illuminated slit in a collimator tube so that it is hard to measure the slit size and its distance from the screen. Using a convex lens one obtains a sharp image of the slit on a screen. The image size is measured to be 4.6 cm. The lens is displaced away from the slit and at a certain location, another sharp image of size 1.7 cm is obtained. Determine the size of the slit.

**Sol.**

**(a)** We are given that distance between the source and the screen, $D = 90$ cm displacement of the lens, $d = 20$ cm




As $f = \dfrac{D^2 - d^2}{4D}$

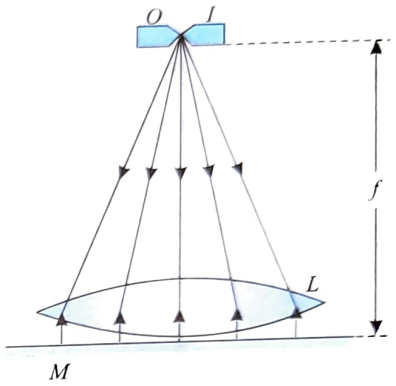
$$f = \frac{(90)^2 - (20)^2}{4 \times 90} = \frac{7700}{360} = 21.4 \text{ cm}$$

**(b)** If $I_1$ and $I_2$ are the sizes of the images of the slit for the two positions of the lens, size of the slit, i.e., object,

$$I_0 = \sqrt{I_1 I_2} = \sqrt{4.6 \times 1.7} = 2.8 \text{ cm}$$

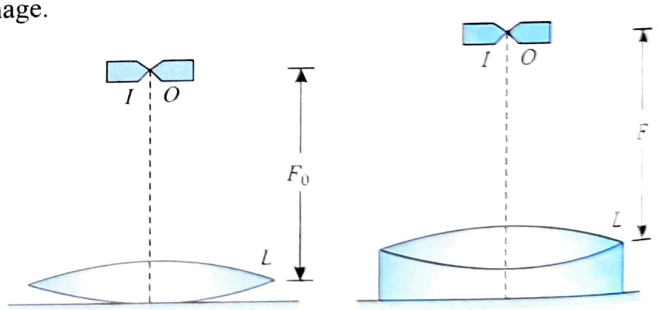
## FINDING FOCAL LENGTH OF A CONVEX LENS

**Plane mirror method:** A convex lens is placed on a plane mirror (figure). Object pin $O$ is moved along the principal axis of the lens. At a certain position, the image coincides with the object. The distance of object pin $O$ from the lens is the focal length of the lens. When an object is placed at the focus of the lens, the refracted rays become parallel to the principal axis. These parallel rays are incident on the mirror normally, therefore, they retrace their path and converge at the focus.




## MEASUREMENT OF REFRACTIVE INDEX OF LIQUID BY A CONVEX LENS

Figure shows an equiconvex lens placed on a plane mirror. An object pin is moved up and down. When the pin lies at the focus of the lens, there is no parallax between the object and the image.




(a) (b)

When a liquid whose refractive index is to be obtained is placed between a plane mirror and a convex lens, object pin $O$ has to be shifted downward so that no parallax exists between it and its image. The position of object $O$ from the lens is now equal to the combined focal length of "lens and liquid" combination. If $f$ is the focal length of the liquid lens, then the combined focal length is given by

$$\frac{1}{F} = \frac{1}{f_0} + \frac{1}{f}$$

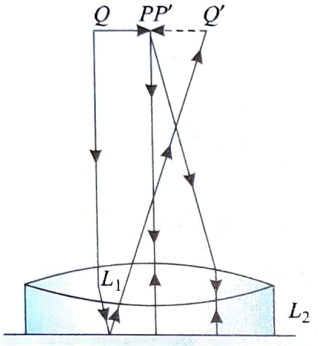
The liquid lens is plano-convex type as its lower surface is the plane surface of the mirror and the upper surface is the curved surface of the convex lens. If $R$ is the radius of curvature of the convex lens, then

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right) \Rightarrow \frac{1}{f}=\frac{(\mu-1)}{R}$$

Thus, $\mu=1+\frac{R}{f}$

**ILLUSTRATION 2.47**

Figure shows an equiconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?




**Sol.** It is clear that, focal length of double convex lens ($L_1$), i.e., $f_1$ = distance of the needle from the lens when liquid is absent = 30 cm focal length of the combination of the double convex lens ($L_1$) and a plano-concave lens ($L_2$) of liquid, i.e., $F$ = distance of the needle from the lens system = 45 cm.

If $f_2$ is the focal length of $L_2$, then as

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\frac{1}{f_2}=\frac{1}{F}-\frac{1}{f_1}=\frac{1}{45}-\frac{1}{30}=-\frac{1}{90}$$

or $f_2=-90$ cm

For lens $L_1$, $\frac{1}{f_1}=({}^{a}\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

As $I_1=+30$ cm, ${}^{a}\mu_g=1.5$, $R_1=R$, $R_2=-R$

$$\frac{1}{+30}=(1.5-1)\left(\frac{1}{R}-\frac{1}{-R}\right)=0.5\times\frac{2}{R}=\frac{1}{R}$$

or $R=30$ cm

For lens $L_2$, $f_2=-90$ cm, $R_1=-30$ cm, $R_2=\infty$

As $\frac{1}{f_2}=({}^{a}\mu_l-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

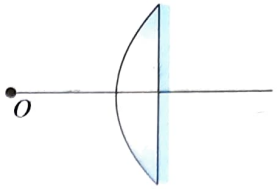
$$\frac{1}{-90}=({}^{a}\mu_l-1)\left(\frac{1}{-30}-\frac{1}{\infty}\right)=({}^{a}\mu_l-1)\left(-\frac{1}{30}\right)$$

or $\frac{1}{90}=({}^{a}\mu_l-1)\left(\frac{1}{30}\right)$

or ${}^{a}\mu_l=1+\frac{1}{3}=1.33$

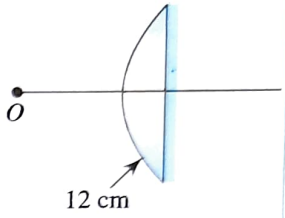
## CONCEPT APPLICATION EXERCISE 2.4

**1.** In figure, if the image of object $O$ has to coincide with itself, then where must the object be placed at a distance from the lens?

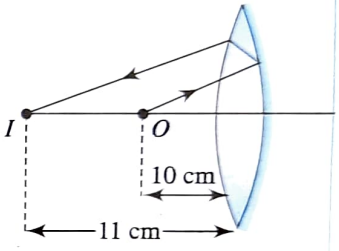



$R = 30$ cm
$\mu = 1.5$

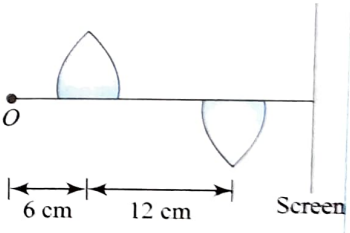
**2.** A plano-convex lens is silvered on its plane side. The radius of curvature of the other face is 12 cm and the refractive index of the material of the lens is 1.5. An object is placed 24 cm in front of the silvered lens. Where will the image be formed?




**3.** A pin is placed 10 cm in front of a convex lens of focal length 20 cm, made of a material having refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature 22 cm. Determine the position of the final image. Is the image real or virtual?




**4.** A convex lens is cut in half along its principal axis and the two halves are separated by a distance of 12 cm. An object is placed 6 cm in front of the lens as shown in figure. Two sharp images are formed on the screen placed 80 cm from the object. What is the focal length of the lens?

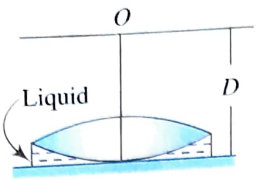



**5.** A concave mirror of focal length 30 cm is placed on the flat horizontal surface with its concave side up. Water with refractive index 1.33 is poured into the lens. Where should an object be placed if its image is to be captured on a screen with a magnification of 2?

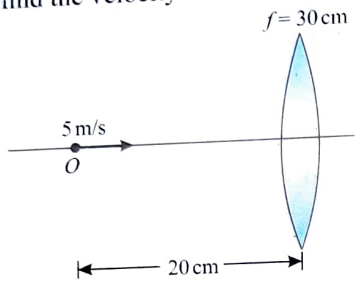
**6.** The convex side of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature of 20 cm. The concave surface has a radius of curvature of 60 cm. What is the focal length of the lens? The convex side is silvered and placed on a horizontal surface. What is the effective focal length of the silvered lens? The concave part is filled with water with refractive index 1.33. What is the effective focal length of the combined glass and water lens? If the convex side is silvered, what is the new effective focal length of the silvered compound lens?

**7.** On a horizontal plane mirror, a thin equiconvex lens of glass is placed and when the space between the lens and

mirror is filled with a liquid, an object held at a distance $D$ which is 30 cm vertically above the lens is found to coincide with its own image as shown in figure. If equiconvex lens of glass has refractive index $\mu = 1.5$ and radius of curvature $R = 20$ cm, then find refractive index of the liquid.




**8.** A point object has velocity $v_0 = 5$ m/s when it is placed at a distance of 20 cm from a lens moving with a speed $v_{lens} = 2$ cm/s towards left. If the focal length of the lens is $f = 30$ cm, find the velocity of the image.




**ANSWERS**

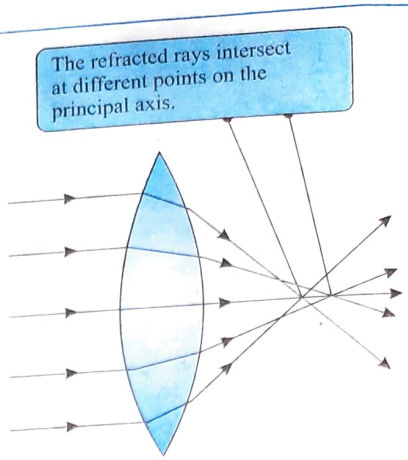
**1.** 60 cm **2.** 24 cm **3.** 11 cm, real
**4.** 19.55 cm **5.** 33.75 cm **7.** 4/3 **8.** 45 cm/s

# LENS ABERRATIONS

A single lens, rather than forming a sharp image, forms an image that is slightly out of focus. This lack of sharpness arises because the rays originating from a single point on the object are not focused to a single point on the image. As a result, each point on the image becomes a small blur. The lack of point-to-point correspondence between object and image is called an **aberration**.

## SPHERICAL ABERRATION

One common type of aberration is **spherical aberration**, and it occurs with converging and diverging lenses made with spherical surfaces. Figure shows how spherical aberration arises with a converging lens. Ideally, all rays traveling parallel to the principal axis are refracted so they cross the axis at the same point after passing through the lens. However, rays far from the principal axis are refracted more by the lens than are those closer in. Consequently, the outer rays cross the axis closer to the lens than do the inner rays, so a lens with spherical aberration does not have a unique focal point. Instead, as the drawing suggests, there is a location along the principal axis where the light converges to the smallest cross-sectional area. This area is circular and is known as the **circle of least confusion**. The circle of least confusion is where the most satisfactory image can be formed by the lens.

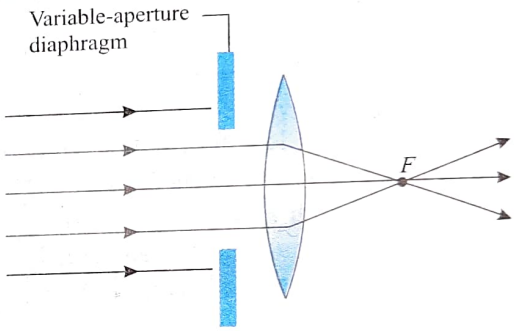



## Methods of Reducing Spherical Aberration

Following are the methods of reducing spherical aberration,

(i) Spherical aberration can be reduced by using a variable-aperture diaphragm or stops in front of the lenses. The stops allow only those rays close to the principal axis to pass through the lens. the paraxial or marginal rays to fall on the lens.

Obviously, by using stops, the brightness of the image is reduced,



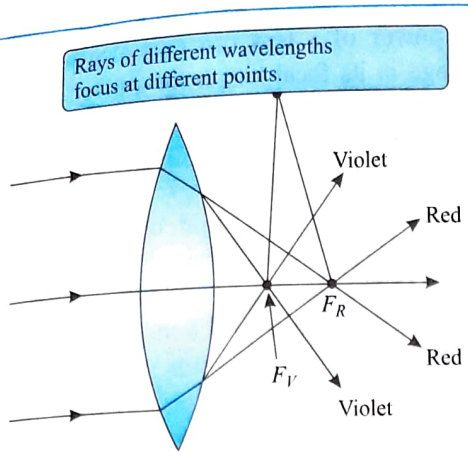

(ii) A plano-convex lens with its curved surface towards the incident beam reduces spherical aberration,

(iii) Lenses with parabolic surfaces are also used to reduce this type of aberration, but they are difficult and expensive to make.

(iv) Spherical aberration can also be reduced to minimum by using two plano-convex lenses separated by a distance equal to the difference in their focal lengths, with their curved surface facing the incident beam.

## CHROMATIC ABERRATION

**Chromatic aberration** also causes blurred images. It arises because the index of refraction of the material from which the lens is made varies with wavelength. Because of this phenomenon, violet rays are refracted more than red rays when white light passes through a lens (figure). The figure shows that the focal length of a lens is greater for red light than for violet light. Other wavelengths have focal points intermediate between those of red and violet, which causes a blurred image and is called **chromatic aberration.** Chromatic aberration for a diverging lens also results in a shorter focal length for violet light than for red light, but on the front side of the lens. Chromatic aberration can be greatly reduced by combining a converging lens made of one type of glass and a diverging lens made of another type of glass.




## Expression for Chromatic Aberration

Let $f_r$, $f_v$ = focal lengths of a lens for red and violet colours respectively,

$\mu_r$, $\mu_v$ = refractive indices of the material of the lens for red and violet colours respectively,

$\mu$, $f$ = mean refractive index of the material of the lens and its mean focal length ,

$\omega$ = dispersive power of the material of the lens,

$R_1$, $R_2$ = radii of curvatures of the surfaces of the lens.

The lens maker's formula: $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \quad \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{(\mu - 1)f} \qquad \ldots(i)$$

For violet colour, $\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(ii)

From (i) and (ii) we get, $\frac{1}{f_v} = \frac{(\mu_v - 1)}{(\mu - 1)f}$ ...(iii)

Similarly, $\frac{1}{f_r} = \frac{(\mu_r - 1)}{(\mu - 1)f}$ ...(iv)

Subtracting Eq. (iv) from Eq. (iii), we get

$$\frac{1}{f_v} - \frac{1}{f_r} = \frac{(\mu_v - 1)}{(\mu - 1)f} - \frac{(\mu_v - 1)}{(\mu - 1)f}$$

Or $$\frac{f_r - f_v}{f_r f_v} = \frac{(\mu_v - 1 - \mu_r + 1)}{(\mu - 1)f}$$

Or $$\frac{f_r - f_v}{f^2} = \frac{\mu_v - \mu_r}{(\mu - 1)} \qquad \left(\text{as } f = \sqrt{f_v f_r}\right)$$

Or $$\frac{f_r - f_v}{f^2} = \frac{\omega}{f}$$

Here $\omega = \frac{\mu_v - \mu_r}{\mu - 1} = \frac{d\mu}{\mu - 1}$ is called dispersive power of the lens.

Thus, $(f_r - f_v) = \omega f$ ...(v)

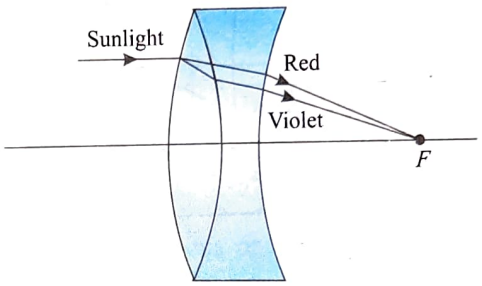
Hence, chromatic aberration = dispersive power × mean focal length

Since focal length of a convex lens is taken as positive and that of a concave lens is taken as negative, therefore, chromatic aberration is positive for a convex lens and negative for a concave lens.

## Achromatism of Lenses

The phenomenon by virtue of which an object seen in white light through an arrangement of lenses appears white, i.e., there is no colour splitting is called **Achromatism**. The arrangement of lenses used for producing achromatism is called an **achromat**.

In case of achromat, as there is no colour splitting, all the colours must be focused at a single point, i.e., there is no chromatic aberration. For achieving this we use a compound lens, such as the combination of a converging lens and a diverging lens shown in figure. Each lens is made from a different type of glass. With this lens combination the red and violet rays almost come to a common focus and, thus, chromatic aberration is reduced. Such a combination is also called an **achromatic combination of lenses**.




### *Condition for Achromatism*

Let us take two lenses $A$ and $B$ for achromatic combination of lenses.

**In case of lens *A***, let $f_r$ and $f_v$ are the focal lengths of a lens for red and violet colours respectively. Also $\mu_r$ and $\mu_v$ refractive indices of the lens materials for red and violet colours respectively.

Let $R_1$ $R_2$ = radii of curvatures of the two surfaces of the lens, and $f$, $\mu$, $\omega$ = mean focal length, refractive index and dispersive power of the material of the lens.

**In case of lens *B***, let the various quantities be represented by the corresponding dashed letters.

The lens maker's formula for lens $A$ can be put as

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{(\mu - 1)f} \qquad \ldots(i)$$

For violet colour, $\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ...(ii)

From (i) and (ii) we get, $\frac{1}{f_v} = \frac{(\mu_v - 1)}{(\mu - 1)f}$ ...(iii)

Similarly, for the lens $B$, we have

$$\frac{1}{f_v'} = \frac{(\mu_v' - 1)}{(\mu' - 1)f'} \qquad \ldots(iv)$$

If $F_v$ represents the focal length of the combination for violet rays, then as

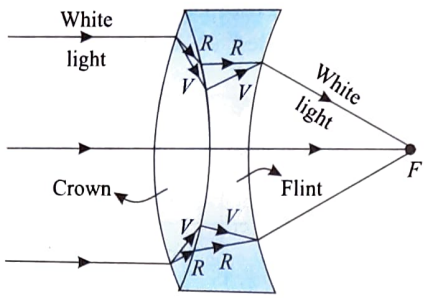
$$\frac{1}{F_v}=\frac{1}{f_v}+\frac{1}{f_v'}$$

$$\frac{1}{F_v}=\frac{(\mu_v-1)}{(\mu-1)f}+\frac{(\mu_v'-1)}{(\mu'-1)f'} \quad ...(v)$$

Similarly, an expression for the focal length of combination for red rays, i.e., $F_r$ can be obtained, viz.

$$\frac{1}{F_r}=\frac{(\mu_r-1)}{(\mu-1)}+\frac{(\mu_r'-1)}{(\mu'-1)f'} \quad ...(vi)$$

For the combination to be achromatic, from figure,




$$F_v=F_r \text{ or } \frac{1}{F_v}=\frac{1}{F_r}$$

From Eqs. (v) and (vi), we obtain

$$\frac{(\mu_v-1)}{(\mu-1)f}+\frac{(\mu_v'-1)}{(\mu'-1)f'}=\frac{(\mu_r-1)}{(\mu-1)f}+\frac{(\mu_r'-1)}{(\mu'-1)f'}$$

or $$\frac{(\mu_v-1)-(\mu_r-1)}{(\mu-1)f}+\frac{(\mu_v'-1)-(\mu_r'-1)}{(\mu'-1)f'}=0$$

or $$\frac{(\mu_v-\mu_r)}{(\mu-1)f}+\frac{(\mu_v'-\mu_r')}{(\mu'-1)f'}=0$$

But as $\frac{(\mu_v-\mu_r)}{(\mu-1)}=\omega$ and $\frac{(\mu_v'-\mu_r')}{(\mu'-1)}=\omega'$,

$$\frac{\omega}{f}+\frac{\omega'}{f'}=0 \quad ...(vii)$$

which is the required condition for a combination of lenses to be achromatic.

We know that $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Differentiating, $-\frac{df}{f^2}=d\mu\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

Or $$-\frac{df}{f^2}=\frac{d\mu}{(\mu-1)f} \qquad \left(\text{as } \frac{1}{R_1}-\frac{1}{R_2}=\frac{1}{(\mu-1)f}\right)$$

Or $$\frac{d\mu}{\mu-1}=-\frac{df}{f}$$

Or $$\omega=-\frac{df}{f} \qquad \left(\text{as } \omega=\frac{d\mu}{\mu-1}\right)$$

**Dispersive power** of a lens can thus also be defined as the relative change in its focal length for a given range of colour corresponding to a change in the refractive index.

**Important Points:**

- The condition (vii) is valid only for removing only longitudinal chromatic aberration. Therefore, lateral chromatic aberration still persists in an achromatic doublet satisfying this condition.
- As in Eq. (vii), $\frac{\omega}{f}=\frac{\omega'}{f'}$

  As $\omega$ and $\omega'$ both are positive, $f$ and $f'$ should be of opposite signs to satisfy the condition for achromatism, i.e., if $f$ is positive, $f'$ should be negative. Thus, the lens A with focal length $f$, say, should be convex and the lens B with focal length $f'$ should be concave.
- In Eq. (vii) if $f<f'$ then $\omega<\omega'$. Actually, dispersive power of crown glass is less than the dispersive power of flint glass. Therefore, convex lens should be of crown glass and concave lens should be of flint glass. The radii of curvatures of the two surfaces which are to be in contact are made equal so that they can be cemented properly.

**ILLUSTRATION 2.48**

Two thin lenses, one of crown glass and the other of flint glass are put in contact to give an achromatic system of focal length 50 cm. Find the focal lengths of the two lenses separately. Dispersive powers of crown glass and flint glass are 0.0163 and 0.0243 respectively.

**Sol.** Here, dispersive power of crown glass, $\omega = 0.0163$
Dispersive power of flint glass, $\omega' = 0.0243$
Focal length of the achromatic system, $F = 50$ cm
Let $f$ and $f'$ be focal lengths of the crown glass and flint glass lenses respectively.

As $$\frac{1}{F}=\frac{1}{f}+\frac{1}{f'}$$

$$\frac{1}{50}=\frac{1}{f}+\frac{1}{f'} \quad ...(i)$$

And for achromatic combination,

$$\frac{\omega}{f}+\frac{\omega'}{f'}=0$$

or $$\frac{0.0163}{f}+\frac{0.0243}{f'}=0$$

$$\Rightarrow \quad f'=-\frac{243}{163}f \quad ...(ii)$$

Putting the value of $f'$ in Eq. (i), we have

$$\frac{1}{50}=\frac{1}{f}-\frac{163}{243f}=\frac{243-163}{243f}=\frac{80}{243f}$$

$$\Rightarrow \quad f=\frac{4000}{243}=16.43 \text{ cm}$$

Putting this value of $f$ in Eq. (ii), we get

$$f' = -\frac{243}{163}\times\frac{4000}{243} = -\frac{4000}{163} = -24.54 \text{ cm}$$

Negative sign shows that the flint glass lens is concave.

## OPTICAL INSTRUMENTS

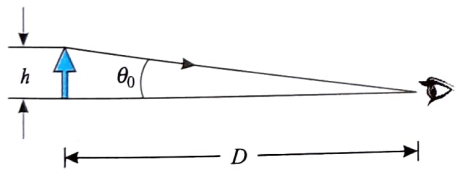
**Human eye** is a natural **optical instrument** provided by nature. But the range of this optical instrument is very limited. Its range can be extended in many ways by optical instruments such as eyeglasses, microscopes, and telescopes.

The mirror and thin-lens formulas can be applied only as approximations to most sophisticated optical instruments as lenses in typical laboratory microscopes are by no means "thin." In most optical instruments the lenses are compound lenses; that is, they are made of several components, the interfaces rarely being exactly spherical. Now we discuss three optical instruments, assuming, for simplicity, that the thin-lens formulas apply.

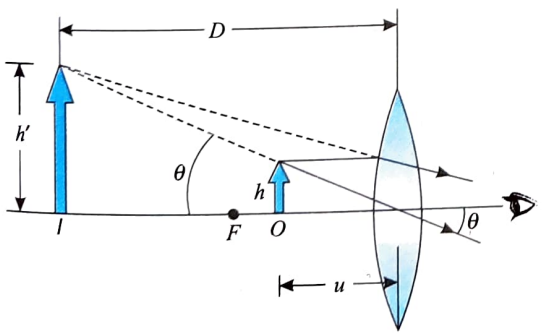
### SIMPLE MICROSCOPE

The simple magnifier, or magnifying glass, consists of a single converging lens of short focal length. This device increases the apparent size of an object. When an object is placed between the lens and its focus, a virtual, erect and magnified image is formed on the same side as the object. The distance of a lens from the object is adjusted till the image is formed at the least distance of distinct vision ($D$ = 25 cm) from the eye. The eye is held close to the lens.

Suppose an object is viewed at some distance $u$ from the eye as illustrated in figure. The size of the image formed at the retina depends on the angle $\theta$ subtended by the object at the eye. As the object moves closer to the eye, $\theta$ increases and a larger image is observed. An average normal human eye, however, cannot focus on an object closer than about $D$ = 25 cm, the near point [Fig. (a)]. Therefore, $\theta$ is maximum at the near point.




(a)




(b)

To further increase the apparent angular size of an object, a converging lens can be placed in front of the eye as in Fig. (b), with the object located at point $O$, immediately inside the focal point of the lens. At this location, the lens forms a virtual, upright, enlarged image. We define **angular magnification** $M$ as the ratio of the angle subtended by an object with a lens in use [angle $\theta$ in Fig. (b)] to the angle subtended by the object placed at the near point with no lens in use [angle $\theta_0$ in Fig. (a)]:

$$M = \frac{\theta}{\theta_0} \qquad \ldots(\text{i})$$

The angular magnification is a maximum when the image is at the near point of the eye, that is, when $v = -D$. The object distance corresponding to this image distance can be calculated from the thin lens equation:

$$\frac{1}{(-D)} - \frac{1}{(-u)} = \frac{1}{(+f)} \Rightarrow u = \frac{Df}{D+f} \qquad \ldots(\text{ii})$$

where $f$ is the focal length of the magnifier in centimeters. If we make the small-angle approximations

$$\tan\theta_0 \approx \theta_0 \approx \frac{h}{D} \quad \text{and} \quad \tan\theta \approx \theta \approx \frac{h}{u} \qquad \ldots(\text{iii})$$

From Eqs. (i), (ii) and (iii),

$$M_{max} = \frac{\theta}{\theta_0} = \frac{h/u}{h/D} = \frac{D}{u} = \frac{D}{Df/(D+f)}$$

$$M_{max} = 1 + \frac{D}{f} \qquad \ldots(\text{iv})$$

Although the eye can focus on an image formed anywhere between the near point and infinity, it is most relaxed when the image is at infinity. For the image formed by the magnifying lens to appear at infinity, the object has to be at the focal point of the lens. In this case, Eq. (ii) become

$$\theta_0 \approx \frac{h}{D} \quad \text{and} \quad \theta \approx \frac{h}{f}$$

and the magnification is

$$M_{min} = \frac{\theta}{\theta_0} = \frac{D}{f} \qquad \ldots(\text{v})$$

Equation (v) gives the magnifying power of a simple microscope, when the image is formed at infinity.

**ILLUSTRATION 2.49**

A detective uses a converging lens of focal length 12 cm to examine the fine details of some cloth fibers found at the scene of a crime.

**(a)** What is the maximum magnification given by the lens?

**(b)** What is the magnification for relaxed eye viewing?

**Sol.**

**(a)** Here, $f$ = 12 cm

The maximum magnification occurs, when the image is formed by the lens at the least distance of distinct vision i.e. at $D$ = 25 cm.

Therefore, maximum magnification given by the lens,

$$M = 1 + \frac{D}{f} = 1 + \frac{25}{12} = 1 + 2.1 = 3.1$$

**(b)** The eye is most relaxed, when the object lies at infinity. Therefore, magnification for relaxed eye viewing,

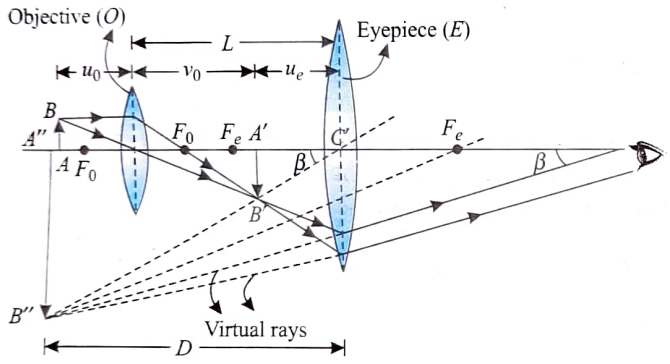
$$M = \frac{D}{f} = \frac{25}{12} = 2.1$$

## COMPOUND MICROSCOPE

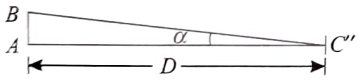
A simple magnifier provides only limited assistance in inspecting minute details of an object. Greater magnification can be achieved by combining two lenses in a device called a **compound microscope** shown in Fig. (a). A compound microscope consists of two convex lenses placed coaxially within two sliding tubes. The lens which faces the object is called the objective. The lens through which the final image is seen is called the **eyepiece.** The objective has a short focal length and short aperture. The eyepiece also has a short focal length (though somewhat more than that of the objective) but large aperture.

The object $AB$ to be magnified is placed a little beyond the principal focus $F_0$ of the objective lens. A real, inverted and magnified image $A'B'$ is formed in front of eyepiece. The position of the eyepiece is so adjusted that the image $A'B'$ falls within its focal length. The eyepiece then merely acts as a magnifying glass for which $A'B'$ is the object. A large, erect and virtual image $A''B''$ is formed on the same side as $A'B'$. By adjusting the position of the eyepiece, $A'B''$ is brought to focus at the distance of distinct vision from the eye, where eye is placed close to the eyepiece.

**Course of rays**: Figure (a) shows the course of rays in a compound microscope.




(a)




(b)

### Angular Magnification or Magnifying Power ($M$)

The **magnifying power** of a compound microscope is defined as the ratio of the angle subtended by the final image at the eye to the angle subtended by the object when both are placed at the distance of distinct vision.

Let $m_o$, $m_e$ = magnifications produced by objective and eyepiece respectively,

$\alpha$, $\beta$ = angles subtended at the eye by the object $AB$ [when placed at $A''B''$ as shown in Fig. (b)] and the image $A''B''$ respectively,

$u_o$, $v_o$ = distances of the object $AB$ and the image $A'B'$ from the objective,

$D$ = least distance of distinct vision for the eye,

$f_0$, $f_e$ = lengths of the objective and eyepiece respectively, and

$M$ = magnifying power of the compound microscope.

By definition, $M = \dfrac{\beta}{\alpha}$

As angles $\alpha$ and $\beta$ are small, $\alpha \approx \tan\alpha$ and $\beta \approx \tan\beta$

Thus, $M = \dfrac{\tan\beta}{\tan\alpha} = \dfrac{A''B''/D}{AB/D} = \dfrac{A''B''}{AB}$

Similarly, $m_0 = \dfrac{A''B''}{AB}$ and $m_e = \dfrac{A''B''}{A'B'}$

Thus, $M = \dfrac{A''B''}{AB} = \dfrac{A'B'}{AB} \times \dfrac{A''B''}{A'B'} = m_0 \times m_e$

But $m_0 = \dfrac{A'B'}{AB} = \dfrac{v_0}{u_0}$

and $m_e = \dfrac{A''B''}{A'B'} = \dfrac{v_e}{u_e} = \dfrac{D}{A'C'}$

($v_e$ is numerically equal to $D$ and $u_e = A'C'$)

According to lens formula, for the eyepiece,

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

Or $$\frac{1}{f_e} = \frac{1}{-D} - \frac{1}{-A'C'}$$

(as $v_e = -D$ because the image $A'B'$ is virtual and $u_e = -A'C'$)

Or $$\frac{D}{f_e} = -1 + \frac{D}{A'C'} \quad \text{or} \quad \frac{D}{f_e} = -1 + m_e$$

Or $$m_e = 1 + \frac{D}{f_e} \qquad \ldots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) \qquad \ldots(iv)$$

In case the final image is formed at infinity, $m_e = \dfrac{D}{f_e}$ and as such

$$M = m_0 m_e = \frac{v_0 D}{u_0 f_e} \qquad \ldots(v)$$

**Important Point:**

Generally, the object lies close to the focus of $O$ (i.e., at $f_0$), $u_o \to f_o$. In such a ease, the image $A'B'$ lies very close to the eyepiece, i.e., $v_o \to L$ (length of the tube). From Eq. (iv),

$$M = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) \qquad \ldots(vi)$$

Thus, to have a large magnifying power, both $f_o$ and $f_e$ should be small.

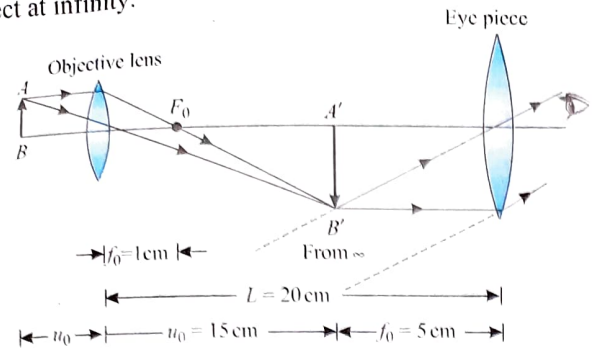
In case the final image is formed at infinity, from Eq. (v),

$$M = \frac{LD}{f_0 f_e} \qquad \ldots(vii)$$

**ILLUSTRATION 2.50**

The focal length of the eye-piece and objective of a compound microscope are 5 cm and 1 cm respectively and the length of the tube is 20 cm. Calculate the magnifying power of the microscope, when the final image is formed at infinity. The value of least distance of distinct vision is 25 cm.

**Sol.** Figure shows the formation of the final image of the object at infinity:




When the object $AB$ is placed at the distance $CA' = u_0$ from the objective, its real image $A'B'$ is formed on the other side of the lens at a distance $CA' = v_0$ from it. The final image will be formed at infinity, if the image lies at distance $C'A' = f_e = 5$ cm from the eye lens. Since the length of the microscope tube is $L = 20$ cm, it follows that

$$v_0 = L - f_e = 20 - 5 = 15\,\text{cm}$$

For objective lens: $\dfrac{1}{u_0} + \dfrac{1}{v_0} = \dfrac{1}{f_0}$

Here, $f_0 = 1$ cm; $v_0 = 15$ cm

$$\therefore \quad \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{15} - \frac{1}{1} = -\frac{14}{15}$$

$$\text{or} \quad u_0 = -\frac{15}{14}\,\text{cm}$$

When the final image is formed at infinity, the magnifying power of the microscope is given by

$$M = \frac{v_0}{u_0} \times \frac{D}{f_e}$$

Here, $D = 25$ cm; $f_e = 5$ cm

$$\therefore \quad M = \frac{15}{-15/14} \times \frac{25}{5} = -70$$

The negative sign shows that the image formed is inverted and real.

**ILLUSTRATION 2.51**

A compound microscope uses an objective lens of focal length 4 cm and eye lens of focal length 10 cm. An object is placed 6 cm from the objective lens.

**(a)** Calculate the magnifying power of the compound microscope if the final image is formed at the near point.

**(b)** Calculate the length of the compound microscope.

**Sol.**

**(a)** Here; $f_o = $ cm, $f_e = 10$ cm, $u_0 = -6$ cm, $v_e = -D = -25$ cm

As $\dfrac{1}{f_e} = \dfrac{1}{v_0} - \dfrac{1}{u_0}, \dfrac{1}{v_0} = \dfrac{1}{f_0} + \dfrac{1}{u_0} = \dfrac{1}{4} + \dfrac{1}{-6} = \dfrac{1}{12}$

Or $v_0 = 12\,\text{cm}$

Magnifying power of the compound microscope when the image is formed at the near point, i.e.,

$$M = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right) = \frac{12}{6}\left(1 + \frac{25}{10}\right) = 7$$

**(b)** As $\dfrac{1}{f_e} = \dfrac{1}{v_e} - \dfrac{1}{u_e}, \dfrac{1}{u_e} = \dfrac{1}{v_e} - \dfrac{1}{f_e} = \dfrac{1}{-25} - \dfrac{1}{10} = -\dfrac{7}{10}$

or $u_e = -\dfrac{50}{7}\,\text{cm} = -7.14$ cm

or $|u_e| = 7.14\,\text{cm}$

Length of the compound microscope,

$$L = v_0 + |u_e| = 12\text{ cm} + 7.14\text{ cm} = 19.14\text{ cm}$$

**ILLUSTRATION 2.52**

A compound microscope consists of an objective lens of focal length 2.0 cm and an eyepiece of focal length 6.25 cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at:

**(a)** the least distance of distinct vision (25 cm)

**(b)** infinity?

What is the magnifying power of the microscope in each case?

**Sol.**

**(a)** Here, $D = 25$ cm, $f_o = 2.0$ cm, $f_e = 6.25$ cm, $L = 15$ cm.

For eyepiece, $\dfrac{1}{f_e} = \dfrac{1}{v_e} - \dfrac{1}{u_e}, \dfrac{1}{u_e} = \dfrac{1}{v_e} - \dfrac{1}{f_e} = \dfrac{1}{-25} - \dfrac{1}{6.25}$

(as $v_e = -D = -25$ cm)

or $\dfrac{1}{u_e} = -\dfrac{1}{5}$ or $u_e = -5$ cm

As $v_o + |u_e| = L = 15$ cm, $v_o = 15 - |u_e| = 15 - 5 = 10$ cm

Again, as for the objective, $\dfrac{1}{f_0} = \dfrac{1}{v_0} - \dfrac{1}{u_0}$,

$$\frac{1}{2} = \frac{1}{10} - \frac{1}{u_0} \quad \text{or} \quad \frac{1}{u_0} = \frac{1}{10} - \frac{1}{2} = -\frac{4}{10}$$

or $u_o = -2.5$ cm

Thus, the object should be placed at a distance 2.5 cm in front of the objective.

Magnifying power of the microscope,

$$M = \frac{v_0}{|u_0|}\left(1 + \frac{D}{f_e}\right) = \frac{10}{2.5}\left(1 + \frac{25}{6.25}\right) = 20$$

**(b)** For the eyepiece, $\dfrac{1}{f_e} = \dfrac{1}{v_e} - \dfrac{1}{u_e}$

or $\dfrac{1}{6.25} = \dfrac{1}{\infty} - \dfrac{1}{u_e}$ (as $v_e = \infty$)

or $u_e = -6.25$ cm

As $v_o + |u_e| = L = 15$ cm

$v_o = 15 - 6.25 = 8.75$ cm

For the objective, $\dfrac{1}{f_0} = \dfrac{1}{v_0} - \dfrac{1}{u_0}$

$$\text{or} \quad \frac{1}{2} = \frac{1}{8.75} - \frac{1}{u_0} \quad \text{or} \quad \frac{1}{u_0} = \frac{1}{8.75} - \frac{1}{2}$$

or $u_o = -2.59$ cm

Distance of the object lying in front of the objective $|u_o| = 2.59$ cm

Magnifying power of the microscope,

$$M = \frac{v_0}{|u_0|}\frac{D}{f_e} = \left(\frac{8.75}{2.59}\right)\left(\frac{25}{6.25}\right) = 13.5$$

**ILLUSTRATION 2.53**

A person with a normal near point (25 cm) using a compound microscope with an objective of focal length 8.0 mm and an eyepiece of focal length 2.5 cm can bring an object placed 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? How much is the magnifying power of the microscope?

**Sol.** With usual notation, we are given that $D = 25$ cm, $f_0 = 8.0$ mm $= 0.8$ cm, $f_e = 2.5$ cm, $u_0 = -9.0$ mm $= -0.9$ cm.

For the objective, $\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$

Or $\frac{1}{0.8} = \frac{1}{v_0} - \frac{1}{-0.9} = \frac{1}{v_0} + \frac{1}{0.9}$

Or $\frac{1}{v_0} = \frac{1}{0.8} - \frac{1}{0.9}$

Or $v_0 = 7.2$ cm

For the eyepiece, $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$

Or $\frac{1}{2.5} = \frac{1}{-2.5} - \frac{1}{u_e}$ (as $v_e = -D = -25$ cm)

Or $u_e = -\frac{25}{11} = -2.27$ cm

Thus, separation between the objective and the eyepiece

$= v_0 + |u_e| = 7.2 + 2.27 = 9.47$ cm

Magnifying power of the microscope, i.e.,

$$M = \frac{v_0}{|u_0|}\left(1 + \frac{D}{f_e}\right) = \frac{7.2}{0.9}\left(1 + \frac{25}{2.5}\right) = 88$$

## TELESCOPE

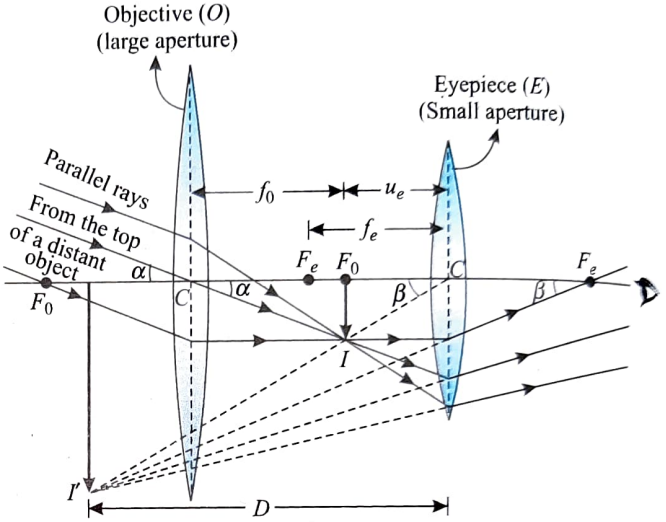
A telescope is an instrument for magnifying distant objects, such as stars and planets. Like a microscope, a telescope consists of an objective and an eyepiece (also called the ocular). It is of two types:

1. **Astronomical telescope:** A telescope used to see the heavenly bodies is called an astronomical telescope. It produces a real and inverted image. As the heavenly bodies are round, the inverted image does not affect the observation.
2. **Terrestrial telescope:** For observing objects on earth, it would be necessary to have an erect image. A telescope used for observing objects on the earth is called a terrestrial telescope.

### Astronomical Telescope

An astronomical telescope consists of two lens systems. The lens system facing the object is called *objective*. It has large aperture and is of *large focal length* ($f_o$). The other lens system is called *eye-piece*. It has *small aperture* and is of *short focal length* ($f_e$). The objective and the eye-piece are mounted coaxially in two metallic tubes. The tube holding the eye-piece can be made to slide inside the tube holding the objective .

Light rays from the distant object form the real and inverted image $I$ at $F_o$, where $F_o$ is the principal focus of the objective $O$. The position of the eyepiece $E$ is so adjusted that the image $I$ falls within its focal length. $F_e$ is the principal focus of $E$. A large, erect and virtual image $I'$ (of $I$) is formed. By adjusting the eyepiece, the final image $I'$ is made to form at the least distance of distinct vision from the eye (figure).




### *Angular Magnification or Magnifying Power (M)*

The magnifying power of a telescope is defined as the ratio of the angle subtended by the final image at the eye as seen through the telescope to the angle subtended by the distant object at the unaided eye.

Let $M$ = magnifying power of the telescope, $\alpha$, $\beta$ = angles subtended by the object and the final image respectively at the unaided eye,

By definition, $M = \frac{\beta}{\alpha}$

As angles $\alpha$ and $\beta$ are small, $\alpha \approx \tan\alpha$ and $\beta \approx \tan\beta$.

Thus, $M = \frac{\tan\beta}{\tan\alpha} = \frac{IF_0 / F_0C'}{IF_0 / F_0C} = \frac{F_0C}{F_0C'}$ ...(i)

i.e., $M = \frac{\text{focal length of the objective}}{\text{distance of the real image from the eye}}$ ...(ii)

(a) **When the final image is formed at infinity (Normal adjustment):** A telescope is said to be in normal adjustment if the final image is formed at infinity.

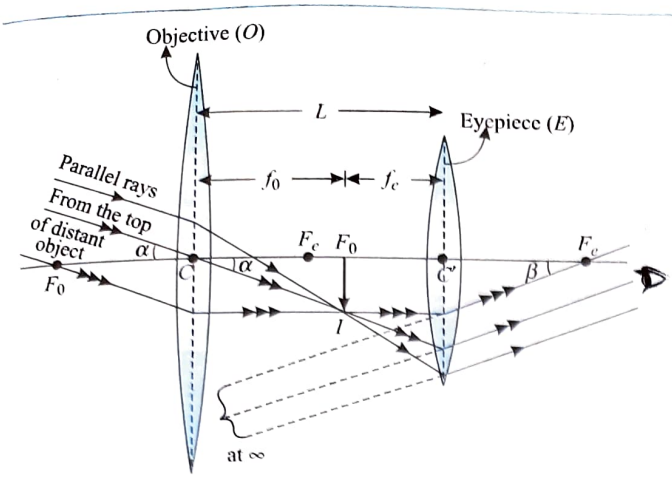
The final image is formed at infinity if the image $IF_0$ is formed at $F_e$ or in other words if $F_o$ and $F_e$ coincide as shown in figure.

But $F_oC = f_o$ (focal length of the objective)

$F_oC = F_eC = f_e$ (focal length of the eyepiece)

From Eq. (ii),

$$M = \frac{f_0}{f_e} = \frac{\text{focal length of the objective}}{\text{focal length of the eyepiece}} \quad \text{...(iii)}$$




In order to increase the magnifying power :
(i) $f_o$ should be large and
(ii) $f_e$ should be small.

**Note:** Length of tube of the telescope, $L = f_o + f_e$.

(b) **When the final image is formed at least distance of distinct vision.**

Equation (ii) can be expressed in another way also as follows.

For $E$, $u_e = -F_0C'$ and $v_e = -D$

Thus, $$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{-D} - \frac{1}{-F_0C'}$$

or $$\frac{1}{F_0C'} = \frac{1}{f_e} + \frac{1}{D}$$

From Eq. (i), $$M = \frac{F_0C}{F_0C'} = F_0C \times \frac{1}{F_0C'} = f_0\left(\frac{1}{f_e} + \frac{1}{D}\right)$$

or $$M = \frac{f_0}{f_e} + \frac{f_0}{D} \quad \ldots(iv)$$

## ILLUSTRATION 2.54

A telescope has an objective of focal length 50 cm and eye-piece of focal length 5 cm. The least distance of distinct vision is 25 cm. The telescope is focused for distinct vision on a scale 200 cm away from the objective. Calculate **(a)** the separation between the objective and eyepiece **(b)** the magnification produced.

**Sol.**

**(a)** Here, $f_e = 5$ cm; $f_o = 50$ cm; $u_o = -200$ cm;

**Image formed by the objective:** If the objective forms the image at a distance $v_0$ from the objective, then

$$\frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{50} + \frac{1}{-200} = \frac{3}{200}$$

or $$v_0 = +\frac{200}{3}\text{cm}$$

**Image formed by the eye-piece:** For eye-piece, the image formed by objective acts as object and its virtual image is produced by the eye-piece at the least distance of distinct vision. If the image formed by objective is at a distance $u_e$ from the eye-piece, then

$$-\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e}$$

Here, $v_e = -D = -25$ cm; $f_e = 5$ cm

$$\therefore \quad \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{5} = -\frac{6}{25}$$

or $$u_e = -\frac{25}{6}\text{cm}$$

Therefore, separation between objective and eye-piece,

$$L = |v_0| + |u_e| = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} = 70.83 \text{ cm}$$

**(b)** Now, $$m_0 = \frac{v_0}{u_0} = \frac{200/3}{-200} = -\frac{1}{3}$$

and $$m_e = \frac{v_e}{u_e} = \frac{-25}{-25/6} = 6$$

Magnification produced by the telescope,

$$m = m_0 \times m_e = -\frac{1}{3} \times 6 = -2$$

The negative sign shows that final image formed is real and inverted.

## ILLUSTRATION 2.55

**(a)** A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?

**(b)** If this telescope is used to view the Moon, what is the diameter of the image of the Moon formed by the objective lens? The diameter of the Moon is $3.48 \times 10^6$ m and the radius of lunar orbit is $3.8 \times 10^8$ m.

**Sol.** Here, $f_o = 15$ m, $f_e = 1.0$ cm $= 0.01$ m

**(a)** Angular magnification, $$M = \frac{f_0}{f_e} = \frac{15\text{m}}{0.01\text{m}} = 1500$$

**(b)** Angle subtended by the image, $IF_o$ (produced by the objective) on the objective, i.e.,

$$\alpha = \frac{IF_0}{F_0C} = \frac{d}{f_0} = \frac{d}{15} \quad \ldots(i)$$

where $IF_o = d$ (diameter of the image of the Moon) and $F_oC = f_o$ (focal length of the objective)

Angle subtended by the moon on the objective, i.e.,

$$\alpha = \frac{\text{diameter of Moon}}{\text{radius of lunar orbit}} \quad \ldots(ii)$$

$$= \frac{3.84 \times 10^6\text{m}}{3.8 \times 10^8\text{m}} = 0.9158 \times 10^{-2} \text{ rad}$$

From Eqs. (i) and (ii), $$\frac{d}{15\text{m}} = 0.9158 \times 10^{-2} \text{ rad}$$

or $d = (0.9158 \times 10^{-2})\ 15 \text{ m} = 13.7 \times 10^{-2} = 13.7$ cm

## ILLUSTRATION 2.56

A telescope objective of focal length 1 m forms a real image of the moon 0.92 cm in diameter. Calculate the diameter of the moon taking its mean distance from the earth to be $38 \times 10^4$ km.

If the telescope uses an eyepiece of 5 cm focal length, what would be the distance between the two lenses for **(i)** the final image to be formed at infinity, **(ii)** the final image (virtual) at 25 cm from the eye.

**Sol.** Here, $f_o = 1$ m; $f_e = 5$ cm $= 0.05$ m

Size of the image of the moon,

$$I = 0.92 \text{ cm} = 0.92 \times 10^{-2} \text{ m}$$

Mean distance of the moon from the earth,

$$u_o = -38 \times 10^4 \text{ km} = -38 \times 10^{-7} \text{ m}$$

Let $d$ be the diameter of the moon.

Both the moon and its image form the same angle($\alpha$) at the objective.

Therefore, the angle subtended by the moon at the objective,

$$\alpha = \frac{d}{|u_0|} \quad \ldots(i)$$

and the angle subtended by the image of the moon at the objective,

$$\alpha = \frac{I}{f_0} \quad \ldots.(ii)$$

From Eqs. (i) and (ii), we have

$$d = 3.496 \times 10^6 \text{ m} = 3{,}496 \text{ km}$$

**(i)** When the final image is formed at infinity, then the distance between the two lenses is equal to sum of their focal lengths i.e., $L = f_o + f_e = 1 + 0.05 = 1.05$ cm $= 105$ cm

**(ii)** For the image formed by eye piece:

$f_e = 5$ cm; $v = -D = -25$ cm

If $u_e$ is the distance of the image of moon formed by the objective from the eye piece, then

$$-\frac{1}{u_e} + \frac{1}{v_e} = \frac{1}{f_e}$$

or $$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{5} = -\frac{6}{25}$$

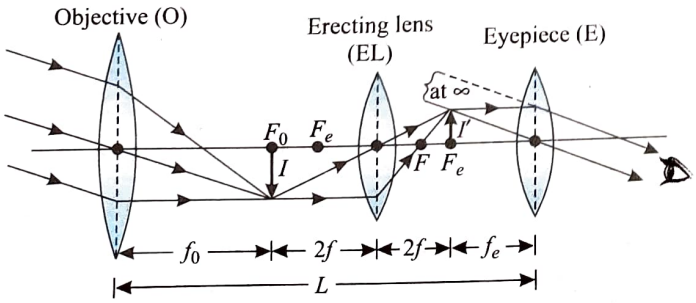
or $$u_e = -\frac{25}{6} = -4.17 \text{ cm}$$

Therefore, the distance between the two lenses,

$$L' = f_0 + |u_e| = 100 + 4.17 = 104.17 \text{ cm}$$

## Terrestrial Telescope

A telescope which forms an erect image is called the terrestrial telescope. An astronomical telescope is not suited for observation of terrestrial objects as the final image is inverted. To get rid of this troublesome drawback, another convergent lens, called the **erecting lens** is inserted between the objective and the eyepiece. In figure, $O$ is the objective,




E is the eyepiece and EL is the erecting lens. These lenses are mounted coaxially.

(i) The image $I$ formed by $O$ is at a distance $2f$ from EL where $f$ is its focal length. EL, therefore, forms a real, inverted image $I'$ of $I$ and having the same size.

(ii) The image $I'$ is thus erect with respect to the object. This image lies at $F_e$ (focus of $E$). Hence, the final image, erect with respect to the object is at infinity.

Thus, the erecting lens merely re-inverts the image without changing the magnification.

Since both $I$ and $I'$ are at distances $2f$ from $EL$, length of the telescope, $L = f_0 + 4f + f_e$, which is longer than an astronomical telescope of the same angular magnification.

### *Angular Magnification of Terrestrial Telescope*

Clearly, $M = \frac{f_0}{f_e}$

(when the final image is formed at infinity)

Or $$M = f_0\left(\frac{1}{f_e} + \frac{1}{D}\right) = \frac{f_0}{f_e} + \frac{f_0}{D}$$

(when the final image is formed at the distance of distinct vision, $D$).

### CONCEPT APPLICATION EXERCISE 2.5

1. The dispersive power of the material of a lens is 0.026 and it produces a chromatic aberration of 0.351 cm. Calculate its focal length for the mean colour.
2. A concave lens is made of crown glass and radii of curvatures equal to 25 cm and 30 cm. Compute the longitudinal chromatic aberration for the lens. Take refractive indices of crown glass for blue and red colours as 1.523 and 1.5204 respectively.
3. A converging lens of focal length 6.25 cm is used as a magnifying glass. If the near point of the observer is 25 cm from the eye and the lens is held close to the eye, calculate: **(i)** the distance of the object from the lens and **(ii)** the angular magnification. Find the angular magnification when the final image is formed at infinity.
4. A person has normal far point (infinity) and near point (25 cm). He intends to read a book using a magnifying glass of focal length 5 cm. What is **(a)** the closest and **(b)** the farthest distance at which he can read the book through the magnifying glass?
5. The magnifying power of an astronomical telescope in the normal adjustment position is 100. The distance between the objective and the eye-piece is 101 cm. Calculate the focal lengths of the objective and the eye-piece.
6. The focal lengths of the objective and eye-piece of a compound microscope are 4 cm and 6 cm respectively. If an object is placed at a distance of 6 cm from the objective, find the distance between the objective and the eye-piece. Distance of distinct vision is 25 cm.
7. The total magnification produced by a compound microscope is 20, while that produced by the eye-piece alone is 5. When the microscope is focused on a certain

object, the distance between objective and eye-piece is 14 cm. Find the focal length of objective and eye-piece, if distance of distinct vision is 20 cm.

8. The magnifying power of an astronomical telescope is 5. When it is set for normal adjustment, the distance between the two lenses is 24 cm. Calculate the focal length of its eyepiece and that of its objective.

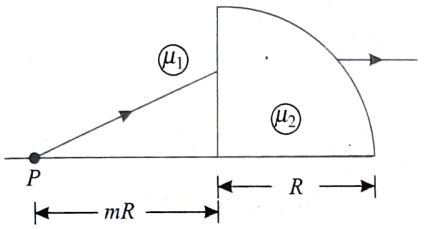
**ANSWERS**

| | | |
|---|---|---|
| 1. 13.5 cm | 2. –0.130 cm | 4. 4.17 cm, 5 cm |
| 5. 100 cm, 1 cm | 6. 16.84 cm | 7. 2 cm, 8 cm |
| 8. 4 cm, 20 cm | | |

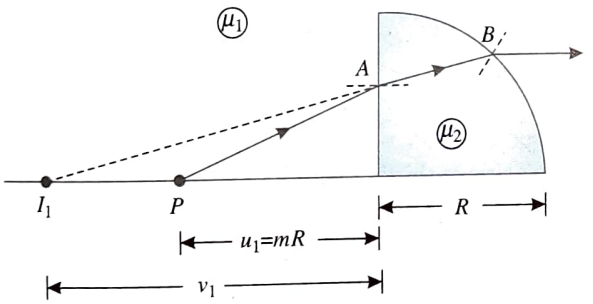
## Solved Examples

### EXAMPLE 2.1

A quarter cylinder of radius $R$ and refractive index 1.5 is placed on a table. A point object $P$ is kept at a distance of $mR$ from it. Find the value of $m$ for which a ray from $P$ will emerge parallel to the table as shown in figure.




**Sol.** Due to refraction from the plane surface of the quarter cylinder, the virtual image $I_1$ of the object $P$ is formed as shown in figure.




For the curved surface of the cylinder, $I_1$ acts as the real object. As the ray emerges parallel to the table, the final image is formed at infinity.

**Refraction from plane surface:** Suppose that the image $I_1$ is formed at a distance $v_1$ from the plane surface of the quarter cylinder. As the object is situated in rarer medium, we have

$$-\frac{1}{u_1}+\frac{\mu}{v_1}=\frac{\mu-1}{R_1}$$

Here, $v_1=-mR; R_1=\infty; \mu=1.5$

**Refraction from curved surface:** As the refracted ray travels in denser medium, we have

$$-\frac{\mu}{u_2}+\frac{1}{v_2}=\frac{1-\mu}{R_2}$$

Here, $u_2=-(v_1+R)=-(1.5mR+R)$

$v_2=\infty, R_2=-R; \mu=1.5$

$$\therefore \quad -\frac{1.5}{-(1.5mR+R)}+\frac{1}{\infty}=-\frac{1-1.5}{-R}$$

or $$\frac{1.5}{1.5mR+R}=\frac{0.5}{R}$$

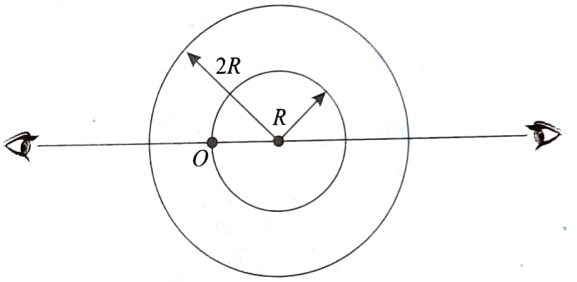
$$1.5mR+R=\frac{1.5}{0.5}R=3R$$

or $$1.5mR=2R$$

or $$m=\frac{2}{1.5}=\frac{4}{3}$$

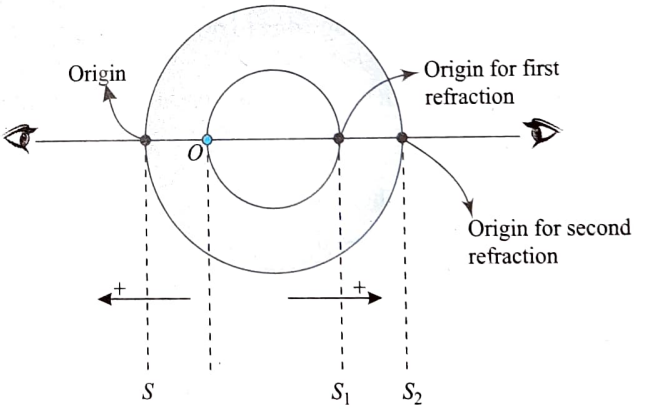
### EXAMPLE 2.2

A glass sphere of radius $2R$, refractive index $n$ has a spherical cavity of radius $R$, concentric with it. A black spot on the inner surface of the hollow sphere is viewed from the left as well as right. Obtain the shift in position of the object.




**Sol.**

**(a) Viewer on the left of hollow sphere:** Single refraction takes place at surface $S$.




From the single surface refraction equation, we have

$\mu_2=1, \mu_1=n, u=-R$, and $R=-2R$

$$\frac{1}{v}-\frac{n}{(-R)}=\frac{(1-n)}{(-2R)}$$

which on solving for $v$ yields, $v=-\left(\frac{2R}{n+1}\right)$

Image is on the right of refracting surface $S$.

Shift = Real depth – Apparent depth

$$=R-\left(\frac{2R}{n+1}\right)=\frac{(n-1)}{(n+1)}R$$

**(b) When the viewer is on the right:** Two refractions take place at surfaces $S_1$ and $S_2$.

$\mu_2 = n,\ \mu_1 = 1,\ u = -2R$, and $R = -R$

For refraction at surface $S_1$: $\dfrac{n}{v_1} - \dfrac{1}{(-2R)} = \dfrac{(n-1)}{(-R)}$

which on solving for $v$ yields, $v_1 = -\dfrac{2nR}{2n-1}$.

The first lies to the left of $S_1$ and acts as object for refraction at the second surface. We have to shift the origin of cartesian coordinate system to the vertex of $S_2$.

The object distance for the second surface is

$$u_2 = -\left[\frac{2nR}{2n-1} + R\right] = -\left(\frac{4n-1}{2n-1}\right)R$$

Here $\mu_2 = 1,\ \mu_1 = n,\ R = -2R$

$$\Rightarrow \quad \frac{1}{v_2} - \frac{n}{-\left[\dfrac{4n-1}{2n-1}\right]R} = \frac{1-n}{-2R}$$

On solving for $v_2$, we get $v_2 = -\dfrac{2(4n-1)}{(3n-1)}R$

The minus sign shows that image is virtual and lies to the left of $S_2$.

Shift = Real depth − Apparent depth

$$= 3R - \frac{2(4n-1)R}{(3n-1)} = \frac{(n-1)}{(3n-1)}R$$

## EXAMPLE 2.3

A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial.

**(a)** Find the position of image due to refraction at first surface and position of the final image.

**(b)** Draw a ray diagram showing the position of both images.

**Sol.**

**(a)** For refraction at spherical surface,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1} \qquad ...(i)$$

For refraction at the first surface,

$\mu_2 = 1,\ \mu_1 = 4/3,\ u = \infty,\ R_1 = +2$ mm, $v = v'$ (say)

Position of image due to refraction at the first surface is given by

$$\frac{1}{v'} - \frac{4/3}{\infty} = \frac{1-(4/3)}{2}$$

This given $v' = -6$ mm

That is the image is formed at a distance of 6 mm to the left of the first surface.

For refraction at the second surface,

$u' = u = -(6+4) = -10$ mm, $\mu_1 = 1,\ \mu_2 = 4/3$

$R_2 = -2$ mm

Substituting these values in Eq. (i), we get

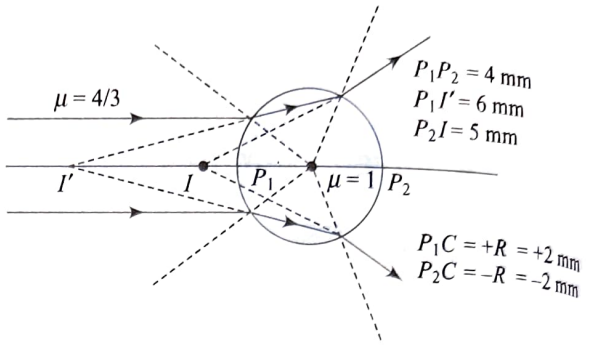
$$\frac{(4/3)}{v} - \frac{1}{(-10)} = \frac{\frac{4}{3}-1}{(-2)}$$

$$\Rightarrow \quad \frac{(4/3)}{v} - \frac{1}{6} - \frac{1}{10} = \frac{-10-6}{60}$$

$$\Rightarrow \quad v = -\frac{60 \times \left(\frac{4}{3}\right)}{16} = -5 \text{ mm}$$

The final image $I$ is at a distance of 5 mm to the left of the second surface.

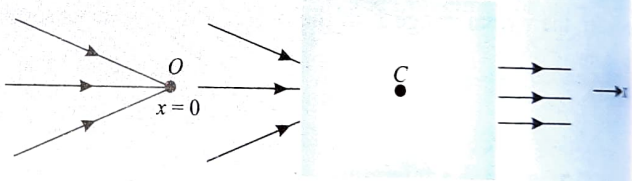
**(b)** The ray diagram is shown in figure




$I'$ is the virtual image formed by thefirst surface and the final image is virtual and is formed at $I$.

## EXAMPLE 2.4

A beam of light rays converges to a point $O(x = 0)$ on $x$-axis as shown. The angle of convergence is small. A cube of glass of refractive index $\mu = 1.5$ and side length 40 cm containing a concentric spherical air cavity of radius 10 cm is to be placed in the path of the converging beam so that the beam emerging from the cube is parallel to $x$-axis.

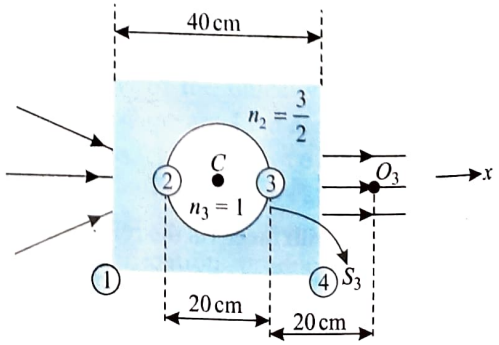



At what point $C$ on $x$-axis should the centre of the cube be placed to achieve this? Give the $x$ coordinate of $C$.

**Sol.** Using the formula for refraction at third surface gives the position of object for this refraction.

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$$

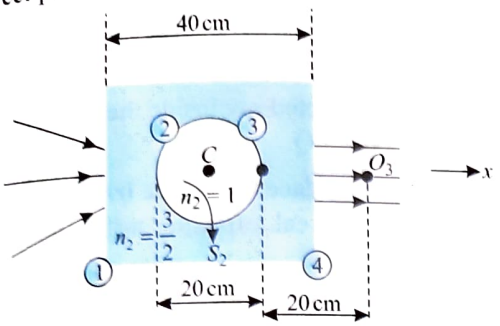
$$\frac{(3/2)}{\infty} - \frac{1}{u_3} = \frac{((3/2)-1)}{(-10)} \Rightarrow u_3 = 20\,\text{cm}$$



For refraction at second surface $S_2$: $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

$$\frac{1}{(+40)}-\frac{3/2}{u_2}=\frac{(1-3/2)}{(+10)} \Rightarrow u_2 = 20 \text{ cm}$$

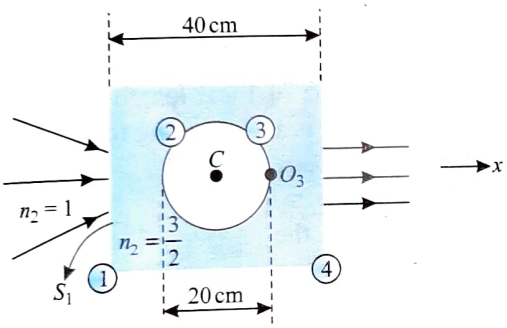
The object position for second surface lies 20 cm to the right of $S_2$.




For refraction at first surface $S_1$: $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

$$\frac{3/2}{(+30)}-\frac{1}{u_1}=\frac{(3/2-1)}{(\infty)} \Rightarrow u_1 = 20\,\text{cm}$$

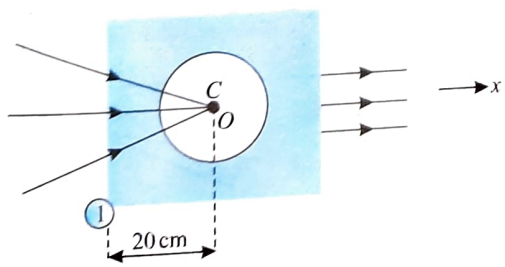
The object position for first surface lies 20 cm to the right of $S_1$




For refraction at second surface $S_1$: $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

$$\frac{3/2}{(+30)}-\frac{1}{u_1}=\frac{(3/2-1)}{(\infty)} \Rightarrow u_1 = 20\,\text{cm}$$

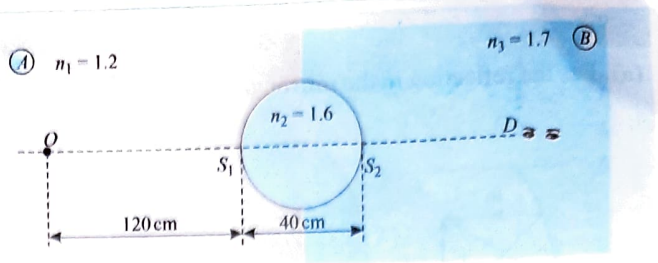
The object position for first surface lies 20 cm to the right of $S_1$.
Hence point $C$ and $O$ must be coincide.
This point $C$ must be place at origin.



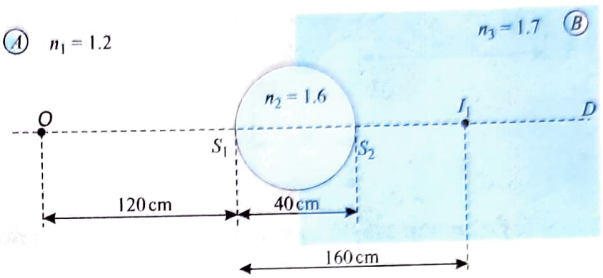

## EXAMPLE 2.5

A transparent sphere of radius 20 cm and refractive index 1.6 is fixed in a hole of the partition separating the two media: $A$ (refractive index $n_1 = 1.2$) and $B$ (refractive index $n_3 = 1.7$).

A luminous point object is placed 120 cm from the surface of the sphere in medium $A$. It is viewed from $D$ in medium $B$ in a direction normal to the sphere. Find the position and nature of the image formed by the rays in the direction $OD$.




**Sol.** We will have to consider two refractions: first at surface $S_1$ and the second at surface $S_2$.




For refraction at surface $S_1$: $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

$$\frac{1.6}{v_1}-\frac{1.2}{(-120)}=\frac{(1.6-1.2)}{20} \Rightarrow v_1 = +160 \text{ cm}$$

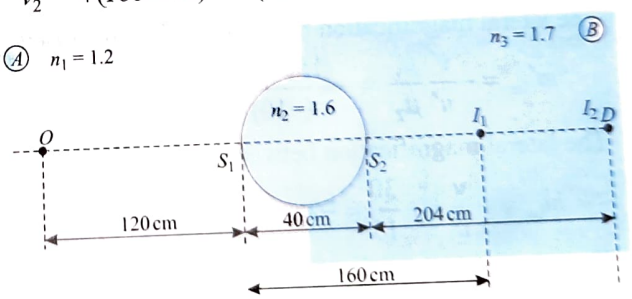
The positive sign implies that first image is formed to the right of $S_1$.

For refraction at surface $S_2$: $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

The first image is object for refraction at second surface.

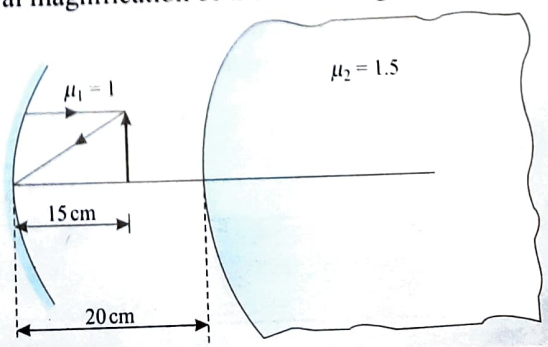
$$u_2 = +(160-40)\,\text{cm}$$

$$\frac{1.7}{v_2}-\frac{1.6}{+(160-40)}=\frac{1.6-1.7}{(-20)} \Rightarrow v_2 = +204\,\text{cm}$$



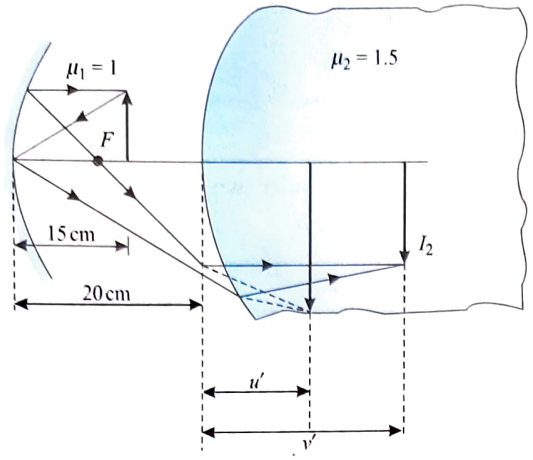

## EXAMPLE 2.6

A thin rod is kept at a distance of 15 cm from a concave mirror of $R = 20$ cm. A convex refracting surface at $R' = 50$ cm and refractive index $\mu = 1.5$ is kept at a distance of 20 cm from the mirror. Find the **(a)** nature and position of the final image **(b)** lateral magnification of the final image.



**Sol.**

**(a)** For the reflection in the mirror $u = 15,\ f = \frac{R}{2} = \frac{20}{2} = 10\,\text{cm}$




Then, $\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{15}$

or $v = 30$ cm

The real image $I_1$ will be formed which behaves as a virtual object for the refracting surface 1. For the refracting surface 1, the virtual object distance is

$\mu_1 = 1,\ \mu_2 = 1.5,\ R' = 50$ cm

$u' = -(30 - 20) = -10\,\text{cm}$

$\frac{\mu_1}{u'} + \frac{\mu_2}{v'} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1}{-10} + \frac{1.5}{v'} = \frac{1.5 - 1}{50}$

or $\frac{1.5}{v'} = \frac{1}{10} + \frac{1}{100} \Rightarrow v' = \frac{150}{11}\,\text{cm}$

**(b)** The lateral magnification between last two images is

$$m'_{\text{lat}} = -\frac{v'}{u'}\frac{\mu_1}{\mu_2} = -\frac{(150/11)}{(-10)} \cdot \frac{1}{(1.5)} = \frac{10}{11}$$

**(c)** The lateral magnification between 1st two images is

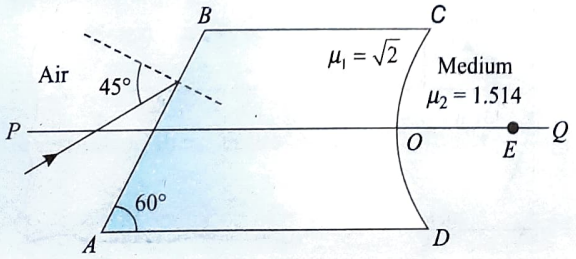
$$m_{\text{lat}} = -\frac{v}{u} = -\frac{30}{15} = -2$$

**(d)** Then the total magnification is $m = m_{\text{lat}} \cdot m'_{\text{lat}}$

$$= (-2)\left(\frac{10}{11}\right) = -\frac{20}{11}$$

## EXAMPLE 2.7

Figure shows an irregular block of material of refractive index $\sqrt{2}$. A ray of light strikes the face *AB*. After refraction, it is incident on a spherical surface *CD* of radius of curvature 0.4 m and enters a medium of refractive index 1.514 to meet *PQ* at *E*. Find the distance *OE* up to two decimal.




**Sol.** Let $r$ be angle of refraction inside the irregular block. According to Snell's law,

$$\mu_1 = \frac{\sin i}{\sin r}$$

or $\sin r = \frac{\sin i}{\mu_1} = \frac{\sin 45^\circ}{\sqrt{2}} = \frac{1/\sqrt{2}}{\sqrt{2}} = \frac{1}{2}$

or $r = 30^\circ$

It follows that the refracted ray inside the irregular block will travel parallel to its axis PQ.

**For refraction at the surface CD of the irregular block:**

For refraction at the spherical refracting surface CD, we have

$$\frac{\mu_1}{v} + \frac{\mu_2}{u} = \frac{\mu_2 - \mu_1}{R}$$

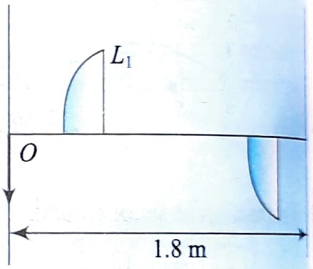
Here, $\mu_1 = \sqrt{2}, \mu_2 = 1.514, u = -\infty, v = +OE,\ R = +0.4\,\text{m}$

$\therefore \quad -\frac{\sqrt{2}}{-\infty} + \frac{1.514}{OE} = \frac{1.514 - \sqrt{2}}{+0.4} = \frac{1.514 - 1.414}{0.4}$

or $OE = \frac{1.514 \times 0.4}{0.1} = 0.06\,\text{m}$

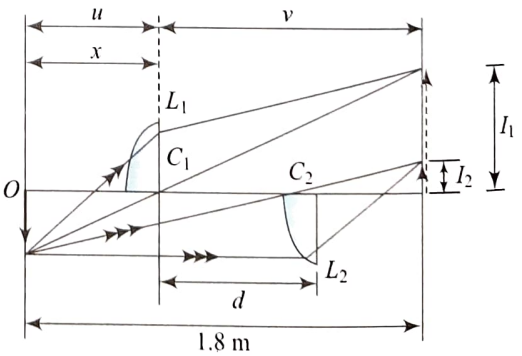
## EXAMPLE 2.8

A thin plano-convex lens of focal length $f$ is split into two halves, one of the halves is shifted along the optical axis. The separation between object and image plane is 1.8 m. The magnification of the image formed by one of the half lenses is 2. Find the focal length of the lens and separation between the halves. Draw the ray diagram for image formation.




**Sol.** Let magnification caused by the first lens be 2 and distance $OL_1 = x$. Distance $v$ of image from first lens $L_1$ is given by

$$m = \frac{v}{u} = 2 \quad \Rightarrow \quad v = 2u = 2x.$$




Clearly, $u + v = 1.8$ m

$\Rightarrow \quad x + 2x = 1.8$ m

or $3x = 1.8$ m

$\Rightarrow \quad x = \frac{1.8}{3} = 0.6$ m

By sign convention, $u = -x = -0.6$ m, $v = 2x = 1.2$ m

Lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ gives

$$\frac{1}{f} = \frac{1}{1.2} + \frac{1}{0.6} = \frac{1+2}{1.2}$$

$\therefore$ Focal length $f = \frac{1.2}{3} = 0.4$ m

For real image, lens formula takes the form

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

Clearly, $u$ and $v$ are interchangeable. Therefore, for lens $L_2$

$u' = v = 1.2$ m and $v' = 0.6$ m

$OL_1 = L_2I_2 = x$

If $d$ is the separation between the lenses, then

$x + d + x = 1.8$ m

$\therefore$ $d = 1.8 - 2x = 1.8 - 2 \times 0.6 = 0.6$ m

**Method-2** Since the magnification for $L_1$ is 2

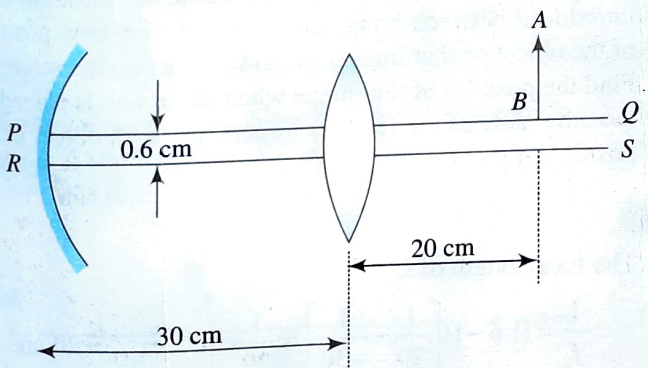
$$\Rightarrow \quad \frac{v}{u} = -2 \quad \Rightarrow \quad \frac{\frac{D+d}{2}}{-\frac{D-d}{2}} = -2$$

$$\Rightarrow \quad \frac{D+d}{D-d} = 2 \quad \Rightarrow \quad D = 1.8 \text{ m}, d = 0.6 \text{ m}$$

$$f = \frac{D^2 - d^2}{4D} = \frac{(1.8+0.6)(1.8-0.6)}{4 \times 1.8} = 0.4 \text{ m}$$

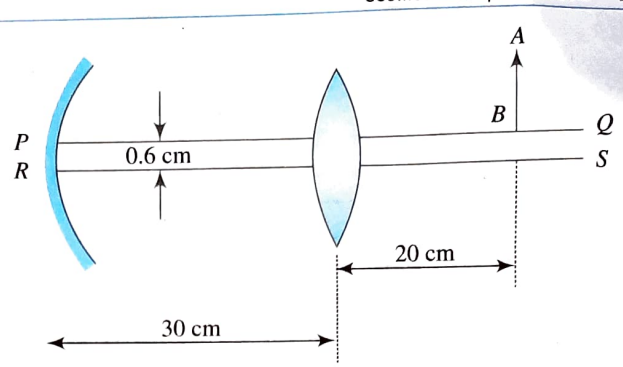
## EXAMPLE 2.9

A convex lens of focal length 15 cm and a concave mirror of focal length 30 cm are kept with their optic axes $PQ$ and $RS$ parallel but separated in vertical direction by 0.6 cm as shown in figure. The distance between the lens and mirror is 30 cm. An upright object $AB$ of height 1.2 cm is placed on the optics axis $PQ$ of the lens at a distance of 20 cm from the lens. If $A'B'$ is the image after refraction from the lens and reflection from the mirror, find the distance of $A'B'$ from the pole of the mirror and obtain its magnification. Also, locate positions of $A'$ and $B'$ with respect to the optic axis $RS$.




**Sol.** For convex lens using sign convection of coordinate geometry,

$u = +20$ cm, $f = -15$ cm

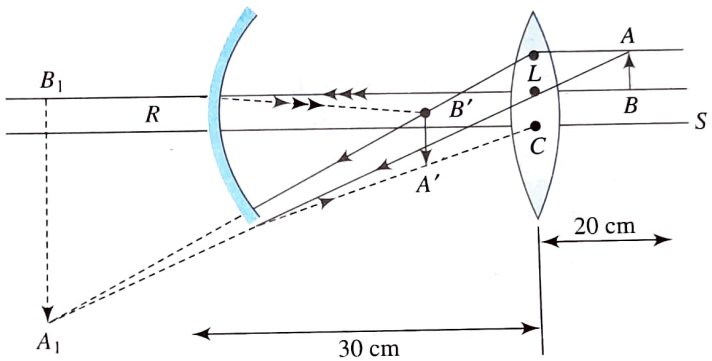



So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \Rightarrow \quad -\frac{1}{15} = \frac{1}{v_1} - \frac{1}{20}$

$$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{20} - \frac{1}{15} = \frac{3-4}{60} \quad \Rightarrow \quad v_1 = -60 \text{ cm}$$

i.e., image is formed at a distance 60 cm to the left of lens $L$.

Magnification, $m_1 = \frac{v_1}{u_1} = -\frac{60}{20} = -3$




This image is real and inverted. It is intercepted by the mirror.

For concave mirror, $u_2 = -60 + 30 = -30$ cm, $f_2 = +30$ cm

So, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ gives

$$\frac{1}{30} = \frac{1}{v_2} - \frac{1}{30} \Rightarrow \frac{1}{v_2} = \frac{1}{30} + \frac{1}{30} = \frac{2}{30} \Rightarrow v_2 = 15 \text{ cm}$$

Magnification, $m_2 = -\frac{v_2}{u_2} = -\frac{15}{-30} = +\frac{1}{2}$

$\therefore$ Net magnification, $m = m_1 \times m_2 = (-3) \times \left(\frac{1}{2}\right) = -1.5$

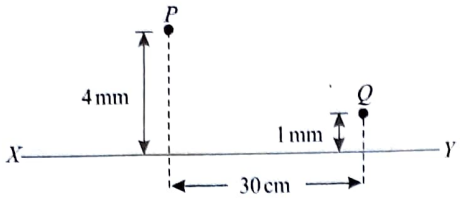
Size of image $A'B' = -1.5 \times 1.2$ cm $= -1.8$ cm.

Magnification of mirror is half and image of $B$ formed by convex lens is 0.6 cm above $RS$, so the length of image will be 1.5 cm below $RS$.

Thus, $B'$ will be 0.3 cm above $RS$ and $A'$ will be 1.5 cm below $RS$.

## EXAMPLE 2.10

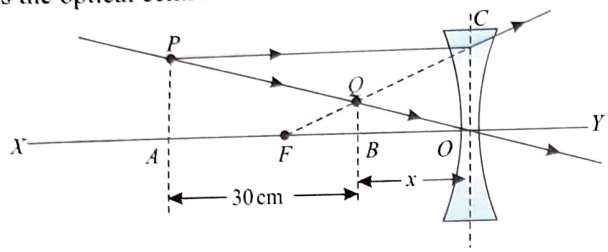
Figure shows the optical axis ($XY$) of a lens, the $P$ is a point source of light and $Q$ is its image. The separation between $P$ and $Q$ are separated by 20 cm along the axis, draw a ray diagram to locate the lens and its focus point and find the focal length of the lens.




**Sol.**

Steps:

(1) Join $P$ to $Q$. Extend the line so that it intersects $XY$ at $O$. $O$ is the optical centre of the lens.




(2) Since image is on same side of the principal axis and closer to it, the lens must be a diverging one.

(3) Draw $PC$ parallel to $XY$.

(4) Joint $C$ to $Q$ and extend. It meets $XY$ at $F$. $F$ is the focus, $\Delta POA \sim \Delta QBO$ are similar triangle

$$\frac{AP}{BQ}=\frac{30+x}{x}$$

$$\therefore \quad 4=\frac{30+x}{x}$$

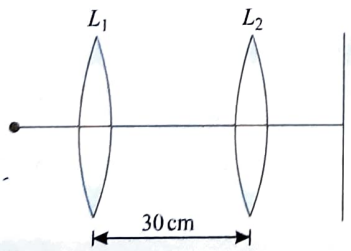
$$\Rightarrow \quad x = 10 \text{ cm}, u = -40 \text{ cm}; v = -10 \text{ cm}; f = ?$$

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{1}{f}=\frac{1}{-10}-\frac{1}{(-40)}=\frac{-4+1}{40}$$

$$f=-\frac{40}{3}\text{cm}$$

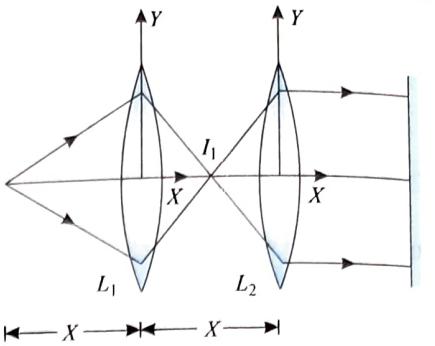
## EXAMPLE 2.11

Two identical convex lenses ($L_1$ and $L_2$) each of focal length 10 cm are separated by 30 cm and placed along common principal axis. A plane mirror is placed behind the two lenses as shown in the figure. Where should a point object be placed in front of the first lens ($L_1$) so that it images on to itself?




**Sol.** Let the object is placed at a distance $x$ from $L_1$.

If the image and object coincide, the ray should retrace its path it must be incident normally on the plane mirror after passing through $L_1$ and $L_2$.




From the diagram we can observe the rays after interaction with $L_1$ should focus at the focal point of $L_2$, i.e., the position of $I_1$ should be 10 cm in front of $L_2$.

Using lens formula for $L_1$, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ ...(i)

From the ray diagram we see that for lens $L_1$,

$$v = 30 - 10 = +20 \text{ cm}, f = +10 \text{ cm}, u = -x$$

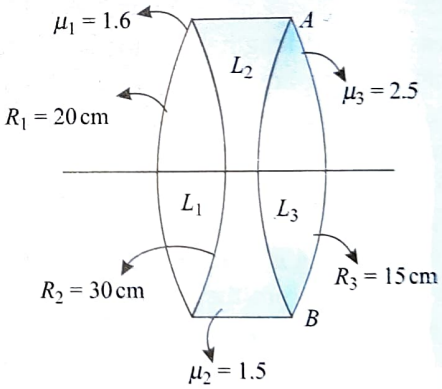
From Eq. (i) we get $\frac{1}{20}-\frac{1}{-x}=\frac{1}{10}$

or $\quad x = 20\,\text{cm}$

## EXAMPLE 2.12

A lens is made of three thin different mediums. Radius of curvature and refractive index of each medium is shown in figure. Surface $AB$ is straight. An object is placed at some distance from the lens by which a real image is formed on the screen placed at a distance 10 cm from the lens. Find

**(a)** the distance of the object from the lens.




**(b)** A slab of thickness 1.5 cm and refractive index 1.5 is introduced between image and the lens, find new position of the object so that image is again formed on the screen.

**(c)** Find the position of the image when same slab is placed on the other side of the lens (as in part (b)) with object in the position of part (a).

**Sol.**

**(a)** The focal length of $L_1$

$$\frac{1}{f_1}=(1.6-1)\left[\frac{1}{20}-\frac{1}{-30}\right]=\frac{1}{20}\text{cm}^{-1} \Rightarrow f_1 = 20 \text{ cm}$$

The focal length of $L_2$

$$\frac{1}{f_2}=(1.5-1)\left[\frac{1}{-30}-\frac{1}{15}\right]=-\frac{1}{20}\text{cm}^{-1} \Rightarrow f_2 = -20 \text{ cm}$$

The focal length of $L_3$

$$\frac{1}{f_3}=(2.5-1)\left[\frac{1}{15}-\frac{1}{\infty}\right]=\frac{1}{10}\text{cm}^{-1}\Rightarrow f_3=10\text{ cm}$$

The equivalent focal length of combination:

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$$

$$\Rightarrow \frac{1}{f_{eq}}=\frac{1}{20}-\frac{1}{20}+\frac{1}{10}=\frac{1}{10}\Rightarrow f_{eq}=10\text{ cm}$$

Since the image is formed on the screen which is 10 cm from the lens. So the object is at infinity.

(b) The shift should be in the direction of ray travelling. Hence in this case the shift of the image will be away from the slab due to introduction of slab is

$$\Delta x=\left(1-\frac{1}{n}\right)t=\left(1-\frac{1}{1.5}\right)\times 1.5=0.5$$

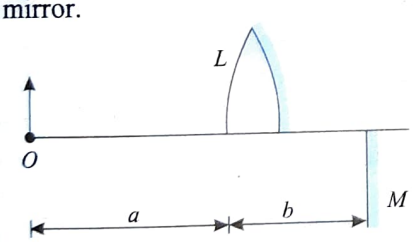
The distance of the object so that image is again formed on the screen

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{9.5}-\frac{1}{u}=\frac{1}{10}\Rightarrow u=190\text{ cm}$$

(c) When the slab is placed on the other side of the lens there will not be any change in the formation.

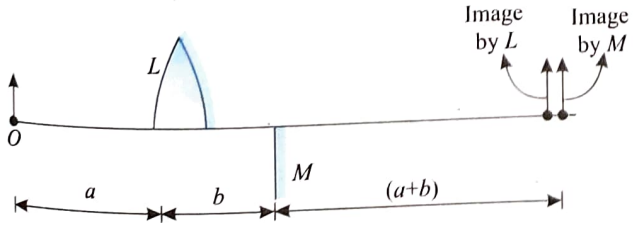
## EXAMPLE 2.13

In figure $L$ is half part of an equiconvex glass lens ($\mu$ = 1.5) whose surface have radius of curvature $r$ = 40 cm and its right surface is silvered. Normal to its principal axis a plane mirror $M$ is placed on right of lens. Distance between lens $L$ and mirror $M$ is $b$. A small object $O$ is placed on left of the lens such that there is no parallax between final images formed by the lens and mirror.




If transverse length of final image formed by lens is twice that of image formed by the mirror, calculate distance $a$ between lens and object and distance $b$.

**Sol.** Since, distance of object $O$ from plane mirror $M$ is $(a+b)$, therefore, it forms a virtual image at a distance $(a+b)$ behind itself or a distance $2(a+b)$ from the object $O$.




Since, there is no parallel between the images formed by the silvered lens $L$ and plane mirror $M$, therefore, two images are formed at the same point.

Hence, distance of image from lens $L$ is $2(a+b)-a=(a+2b)$ behind the lens.

Since, length of image formed by $L$ is twice the length of image formed by the mirror $M$ and length of image formed by a plane mirror is always equal to length of the object, therefore modulus of transverse magnification produced by the lens $L$ is equal to 2.

Since, distance of object from $L$ is $a$, therefore of image from $L$ must be equal to $2a$.

$$(a+2b)=1a\quad\text{or,}\quad b=\frac{a}{2}\qquad\ldots(i)$$

Power of the equivalent mirror formed

$$(P_M)_{eq}=2P_l+P_m\qquad\ldots(ii)$$

Focal length of the lens

$$\frac{1}{f_l}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1.5-1)\left(\frac{1}{40}-\frac{1}{(-40)}\right)$$

$$f_l=40\,\text{cm}=\frac{2}{5}\text{m}$$

Power of the lens $P_l=\frac{1}{f_l}=\frac{5}{2}\text{D}$

Focal length of the mirror surface

$$f_m=\frac{R}{2}=\frac{(-40)}{2}=-20\,\text{cm}=-\frac{1}{5}\text{m}$$

Power of the mirror $P_m=-\frac{1}{f_m}$

$$\Rightarrow\quad P_m=-\frac{1}{\left(\frac{-1}{5}\right)}=5\text{D}$$

Hence power of equivalent mirror $(P_M)_{eq}=2\times\frac{5}{2}+5=10\text{D}$

The focal length of equivalent mirror

$$(f_M)_{eq}=-\frac{1}{(P_M)_{eq}}=-\frac{1}{(10)}=-\frac{1}{10}\text{m}=-10\text{ cm}$$

Hence, for the combination, $u=-a$, $v=+2a$, $F=-10\,\text{cm}$

Using mirror formula, $\frac{1.5}{v}=\frac{1}{-\infty}=0$

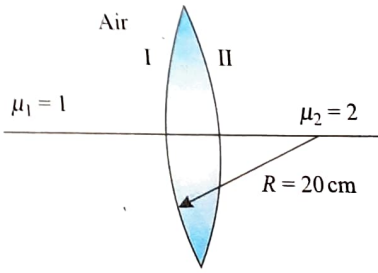
$a$ = 5 cm

Substituting $a$ = 5 cm in Eq. (i), $b$ = 2.5 cm

## EXAMPLE 2.14

Focal length of a thin convex lens in air is 10 cm. Now, medium on one side of the lens is replaced by a medium of refractive index $\mu_2$ = 2. The radius of curvature of the surface of lens, in contact with the medium, is 20 cm. Find the new focal length.

**Sol.** Let radius of surface 1 be $R_1$ and refractive index of lens be $\mu$. For finding focal length of lens, let us assume parallel rays be incident on the lens from left.




Applying refraction formula at the first surface,

$$\frac{\mu}{v_1}-\frac{1}{\infty}=\frac{\mu-1}{(+R_1)}\qquad\ldots(i)$$

The image formed by first surface acts as object for second surface

At the second surface: $\frac{2}{f'}-\frac{\mu}{v_1}=\frac{2-\mu}{(-20)}$ ...(ii)

Here $f'$ is the new focal length

Adding (i) and (ii), we get $\frac{\mu}{v_1}-\frac{1}{\infty}+\frac{2}{f'}-\frac{\mu}{v_1}=\frac{\mu-1}{R_1}+\frac{2-\mu}{-20}$

$$\frac{2}{f'}=\frac{\mu-1}{R_1}-\frac{2-\mu}{20} \quad \text{...(iii)}$$

The focal length of the lens in air, $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{-20}\right)$

$$\frac{1}{+10}=\frac{(\mu-1)}{R_1}+\frac{(\mu-1)}{20} \quad \text{...(iv)}$$

From Eqs. (iii) and (iv) [Eq.(iii) –Eq.(iv)]

$$\frac{2}{f'}-\frac{1}{10}=\left(\frac{\mu-1}{R_1}-\frac{2-\mu}{20}\right)-\left(\frac{(\mu-1)}{R_1}+\frac{(\mu-1)}{20}\right)=-\frac{2}{20}+\frac{1}{20}$$

$$\frac{2}{f'}-\frac{1}{10}=-\frac{1}{20}\Rightarrow f'=40 \text{ cm}$$

**2nd approach**

The refractive index of the medium on both sides of the lens is different,

Using the equation $\frac{\mu_b}{v}-\frac{\mu_a}{u}=\frac{\mu_a-\mu}{R_1}+\frac{\mu_b-\mu}{R_2}$

$u=\infty, R_1=+R, R_2=-20\,\text{cm}, \mu_a=1, \mu=\mu$ and $\mu_b=2$

$$\frac{2}{f'}-\frac{1}{\infty}=\frac{\mu-1}{(+R_1)}+\frac{2-\mu}{(-20)}$$

$$\frac{2}{f'}=\frac{\mu-1}{R_1}-\frac{2-\mu}{20}$$

The focal length of the lens in air, $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{-20}\right)$

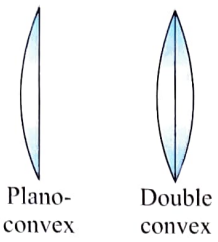
$$\frac{1}{+10}=\frac{(\mu-1)}{R_1}+\frac{(\mu-1)}{20}$$

From Eqs. (i) and (ii), we get $f'=40\,\text{cm}$

# Exercises

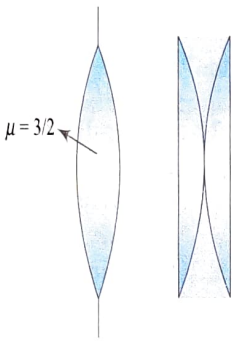
## Single Correct Answer Type

1. You are given two identical plano-convex lenses. When you place an object 20 cm to the left of a single plano-convex lens, the image appears 40 cm to the right of the lens. You then arrange the two plano-convex lenses back to back to form a double convex lens. If the object is 20 cm to the left of this new lens, what is the approximate location of the image?



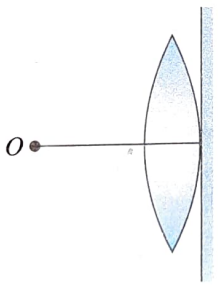

(1) 10 cm to the right of the lens
(2) 20 cm to the right of the lens
(3) 80 cm to the right of the lens
(4) 80 cm to the left of the lens

2. A thin equiconvex lens ($\mu = 3/2$) of focal length 10 cm is cut and separated and a material of refractive index 3 is filled between them. What is the focal length of the combination?



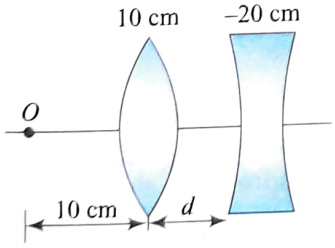

(1) –10 cm
(2) –10/4 cm
(3) –10/3 cm
(4) None of these

3. Behind a thin converging lens having both the surfaces of the same radius 10 cm, a plane mirror has been placed. The image of an object at a distance of 40 cm from the lens is formed at the same position. What is the refractive index of the lens ?




(1) 1.5 (2) 5/3
(3) 9/8 (4) None of these

4. What should be the value of distance $d$ so that final image is formed on the object itself. (Focal lengths of the lenses are written on the lenses.)



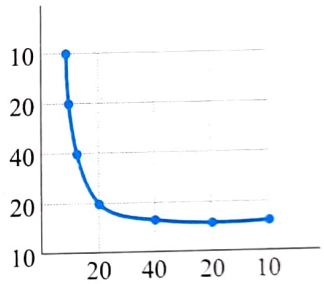

(1) 10 cm (2) 20 cm
(3) 5 cm (4) None of these

5. Consider an equiconvex lens of radius of curvature $R$ and focal length $f$. If $f > R$, the refractive index $\mu$ of the material of the lens
(1) is greater than zero but less than 1.5
(2) is greater than 1.5 but less than 2.0
(3) is greater than 1.0 but less than 1.5
(4) none of these

6. A plano-convex lens when silvered on the plane side behaves like a concave mirror of focal length 60 cm. However, when silvered on the convex side, it behaves like a concave mirror of focal length 20 cm. Then, the refractive index of the lens is
(1) 3.0 (2) 1.5
(3) 1.0 (4) 2.0

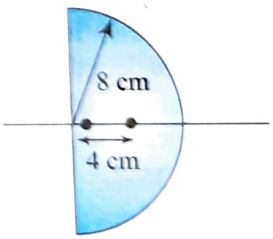
7. Two thin lenses are placed 5 cm apart along the same axis and illuminated with a beam of light parallel to that axis. The first lens in the path of the beam is a converging lens of focal length 10 cm whereas the second is a diverging lens of focal length 5 cm. If the second lens is now moved toward the first, the emergent light
(1) remains parallel
(2) remains convergent
(3) remains divergent
(4) changes from parallel to divergent

8. A lens forms a real image of an object. The distance from the object to the lens is $x$ cm and that from the lens to the image is $y$ cm. The graph (see figure) shows the variation of $y$ with $x$.



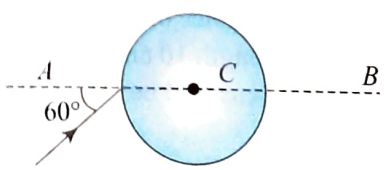

It can be deduced that the lens is
(1) converging and of focal length 10 cm
(2) converging and of focal length 20 cm
(3) converging and of focal length 40 cm
(4) diverging and of focal length 20 cm

9. A plastic hemisphere has a radius of curvature of 8 cm and an index of refraction of 1.6. On the axis halfway between the plane surface and the spherical one (4 cm from each) is a small object $O$.




The distance between the two images when viewed along the axis from the two sides of the hemisphere is approximately
(1) 1.0 cm (2) 1.5 cm
(3) 3.75 cm (4) 2.5 cm

10. A ray of light falls on a transparent sphere with centre at $C$ as shown in figure.



The ray emerges from the sphere parallel to line $AB$. The refractive index of the sphere is

(1) $\sqrt{2}$ (2) $\sqrt{3}$

(3) 3/2 (4) 1/2

**11.** A ray of light is incident on a glass sphere of refractive index 3/2. What should be the angle of incidence so that the ray which enters the sphere does not come out of the sphere?

(1) $\tan^{-1}(2/3)$ (2) $60^\circ$

(3) $90^\circ$ (4) $30^\circ$

**12.** Two identical glass ($\mu_g = 3/2$) equiconvex lenses of focal length $f$ are kept in contact. The space between the two lenses is filled with water ($\mu_w = 4/3$). The focal length of the combination is

(1) $f$ (2) $\frac{f}{2}$

(3) $\frac{4f}{3}$ (4) $\frac{3f}{4}$

**13.** An object is kept at a distance of 16 cm from a thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens, the image formed is virtual. If the sizes, of the images formed are equal, the focal length of the lens will be

(1) 15 cm (2) 17 cm

(3) 21 cm (4) 11 cm

**14.** Point object $O$ is placed on the principal axis of a convex lens of focal length 20 cm at a distance of 40 cm to the left of it. The diameter of the lens is 10 cm. If the eye is placed 60 cm to the right of the lens at a distance $h$ below the principal axis, then the maximum value of $h$ to see the image will be

(1) 0 (2) 5 cm

(3) 2.5 cm (4) 10 cm

**15.** A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids $L_1$ or $L_2$ having refractive indices $n_1$ and $n_2$, respectively ($n_2 > n_1 > 1$). The lens will diverge parallel beam of light if it is filled with

(1) air and placed in air (2) air and immersed in $L_1$

(3) $L_2$ and immersed in $L_2$ (4) $L_2$ and immersed in $L_1$

**16.** A convex lens of focal length 20 cm and a concave lens of focal length $f$ are mounted coaxially 5 cm apart. Parallel beam of light incident on the convex lens emerges from the concave lens as a parallel beam. Then, $f$ in cm is

(1) 35 (2) 25

(3) 20 (4) 15

**17.** An equiconvex lens is made from glass of refractive index 1.5. If the radius of each surface is changed from 5 cm to 6 cm, then the power

(1) remains unchanged (2) increases by 3.33 D

(3) decreases by 3.33 D (4) decreases by 5.5 D

**18.** An object is put at a distance of 5 cm from the first focus of a convex lens of focal length 10 cm. If a real image is formed, its distance from the lens will be

(1) 15 cm (2) 20 cm

(3) 25 cm (4) 30 cm

**19.** Refraction takes place at a convex spherical boundary separating glass–air medium. For the image to be real, the object distance ($\mu_g = 3/2$) should be/is

(1) greater than three times the radius of curvature of the refracting surface

(2) greater than two times the radius of curvature of the refracting surface

(3) greater than the radius of curvature of the refracting surface

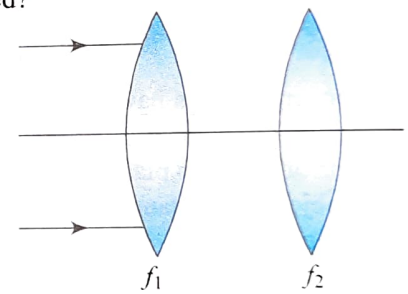
(4) independent of the radius of curvature of the refracting surface

**20.** A plano-convex lens fits exactly into a plano-concave lens. Their plane surfaces are parallel to each other. If the lenses are made of different material of refractive indices $\mu_1$ and $\mu_2$ and $R$ is the radius of curvature of the curved surface of the lenses, then focal length of the combination is

(1) $\frac{R}{\mu_1 - \mu_2}$ (2) $\frac{2R}{\mu_2 - \mu_1}$

(3) $\frac{R}{2(\mu_1 - \mu_2)}$ (4) $\frac{R}{2 - (\mu_1 + \mu_2)}$

**21.** A parallel beam of light is incident on the system of two convex lenses of focal lengths $f_1 = 20$ cm and $f_2 = 10$ cm. What should be the distance between the two lenses so that rays after refraction from both the lenses pass undeviated?




(1) 60 cm (2) 30 cm

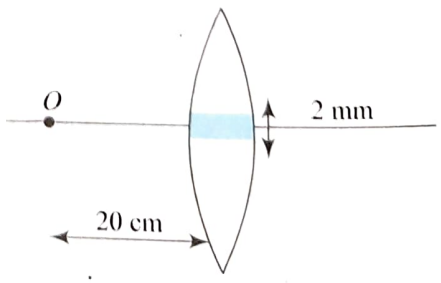
(3) 90 cm (4) 40 cm

**22.** A point object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. If a glass slab of thickness $t$ and refractive index 1.5 is inserted between the lens and the object, the image is formed at infinity. The thickness $t$ is

(1) 10 cm (2) 5 cm

(3) 20 cm (4) 15 cm

**23.** A convex lens of focal length 10 cm is painted black at the middle portion as shown in figure. An object is placed at a distance of 20 cm from the lens.




Then,

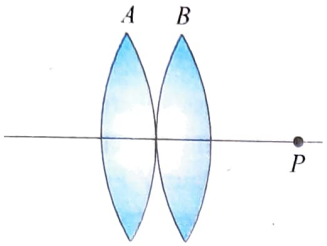
(1) only one image will be formed by the lens

(2) the distance between the two images formed by such a lens is 6 mm

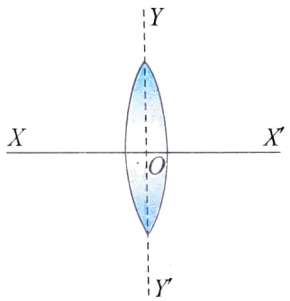
(3) the distance between the images is 4 mm
(4) the distance between the images is 2 mm

24. A lens forms a virtual, diminished image of an object placed at 2 m from it. The size of image is half of the object. Which one of the following statements is correct regarding the nature and focal length of the lens?
(1) Concave, $|f| = 1$ m (2) Convex, $|f| = 1$ m
(3) Concave, $|f| = 2$ m (4) Convex, $|f| = 2$ m

25. Two convex lenses placed in contact form the image of a distant object at $P$. If the lens $B$ is moved to the right, the image will
(1) move to the left
(2) move to the right
(3) remain at $P$
(4) move either to the left or right, depending upon focal lengths of the lenses




26. An equiconvex lens is cut into two halves along (i) $XOX'$ and (ii) $YOY'$ as shown in figure. Let $f, f', f''$ be the focal length of the lens, of each half in case (i), and of each half in case (ii), respectively.
Choose the correct statement from the following:
(1) $f' = f$, $f'' = 2f$
(2) $f' = 2f$, $f'' = f$
(3) $f' = f$, $f'' = f$
(4) $f' = 2f$, $f'' = 2f$




27. An object 15 cm high is placed 10 cm from the optical center of a thin lens. Its image is formed 25 cm from the optical center on the same side of the lens as the object. The height of the image is
(1) 2.5 cm (2) 0.2 cm
(3) 16.7 cm (4) 37.5 cm

28. A convex lens of power +6 dioptre is placed in contact with a concave lens of power –4 dioptre. What will be the nature and focal length of this combination?
(1) Concave, 25 cm
(2) Convex, 50 cm
(3) Concave, 20 cm
(4) Convex, 100 cm

29. A convex lens of focal length 1.0 m and a concave lens of focal length 0.25 m are 0.75 m apart. A parallel beam of light is incident on the convex lens. The beam emerging after refraction from both lenses is
(1) parallel to the principal axis
(2) convergent
(3) divergent
(4) none of the above

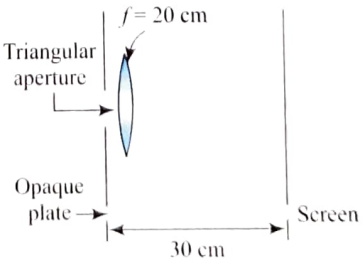
30. A convex lens $A$ of focal length 20 cm and a concave lens $G$ of focal length 5 cm are kept along the same axis with the distance $d$ between them. If a parallel beam of light falling on $A$ leaves $B$ as a parallel beam, then distance $d$ in cm will be
(1) 25 (2) 15
(3) 30 (4) 50

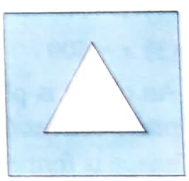
31. A convex lens forms an image of an object placed 20 cm away from it at a distance of 20 cm on the other side of the lens. If the object is moves 5 cm toward the lens, the image will be
(1) 5 cm toward the lens
(2) 5 cm away from the lens
(3) 10 cm toward the lens
(4) 10 cm away from the lens

32. An object is placed at a distance of 15 cm from a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at its focus such that the image formed by the combination coincides with the object itself. The focal length of the convex mirror is
(1) 20 cm (2) 10 cm
(3) 15 cm (4) 30 cm

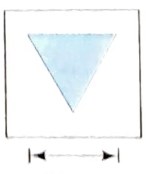
33. The lateral magnification of the lens with an object located at two different positions $u_1$ and $u_2$ are $m_1$ and $m_2$, respectively. Then the focal length of the lens is
(1) $f = \sqrt{m_1 m_2}\,(u_2 - u_1)$
(2) $f = \sqrt{m_1 m_2}\,(u_2 + u_1)$
(3) $\dfrac{(u_2 - u_1)}{\sqrt{m_2 m_1}}$
(4) $\dfrac{(u_2 - u_1)}{(m_2)^{-1} - (m_1)^{-1}}$

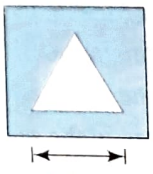
34. A parallel beam of light falls axially on a thin converging lens of focal length 20 cm. The emergent light falls or a screen placed 30 cm beyond the lens. An opaque plate with a triangular aperture, side 1 cm, is in contact with the lens [see Figs. (a) and (b)].

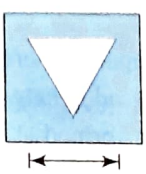


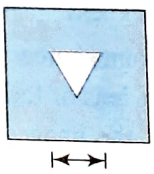




Which one of the following diagrams best shows to appearance of the patch of light seen on the screen?

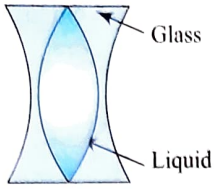
(1)



(2)



(3)


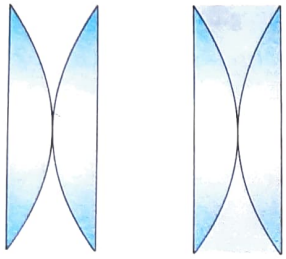

(4)


**35.** A liquid of refractive index 1.6 is contained in the cavity of a glass specimen of refractive index 1.5 as shown in figure. If each of the curved surfaces has a radius of curvature of 0.20 m, the arrangement behaves as a




(1) converging lens of focal length 0.25 m
(2) diverging lens of focal length 0.25 m
(3) diverging lens of focal length 0.17 m
(4) converging lens of focal length 0.72 m

**36.** Figure (a) shows two plano-convex lenses in contact as shown. The combination has focal length 24 cm. Figure (b) shows the same with a liquid introduced between them. If refractive index of glass of the lenses is 1.50 and that of the liquid is 1.60, the focal length of system in Fig. (b) will be



(a) (b)

(1) –120 cm (2) 120 cm
(3) –24 cm (4) 24 cm

**37.** In a slide show programme, the image on the screen has an area 900 times that of the slide. If the distance between the slide and the screen is $x$ times the distance between the slide and the projector lens, then
(1) $x = 30$ (2) $x = 31$
(3) $x = 500$ (4) $x = 1/30$

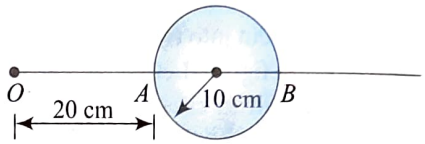
**38.** An object is placed 1 m in front of the curved surface of a plano-convex lens whose plane surface is silvered. A real image is formed in front of the lens at a distance of 120 cm. Then, the focal length of the lens is
(1) 100 cm (2) 120 cm
(3) 109.1 cm (4) 110.0 cm

**39.** The apparent thickness of a thick plano-convex lens is measured once with the plane face upward and then with the convex face upward. The value will be
(1) more in the first case
(2) same in the two cases
(3) more in the second case
(4) any of the above depending on the value of its actual thickness

**40.** Two point sources $S_1$ and $S_2$ are 24 cm apart. Where should a convex lens of focal length 9 cm be placed in between them so that the images of both sources are formed at the same place?
(1) 6 cm from $S_1$
(2) 15 cm from $S_1$
(3) 10 cm from $S_1$
(4) 12 cm from $S_1$

**41.** Two thin symmetrical lenses of different nature and of different material have equal radii of curvature $R = 15$ cm. The lenses are put close together and immersed in water ($\mu_w = 4/3$). The focal length of the system in water is 30 cm. The difference between refractive indices of the two lenses is
(1) 1/2 (2) 1/4
(3) 1/3 (4) 3/4

**42.** A glass sphere of radius $R = 10$ cm is kept inside water. A point object $O$ is placed at 20 cm from $A$ as shown in the figure. Find the position and nature of the image when seen from other side of the sphere. Given $\mu_g = 3/2$ and $\mu_w = 4/3$.




(1) 200 cm, virtual (2) 100 cm, real
(3) 100 cm, virtual (4) 300 cm, virtual

**43.** A spherical convex surface separates object and image spaces of refractive indices 1.0 and 4/3. If radius of curvature of the surface is 10 cm, find its power.
(1) 3.5 D (2) 2.5 D
(3) 25 D (4) 1.5 D

**44.** A luminous object and a screen are at a fixed distance $D$ apart. A converging lens of focal length $f$ is placed between the object and screen. A real image of the object in formed on the screen for two lens positions if they are separated by a distance $d$ equal to
(1) $\sqrt{D(D+4f)}$ (2) $\sqrt{D(D-4f)}$
(3) $\sqrt{2D(D-4f)}$ (4) $\sqrt{D^2+4f}$

**45.** For the same statement as above, the ratio of the two image sizes for these two positions of the lens is
(1) $\left[\frac{D-d}{D+d}\right]^2$ (2) $\left[\frac{D+d}{D-d}\right]^2$
(3) $\left[\frac{D-2d}{D+2d}\right]^2$ (4) $\left[\frac{D+2d}{D-2d}\right]^2$

**46.** For statement of question 44, if the heights of the two images are $h_1$ and $h_2$, respectively, then the height of the object ($h$) is
(1) $h_1 + h_2$ (2) $h_1 h_2$
(3) $\sqrt{h_1 h_2}$ (4) $h_1/h_2$

**47.** The focal length of the lens used in question 44 is
(1) $\frac{D^2+d^2}{2D}$ (2) $\frac{D^2-d^2}{4D}$

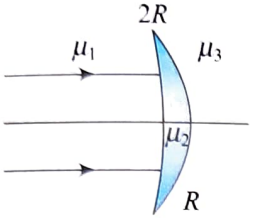
(3) $\frac{D^2-d^2}{2D}$ (4) $\frac{D^2+d^2}{d}$

48. In question 44, if $m_1$ and $m_2$ are the magnifications for two positions of the lens, then

(1) $f=\frac{d}{m_1+m_2}$ (2) $f=\frac{2d}{m_1+m_2}$

(3) $f=\frac{3d}{m_1-m_2}$ (4) $f=\frac{d}{m_1-m_2}$

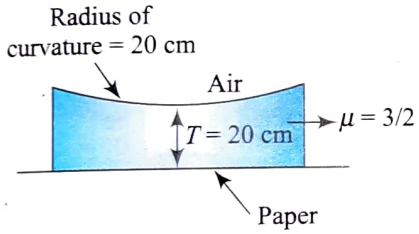
49. Figure shows a concavo-convex lens $\mu_2$. What is the condition on the refractive indices so that the lens is diverging?




(1) $2\mu_2 < \mu_1 + \mu_2$ (2) $2\mu_2 > \mu_1 + \mu_2$

(3) $\mu_2 > 2(\mu_1 - \mu_2)$ (4) None of these

50. A plano-concave lens is placed on a paper on which a flower is drawn. How far above its actual position does the flower appear to be?




(1) 10 cm (2) 15 cm

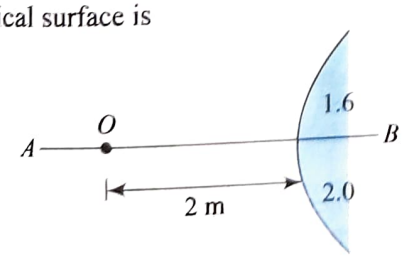
(3) 50 cm (4) None of these

51. The distance between an object and the screen is 100 cm. A lens produces an image on the screen when the lens is placed at either of the positions 40 cm apart. The power of the lens is nearly

(1) 3 diopter (2) 5 diopter

(3) 2 diopter (4) 9 diopter

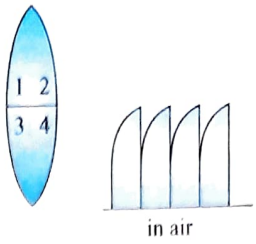
52. In figure, a point object $O$ is placed in air. A spherical boundary separates two media. $AB$ is the principal axis. The refractive index above $AB$ is 1.6 and below $AB$ is 2.0. The separation between the images formed due to refraction at the spherical surface is




(1) 12 m (2) 20 m

(3) 14 m (4) 10 m

53. The given lens is broken into four parts rearranged as shown. If the initial focal length is $f$, then after rearrangement the equivalent focal length is




(1) $f$ (2) $f/2$

(3) $f/4$ (4) $4f$

54. A concave refractive surface of a medium having refractive index $\mu$ produces a real image of an object (located outside the medium) irrespective of its location. Choose the correct option from the following.

(1) Always

(2) May be if refractive index of surrounding medium is greater than $\mu$

(3) May be if refractive index of surrounding medium is less than $\mu$

(4) None of the above

55. A convex spherical refracting surface with radius $R$ separates a medium having refractive index 5/2 from air. As an object is moved towards the surface from far away from the surface along the principal axis, its image

(1) changes from real to virtual when it is at a distance $R$ from the surface

(2) changes from virtual to real when it is at a distance $R$ from the surface

(3) changes from real to virtual when it is at a distance $2R/3$ from the surface

(4) changes from virtual to real when it is at a distance $2R/3$ from the surface

56. A concave spherical refractive surface with radius $R$ separates a medium of refractive index 5/2 from air. As an object is approaching the surface from far away along the central axis, its image

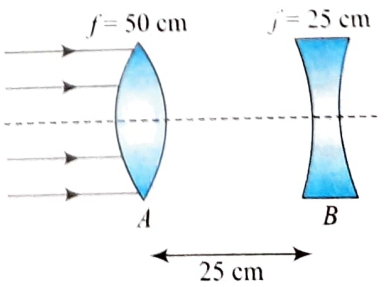
(1) always remains real

(2) always remains virtual

(3) changes from real to virtual at a distance $2R/3$ from the surface

(4) changes from virtual to real at a distance $2R/3$ from the surface

57. Two lenses shown in figure are illuminated by a beam of parallel light from the left. Lens $B$ is then moved slowly toward lens $A$. The beam emerging from lens $B$ is



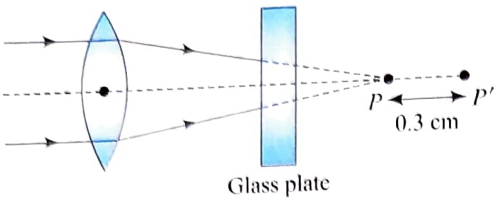

(1) always diverging

(2) initially parallel and then diverging

(3) always parallel

(4) initially converging and then parallel

**58.** Rays from a lens are converging toward a point $P$, as shown in figure. How much thick glass plate having refractive index 1.6 must be located between the lens and point $P$, so that the image will be formed at $P'$?




(1) 0.8 cm (2) 1.6 cm

(3) 5 cm (4) 2.4 cm

**59.** When an object is placed 15 cm from a lens, a virtual image is formed. Mark the correct statements.

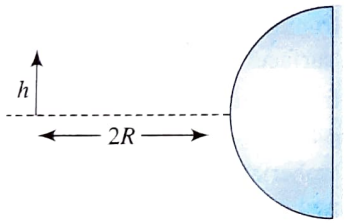
(1) The lens may be convex or concave.

(2) If the lens is diverging, the image distance has to be less than 15 cm.

(3) If the lens is converging, then its focal length has to be greater than 15 cm.

(4) All of these.

**60.** A glass hemisphere of radius $R$ and of material having refractive index 1.5 is silvered on its flat face as shown in figure. A small object of height $h$ is located at a distance $2R$ from the surface of hemisphere as shown in the figure. The final image will form




(1) at a distance of $R$ from silvered surface, on the right side

(2) on the object itself

(3) at hemisphere surface

(4) at a distance of $2R$ from the silvered surface, on left side

**61.** A lens of focal length 20.0 cm and aperture radius 2.0 cm is placed at a distance 30.0 cm from a point source of light. On the other side a screen is placed at a distance 50.0 cm from the lens. The radius of spot of light formed on screen is (neglect spherical abberation through lens)

(1) 1/2 cm (2) 1/3 cm

(3) 1/5 cm (4) 1.0 cm

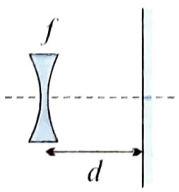
**62.** A diverging lens of focal length 10 cm is placed 10 cm away in front of a plane mirror as shown in figure. Light from a very far away source falls on the lens. An observer is situated some where in between lens and mirror, and observer is looking towards mirror. The observer will see the image behind the mirror at a distance

(1) 10 cm from the mirror

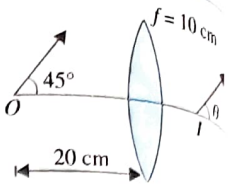
(2) infinte away from mirror

(3) 20 cm from mirror

(4) no image will be formed behind mirror.




**63.** An object starts moving at an angle of 45° with the principal axis as shown in figure in front of a biconvex lens of focal length +10 cm. If $\theta$ denotes the angle at which image starts to move with principal axis, then




(1) $\theta = \frac{3\pi}{4}$ (2) $\theta = \frac{\pi}{2}$

(3) $\theta = \frac{\pi}{4}$ (4) $\theta = -\frac{\pi}{4}$

**64.** A converging beam of rays is incident on a diverging thin lens. Having passed through the lens the rays intersect at a point 15 cm from the lens. If lens is removed the point where the rays meet will move 5 cm closer towards the mounting that holds the lens. The focal length of lens is

(1) $f = 10$ cm (2) $f = 15$ cm

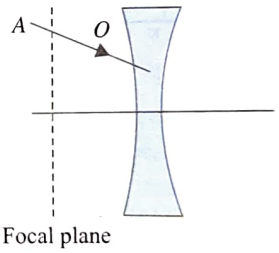
(3) $f = 30$ cm (4) $f = 40$ cm

**65.** Consider a sphere of radius $R$ made of glass of refractive index $\mu$. A small object moves along the diameter with a constant velocity $u$. Find the velocity of the image as seen by an observer outside when the object passes through center.

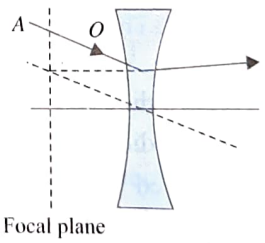
(1) $u$ (2) $\mu u$

(3) $u/\mu$ (4) zero

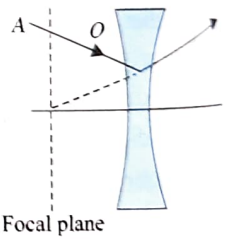
**66.** A ray of light $AO$ is incident on a diverging lens as shown in figure. The focal plane of the lens is also given. The possible direction of the refracted ray will be



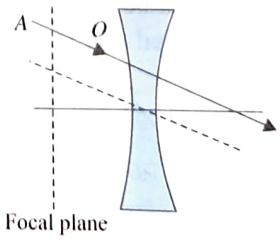

(1)




(2)

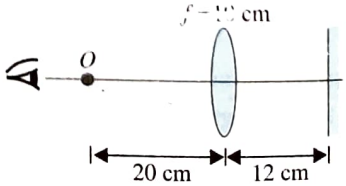



(3)



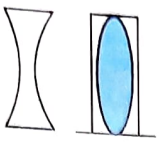

(4) None of these

**67.** In the arrangement shown in figure, $O$ is the object. An observer is to the left of convex lens and looking towards lens. The observer will see an image



(1) behind the mirror
(2) between lens and mirror
(3) to the left of lens
(4) no image will be seen

68. An equiconcave diverging lens of focal length $F$ is cut into two equal halves. The two halves are turned around and joined with some liquid between them. The lens obtained is converging with focal length $F$. If the refractive index of the liquid is 3 then what is the refractive index of the lens?

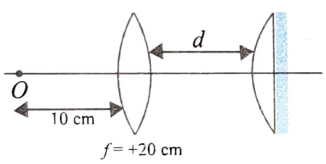


(1) 4 (2) 2
(3) 5 (4) 1.5

69. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance $u$ and the image distance $v$, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the $x$-axis meets the experimental curve at $P$. The coordinates of $P$ will be

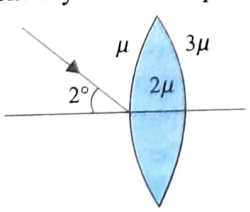
(1) $(2f, 2f)$ (2) $\left(\frac{f}{2}, \frac{f}{2}\right)$
(3) $(f, f)$ (4) $(4f, 4f)$

70. A convex lens of focal length 20 cm and another plano convex lens of focal length 40 cm are placed co-axially (see figure). The plano convex lens is silvered on plane surface. What should be the distance $d$ (in cm) so that final image of the object '$O$' is formed on $O$ itself.




(1) 10 cm (2) 40 cm
(3) 20 cm (4) 80 cm

71. A light ray hits the pole of a thin biconvex lens as shown in figure. The angle made by the emergent ray with the optic axis will be approximately




(1) 0°
(2) (1/3)°
(3) (2/3)°
(4) 2°

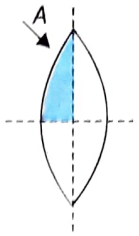
72. A lens forms a diminished and erect image of an object. The magnification is 1/4. Find ratio of distances between object and focus and focus and image.

(1) 4 : 1 (2) 1 : 4
(3) 8 : 1 (4) 2 : 1

73. When an object is placed on the principal axis of a convex lens at two different positions, it produces the images with magnifications +2 and –4 respectively. How many times more away from the lens the image will be formed in the second position as compared to the first position?

(1) 2 (2) 4
(3) 5 (4) 10

74. A convex lens of focal length $f$ cut into parts first horizontally and then vertically. Find the focal length of part $A$ of the lens, as shown.

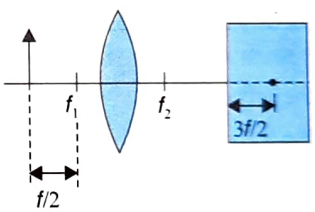



(1) $f/2$ (2) $f$
(3) $4f$ (4) $2f$

75. Find the distance of object placed in the slab of refractive index $\mu$ from point $P$ of the curved surface of radius $R$ so that image is formed at infinity.

(1) $\frac{(\mu-1)R}{\mu}$ (2) $\frac{\mu R}{(\mu-1)}$
(3) $\frac{R}{(\mu-1)}$ (4) $\frac{(\mu-1)R}{2\mu}$

76. An object is placed at $f/2$ away from first focus of a convex lens where $f$ is the focal length of the lens. Its image is formed at a distance $3f/2$ in a slab of refractive index 3/2, from the face of the slab facing the lens. Find the distance of this face of the slab from the second focus of the lens.

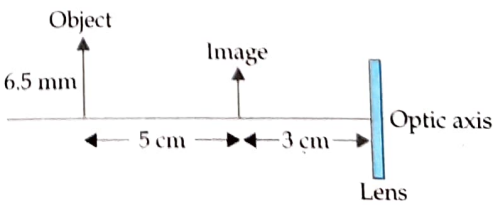



(1) $f/2$ (2) $3f/2$
(3) $2f$ (4) $f$

77. A fish in a lake (refractive index 4/3 for water) is viewed through a convex lens. From water surface, the lens is placed in air at half of the distance of the fish from the water surface, so that the image is formed at the fish itself. The focal length of the lens is how many times the depth of fish in water?

(1) 4 (2) 6
(3) 7.5 (4) 9

78. Figure shows an object and its image formed by a thin lens. Then the nature and focal length of lens is




(1) $f = 4.8$ cm, converging lens
(2) $f = -4.8$ cm, diverging lens
(3) $f = 2.18$ cm, converging lens
(4) $f = -2.18$ cm, diverging lens

**79.** A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids $L_1$ or $L_2$ having refractive indices $\mu_1$ and $\mu_2$ respectively $(\mu_2 > \mu_1 > 1)$. The lens will diverge a parallel beam of light if it is filled with

(1) air and placed in air

(2) air and immersed in $L_1$

(3) $L_1$ and immersed in $L_2$

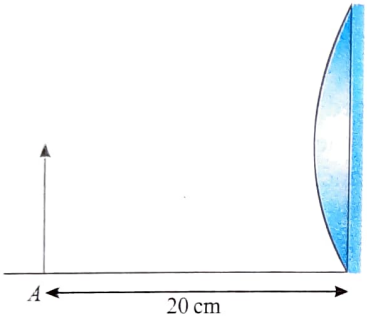
(4) $L_2$ and immersed in $L_1$.

**80.** The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image.

(1) 1.25 cm (2) 2.5 cm

(3) 1.05 cm (4) 2 cm

**81.** Focal length of the plano-convex lens is 15 cm. A small object is placed at $A$ as shown in figure. The plane surface is silvered. The image will form at



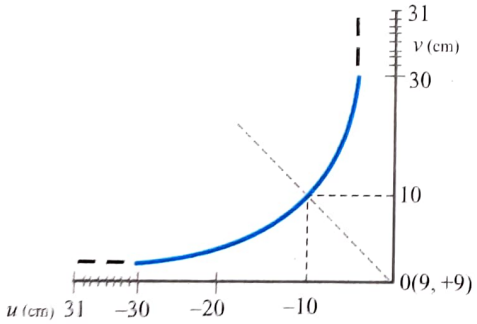

(1) 60 cm to the left of lens

(2) 12 cm to the left of lens

(3) 60 cm to the right of lens

(4) 30 cm to the left of lens

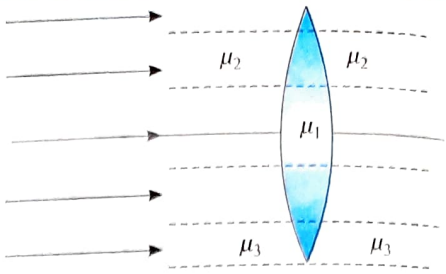
**82.** The graph shows relationship between object distance and image distance for an equiconvex lens. Then, focal length of the lens is




(1) $0.50 \pm 0.05$ cm (2) $0.50 \pm 0.10$ cm

(3) $5.00 \pm 0.05$ cm (4) $5.00 \pm 0.10$ cm

**83.** A double convex lens which is made of a material of refractive index $\mu_1$, is placed inside two liquids of refractive indices $\mu_2$ and $\mu_3$. The upper half of the lens is submerged in liquid of refractive index $\mu_2$ while the lower half is submerged in liquid of refractive index $\mu_3$ as shown. It is given, $\mu_2 > \mu_1 > \mu_3$. A wide, parallel beam of light is incident on the lens from the left. The lens will give rise to



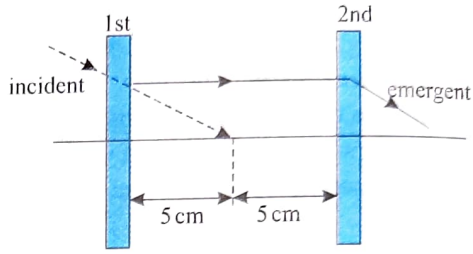

(1) a single convergent beam

(2) two different convergent beams

(3) two different divergent beams

(4) a convergent and a divergent beam

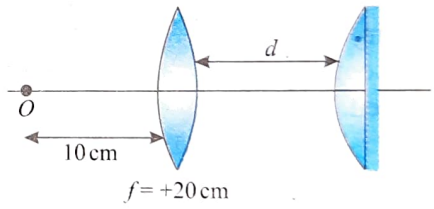
**84.** Look at the ray diagram shown, what will be the focal length of the 1st and the 2nd lens, if the incident light ray passes without any deviation ?




(1) −5 cm and +10 cm (2) +5 cm and +10 cm

(3) −5 cm and +5 cm (4) +5 cm and − 5 cm

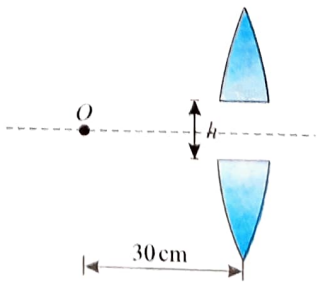
**85.** A convex lens of focal length 20 cm and another plano convex lens of focal length 40 cm are placed co-axially (see figure). The plano convex lens is silvered on plane surface. What should be the distance $d$ (in cm) so that final image of the object '$O$' is formed on $O$ itself.

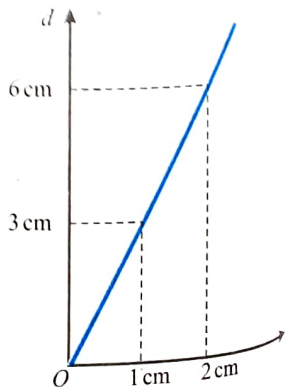



(1) 10 cm (2) 40 cm

(3) 20 cm (4) 80 cm

**86.** A convex lens cut symmetrically into two equal halves and separated laterally by a distance $h$ as shown in figure. A point object $O$ is placed at a distance 30 cm, from the lens halves. The cut parts of the lens forms two real images separated by a distance $d$. A graph $d$ versus $h$ is plotted as shown in figure. The focal length of the lens should be



(1) 60 cm (2) 50 cm
(3) 40 cm (4) 30 cm

87. A diminished image of an object is to be obtained on a screen 1.0 m from it. This can be achieved by appropriately placing
(1) a concave mirror of suitable focal length
(2) a convex mirror of suitable focal length
(3) a convex lens of focal length less than 0.25 m
(4) a concave lens of suitable focal length.

## Multiple Correct Answers Type

1. In displacement method, the distance between object and screen is 96 cm. The ratio of lengths of two images formed by a converging lens placed between them is 4. Then,
(1) ratio of the length of object to the length of shorter image is 2
(2) distance between the two positions of the lens is 32 cm
(3) focal length of the lens is 64/3 cm
(4) when the shorter image is formed on screen, distance of the lens from the screen is 32 cm

2. The distance between an electric lamp and a screen is $d = 1$ m. A convergent lens of focal length $f = 21$ cm is placed between the lamp and the lens such that a sharp image of the lamp filament is formed on the screen.
(1) The positions of the lens from the lamp for which sharp images are formed on the screen are 35 cm and 65 cm
(2) The positions of the lens from the lamp for which sharp images are formed on the screen are 30 cm and 70 cm
(3) Magnitude of the difference in magnification is 40/21
(4) The size of the lamp filament for which there are two sharp images of 4.5 cm and 2 cm, is 3 cm

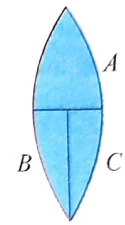
3. A lens of focal length $f$ is placed in between an object and screen at a distance $D$. The lens forms two real images of object on the screen for two of its different positions, a distance $x$ apart. The two real images have magnifications $m_1$ and $m_2$, respectively ($m_1 > m_2$). Then,
(1) $f = x/(m_1 - m_2)$ (2) $m_1 m_2 = 1$
(3) $f = (D^2 - x^2)/4D$ (4) $D \geq 4f$

4. An image of a bright square is obtained on a screen with the aid of a convergent lens. The distance between the square and the lens is 40 cm. The area of the image is nine times larger than that of the square. Select the correct statement(s).
(1) Image is formed at a distance of 120 cm from the lens.
(2) Image is formed at a distance of 360 cm from the lens.
(3) Focal length of the lens is 30 cm.
(4) Focal length of the lens is 36 cm.

5. When a real object is placed 25 cm from a lens, a real image is formed. Mark the correct statement(s) from the following.
(1) The lens is a converging lens.
(2) The image may be magnified or diminished.
(3) The focal length of the lens is less than 25 cm.
(4) The focal length of the lens may be greater than 25 cm.

6. Two converging lenses of focal lengths $f_1 = 10$ cm and $f_2 = 20$ cm are placed at some separation. A parallel beam of light is incident on first lens. Then,
(1) for emergent beam from second lens to be parallel, the separation between the lenses has to be 30 cm
(2) for emergent beam from second lens to be parallel, the separation between the lenses has to be 60 cm
(3) if lenses are placed at such a separation that emergent beam from second lens is parallel, then the emergent beam width is 2 cm if original beam has a width of 1 cm
(4) if lenses are placed at such a separation that emergent beam from second lens is parallel, then the emergent beam width is 4 cm if original beam width is 1 cm

7. A thin, symmetric double convex lens of power $P$ is cut into three parts $A$, $B$, and $C$ as shown in figure. The power of

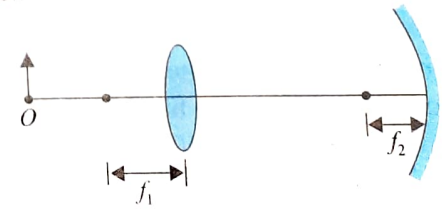



(1) $A$ is $P$ (2) $A$ is $2P$
(3) $B$ is $P/2$ (4) $C$ is $P/4$

8. A diverging lens of focal length $f_1$ is placed in front of and coaxially with a concave mirror of focal length $f_2$. Their separation is $d$. A parallel beam of light incident on the lens returns as a parallel beam from the arrangement. Then,
(1) the beam diameters of the incident and reflected beams must be the same
(2) $d = 2|f_2| - |f_1|$
(3) $d = |f_2| - |f_1|$
(4) if the entire arrangement is immersed in water, the conditions will remain unaltered

9. A converging lens of focal length $f_1$ is placed in front of and coaxially with a convex mirror of focal length $f_2$. Their separation is $d$. A parallel beam of light incident on the lens returns as a parallel beam from the arrangement. Then,
(1) the beam diameters of the incident and reflected beams must be the same
(2) $d = f_2 - 2|f_2|$
(3) $d = f_1 - |f_2|$
(4) if the entire arrangement is immersed in water, the conditions will remain unaltered

10. An object is placed in front of a converging lens at a distance equal to twice the focal length $f_1$ of the lens. On the other side of the lens is a concave mirror of focal length $f_2$ separated from the lens by a distance $2(f_1 + f_2)$. Light from the object passes rightward through the lens, reflects from the mirror, passes leftward through the lens, and forms a final image of the object.



(1) The distance between the lens and the final image is equal to $2f_1$.

(2) The distance between the lens and the final image is equal to $2(f_1 + f_2)$.

(3) The final image is real, inverted and of same size as that of the object.

(4) The final image is real, erect and of same size as that of the object.

**11.** A convex lens made of glass ($\mu_g = 3/2$) has focal length $f$ in air. The image of an object placed in front of it is inverted, real and magnified. Now the whole arrangement is immersed in water ($\mu_w = 4/3$) without changing the distance between object and lens. Then

(1) the new focal length will become $4f$

(2) the new focal length will become $f/4$

(3) new image will be virtual and magnified

(4) new image will be real, inverted and smaller in size

**12.** A plano-convex lens of focal length 20 cm has its plane side silvered.

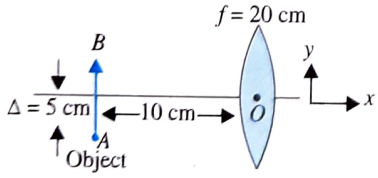
(1) The radius of curvature of curved surface of given plano-convex lens is equal to half of radius of curvature of a surface of equiconvex lens of focal length 20 cm

(2) An object placed at 15 cm on the axis on the convex side of silvered plano-convex lens gives rise to an image at a distance of 30 cm from it

(3) An object placed at a distance of 20 cm on the axis on the convex side of silvered plano-convex lens gives rise to an image at 40 cm from it

(4) Silvered plano-convex lens acts as a concave mirror of focal length 10 cm.

**13.** A linear object of size 1.5 cm placed at 10 cm from a lens of focal length 20 cm. The optic centre of lens and the object are displaced a distance $\Delta$. The magnification of the image formed is $m$ (Take optic centre as origin). The coordinates of image of $A$ and $B$ are $(x_1, y_1)$ and $(x_2, y_2)$ respectively. Then




(1) $(x_1, y_1) = (-20\text{ cm}, -1\text{ cm})$ (2) $(x_2, y_2) = (-20\text{ cm}, 2\text{ cm})$

(3) $m = 3$ (4) $m = 2$

**14.** A combination of two thin convex lenses of equal focal lengths, is kept separated along the optical axis by a distance of 20 cm between them. The combination behaves as a lens system of infinite focal length,

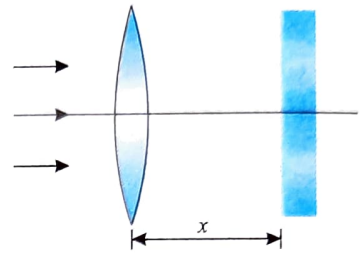
(1) if an object is kept on the common optic axis at a distance of 10 cm from the first lens, its final image will be formed at a distance of 10 cm from the next lens.

(2) if an object is kept on the common optic axis at a distance of 10 cm from the first lens, its final image will be formed at a distance of 30 cm from the next lens.

(3) if an object is kept on the common optic axis at a distance of 30 cm from the first lens, its final image will be formed at a distance of 10 cm from the next lens.

(4) if an object is kept on the common optic axis at a distance of 30 cm from the first lens, its final image will be formed at a distance of 20 cm from the next lens.

**15.** A parallel beam of light is incident on a lens of focal length 10 cm. A parallel slab of refractive index 1.5 and thickness 3 cm is placed on the other side of the lens. The nature of the final image from the lens slab combination is (Assume rays to be paraxial)



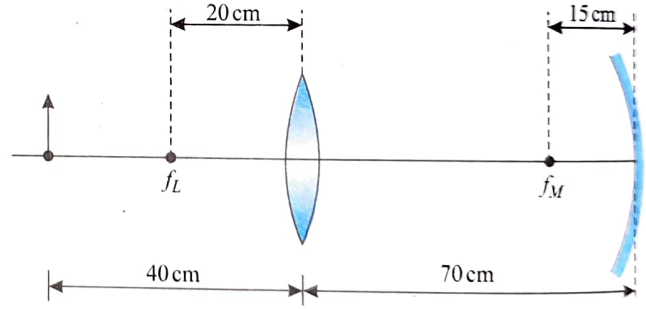

(1) at distance of 11 cm and image is virtual if $x = 12$ cm.

(2) at distance of 11 cm and image is real if $x = 14$ cm.

(3) at the distance of 11 cm and image is virtual if $x = 16$ cm

(4) at distance of 11 cm and image is real if $x = 5$ cm.

**16.** An upright object is placed at a distance in front of a converging lens equal to twice the focal length 20 cm of the lens. On the other side there is a concave mirror of focal length 15 cm separated from the lens by a distance of 70 cm. Then select the correct statements from the following




(1) Magnification for the system is $-1/2$

(2) Magnification for the system is $-1$

(3) Final image by the system will real and at distance of 110 cm from spherical mirror

(4) Magnification for the system is $+1/2$

**17.** A convex lens forms inverted image of a real object on a fixed screen. The size of image is 12 cm When lens is displaced 20 cm along principal axis it again forms a real image of size 3 cm on the screen. Choose the correct options. (Assume image formation only by paraxial rays)

(1) Focal length of the lens is $\frac{40}{3}$ cm

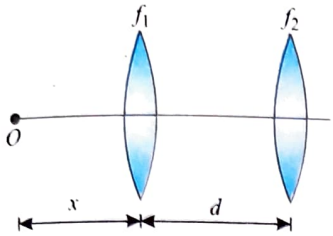
(2) Focal length of the lens is $\frac{80}{3}$ cm

(3) Size of object is 6 cm

(4) Size of object is 9 cm

**18.** Two converging lenses have focal length $f_1$ and $f_2$ $(f_1 > f_2)$. The optical axis of the two lenses coincide. This lens system

is used to form an image of real object. It is observed that final magnification of the image does not depend on the distance $x$. Whole arrangement is shown in figure. Choose the correct options.




(1) Final magnification is $-\frac{f_1}{f_2}$

(2) Final magnification is $-\frac{f_2}{f_1}$

(3) Distance between the lenses is $\frac{f_1 f_2}{f_1 + f_2}$

(4) Distance between the lenses is $f_1 + f_2$

19. A plano-convex lens is made of a material of refractive index $n$. When a small object is placed 30 cm away from the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement (s) is(are) true?

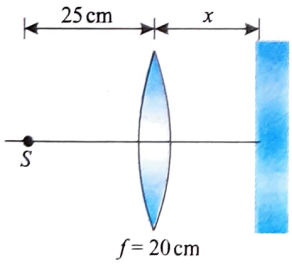
(1) The refractive index of the lens is 2.5.

(2) The radius of curvature of the convex surface is 45 cm.

(3) The faint image is erect and real.

(4) The focal length of the lens is 20 cm.

20. A point source of light $S$ is placed on the axis of a lens of focal length 20 cm as shown. A screen is placed normal to the axis of lens at a distance $x$ from it. Treat all rays as paraxial. Mark the correct statements.



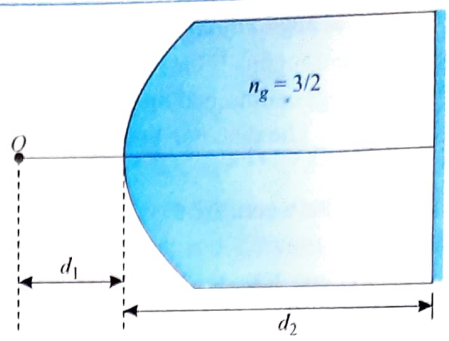

(1) As $x$ is increased from zero, intensity continuously decreases.

(2) As $x$ is increased from zero, intensity first increases and then decreases.

(3) Intensity at centre of screen for $x = 90$ cm and $x = 110$ cm is same.

(4) Radius of bright circle obtained on screen is equal to 1 cm for $x = 200$ cm.

21. In the figure shown a point object $O$ is placed in air on the principal axis. The radius of curvature of the spherical surface is 60 cm. $I_f$ is the final image formed after all the refractions and reflections.



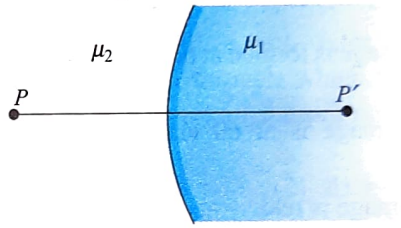

(1) If $d_1 = 120$ cm, then '$I_f$' is formed on '$O$' for any value of $d_2$.

(2) If $d_1 = 240$ cm, then '$I_f$' is formed on '$O$' only if $d_2 = 360$ cm.

(3) If $d_1 = 240$ cm, then '$I_f$' is formed on '$O$' for all values of $d_2$.

(4) If $d_1 = 240$ cm, then '$I_f$' cannot be formed on '$O$'.

22. Two refracting media are separated by a spherical interface as shown in the figure. $PP'$ is the principal axis, $\mu_1$ and $\mu_2$ are the refractive indices of medium of incidence and medium of refraction respectively. If



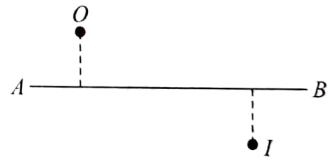

(1) $\mu_2 > \mu_1$, there cannot be a real image of real object.

(2) $\mu_2 > \mu_1$, there cannot be a real image of virtual object.

(3) $\mu_1 > \mu_2$, there cannot be a virtual image of virtual object.

(4) $\mu_1 > \mu_2$, there cannot be a real image of real object.

23. A luminous point object is placed at $O$, whose image is formed at $I$ as shown in the figure. $AB$ is the optical axis. Which of the following statements are correct?




(1) If a lens is used to obtain an image, the lens must be converging.

(2) If a mirror is used to obtain an image, the mirror must be a convex mirror having pole at the point of intersection of lines $OI$ and $AB$.

(3) Position of principal focus of mirror cannot be found.

(4) $I$ is a real image.

24. A convex lens is placed in the $XYZ$ coordinate system such that its optical centre is at the origin and principal axis is along the $x$-axis. The focal length of the lens is 20 cm. A point object has been placed at the point (–40 cm, +1 cm, –1 cm.). Which of the following are correct about coordinates of the image?

(1) $x = 40$ cm (2) $y = +1$ cm

(3) $z = +1$ cm (4) $z = -1$ cm

**25.** A convex lens forms an image of an object on a screen. The height of the image is 9 cm. The lens is now displaced until an image is again obtained on the screen. The height of this image is 4 cm. The distance between the object and the screen is 90 cm.

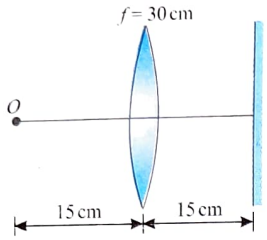
(1) The distance between the two positions of the lens is 30 cm.

(2) The distance of the object from the lens in its first position is 36 cm.

(3) The height of the object is 6 cm.

(4) The focal length of the lens is 21.6 cm.

**26.** An object $O$ is kept in front of a converging lens of focal length 30 cm behind which there is a plane mirror at 15 cm from the lens. Choose the correct statements.




(1) The final image is formed at 60 cm from the lens towards right of it.

(2) The final image is at 60 cm from lens towards left of it.

(3) The final image is real.

(4) The final image is virtual.

**27.** A convex lens forms an image of an object on a screen. The height of the image is 9 cm. The lens is now displaced until an image is again obtained on the screen. Then height of this image is 4 cm. The distance between the object and the screen is 90 cm.

(1) The distance between the two positions of the lens is 30 cm.

(2) The distance of the object from the lens in its first position is 36 cm.

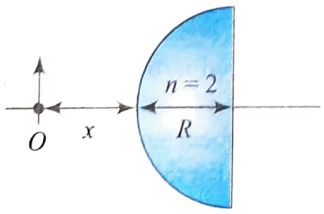
(3) The height of the object is 6 cm.

(4) The focal length of the lens is 21.6 cm.

## Linked Comprehension Type

### For Problems 1–3

Consider a transparent hemisphere ($n = 2$) in front of which a small object is placed in air ($n = 1$) as shown in figure.




**1.** For which value of $x$, of the following, will the final image of the object at $O$ be virtual?

(1) $2R$ (2) $3R$

(3) $R/2$ (4) $1.5R$

**2.** What is the nature of final image of the object when $x = 2R$?

(1) Erect and magnified.

(2) Inverted and magnified.

(3) Erect and same size.

(4) Inverted and same size.

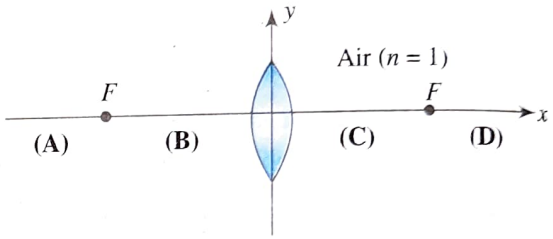
**3.** Consider a ray starting from $O$ which strikes the spherical surface at grazing incidence ($i = 90°$). Taking $x = R$, what will be the angle (from normal) at which the ray may emerge from the plane surface.

(1) 90° (2) 0°

(3) 30° (4) 60°

### For Problems 4–6

This question concerns a symmetrical lens shown, along with its two focal points. It is made of plastic with $n = 1.2$ and has focal length $f$. Four different regions are shown:




Here,

(A) $-\infty < x < -f$ (B) $-f < x < 0$

(C) $0 < x < f$ (D) $f < x < \infty$

**4.** If an object is placed somewhere in region (A), in which region does the image appear?

(1) A (2) B

(3) C (4) D

**5.** If incident rays are converging, then in which region does the image appear?

(1) A (2) B

(3) C (4) D

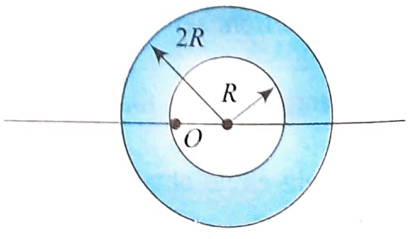
**6.** A second lens is now placed adjacent to the first in contact with it. The new lens is concave, with a focal length $f = -3f$. What is the focal length of the two-lens system? Treat the lenses as thin.

(1) A (2) B

(3) C (4) D

### For Problems 7–8

A glass sphere of radius $2R$ and refractive index $n$ has a spherical cavity of radius $R$, concentric with it.




**7.** When viewer is on left side of the hollow sphere, what will be the shift in position of the object?

(1) $\frac{(n+1)}{(n-1)}R$, right (2) $\frac{(n-1)}{(n+1)}R$, right

(3) $\frac{(2n-1)}{(2n+1)}R$, left (4) $\frac{2(n-1)}{(n+1)}R$, left

8. When viewer is on right side of the hollow sphere, what will be the apparent change in position of the object?

(1) $\frac{(n-1)}{(3n+1)}R$, toward left

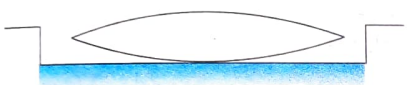
(2) $\frac{(n-1)}{(3n-1)}R$, toward left

(3) $\frac{(n+1)}{(3n+1)}R$, toward right

(4) $\frac{(n+1)}{(3n+1)}R$, toward right

**For Problems 9–10**

A thin equiconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in figure. The space between the lens and the mirror is filled with a liquid of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image.



9. The radius of curvature of the convex surface is

(1) 10 cm (2) 15 cm

(3) 20 cm (4) 25 cm

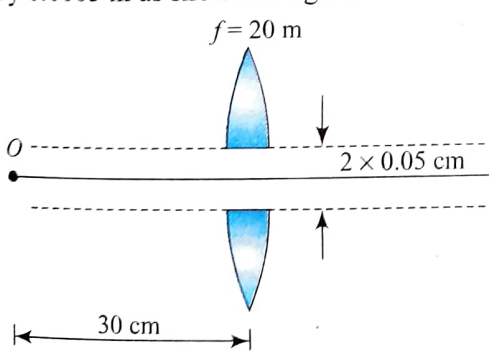
10. If another liquid is filled instead of water, the object and the image coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid.

(1) 1.6 (2) 2.6

(3) 2.8 (4) 3.2

**For Problems 11–13**

A point object $O$ is placed at a distance of 0.3 m from a convex lens (focal length 0.2 m) cut into two halves each of which is displaced by 0.0005 m as shown in figure.




11. What will be the location of the image?

(1) 30 cm, right of lens (2) 60 cm, right of lens

(3) 70 cm, left of lens (4) 40 cm, left of lens

12. If this arrangement will generate more than one image, then what will be the total number of images?

(1) 2 (2) 4

(3) 6 (4) 5

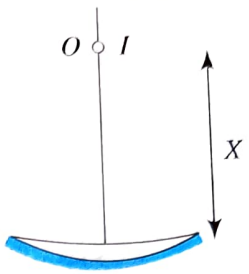
13. Find the spacing between the images so formed.

(1) 0.1 cm (2) 0.5 cm

(3) 0.3 cm (4) 1 cm

**For Problems 14–15**

The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface.




14. Where should a pin be placed on the optic axis such that its image is formed at the same place?

(1) $x = 5$ cm (2) $x = 20$ cm

(3) $x = 15$ cm (4) $x = 25$ cm

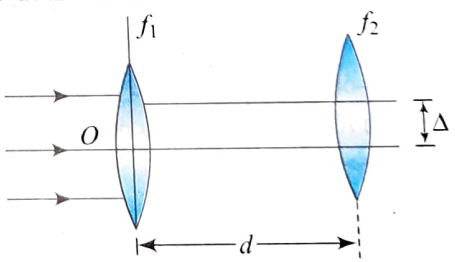
15. If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved so that the image of the pin again coincides with the pin?

(1) $\Delta x = 1.15$ cm, up (2) $\Delta x = 3.15$ cm, down

(3) $\Delta x = 0.05$ cm, up (4) $\Delta x = 0.15$ cm, down

**For Problems 16–17**

Two thin convex lenses of focal lengths $f_1$ and $f_2$ are separated by a horizontal distance $d$ ($d < f_1$ and $d < f_2$) and their centers are displaced by a vertical separation as shown in the figure. A parallel beam of rays coming from left. Take the origin of coordinates $O$ at the center of first lens.




16. Find the $y$-coordinate of the focal point of this lens system.

(1) $\frac{(f_1+d)\Delta}{(f_1+f_2-d)}$ (2) $\frac{2(f_1+d)\Delta}{(f_1+f_2-d)}$

(3) $\frac{2(f_1-d)\Delta}{(f_1+f_2+d)}$ (4) $\frac{(f_1-d)\Delta}{(f_1+f_2-d)}$

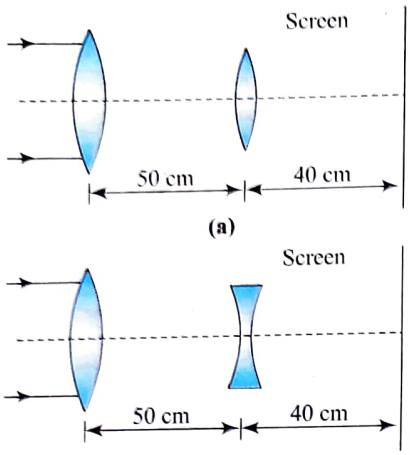
17. Find the $x$-coordinate in the same problem from the focal point of this lens system.

(1) $\frac{d(f_1-d)+f_1f_2}{(f_1+f_2-d)}$ (2) $\frac{f_1f_2}{(f_1+f_2-d)}$

(3) $\frac{d(f_1-d)}{(f_1+f_2-d)}$ (4) $\frac{2d(f_1+d)-f_1f_2}{(f_1+f_2-d)}$

**For Problems 18–19**

A parallel beam of light falls successively on a thin convex lens of focal length 40 cm and then on a thin convex lens of focal length 10 cm as shown in Fig. (a).




In Fig. (b), the second lens is an equiconcave lens of focal length 10 cm and made of a material of refractive index 1.5. In both the cases, the second lens has an aperture equal to 1 cm.

**18.** Compare the area illuminated by the beam of light on the screen, which passes through the second lens in the two cases. The ratio $(A_2 / A_1)$ will be

(1) 72/5 (2) 81/4

(3) 56/3 (4) 29/2

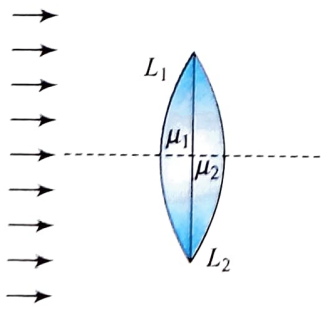
**19.** Now, a liquid of refractive index $\mu$ is filled to the right of the second lens in case (b) such that the area illuminated in both the cases is the same. Determine the refractive index of the liquid.

(1) 1 (2) 2.5

(3) 3 (4) 1.5

**For Problems 20–21**

Two identical plano-convex lenses $L_1$ ($\mu_1 = 1.4$) and $L_2$ ($\mu_2 = 1.5$) of radii of curvature $R = 20$ cm are placed as shown in figure.

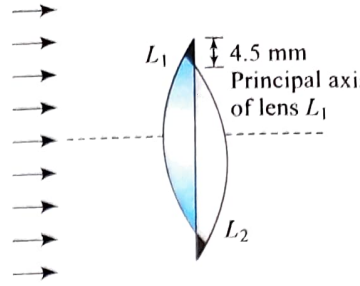



**20.** Find the position of the image of the parallel beam of light relative to the common principal axis.

(1) 100/7 cm (2) 200/9 cm

(3) 31.2 cm (4) 21.8 cm

**21.** Now, the second lens is shifted vertically downward by a small distance 4.5 mm and the extended parts of $L_1$ and $L_2$ are blackened as shown in figure. Find the new position of the image of the parallel beam.




(1) 200/9 cm behind the lens 2.5 mm below the principal axis of $L_1$

(2) 100/9 cm in front of the lens 2 mm below the principal axis of $L_1$

(3) 200/9 cm in front of the lens 2.5 mm below the principal axis of $L_1$

(4) 100/9 cm behind the lens 2 mm below the principal axis of $L_1$

**For Problems 22–23**

A beam of light converges towards a point $O$, behind a convex mirror of focal length 20 cm.

**22.** Find the magnification and nature of the image when point $O$ is 10 cm behind the mirror.

(1) 2(virtual, inverted) (2) 3 (real, inverted)

(3) 5 (real, erect) (4) 2 (real, erect)

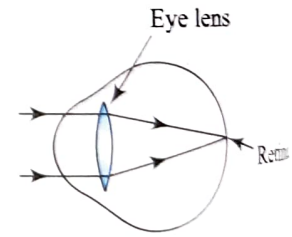
**23.** Similarly, as in above question when point $O$ is 30 cm behind the mirror.

(1) 2 (virtual, inverted) (2) 3 (real, inverted)

(3) 3 (virtual, enlarged) (4) 1 (real, enlarged)

**For Problems 24–28**

The ciliary muscles of eye control the curvature of the lens in the eye and hence can alter the effective focal length of the system. When the muscles are fully relaxed, the focal length is maximum. When the muscles are strained, the curvature of lens increases. That means, radius of curvature decreases and focal length decreases. For a clear vision, the image must be on the retina. The image distance is therefore fixed for clear vision and it equals the distance of retina from eye lens. It is about 2.5 cm for a grown up person.




A person can theoretically have clear vision of an object situated at any large distance from the eye. The smallest distance at which a person can clearly see is related to minimum possible focal length. The ciliary muscles are most strained in this position. For an average grown up person, minimum distance of the object should be around 25 cm.

A person suffering from eye defects uses spectacles (eye glass). The function of lens of spectacles is to form the image of the objects within the range in which the person can see clearly. The image of the spectacle lens becomes object for the eye lens and whose image is formed on the retina.

The number of spectacle lens used for the remedy of eye defect is decided by the power of the lens required and the number

of spectacle lens is equal to the numerical value of the power of lens with sign. For example, if power of the lens required is +3D (converging lens of focal length 100/3 cm), then number of lens will be +3.

For all the calculations required, you can use the lens formula and lensmaker's formula. Assume that the eye lens is equiconvex lens. Neglect the distance between the eye lens and the spectacle lens.

24. Minimum focal length of eye lens of a normal person is
(1) 25 cm (2) 2.5 cm
(3) $\frac{25}{9}$ cm (4) $\frac{25}{11}$ cm

25. Maximum focal length of a eye lens of a normal person is
(1) 25 cm (2) 2.5 cm
(3) $\frac{25}{9}$ cm (4) $\frac{25}{11}$ cm

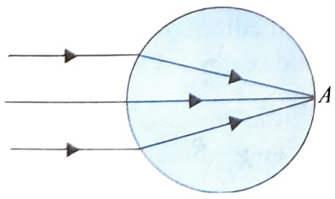
26. A near sighted man can clearly see objects only upto a distance of 100 cm and not beyond this. The number (power) of the spectacle lenses necessary for the remedy of this defect will be
(1) +1 (2) −1
(3) +3 (4) −3

27. A far sighted man cannot see objects clearly unless they are at least 100 cm from his eyes. The number (power) of the spectacle lenses that will make his range of clear vision equal to an average grown up person will be
(1) +1 (2) −1
(3) +3 (4) −3

28. A person can see objects clearly from distance 10 cm to ∞. Then, we can say that the person is
(1) normal sighted person
(2) near-sighted person
(3) far-sighted person
(4) a person with exceptional eye having no eye defect

**For Problems 29–31**

A parallel beam of light falls on a solid transparent sphere.



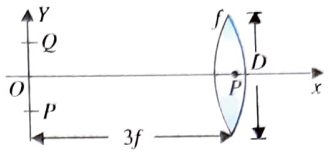

29. Which is correct?
(1) If the beam is thick, then whole beam can be focussed at $A$.
(2) The whole beam can be focussed at $A$ only if the beam is thin enough.
(3) If the beam is thin, then the beam can't be focussed before $A$.
(4) None of these

30. If however thin beam is focussed at $A$, then find the refractive index of the sphere.
(1) 1.5 (2) 1.7
(3) 2.0 (4) 2.5

31. For what value of refractive index $\mu$, the thin beam can be focussed at centre of sphere.
(1) 1.5 (2) 2
(3) 2.5 (4) None of these

**For Problems 32–34**

An object kept near a convex lens of focal length $f$, executes SHM between $P$ and $Q$ according to the equation $y = A \sin \omega t$, $O$ being the mean position. Take $x$-axis as the principal axis of the lens and $A << D$ to answer the following questions:



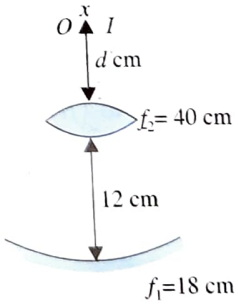

32. The phase difference between the object and the image oscillations is
(1) $\pi/2$
(2) $\pi$
(3) 0
(4) None of these

33. The amplitude of oscillation of the image is
(1) $A$ (2) $2A$
(3) $A/2$ (4) Image does not oscillate

34. The velocity of the image when the object crosses the mean position and goes towards $Q$ is
(1) $A\omega$ (2) $-A\omega$
(3) $A\omega/2$ (4) None of these

**For Problems 35–37**

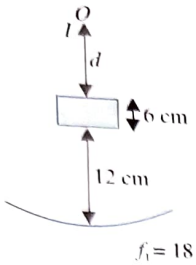
A convex lens of focal length 40 cm is held at a distance 12 cm coaxially above a concave mirror of focal length 18 cm.




35. A luminous point object placed $d$ cm above the lens on its axis gives rise to an image coincident with itself, then the value of $d$ is equal to
(1) 15 cm (2) 30 cm
(3) 24 cm (4) 18 cm

36. If the convex lens is replaced by a concave lens of focal length 60 cm, it gives rise to an image coincident with itself. Then the value of $d$ is
(1) 20 cm (2) 30 cm
(3) 40 cm (4) 60 cm

37. If the convex lens is replaced by a glass plate of thickness 6 cm, refractive index $\mu = \frac{3}{2}$ and gives rise to an image coincident with itself, then the value of $d$ is equal to

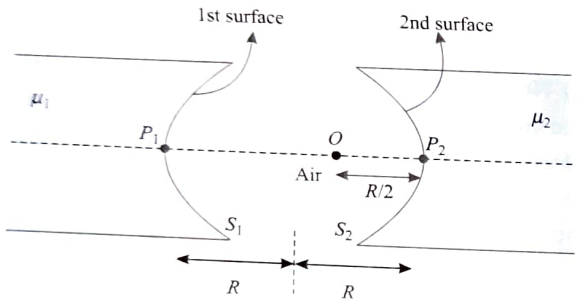



$f_1 = 18$ cm

(1) 20 cm (2) 30 cm
(3) 40 cm (4) 60 cm

**For Problems 38–39**

Two concave refracting surfaces $S_1$ and $S_2$ each with radius of curvature $R = 35$ cm and refractive indices $\mu_1$ and $\mu_2$ respectively, are placed facing each other in air as shown in figure. A point object '$O$' is placed at a distance $R/2$ from one of the surface as shown. (Consider image formation only by paraxial rays)




38. Distance of image after refraction from 1st surface $(\mu_1 = 1.5)$ is equal to
(1) 45 cm towards left of $P_1$
(2) 45 cm towards right of $P_1$
(3) 30 cm towards left of $P_1$
(4) 30 cm towards right of $P_1$

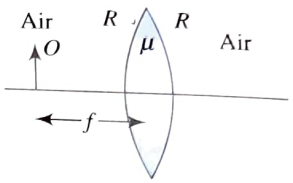
39. Separation between the images of '$O$' formed by each refracting surface is equal to $(\mu_2 = 1.5)$
(1) 8 cm (2) 4 cm
(3) 2 cm (4) cannot be determined

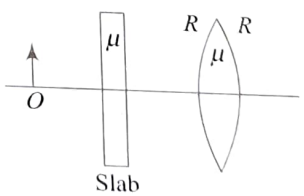
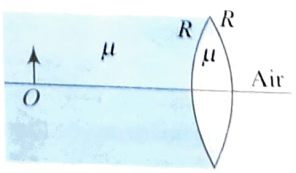
## Matrix Match Type

1. Match the following

| Column I | Column II |
|---|---|
| i. Object placed between optic center and 1st principal focus in a diverging lens | a. Image is inverted |
| ii. Object placed between optic center and 1st principal focus of a converging lens | b. Image is erect |
| iii. Object placed between optic center and 2nd principal focus of a diverging lens | c. Image is of greater size than the object |
| iv. Object placed between optic center and 2nd principal focus of a converging lens | d. Image is of smaller size than the object |

2. An object $O$ (real) is placed at focus of an equi-biconvex lens as shown in figure. The refractive index of the lens is $\mu = 1.5$ and the radius of curvature of either surface of lens is $R$. The lens is surrounded by air. In each statement of column I, some changes are made to situation given above and information regarding final image formed as a result is given in column II. The distance between lens and object is unchanged in all statements of column I. Match the statements in columnI with resulting image in column II.



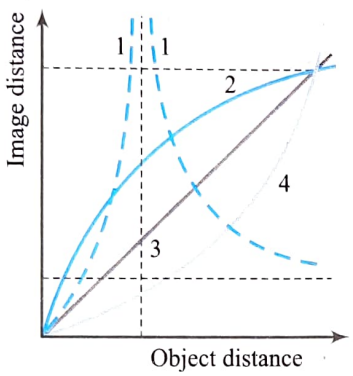

| Column I | Column II |
|---|---|
| i. If the refractive index of the lens is doubled (that is, made $2\mu$,) then | a. final image is real |
| ii. If the radius of curvature is doubled (that is, made $2R$,) then | b. final image is virtual |
| iii. If a glass slab of refractive index $\mu = 1.5$ is introduced between the object and lens as shown, then   | c. final image becomes smaller in size in comparison to size of image before the change was made |
| iv. If the left side of lens is filled with a medium of refractive index $\mu = 1.5$ as shown, then   | d. final image is of same size as the object |

3. Match the following:

| Column I | Column II |
|---|---|
| i. A convex lens in a denser medium will behave like a | a. converging lens |
| ii. A concave lens in a rarer medium will behave like a | b. diverging lens |

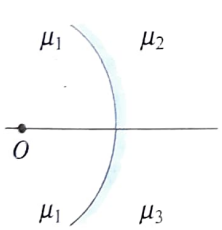
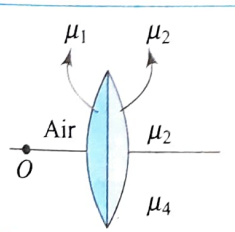
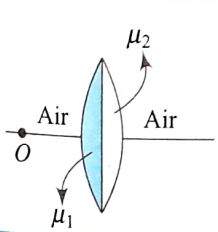
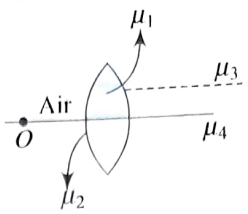
| | |
|---|---|
| **iii.** A plano-convex lens silvered on its curved surface and placed in air will behave like a | **c.** concave mirror |
| **iv.** A planoconcave lens silvered on its plane surface and placed in air will behave like a | **d.** convex mirror |

4. A small particle is placed at the pole of a concave mirror and then moved along the principal axis to a large distance. During the motion, the distance between the pole of the mirror and the image is measured. The procedure is then repeated with a convex mirror, a concave lens, and a convex lens. The graph is plotted between image distance and object distance. Match the curves shown in the graph with the mirror or lens that is corresponding to it. (Curve 1 has two segments.)

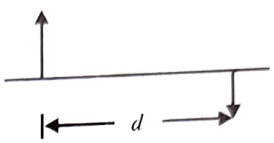



| Column I (Lens/Mirror) | Column II (Curve) |
|---|---|
| **i.** Converging lens | **a.** 1 |
| **ii.** Converging mirror | **b.** 2 |
| **iii.** Diverging lens | **c.** 3 |
| **iv** Diverging mirror | **d.** 4 |

5. Match the entries of column I with entries of column II:

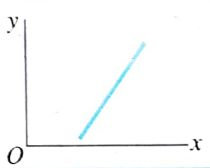
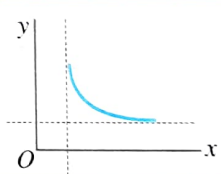
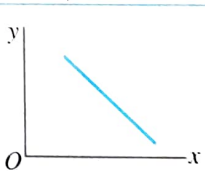
| Column I | Column II |
|---|---|
| **i.**   | **a.** Number of images formed is 1. |
| **ii.**   | **b.** Number of images formed is 2. |
| **iii.**   | **c.** Number of images formed is 3. |
| **iv.**   | **d.** Number of images formed is 4. |

6. Real and inverted image of an object is formed at a distance $d = 40$ cm from the object. The size of the image is half of that of object.

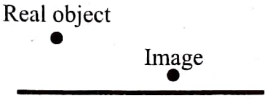



| Column I | Column II |
|---|---|
| **i.** The nature of lens used should be | **a.** concave |
| **ii.** focal length of the lens | **b.** convex |
| **iii.** distance of object from the lens | **c.** 80/3 cm |
| **iv.** If mirror were used, then nature of mirror should be | **d.** 80/9 cm |

7. The graphs given apply to a convex lens of focal length $f$, producing a real image at a distance $v$ from the optical centre when self luminous object is at distance $u$ from the optical centre. The magnitude of magnification is $m$. Identify the following graphs with the first named quantity being plotted along $y$-axis. Assume object distance greater than focal length.

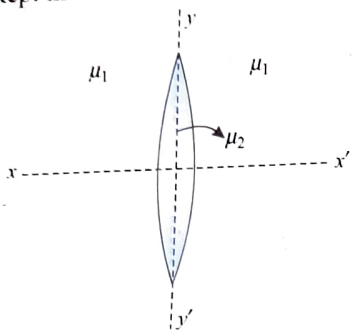
| Column I | Column II |
|---|---|
| **i.** $v$ against $u$ | **a.**   |
| **ii.** $\frac{1}{v}$ against $\frac{1}{u}$ | **b.**   |
| **iii.** $m$ against $v$ | **c.**   |

8. In the column I, optical systems are given. Match the optical system with the image and object combination possible. The nature of image is not known and the optical system can be at any position.

| Column I | Column II |
|---|---|
| **i.** Convex mirror | **a.**   |

| | |
|---|---|
| ii. Concave mirror | b. Virtual object, Image, P axis |
| iii. Converging lens | c. Virtual object, P axis, Image |
| iv. Diverging lens | d. Real object, P axis, Image |

9. A equi-convex lens of refractive index $\mu_2$ and focal length $f$ (in air) is kept in medium of refractive index $\mu_1$.




| Column I | Column II |
|---|---|
| i. If lens is cut in two equal parts by a plane $yy'$. ($\mu_1 = 1$) | a. Lens will act as a glass slab |
| ii. If lens is cut in two equal parts by a plane $xx'$. ($\mu_1 = 1$) | b. Lens will be converging and focal length will change |
| iii. If $\mu_1 = \mu_2$ | c. Lens will be converging and focal length will remain same |
| iv. If $\mu_1 > \mu_2$ | d. Lens will be diverging and focal length will change. |

**Codes:**

| | i. | ii. | iii. | iv. |
|---|---|---|---|---|
| (1) | b. | d. | a. | c. |
| (2) | b. | c. | a. | d. |
| (3) | c. | d. | b. | a. |
| (4) | a. | b. | c. | d. |

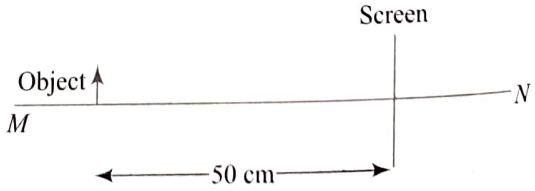
10. Match the columns

| Column I | Column II |
|---|---|
| i. $\mu_2$ $\mu_1$ $\mu_2$, If $\mu_2 > \mu_1$ | a. Optical power will be positive |
| ii. $\mu_1$ $\mu_2$ $\mu_1$, If $\mu_2 > \mu_1$ | b. Optical power will be negative |
| iii. $\mu_2$ $\mu_1$ $\mu_1$, If $\mu_2 < \mu_1$ | c. System will converge a parallel beam of light incident on it |
| iv. $\mu_1$ $\mu_2$ Concave mirror silver polish, $\mu_2 = \mu_2$ | d. Focal length will be positive |
| | e. Focal length will be negative |

(1) (i) – b., e.; (ii) – a., c., d.; (iii) –a., c., d.; (iv) – a., c., e.
(2) (i) – b.; (ii) – a., c., d.; (iii) –a., c., d.; (iv) – a., c., d., e.
(3) (i) – b., d.; (ii) – a., c.; (iii) –a., c., d.; (iv) – a., c.
(4) (i) – a., b.; (ii) – a., c., d.; (iii) –c., d.; (iv) – a., c., d., e.

## Numerical Value Type

1. The magnification of an object placed in front of a convex lens is +2. The focal length of the lens is 2.0 metres. Find the distance by which object has to be moved to obtain a magnification of –2 (in metres).
2. A point object is placed at a distance of 25 cm from a convex lens. Its focal length is 22 cm. A glass slab of refractive index 1.5 is inserted between the lens and the object, then the image is formed at infinity. Find the thickness of the glass slab (in cm).
3. Where should a convex lens of focal length 9 cm be placed (in cm) between two point sources $S_1$ and $S_2$ which are 24 cm apart, so that images of both the sources are formed at the same place. You have to find distance of lens from $S_1$ or $S_2$ whichever is lesser.
4. An object is placed 50 cm from a screen as shown in figure.

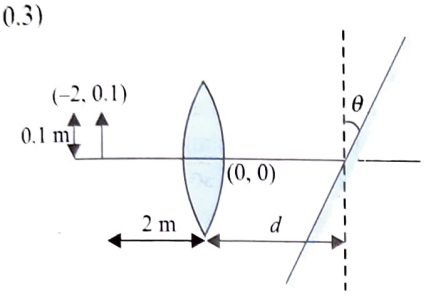



A converging lens is moved such that line $MN$ is its principal axis. Sharp images are formed on the screen in two positions of lens separated by 30 cm. Find the focal length of the lens in cm.

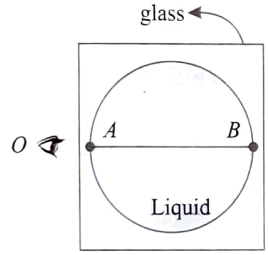
5. An extended object of size 2 mm is placed on the principal axis of a converging lens of focal length 10 cm. It is found that when the object is placed perpendicular to the principal axis the image formed is 4 mm in size. The size of image when it is placed along the principal axis is _____ mm.

6. A point object is placed at a distance 25 cm from a convex lens of focal length 20 cm. If a glass slab of thickness $t$ and refractive index 1.5 is inserted between the lens and object, image is formed at $\infty$. Thickness $t$ is found to be $K$ times of 5 cm. Find $K$.

7. A convex lens of focal length 1.5 m is placed in a system of coordinate axis such that its optical centre is at origin and principal axis coinciding with the $x$-axis. An object and a plane mirror are arrange on the principal axis as shown in figure. Find value of $d$ (in m) so that $y$-coordinate of image (after refraction and reflection) is 0.3 m. (Take $\tan\theta = 0.3$)




8. The observer '$O$' sees the distance $AB$ as infinitely large. If refractive index of liquid is $\mu_1$ and that of glass is $\mu_2$, then $\frac{\mu_1}{\mu_2}$ is

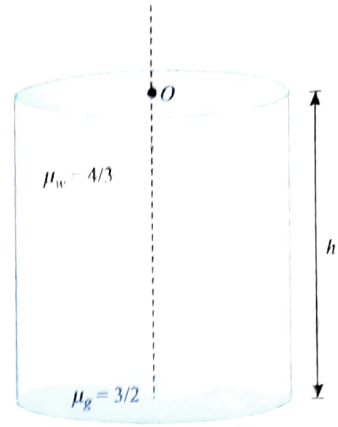



9. A converging beam of rays is incident on a diverging lens. Having passed through the lens the rays intersect at a point 10 cm from the lens. If the lens is removed, the point where the rays meet will move 20/3 cm closer to the mounting that holds. Find the focal length (in cm) of the lens without sign.

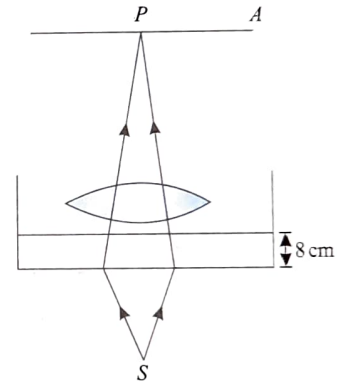
10. A point object is placed at a distance of 25 cm from a convex lens of focal length 20 cm. If a glass slab of thickness $t$ and refractive index 1.5 is inserted between the lens and the object, the image is formed at infinity. Find the thickness $t$?

11. An object is kept at a distance of 16 cm from a thin lens and the image formed is real. If the object is kept at a distance of 6 cm from the same lens, the image formed is virtual. If the size of the images formed are equal, then find the focal length of the lens in cm.

12. Bottom of a glass beaker is made of a thin equi-convex lens having bottom side silver polished as shown in the figure. Water is filled in the beaker upto a height of $h = 4$ m. The image of point object, floating at middle point of beaker at the surface of water coincides with it. Find out the radius of curvature of the lens in m. $\left({}_a\mu_g = 3/2,\ {}_a\mu_w = 4/3\right)$

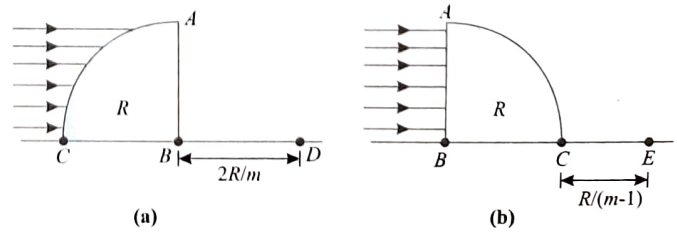



13. A thin convex lens of refractive index $\mu = 1.5$ is placed between a point source of light $S$ and a screen $A$, as shown in the figure. Light rays from the source $S$ are brought to focus on the screen $A$, forming a point image $P$. The distance $SP$ is equal to 50 cm. Water $\left(\mu = \frac{4}{3}\right)$ is now poured into a vessel interposed between the object and the lens, and it is observed that when the water level is 8 cm the screen has to be moved up by a distance of 6 cm in order to get a sharp image. Find the focal length of the lens in cm.

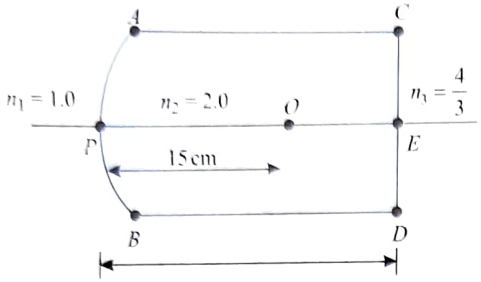



14. A pin is placed 10 cm in front of a convex lens of focal length 20 cm, made of material having refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature 22 cm. Determine the position of the final image from lens (in cm).

15. Quarter part of a transparent cylinder $ABC$ of radius $R$ is kept on a horizontal floor and a horizontal beam of light falls on the cylinder in the two different arrangement of cylinder as shown in Figs. (a) and (b). In arrangement (a) light converges at point $D$, which is at a distance $2R/m$ from $B$. And in arrangement (b) light converges at point $E$, which is at a distance $R/(m-1)$ from $C$. Find out the refractive index of the material.



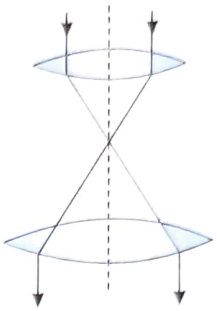
16. A slab of material of refractive index 2 shown in figure has a curved surface $APB$ of radius of curvature 10 cm and a plane surface $CD$. On the left of $APB$ is air and on the right of $CD$ is water with refractive indices as given in the figure. An object $O$ is placed at a distance of 15 cm from the pole $P$ as shown. The distance of the final image of $O$ from $P$ (in cm), as viewed from the left is _____.




17. A parallel beam of light is incident on a transparent sphere of refractive index '$n$'. If the beam finally gets focussed at a point situated at a distance = 2 × (radius of sphere) from the centre of the sphere, then find $n$.

18. A lens placed between a candle and a screen forms a real triply magnified image of the candle on the screen. When the lens is moved away from the candle by 0.8 m without changing the position of the candle, a real image one–third the size of the candle is formed on the screen. Determine the focal length of the lens (in cm).

19. Consider a 'beam expander' which consists of two converging lenses of focal lengths 40 cm and 100 cm having a common optical axis. A laser beam of diameter 4 mm is incident on the 40 cm focal length lens. Then what is the diameter of the final beam (in cm) (see figure)?



20. Two thin similar watch glass pieces are joined together, front to front, with rear portion silvered and the combination of glass pieces is placed at a distance $a = 60$ cm from a screen. A point object is placed on optical axis of the combination such that its two times magnified image is formed on the screen. If air between the glass pieces is replaced by water ($\mu = 4/3$), calculate the distance through which the object must be displaced (in cm) so that a sharp image is again formed on the screen.

# Archives

## JEE Main

### Single Correct Answer Type

1. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50, is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film?
(1) 7.2 m (2) 2.4 m
(3) 3.2 m (4) 5.6 m **(AIEEE 2012)**

2. Diameter of plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is $2 \times 10^8$ m/s, the focal length of the lens is
(1) 20 cm (2) 30 cm
(3) 10 cm (4) 15 cm
**(JEE Main 2013)**

3. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length $f$. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\sqrt{\frac{3}{2}}$, it has the focal lengths $f_1$ and $f_2$ respectively. The correct relation between the focal lengths is
(1) $f_2 > f$ and $f_1$ becomes negative
(2) both $f_1$ and $f_2$ become negative
(3) $f_1 = f_2 < f$
(4) $f_1 > f$ and $f_2$ becomes negative **(JEE Main 2014)**

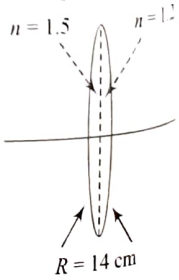
4. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears
(1) 10 times taller (2) 10 times nearer
(3) 20 times taller (4) 20 times nearer
**(JEE Main 2016)**

## JEE Advanced

### Single Correct Answer Type

1. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is
(1) virtual and at a distance of 16 cm from the mirror
(2) real and at a distance of 16 cm from the mirror
(3) virtual and at a distance of 20 cm from the mirror
(4) real and at a distance of 20 cm from the mirror
**(IIT-JEE 2010)**

2. A biconvex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index $n$ of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature $R = 14$ cm. For this biconvex lens, for an object distance of 40 cm, the image distance will be



(1) −280.0 cm (2) 40.0 cm
(3) 21.5 cm (4) 13.3 cm

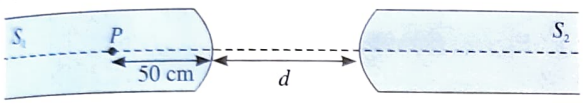
**(IIT-JEE 2012)**

3. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is 2/3 times the wavelength in free space. The radius of the curved surface of the lens is

(1) 1 m (2) 2 m
(3) 3 m (4) 4 m

**(JEE Advanced 2013)**

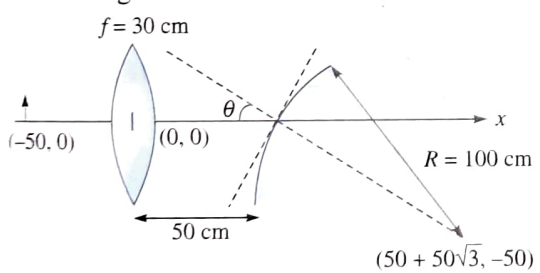
4. Two identical glass rods $S_1$ and $S_2$ (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance $d$ as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light $P$ is placed inside rod $S_1$ on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside $S_2$. The distance $d$ is




(1) 60 cm (2) 70 cm
(3) 80 cm (4) 90 cm

**(JEE Advanced 2015)**

5. A smaller object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle $\theta = 30°$ to the axis of the lens, as shown in the figure.




(1) $(125/3, 25/\sqrt{3})$ (2) $(25, 25\sqrt{3})$
(3) $(50-25\sqrt{3}, 25)$ (4) $(0, 0)$

**(JEE Advanced 2016)**

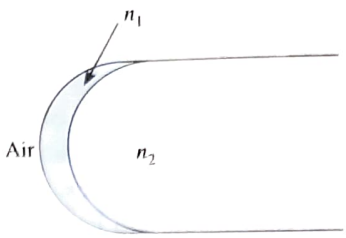
## Multiple Correct Answers Type

1. A student performed the experiment of determination of focal length of a concave mirror by $u$–$v$ method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of $(u, v)$ values recorded by the student (in cm) are: (42, 56), (48, 48), (60, 40), (66, 33), and (78, 39). The data set(s) that *cannot* come from experiment and is (are) incorrectly recorded, is (are)

(1) (42, 56) (2) (48, 48)
(3) (66, 33) (4) (78, 39)

**(IIT-JEE 2009)**

2. A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius $R$ at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance $f_1$ from the film, while rays of light traversing from glass to air get focused at distance $f_2$ from the film. Then




(1) $|f_1| = 3R$ (2) $|f_1| = 2.8R$
(3) $|f_2| = 2R$ (4) $|f_2| = 1.4R$

**(JEE Advanced 2014)**

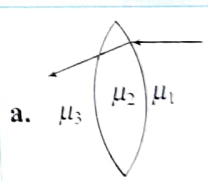
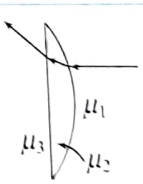
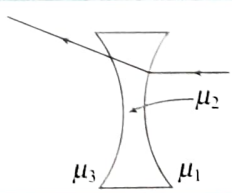
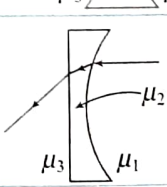
3. A plano-convex lens is made of a material of refractive index $n$. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is (are) true?

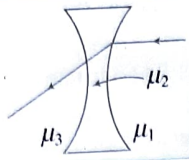
(1) The refractive index of the lens is 2.5
(2) The radius of curvature of the convex surface is 45 cm
(3) The faint image is erect and real
(4) The focal length of the lens is 20 cm

**(JEE Advanced 2016)**

## Matrix Match Type

1. Two transparent media of refractive indices $\mu_1$ and $\mu_3$ have a solid lens shaped transparent material of refractive index $\mu_2$ between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between $\mu_1$, $\mu_2$ and $\mu_3$ are given. Match them to the ray diagram shown in Column II.

| Column I | Column II |
|---|---|
| i. $\mu_1 < \mu_2$ | a.  |
| ii. $\mu_1 > \mu_2$ | b.  |
| iii. $\mu_2 = \mu_3$ | c.  |
| iv. $\mu_2 > \mu_3$ | d.  |

e. 


**(IIT-JEE 2010)**

2. Four combinations of two thin lenses are given in List I. The radius of curvature of all curved surfaces is $r$ and the refractive index of all the lenses is 1.5. Match lens combinations in List I with their focal length in List II and select the correct answer using the code given below the lists.

| List I | List II |
|---|---|
| i. | a. $2r$ |
| ii. | b. $r/2$ |
| iii. | c. $-r$ |
| iv. | d. $r$ |

**Code:**

(1) i-a, ii-b, iii-c, iv-d (2) i-b, ii-d, iii-c, iv-a

(3) i-d, ii-a, iii-b, iv-c (4) i-b, ii-a, iii-c, iv-d

**(JEE Advanced 2014)**

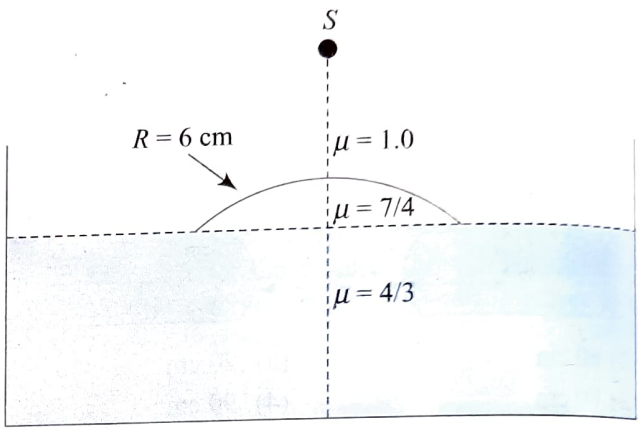
## Numerical Value Type

1. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from $m_{25}$ to $m_{50}$. The ratio $m_{25}/m_{50}$ is

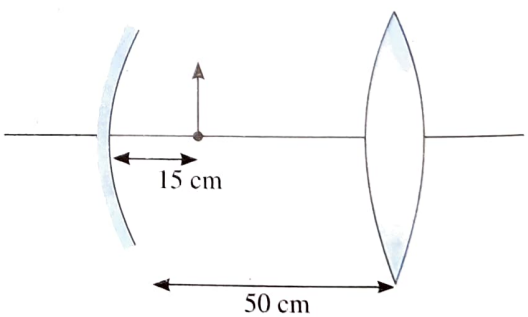
**(IIT-JEE 2010)**

2. Water (with refractive index = 4/3) in a tank is 18 cm deep. Oil of refractive index 7/4 lies on water making a convex surface of radius of curvature $R$ = 6 cm as shown. Consider oil to act as a thin lens. An object $S$ is placed 24 cm above water surface. The location of its image is at '$x$' cm above the bottom of the tank. Then $x$ is

**(IIT-JEE 2011)**




3. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification $M_1$. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes $M_2$. The magnitude $\left|\frac{M_2}{M_1}\right|$ is _____. **(JEE Advanced 2015)**




# Answers Key

## EXERCISES

### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| 1. (1) | 2. (3) | 3. (3) | 4. (1) | 5. (3) |
| 6. (2) | 7. (4) | 8. (1) | 9. (4) | 10. (2) |
| 11. (3) | 12. (4) | 13. (4) | 14. (3) | 15. (4) |
| 16. (4) | 17. (3) | 18. (4) | 19. (1) | 20. (1) |
| 21. (2) | 22. (4) | 23. (1) | 24. (3) | 25. (2) |
| 26. (1) | 27. (4) | 28. (2) | 29. (1) | 30. (2) |
| 31. (4) | 32. (2) | 33. (4) | 34. (4) | 35. (2) |
| 36. (1) | 37. (2) | 38. (3) | 39. (3) | 40. (1) |
| 41. (3) | 42. (3) | 43. (2) | 44. (2) | 45. (2) |
| 46. (3) | 47. (2) | 48. (4) | 49. (2) | 50. (1) |
| 51. (2) | 52. (1) | 53. (2) | 54. (4) | 55. (3) |
| 56. (2) | 57. (2) | 58. (1) | 59. (4) | 60. (2) |
| 61. (2) | 62. (3) | 63. (4) | 64. (3) | 65. (2) |

66. (1) 67. (2) 68. (2) 69. (1) 70. (3)
71. (3) 72. (3) 73. (3) 74. (4) 75. (2)
76. (4) 77. (3) 78. (2) 79. (4) 80. (2)
81. (2) 82. (3) 83. (4) 84. (3) 85. (3)
86. (1) 87. (3)

### Multiple Correct Answers Type

1. (1),(2),(3),(4) 2. (2),(3) 3. (1),(2),(3),(4)
4. (1),(3) 5. (1),(2),(3) 6. (1),(3)
7. (1),(3) 8. (1),(2) 9. (1),(2)
10. (1),(3) 11. (1),(3) 12. (1),(2),(4)
13. (1),(2),(4) 14. (1),(3) 15. (1),(3),(4)
16. (2),(3) 17. (1),(3) 18. (2),(4)
19. (1),(4) 20. (2),(3),(4) 21. (1),(2)
22. (1),(3) 23. (1),(4) 24. (1),(3)
25. (2),(3),(4) 26. (2),(3) 27. (2),(3),(4)

### Linked Comprehension Type

1. (3) 2. (4) 3. (1) 4. (4) 5. (3)
6. (4) 7. (2) 8. (2) 9. (1) 10. (1)
11. (2) 12. (1) 13. (3) 14. (3) 15. (1)
16. (4) 17. (1) 18. (2) 19. (3) 20. (2)
21. (1) 22. (4) 23. (1) 24. (4) 25. (2)
26. (2) 27. (3) 28. (4) 29. (2) 30. (3)
31. (4) 32. (2) 33. (3) 34. (3) 35. (1)
36. (3) 37. (1) 38. (2) 39. (1)

### Matrix Match Type

1. i. → b., c.; ii. → b., c.; iii. → b., d.; iv. → b., d.
2. i. → a., c.; ii. → b., c., iii. → b., c.; iv. → b., c.
3. i. → b.; ii. → b.; iii. → c.; iv. → d.
4. i. → a.; ii. → a.; iii. → b.; iv. → b.
5. i. → a.; ii. → b.; iii. → a.; iv. → c.
6. i. → b.; ii. → d.; iii. → c.; iv. → a.
7. i. → b.; ii. → c.; iii. → a.
8. i. → a., c.; ii. → b., d.; iii. → b., d.; iv. → a., c.
9. (2) 10. (1)

### Numerical Value Type

1. (2) 2. (9) 3. (6) 4. (8) 5. (8)
6. (3) 7. (5) 8. (2) 9. (5) 10. (15)
11. (11) 12. (5) 13. (12) 14. (11) 15. (1.5)
16. (30) 17. (1.33) 18. (30) 19. (1) 20. (15)

## ARCHIVES

### JEE Main

#### Single Correct Answer Type

1. (4) 2. (2) 3. (4) 4. (3)

### JEE Advanced

#### Single Correct Answer Type

1. (2) 2. (2) 3. (3) 4. (2) 5. (2)

#### Multiple Correct Answers Type

1. (3),(4) 2. (1),(3) 3. (1),(4)

#### Matrix Match Type

1. i. → a., c.; ii. → b., d., e.; iii. → a., c., e.; iv. → b., d.
2. (2)

#### Numerical Value Type

1. (6) 2. (2) 3. (7)

# 3 Wave Optics

## HUYGENS' WAVE THEORY

Huygens assumed that a body emits light in the form of waves. According to him, each point source of light is a centre of disturbance from which waves spread in all directions.

The locus of all the particles of the medium vibrating in the same phase at a given instant is called the wavefront.

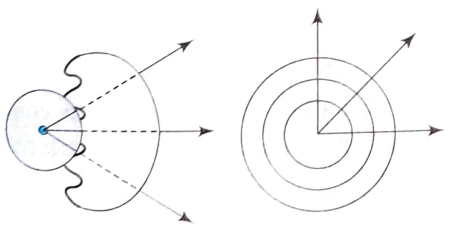
### WAVEFRONTS

Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillate in phase is an example of a wavefront.

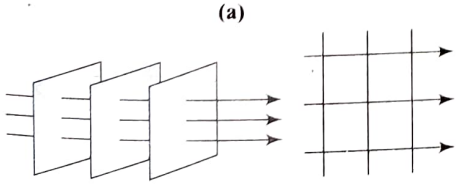
A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outward from the source is called the phase speed. The energy of the wave travels in a direction perpendicular to the wavefront.

Figure (a) shows light waves from a point source forming a spherical wavefront in three-dimensional space. The energy travels outward along straight lines emerging from the source, i.e., radii of the spherical wavefront. These lines are the rays. Note that when we measure the spacing between a pair of wavefronts along any ray, the result is a constant. This example illustrates two important general principles which we will use later:

1. Rays are perpendicular to wavefronts.
2. The time taken by light to travel from one wavefront to another is the same along any ray.



(a)



(b)

**Wavefronts and the corresponding rays in two cases: (a) diverging spherical wave and (b) plane wave. The figure on the left shows a wave (e.g., light) in three dimensions. The figure on the right shows a wave in two dimensions (e.g., a water surface).**

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Fig. (b).

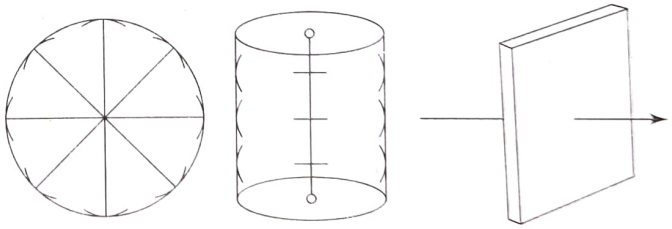
A linear source such as a slit illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at larger distance from the source, these wavefronts may be regarded as planar.

A wavefront is a surface joining the points of same phase. For a point source, the wavefronts are spherical which become almost plane at a very large distance. For example, wavefronts of sunlight are plane.

**Important Points:**

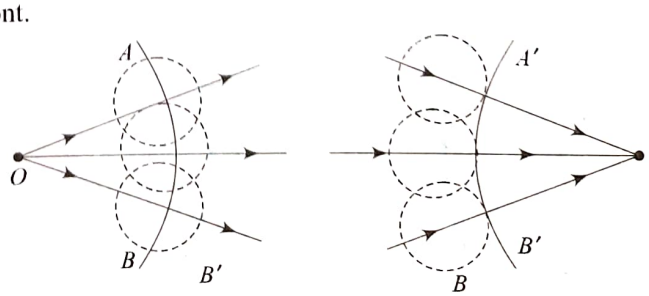
- Every point on a wavefront is an independent source to produce secondary wavefronts.
- All points on a wavefront vibrate in same phase with same frequency.
- Wavefronts move with the velocity of wave in that medium.

The shape of wavefront depends on the source producing the waves and is usually spherical, cylindrical, or plane as shown in the figure.

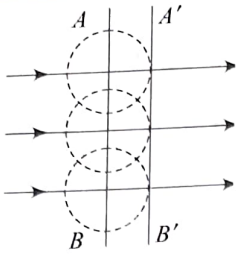


**(a) Point source** **(b) Line source** **(c) Source at ∞**

Each point on a wavefront is a source of new disturbance, called secondary wavelets. These wavelets are spherical and travel with speed of light in that medium. The forward envelope of the secondary wavelets at any instant gives the new wavefront.




(a) (b)

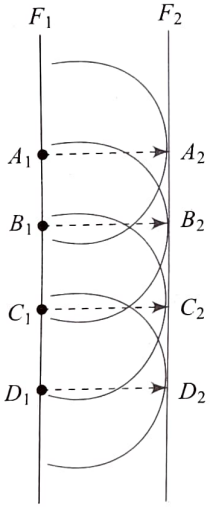



(c)

## HUYGENS' CONSTRUCTION

Huygens, the Dutch physicist and astronomer of the seventeenth century, gave a beautiful geometrical description of wave propagation. A stick placed in water and oscillated up and down becomes a source of waves. Since the surface of water is two dimensional, the resulting wavefronts would be circles instead of spheres. At each point on such a circle, the water level moves up and down. Huygens' idea is that we can think of every such oscillating point on a wavefront as a new source of waves. According to Huygens' principle, what we observe is the result of adding up the waves from all these different sources. These are called secondary waves or wavelets.

Huygens' principle is illustrated in figure in the simple case of a plane wave.

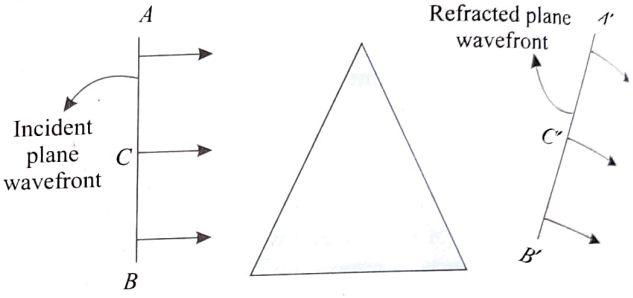



1. At time $t = 0$, we have a wavefront $F_1$. $F_1$ separates these parts of the medium which are undisturbed from those where the wave has already reached.
2. Each point on $F_1$ acts like a new source and sends out a spherical wave. After a time $t$, each of these will have radius $vt$. These spheres are the secondary wavelets.
3. After a time $t$, the disturbance would now have reached all points within the region covered by all these secondary waves. The boundary of this region is the new wavefront $F_2$. Note that $F_2$ is a surface tangent to all the spheres. It is called the forward envelope of these secondary wavelets.
4. The secondary wavelet from point $A_1$ on $F_1$ touches $F_2$ at $A_2$. Draw the line connecting any point $A_1$ on $F_1$ to the corresponding point $A_2$ on $F_2$. According to Huygens, $A_1A_2$ is a ray. It is perpendicular to the wavefronts $F_1$ and $F_2$ and has length $vt$. This implies that rays are perpendicular to wavefronts. Further, the time taken for light to travel between two wavefronts is the same along any ray. In our example, the speed $v$ of the wave has been taken to be the same at all points in the medium. In this case, we can say that the distance between two wavefronts is the same measured along any ray.
5. This geometrical construction can be repeated starting with $F_2$ to get the next wavefront $F_3$ a time $t$ later, and so on. This is known as Huygens' construction.

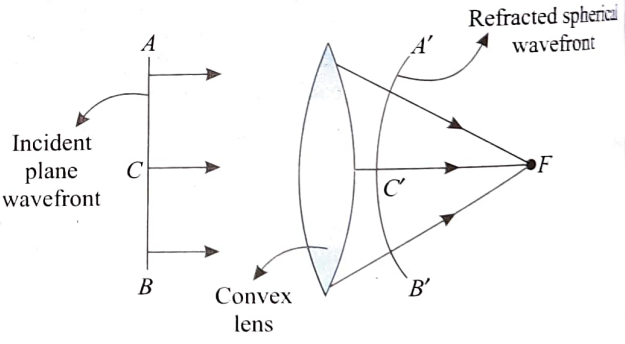
Huygens' construction can be understood physically for waves in a material medium, like the surface of water. Each oscillating particle can set its neighbours into oscillation, and therefore acts as a secondary source. But what if there is no medium, as for light travelling in vacuum? The mathematical theory, which cannot be given here, shows that the same geometrical construction works in this case as well.

## BEHAVIOUR OF PLANE WAVEFRONTS ON REFLECTION AND REFRACTION

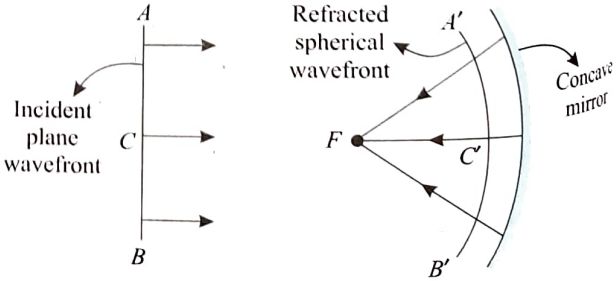
(a) **Refraction of a plane wavefront by a thin prism:** Since the speed of light wave is less in glass, the lower portion ($B$) of the incident plane wavefront which travels through the greatest thickness of glass is tilted towards the prism as it gets delayed as compared to its upper portion therefore, emerges as $A'B'$; $B'$ closer to the prism than $A'$ (shown in figure).




(b) **Refraction of a plane wavefront by a lens:** The central portion $C$ of the incident plane wavefront $ABC$ covers the greatest thickness through the glass and is delayed the most while reaching $C'$. Other portions are delayed depending on the thickness of the glass dies traverse. The refracted wavefront ($A'B'C'$) is spherical and the corresponding refracted rays converges to a point $F$ which is called the focus of the lens (shown in figure).




(c) **Reflection of a plane wavefront by a concave mirror:** The incident wavefront $ABC$ on getting reflected from the concave mirror becomes a spherical wavefront $A'B'C'$. The corresponding reflected rays converge to $F$, the focus of the concave mirror (shown in figure).



**Wavefront in different optical systems**

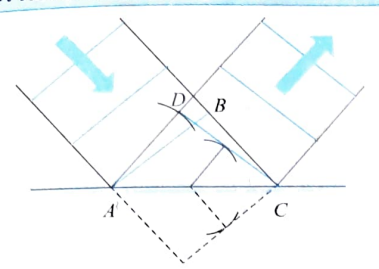
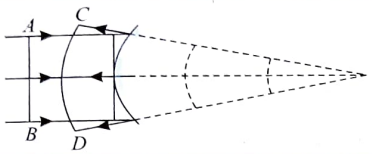
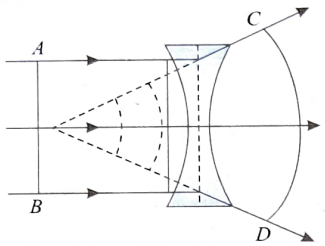
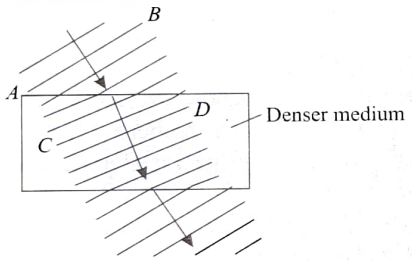
| Mirror/Lens/Slab Prism | Wavefront<br>AB: Incident wavefront<br>CD: Reflected/Refracted wavefront |
|---|---|
| Plain mirror |  |
| Convex mirror |  |
| Concave lens |  |
| Plane slab |  |

**ILLUSTRATION 3.1**

What is the shape of the wavefront on earth for sunlight?

**Sol.** The sun is at very large distance from the earth. Assuming sun as spherical, it can be considered as point source situated at infinity. We can treat it like a point object as seen from the surface of earth.

Because of large distance the radius of wavefront can be considered as large (infinity) and hence, wavefront is almost plane.

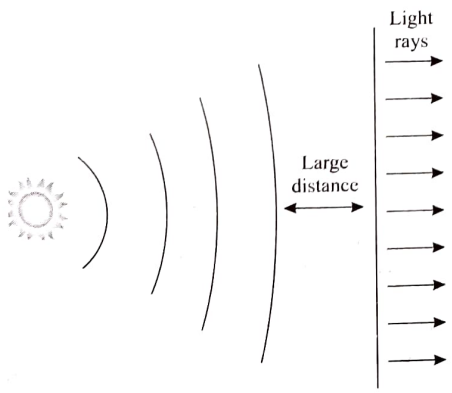
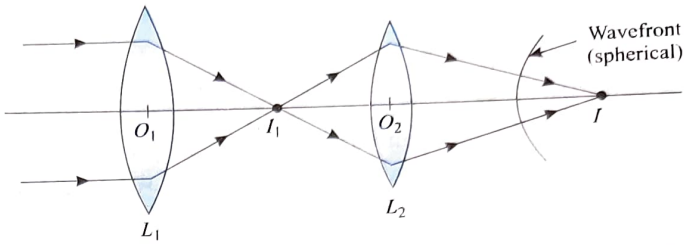



**ILLUSTRATION 3.2**

Consider a point at the focal point of a convergent lens. Another convergent lens of short focal length is placed on the other side. What is the nature of the wavefronts emerging from the final image?

**Sol.** Orientation of wave front is perpendicular to ray. The ray diagram of the situation is as shown in figure.




Parellel rays incident on lens $L_1$ forms the image $I_1$ at the focal point of the lens. This image acts as object for the lens $L_2$ Now, due to the converging lense $L_2$, let final image formed is $I$ which is point image. Hence the wavefront for this image will be of spherical symmetry.

## GEOMETRICAL AND OPTICAL PATHS

Consider a light wave travelling in a medium of refractive index $\mu$. Its equation may be written as

$$E = E_0 \sin\omega\left(t - \frac{x}{v}\right) = E_0 \sin\omega\left(t - \frac{x}{c/\mu}\right)$$

$$\Rightarrow \quad E = E_0 \sin\left(\omega t - \omega\frac{\mu x}{c}\right) = E_0 \sin(\omega t - \delta)$$

If the light wave travels a distance $\Delta x$ in vacuum, the phase changes by

$$\delta_1 = \frac{\omega}{c}(\mu\Delta x) \qquad \ldots(i)$$

Instead, if the light wave travels in vacuum, its equation will be

$$E = E_0 \sin\omega\left(t - \frac{x}{c}\right) = E_0 \sin\left(\omega t - \omega\frac{x}{c}\right) = E_0 \sin(\omega t - \delta)$$

If the light travels through a distance $\mu\Delta x$, the phase changes by

$$\delta_2 = \frac{\omega}{c}(\mu\Delta x) \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we see that a wave travelling through a distance $\Delta x$ in a medium of refractive index $\mu$ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum. In other words, a path length of $\Delta x$ in a medium of refractive index $\mu$ is equivalent to a path length of $\mu\Delta x$ in vacuum. The quantity $\mu\Delta x$ is called the **optical path** of the light. In dealing with interference of two waves, we need the difference between the optical paths travelled by the waves. The geometrical path and the optical path are equal only when light travels in vacuum or in air where the refractive index is close to 1.

The concept of optical path may also be introduced in terms of the change in wavelength as the wave changes its medium. The frequency of a wave is determined by the frequency of the source and is not changed when the wave enters in a new medium. If the wavelength of light in vacuum is $\lambda_0$ and that in the medium is $\lambda_n$, then

$$\lambda_0 = \frac{c}{f},\ f = \text{frequency of the wave}$$

and $$\lambda_n = \frac{v}{f} = \frac{c/\mu}{f} = \frac{1}{\mu}\frac{c}{f} = \frac{\lambda_0}{\mu}$$

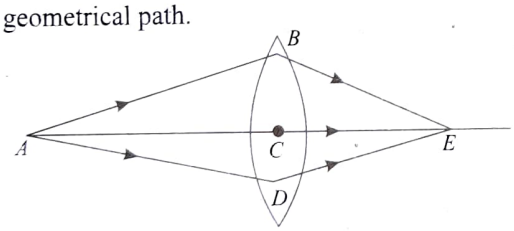
At any given instant, the points differentiated by one wavelength have same phase of vibration. Thus, the points at separation $\lambda_n$ in the medium have same phase of vibration. On the other hand, in vacuum, points at separation $\lambda_0$ will have same phase of vibration. Thus, a path $\lambda_n$ in a medium is equivalent to a path $\lambda_0 = \mu\lambda_n$ in vacuum. In general, a path $\Delta x$, in a medium of refractive index $\mu$, is equivalent to a path $\Delta x$ in vacuum which is called the optical path.

It means that, the length of the path covered by light in a medium in a given time known as geometrical path is obviously lesser than in vacuum (or air) during the given time known as optical path. In other words

$$(\text{Optical path}) = \mu \times (\text{Geometrical path in a medium})$$

$\mu$ = Refractive index of the medium

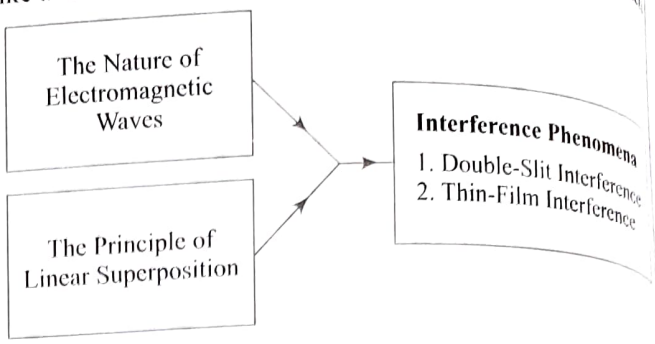
**Note:** Consider the situation in the figure. The geometrical paths *ABE*, *ACE*, and *ADE* are different, but the optical paths are equal. This is because each path leads to the same phase difference. Note that the ray having longer geometrical path covers less distance in the lens as compared to the ray having shorter geometrical path.




## PRINCIPLE OF LINEAR SUPERPOSITION

The pressure disturbance that results is governed by the principle of linear superposition, which states that the resultant disturbance is the sum of the disturbances from the individual waves. Light is also a wave, an electromagnetic wave, and it too obeys the superposition principle. When two or more light waves pass through a given point, their electric fields combine according to the principle of linear superposition and produce a resultant electric field. According to theory of electromagnetic waves, the square of the electric field strength is proportional to the intensity of the light, which, in turn, is related to its brightness. Thus, interference can and does alter the brightness of light, just as it affects the loudness of sound.

**Optical wave:** The principle of linear superposition explains various interference phenomena associated with light waves (see figure). The beautiful iridescent colors on the wings of tropical butterfly are due to thin-film interference of light. A transparent cuticle-like material on the upper wing surface acts like a thin film.




**Sound wave:** Figure indicates that transverse and longitudinal standing waves are related to the principle of linear superposition. Guitars produce sound by utilizing the transverse standing waves that form on each plucked string.

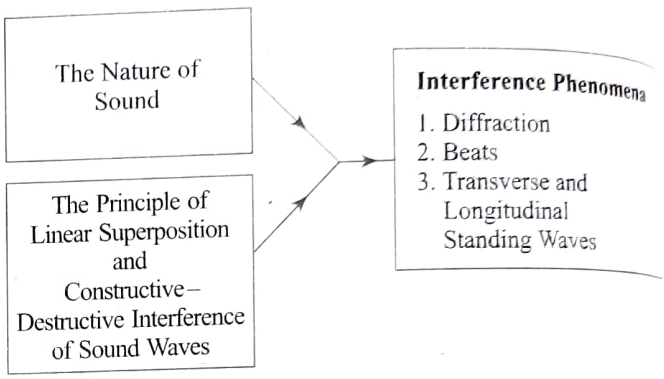
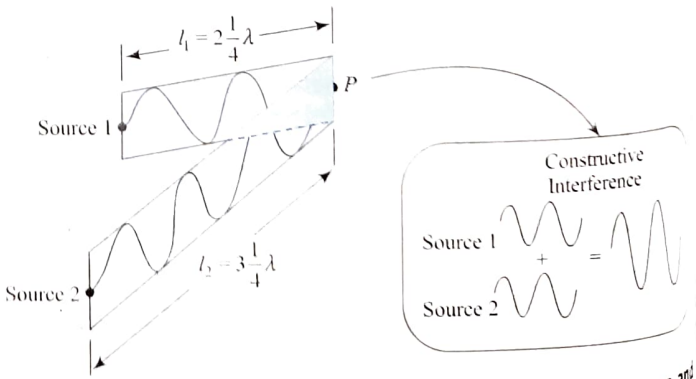
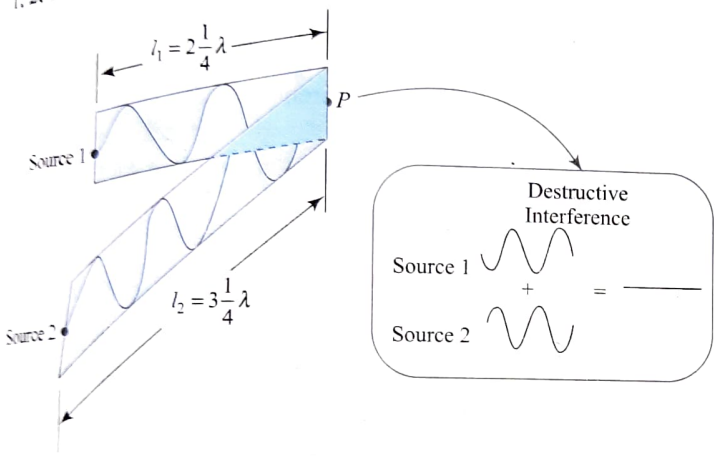



Figure illustrates what happens when two identical waves (same wavelength $\lambda$ and same amplitude) arrive at point $P$ in phase, that is, crest to crest and trough to trough. According to the principle of linear superposition, the waves reinforce each other and constructive interference occurs. The resulting total wave at $P$ has an amplitude that is twice the amplitude of either individual wave, and in the case of light waves, the brightness at $P$ is greater than that due to either wave alone. The waves start out in phase and are in phase at $P$ because the distances $l_1$ and $l_2$, between this spot and the sources of the waves differ by one wavelength. In general, when the waves start out in phase, constructive interference will result at $P$ whenever the distances are the same or differ by any integer number of wavelengths; that is, in other words, assuming $l_2$ is the larger distance, whenever $l_2 - l_1 = m\lambda$, where $m = 0, 1, 2, 3, \ldots$ .




**The waves emitted by source 1 and source 2 start out in phase and arrive point at *P* in phase, leading to constructive interference at that point.**

Figure shows what happens when two identical waves arrive at point $P$ out of phase with one another, or crest to trough. Now, the waves mutually cancel, according to the principle of linear superposition, and destructive interference results. With light waves, this would mean that there is no brightness. The waves begin with the same phase but are out of phase at $P$ because the distances through which they travel in reaching this spot differ by one-half of a wavelength. For waves that start out in phase, destructive interference will take place at $P$ whenever the distances differ by any odd integer number of half-wavelengths, that is, whenever $l_2 - l_1 = \frac{1}{2}\lambda, \frac{3}{2}\lambda, \frac{5}{2}\lambda$ .... where $l_2$ is the larger distance. This is equivalent to $l_2 - l_1 = \left(m + \frac{1}{2}\right)\lambda$, where $m = 0, 1, 2, 3, \ldots$ .




**The waves emitted by two sources have the same phase to begin with, but they arrive at point *P*, out of phase. As a result, destructive interference occurs at *P*.**

## CONDITIONS OF INTERFERENCE

We know that the superpositions of two mechanical waves can be constructive or destructive. In constructive interference, the amplitude of the resultant wave at a given position or time is greater than that of either individual wave.

If two light bulbs are placed side by side, no interference effects are observed because the light waves from one bulb are emitted independently of those from the other bulb. The emissions from the two light bulbs do not maintain a constant phase relationship with each other over time. Light waves from an ordinary source such as a light bulb undergo random phase changes in time intervals less than a nanosecond. Therefore, the conditions for constructive interference, destructive interference, or some intermediate state are maintained only for such short time intervals. Because the eye cannot follow such rapid changes, no interference effects are observed. Such light sources are said to be incoherent.

In order to observe interference in light waves, the following conditions must be met:

1. The sources must be coherent—that is, they must maintain a constant phase with respect to each other.
2. The source should be monochromatic—that is, of a single wavelength.

## COHERENT SOURCES

Two sources are said to be coherent if they produce waves of same frequency with a constant phase difference. Unlike sound waves, two independent sources of light cannot be coherent. Since sound is a bulk property of matter, therefore, two independent sources of sound can be identical in all respects and can produce coherent waves. On the contrary, light is not a bulk property of matter, it is a property of each individual atom. As the individual atoms emit light randomly and independently, therefore, two independent sources of light cannot be coherent.

Coherent sources can be obtained by splitting a light beam from a source into two. This can be done in two ways:

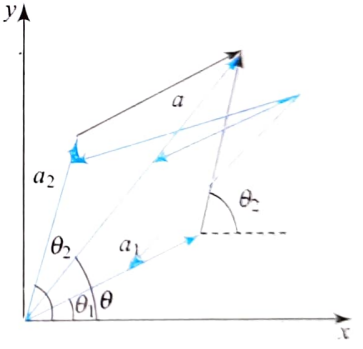
1. **Division of wavefront:** It is done in Young's double-slit experiment, Fresnel's biprism, Lloyd's mirror, etc.
2. **Division of amplitude:** It is usually done by partial reflection and transmission at a boundary, as it occurs in thin film interference, Newton's rings, etc.

## INTERFERENCE

We have learned that in the superposition of waves there is interference of two sinusoidal waves. Let the two sinusoidal waves be

$$y_1 = a_1 \cos(\omega t + \theta_1) \quad \ldots(i)$$

$$y_2 = a_2 \cos(\omega t + \theta_2) \quad \ldots(ii)$$




According to the principle of superposition, the resultant displacement would be given by

$$y(t) = a \cos(\omega t + \theta) \quad \ldots(iii)$$

where

$$a \cos\theta = a_1 \cos\theta_1 + a_2 \cos\theta_2 \quad \ldots(iv)$$

$$a \sin\theta = a_1 \sin\theta_1 + a_2 \sin\theta_2 \quad \ldots(v)$$

The resultant wave is also sinusoidal, with same frequency but different amplitude. On squaring and adding Eqs. (iv) and (v), we get

$$a = \left[a_1^2 + a_2^2 + 2a_1a_2\cos(\theta_1 - \theta_2)\right]^{1/2} \quad \ldots(vi)$$

$$\tan\theta = \frac{a_1 \sin\theta_1 + a_2 \sin\theta_2}{a_1 \cos\theta_1 + a_2 \cos\theta_2} \quad \ldots(vii)$$

If $\theta_1 = 0$ and $\theta_2 = \theta$, we have

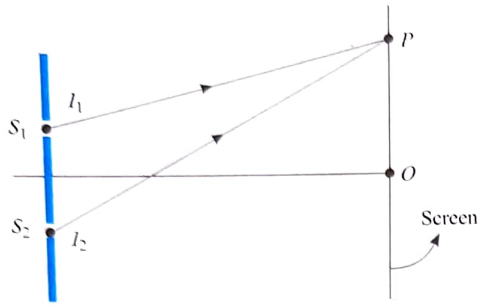
$$a = \left[a_1^2 + a_2^2 + 2a_1a_2\cos\theta\right]^{1/2} \quad \ldots(viii)$$

$$\therefore \quad \tan\theta = \frac{\sin}{a_1 + a_2 \cos} \quad \ldots(ix)$$

From Eq. (viii), we find that if $\theta = 0, 2\pi, 4\pi, \ldots$, then $a = a_1 + a_2$. When two displacements are in phase, then the resultant amplitude will be the sum of the two amplitudes; this is constructive interference.

Similarly, if $\theta = \pi, 3\pi, 5\pi, \ldots$, then $a = a_1 - a_2$.

When superposing waves are in opposite phase, the resultant amplitude is the difference of the two amplitudes; this is destructive interference.




The difference in the distances from $S_1$ and $S_2$ to a given point is called the path difference. A path difference of one wavelength corresponds to a phase difference of $2\pi$ rad because a shift in position of one wavelength along the wave changes its phase by a complete cycle.

For a path difference $\Delta x$, the phase difference (in radians) is

$$\delta = \frac{2\pi}{\lambda}\ (\text{path difference}) = \frac{2\pi}{\lambda}(\Delta x)$$

The factor $\Delta x/\lambda$ is number of wavelengths corresponding to path difference. Each wavelength corresponds to a phase difference of $\pi$.

The intensity of a wave is proportional to square of amplitude, therefore, Eq. (viii) reduces to

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\delta$$

where $\delta = \dfrac{2\pi}{\lambda}(\Delta x)$

As the maximum and minimum values of $\cos\delta$ are +1 and −1, respectively, the maximum and minimum values of $I$ are given by

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \quad \text{and} \quad I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

The conditions for maximum intensity are

$\delta = 2n\pi, \quad n = 0, 1, 2, \ldots$

or $\Delta x = n\lambda$

The conditions for minimum intensity are

$\delta = (2n-1)\pi, \quad n = 0, 1, 2, 3, \ldots$

or $\Delta x = (n - 1/2)\lambda, \quad n = 0, 1, 2, 3, \ldots$

When $I_1 = I_2 = I_0$,

then $I = 4I_0 \cos^2 \delta/2$

Clearly, if $\delta = \pm\pi, \pm 3\pi, \ldots$, the resultant intensity will be zero and we will have a minimum. When $\phi = 0, \pm 2\pi, \pm 4\pi, \ldots$, the intensity will be maximum ($I = 4I_0$).

**Note:** In interference, energy is neither created nor destroyed but it is conserved.

In case of interference, intensity at a point is given by

$$I = I_1 + I_2 + 2(\sqrt{I_1 I_2})\ \cos\delta$$

Now, as at a given point phase difference can have any value between 0 and $2\pi$, the average value of intensity will be

$$I_{av} = \left[\frac{\int_0^{2\pi} I d\delta}{\int_0^{2\pi} d\delta}\right] = \frac{1}{2\pi}\int_0^{2\pi}(I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta)d\delta$$

$$= I_1 + I_2 \qquad [\text{as} \int_0^{2\pi}(\cos\delta)d\delta = 0]$$

As the average value of intensity is equal to the sum of individual intensities, in interference, energy is neither created nor destroyed but is redistributed and, hence, in interference, energy is conserved.

## ILLUSTRATION 3.3

Light waves from two coherent sources superimpose at a point. The waves, at this point, can be expressed as $y_1 = a \sin[10^{15}\ \pi t]$ and $y_2 = 2a \sin[10^{15}\ \pi t + \phi]$. Find the resultant amplitude if phase difference $\phi$ is

**(a)** zero **(b)** $\pi/3$ **(c)** $\pi$

Also find the frequency (Hz) of resultant wave in each case.

**Sol.** Resultant amplitude can be obtained from the relation

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi \qquad [I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi]$$

$A_1$ and $A_2$ are the amplitudes of interfering waves, $\phi$ is the phase difference at the given point and $A$ is the resultant amplitude. Here, $A_1 = a$ and $A_2 = 2a$.

**(a)** $\phi = 0$; $A^2 = a^2 + 4a^2 + 2a(2a)\cos 0° = 9a^2 \Rightarrow A = 3a$

**(b)** $\phi = \phi/3$; $A^2 = a^2 + 4a^2 + 2a(2a)\cos\pi/3 = 7a^2 \Rightarrow A = \sqrt{7}a$

**(c)** $\phi = \pi$; $A^2 = a^2 + 4a^2 + 2a(2a)\cos\pi = a^2 \Rightarrow A = a$

Interference results due to superposition of waves of same frequency and frequency of the resultant wave also has the same value. Here, this frequency is

$\omega = 10^{15}\ \pi$

or $2\pi f = 10^{15}\pi$

$\therefore \quad f = 5 \times 10^{14}$ Hz

## ILLUSTRATION 3.4

In Young's experiment, the interfering waves have amplitudes in the ratio 3:2. Find the ratio of amplitudes and intensities, between the bright and dark fringes.

**Sol.** Here, we have to obtain the ratio $\dfrac{A_{max}}{A_{min}} = \dfrac{A_1 + A_2}{A_1 - A_2}$ and also the corresponding ratio of intensities

$$\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$$

But $\dfrac{A_1}{A_2} = \dfrac{3}{2}$ (given)

By componendo and dividendo, $\dfrac{A_1 + A_2}{A_1 - A_2} = \dfrac{3+2}{3-2} = 5$

Hence, $\dfrac{A_{max}}{A_{min}} = 5$ and $\dfrac{I_{max}}{I_{min}} = \dfrac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = 25$

### ILLUSTRATION 3.5

Two coherent sources emit light waves which superimpose at a point where these can be expressed as

$E_1 = E_0 \sin(\omega t + \pi/4)$

$E_2 = 2E_0 \sin(\omega t - \pi/4)$

Here, $E_1$ and $E_2$ are the electric field strengths of the two waves at the given point.

If $I$ is the intensity of wave expressed by field strength $E_1$, find the resultant intensity.

**Sol.** Intensity of wave expressed by field strength $E_1$, $I \propto E_0^2$

[intensity $\propto$ (amplitude)$^2$]

Intensity of wave expressed by $E_2$, $I' \propto (2E_0)^2$

$$\frac{I'}{I} = 4 \quad \text{or} \quad I' = 4I$$

Phase difference between the two waves is

$$\phi = (\omega t + \pi/4) - (\omega t - \pi/4) = \frac{\pi}{2}$$

Resultant intensity is given by

$$I_R = I + I' + 2\sqrt{II'}\cos\phi \qquad [I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi]$$

$$I_R = I + 4I + 2\sqrt{I(4I)}\cos\pi/2$$

$$\Rightarrow \quad I_R = 5I$$

Thus, resultant intensity is five times the intensity of wave expressed by $E_1$.

### ILLUSTRATION 3.6

Determine the resultant of two waves given by $y_1 = 4\sin(200\pi t)$ and $y_2 = 3\sin(200\pi t + \pi/2)$.

**Sol.** In general, resultant of two waves given by $y_1 = A_1 \sin\omega t$ and $y_2 = A_2 \sin(\omega t + \phi)$ can be obtained by the principle of superposition, i.e.,

$$y = y_1 + y_2 = A_1 \sin\omega t + A_2 \sin(\omega t + \phi) = A\sin(\omega t + \alpha)$$

where $A = [A_1^2 + A_2^2 + 2A_1A_2\cos\phi]^{1/2}$

$$\alpha = \tan^{-1}\left[\frac{A_2\sin\phi}{A_1 + A_2\cos\phi}\right]$$

In the present case, $y_1 = 4\sin 200\pi t$ and $y_2 = 3\sin\left(200\pi t + \frac{\pi}{2}\right)$

Here, $A_1 = 4$, $A_2 = 3$, and $\phi = \pi/2$.

Resultant wave will be $y = A\sin(200\pi t + \alpha)$ with

$$A = [A_1^2 + A_2^2 + 2A_1A_2\cos\phi]^{1/2}$$

$$\Rightarrow \quad A = [4^2 + 3^2 + 2(4)(3)\cos\pi/2]^{1/2}$$

$$\Rightarrow \quad A = 5$$

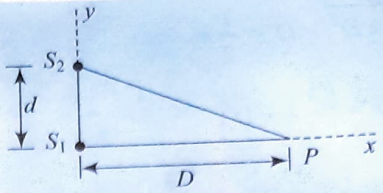
$$\tan\alpha = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi} = \frac{3\sin\pi/2}{4 + 3\cos\pi/2} = \frac{3}{4}$$

or $\alpha = 37^\circ \approx 0.2\pi$ rad

Hence, the resultant wave is $y = 5\sin(200\pi t + 0.2\pi)$.

### ILLUSTRATION 3.7

Two sources $S_1$ and $S_2$ emitting light of wavelength 600 nm are placed a distance $1.0 \times 10^{-2}$ cm apart. A detector can be moved on line $S_1P$ which is perpendicular to $S_1S_2$.

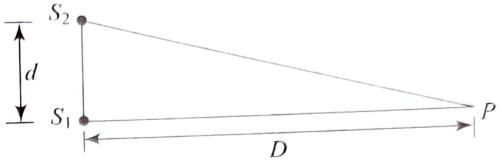



**(a)** What would be the minimum and maximum path difference at the detector as it moves along line $S_1P$?

**(b)** Locate the position of the farthest minima detected.

**Sol.**

**(a)** Path difference at $P$, $\Delta x = S_2P - S_1P$




Minimum path difference will occur when the observer will be at a large distance from the sources. The path difference will be maximum when the observer is near source $S_1$. As observer moves from $S_1$ toward $+x$-direction, the path difference decreases from $d$ to zero.

$(\Delta x)_{max}$: when $S_1P = 0$, i.e., detector is at position $S_1$.

$\therefore \quad (\Delta x)_{max} = S_1S_2 = d = 1.0 \times 10^{-2}$ cm

$(\Delta x)_{min}$: when detector is at large distance and $(\Delta x)_{min}$ will be approximately zero.

**(b)** Let the farthest minimum occur at $P$. As we move away, the path difference decreases, order of maxima or minima increases as we move toward $S_2$. At the position of the farthest minimum path difference is

$$S_2P - S_1P = \lambda/2$$

$$(D^2 + d^2)^{1/2} - D = \lambda/2$$

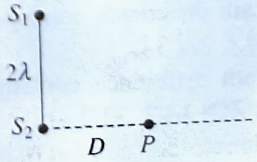
$$\left[D\left(1 + \frac{d^2}{D^2}\right)^{1/2}\right] - D = \lambda/2$$

$$\frac{d^2}{2D} = \frac{\lambda}{2}$$

$$\Rightarrow \quad D = \frac{d^2}{\lambda} = \frac{(1\times10^{-4})^2}{(600\times10^{-9})}\text{ m} = 1.7\text{ cm}$$

### ILLUSTRATION 3.8

Two coherent point sources $S_1$ and $S_2$ vibrating in phase emit light of wavelength $\lambda$. The separation between the sources is $2\lambda$. Consider a line passing through $S_2$ and perpendicular to line $S_1S_2$. Find the position of farthest and nearest minima.




**Sol.** $\Delta x_{min} = (2n - 1)\frac{\lambda}{2}$

The farthest minima has path difference $\lambda/2$ while the nearest minima has path difference $(3/2)\lambda$. For the nearest minima,

$$S_1P - S_2P = \frac{3}{2}\lambda \qquad \text{[as maximum path difference is } 2\lambda\text{]}$$

$$\Rightarrow \quad \sqrt{(2\lambda)^2 + D^2} - D = \frac{3}{2}\lambda$$

$$\Rightarrow \quad (2\lambda)^2 + D^2 = \left(\frac{3}{2}\lambda + D\right)^2$$

$$\Rightarrow \quad 4\lambda^2 + D^2 = \frac{9}{4}\lambda^2 + D^2 \times 2 \times \frac{3}{2}\lambda \times D$$

$$\Rightarrow \quad 3D = 4\lambda - \frac{9\lambda}{4} = \frac{7\lambda}{4} \quad \Rightarrow \quad D = \frac{7}{12}\lambda$$

For the farthest minima,

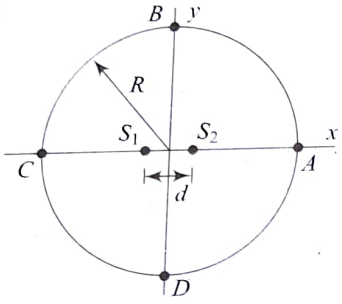
$$S_1P - S_2P = \frac{\lambda}{2}$$

$$\Rightarrow \quad \sqrt{4\lambda^2 + D^2} - D = \frac{\lambda}{2}$$

$$\Rightarrow \quad 4\lambda^2 + D^2 = \frac{\lambda^2}{4} + D^2 + D\lambda \quad \Rightarrow \quad D = 4\lambda - \lambda/4 = \frac{15\lambda}{4}$$

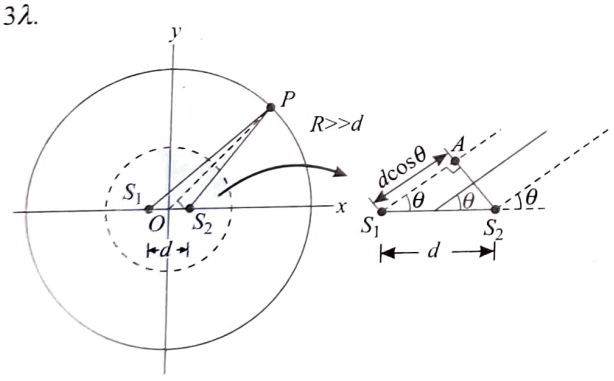
### ILLUSTRATION 3.9

Two sources are placed on $x$-axis at a separation $d = 3\lambda$. An observer starts moving from $A$ on a circular track of radius $R$ ($R \gg d$). How many bright points and dark points will he observe? Find the angular positions of maxima and minima.




**Sol.** The observer will have maximum path difference at positions $A$ and $C$ equal to $3\lambda$. Minimum path difference equals to zero at positions $B$ and $D$.

In each quarter circle, he will have path difference between 0 and $3\lambda$.




Path difference corresponding to maxima in our given range is $0, \lambda, 2\lambda, 3\lambda$.

Path difference corresponding to minima in our given range is $\lambda/2, 3\lambda/2, 5\lambda/2$.

Hence, total number of bright points in the complete circular path is $4 \times 2 + 4 = 12$.

Total number of dark points on circular path is $4 \times 3 = 12$.

Hence, the observer will find 12 bright points and 12 dark points in the circular track.

**Finding angular positions of maxima and minima:**

Path difference at point $P$, $S_1P - S_2P = \Delta x$

As $R >> d$, we can consider the lines $S_1P$ approximately parallel. If we drop a perpendicular from centre of circle to the line $S_1P$, the length $S_1A$ should be equal to the path difference

$$S_1A = \Delta x$$

$$\Rightarrow \quad d\cos\theta = \Delta x$$

Hence, $\cos\theta = \dfrac{\Delta x}{d}$

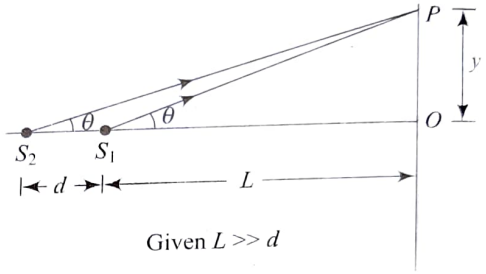
$$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{\Delta x}{d}\right)$$

For maxima, $\Delta x = n\lambda$

For minima, $\Delta x = (2n - 1)\dfrac{\lambda}{2}$

### ILLUSTRATION 3.10

Two point sources are placed on a straight line separated by a distance $d = 3\lambda$. Both the sources are placed at a distance $L$ from a wall which is perpendicular to the straight line. Both the sources are sending waves of equal intensity.




Given $L >> d$

Find:

**(a)** Locus of the points on the wall having equal intensity.

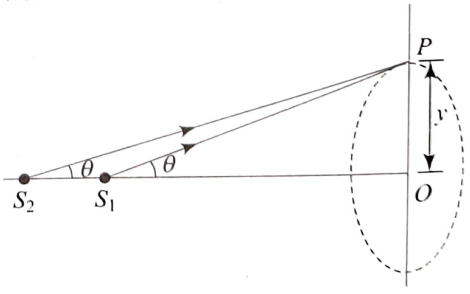
**(b)** Maximum and minimum path difference observed on the wall.

**Sol.**

**(a)** As shown in the figure, path difference between the waves is same in the circular path on the wall. Hence, locus of the points of same intensity is a circle.

**(b)** Maximum path difference will be at $O = 3\lambda$.

Minimum path difference will be at a large distance from $O$ and it will be zero.




### ILLUSTRATION 3.11

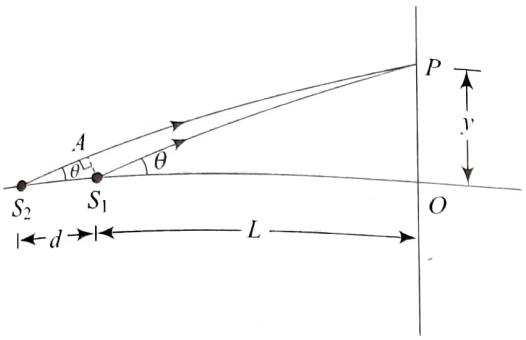
**(a)** In Illustration 3.10, how many dark rings will be observed on the wall? **(b)** What is the path difference at point $P$?

**Sol.**

**(a)** Path difference corresponding to minima in our given range is $\lambda/2, 3\lambda/2, 5\lambda/2$.

Hence, three dark rings will be observed on the wall.

(b) Path difference between the waves reaching at $P$, from figure, $\Delta x = S_2A$




The optical path difference is $\Delta x = S_1P - S_2P$.
Since $L >> d$,

$$\Delta x = d\cos\theta = d\left(1-2\sin^2\frac{\theta}{2}\right) = d\left(1-\frac{\theta^2}{2}\right)$$

(As $\theta$ is small),

$$\Rightarrow \quad \Delta x = d\left(1-\frac{y^2}{2L^2}\right)\left(\because \theta = \frac{y}{L}\right) \qquad ...(i)$$

Here $y$ is the radius of any dark or bright ring.
For maxima $\Delta x = n\lambda$

And for minima $\Delta x = (2n-1)\dfrac{\lambda}{2}$

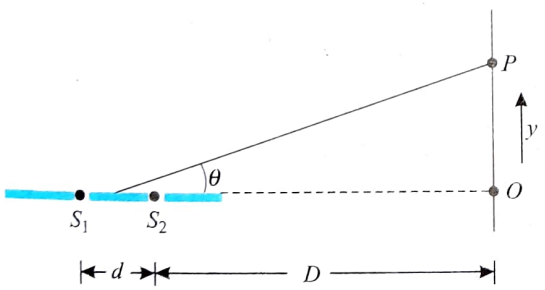
On substituting the value $\Delta x$ in Eq. (i) we can find the radius of any dark or bright ring.

## ILLUSTRATION 3.12

Two monochromatic coherent sources of wavelength 5000 Å are placed along the line normal to the screen. Determine the:

(a) condition of maxima at $P$ (given $\theta$ is small),

(b) order of central bright fringe, if $d = 0.5$ nm and $D = 1$ m.




**Sol.**

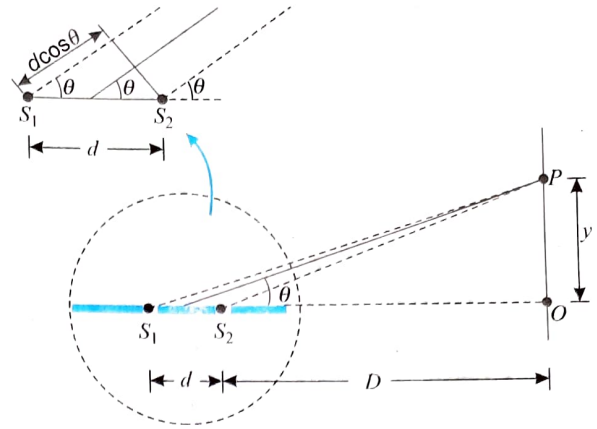
(a) The optical path difference is $\Delta x = S_1P - S_2P$.
Since $D >> d$,

$$\Delta x = d\cos\theta = d\left(1-2\sin^2\frac{\theta}{2}\right) = d\left(1-\frac{\theta^2}{2}\right)$$

(As θ is small),

$$\Rightarrow \Delta x = d\left(1-\frac{y^2}{2D^2}\right) \qquad \left(\because \theta = \frac{y}{D}\right)$$

For maxima, $\Delta x = n\lambda$




Or $$d\left(1-\frac{y^2}{2D^2}\right) = n\lambda$$

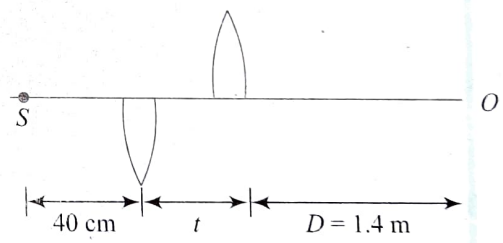
Or $$y = D\sqrt{2\left(1-\frac{n\lambda}{d}\right)}$$

(b) For central maxima $y = 0$ or $n = \dfrac{d}{\lambda}$

$$\Rightarrow \quad n = \frac{0.5}{0.5\times10^{-3}} = 1000$$

## ILLUSTRATION 3.13

A convex lens of focal length 20 cm is cut along its diameter and the two halves are displaced by $t = 1$ cm along the principal axis. A monochromatic point source of wavelength 476 nm is placed at a distance 40 cm from the first half, as shown in figure. Find the position of the first maximum.

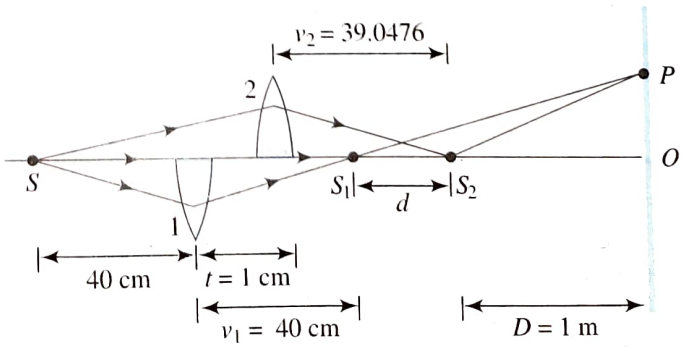



**Sol.** Using the formula

$$v_1 = \frac{fu_1}{u_1 - f} = \frac{(20)(40)}{(40-20)} = 40 \text{ cm}$$

and $$v_2 = \frac{u_2 f}{u_2 - f} = \frac{(41)(20)}{41-20} = 39.0476 \text{ cm}$$

$$d = v_2 - v_1 = 0.0476 \text{ cm} = 0.476 \text{ mm}$$



The optical path difference at $P$ is

$$\Delta x_p = (SS_2 + S_2P) - (SS_1 + S_1P)$$
$$= (SS_2 - SS_1) - (S_1P - S_2P)$$
$$= d - (S_1P - S_2P)$$

We know that $S_1P - S_2P = d\left[1 - \frac{y^2}{2D^2}\right]$

[As we did in previous illustration]

$$\Delta x_p = d - d\left[1 - \frac{y^2}{2D^2}\right] = \frac{y^2 d}{2D^2}$$

The position of the $n$th maximum is given by

$$\frac{y^2 d}{2D^2} = n\lambda \quad \text{or} \quad y = D\sqrt{\frac{2n\lambda}{d}}$$

The position of the first maximum is given by

$$y = 10^3\sqrt{\frac{2(1)(0.476\times10^{-3})}{0.476}} = 44.72 \text{ mm}$$

## ILLUSTRATION 3.14

Two parallel beams of light $P$ and $Q$ are incident normally on a prism and the transmitted rays are brought to focus with the help of a convergent lens as shown in figure. If the intensities of the upper and lower beams immediately after transmission from face $AC$ are $4I$ and $I$, respectively, find the resultant intensity at the focus.

**Sol.** In case of interference, the resultant intensity

$$I = I_1 + I_2 + 2(\sqrt{I_1 I_2})\cos\phi$$

Here, $I_1 = I$ and $I_2 = 4I$

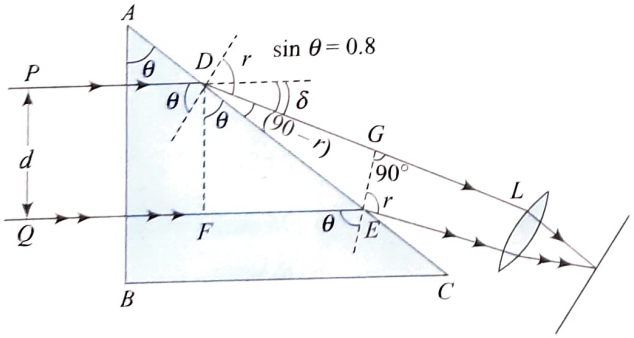
So, $I_R = 5I + 4I\cos\phi$

$$= I\left[1 + 8\cos^2\left(\frac{\phi}{2}\right)\right] \quad ...(i)$$

with $\phi = \frac{2\pi}{\lambda}(\Delta x)$ ...(ii)

where $\Delta x = [DG - EF \text{ (in medium)}]$ ...(iii)

Now, from figure,




$FE$ (in medium) $= \mu(FE) = \mu d\tan\theta$ [as $FE = d\tan\theta$] ...(iv)

$DG = DE\cos(90° - r) = DE\sin r$

or $DG = (d\sin r/\cos\theta)$ [as $DE = d/\cos\theta$]

or $DG = \mu d\tan\theta$ [as at $D$, $\mu\sin\theta = 1\sin r$] ...(v)

Substituting the value of $FE$ and $DG$ from Eqs. (iv) and (v) in Eq. (iii), we get

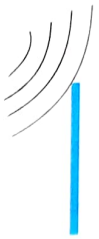
$$\Delta x = \mu d\tan\theta - \mu d\tan\theta = 0$$

So, Eq. (ii) gives $\phi = 0°$.

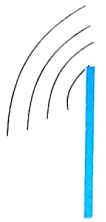
Hence, from Eq. (i), $I_R = I[1 + 8\cos^2 0°] = 9I$

## CONCEPT APPLICATION EXERCISE 3.1

**1.** Spherical wavefronts shown in figure, strike a plane mirror. Reflected wavefront will be as shown in

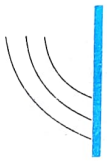


(a)

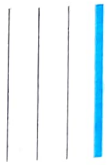


(b)

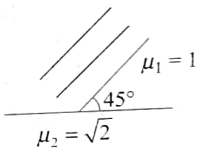
(c)



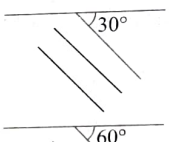
(d)



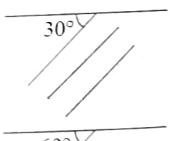
**2.** Wavefronts incident on an interface between the media are shown in the figure. The refracted wavefront will be as shown in




(a)



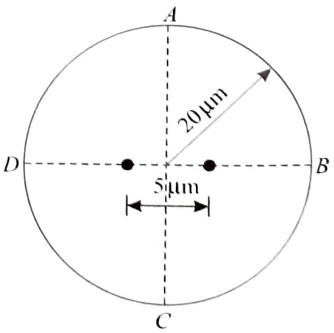

(b)




(c)

(d)

**3.** Two point source separated by $d = 5$ μm emit light of wavelength $\lambda = 2$ μm in phase. A circular wire of radius 20 μm is placed around the source as shown in figure.




(a) Points $A$ and $B$ are dark and points $C$ and $D$ are bright.

(b) Points $A$ and $B$ are bright and point $C$ and $D$ are dark.

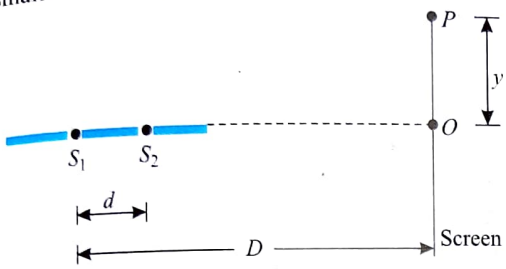
(c) Points $A$ and $C$ are dark and points $B$ and $D$ are bright.

(d) Points $A$ and $C$ are bright and points $B$ and $D$ are dark.

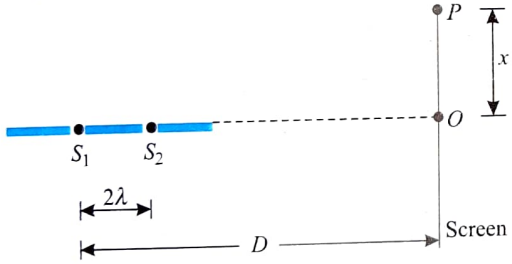
4. Two beams of light having intensities $I$ and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beam is $\pi/2$ at point $A$ and $2\pi$ at point $B$. Then find out the difference between the resultant intensities at $A$ and $B$.

5. Waves emitted by two identical sources produces intensity of $K$ unit at a point on screen where path difference between these waves is $\lambda$, calculate the intensity at that point on screen at which path difference is $\lambda/4$.

6. Two coherent narrow slits emitting light of wavelength $\lambda$ in the same phase are placed parallel to each other at a small separation of $3\lambda$. The light is collected on a screen $S$ which is placed at a distance $D$ ($>> \lambda$) from the slits. Find the smallest distance $y$ such that the $P$ is a maxima.




7. Two coherent narrow slits emitting light of wavelength $\lambda$ in the same phase are placed parallel to each other at a small separation of $2\lambda$. The light is collected on a screen $S$ which is placed at a distance $D$ ($>> \lambda$) from the slit $S_1$ as shown in figure. Find the finite distance $x$ such that the intensity at $P$ is equal to intensity at $O$.




**ANSWERS**

1. (c) **2.** (b) **3.** (d) **4.** $4I$ **5.** $\frac{K}{2}$ unit
**6.** $2\sqrt{2}D$ **7.** $\sqrt{3}D$

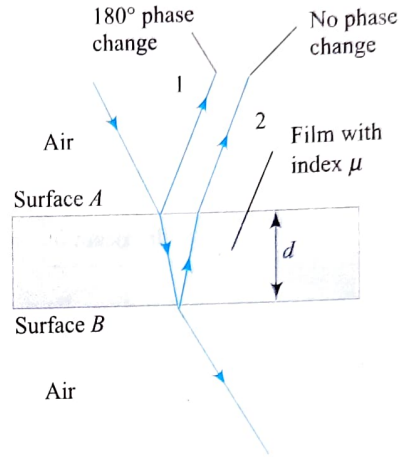
## THIN-FILM INTERFERENCE

In YDSE, we obtained two coherent sources from a single (incoherent) source by division of wavefront. Here, we do the same by division of amplitude (into reflected and refracted waves).

When a plane wave (parallel rays) is incident normally on a thin film of uniform thickness $d$, then waves reflected from the upper surface interfere with waves reflected from the lower surface. Clearly, the waves reflected from the lower surface travel an extra optical path of $2\mu d$, where $\mu$ is refractive index of the film. Further, if the film is placed in air the wave reflected from the upper surface (from a denser medium) suffers a sudden phase change of $\pi$, while the wave reflected from the lower surface (from a rarer medium) suffers no such phase change.

It is useful to draw an analogy between reflected light waves and the reflections of a transverse wave on a stretched string when the wave meets a boundary, as in shown. The reflected pulse on a string undergoes a phase change of 180° when it is reflected from the boundary of denser string, or from a rigid barrier, and no phase change when it is reflected from the boundary of a less dense string. Similarly, an electromagnetic wave undergoes a 180° phase change when reflected from the boundary of a medium of higher index of refraction than the one in which it has been travelling. There is no phase change when the wave is reflected from a boundary leading to a medium of lower index of refraction. The transmitted wave that crosses the boundary also undergoes no phase change.

Figure (a) shows that ray reflecting from a medium of higher refractive index undergoes a 180° phase change. The right side shows the analogy with a reflected pulse on a string.



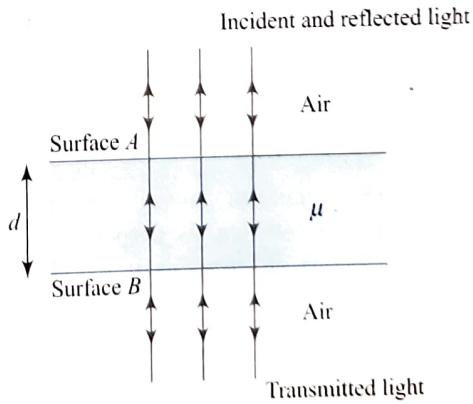

**(a)**

Figure (b) shows that a ray reflecting from a medium of lower refractive index undergoes no phase change. Consequently, condition for constructive and destructive interference in the reflected light is given by

$$2\mu d = n\lambda \quad \text{(for destructive interference)}$$

$$= \left(n + \frac{1}{2}\right)\lambda \quad \text{(for constructive interference)} \qquad ...(i)$$

where $n = 0, 1, 2, \ldots$ and $\lambda$ = wavelength in free space.




**(b)**

Interference will also occur in the transmitted light and here conditions of constructive and destructive interference will be the reverse of Eq. (i), i.e.,

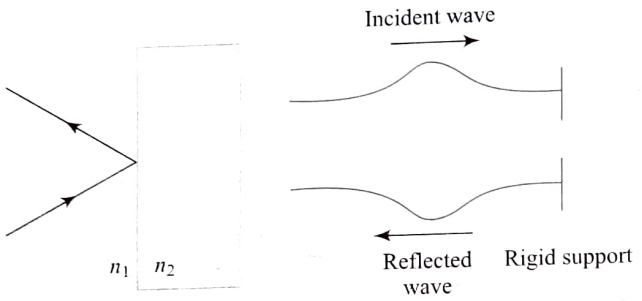
$$2\mu d = \begin{cases} n\lambda & \text{(for constructive interference)} \\ \left(n + \frac{1}{2}\lambda\right) & \text{(for destructive interference)} \end{cases} \qquad ...(ii)$$

This can easily be explained by energy conservation (when intensity is maximum in reflected light, it has to be minimum in transmitted light). However, the amplitude of the directly transmitted wave and the wave transmitted after one reflection differ substantially and hence the fringe contrast in transmitted light is poor. It is for this reason that the thin film interference is generally viewed only in the reflected light.

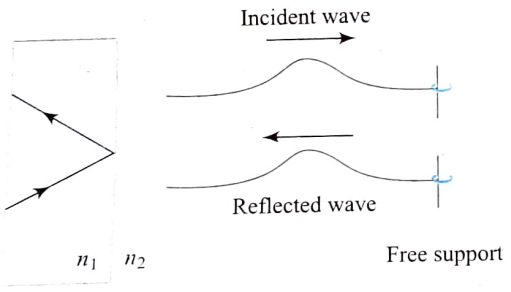
In deriving Eq. (i), we assumed that the media surrounding the thin film on both sides are rarer compared to the medium of thin film.

If media on both sides are denser, then there is no sudden phase change in the wave reflected from the upper surface, but there is a sudden phase change of $\pi$ in waves reflected from the lower surface. The conditions for constructive and destructive interference in reflected light would still be given by Eq. (i).

However, if medium on one side of the film is denser and that on the other side is rarer, then either there is no sudden phase in any reflection, or there is a sudden phase change of $\pi$ in both reflection from upper and lower surfaces.




**(a) 180° phase change**




**(b) No phase change**

## ILLUSTRATION 3.15

White light, with a uniform intensity across the visible wavelength range 430–690 nm, is perpendicularly incident on a water film of index of refraction $\mu = 1.33$ and thickness $d = 320$ nm, that is suspended in air. At what wavelength $\lambda$ is the light reflected by the film brightest to an observer?

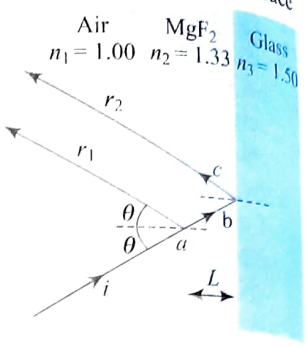
**Sol.** In this situation, Eq. (i) gives the interference maxima. Solving for $\lambda$ and inserting the given data, we obtain

$$\lambda = \frac{2\mu d}{n + 1/2} = \frac{(2)(1.33)(320\text{ nm})}{n + 1/2} = \frac{851\text{ nm}}{n + 1/2}$$

For $m = 0$, this give us $\lambda = 1700$ nm, which is in the infrared region. For $n = 1$, we find $\lambda = 567$ nm, which is yellow–green light, near the middle of the visible spectrum. For $n = 2$, $\lambda = 340$ nm, which is in the ultraviolet region. So, the wavelength at which the light seen by the observer is brightest is $\lambda = 567$ nm.

## ILLUSTRATION 3.16

A glass lens is coated on one side with a thin film of magnesium fluoride ($MgF_2$) to reduce reflection from the lens surface (see figure). The index of refraction of $MgF_2$ is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum ($\lambda = 550$ nm)? Assume that the light is approximately perpendicular to the lens surface.




**Sol.** This situation here is different from previous problem in that $n_3 > n_2 > n_1$. The reflection at point $a$ still introduces a phase difference of $\pi$ but now the reflection at point $b$ also does the same (see figure). Unwanted reflections from glass can be suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of a properly chosen thickness which introduces a phase change of half a wavelength. For this, the path length difference $2L$ within the film must be equal to an odd number of half wavelengths:

$$2n_2 L = (n + 1/2)\lambda$$

We want the least thickness for the coating, that is, the smallest $L$. Thus, we choose $n = 0$, the smallest value of $n$. Solving for $L$ and inserting the given data, we obtain

$$L = \frac{\lambda}{4n_2} = \frac{550\text{ nm}}{(4)(1.38)} = 96.6\text{ nm}$$

## ILLUSTRATION 3.17

White light is incident normally on a glass plate of thickness $0.50 \times 10^{-6}$ and index of refraction 1.50. Which wavelengths in the visible region (400 nm–700 nm) are strongly reflected by the plate?

**Sol.** The light of wavelength $\lambda$ is strongly reflected if

$$2\mu d = \left(n + \frac{1}{2}\right)\lambda \quad \ldots(i)$$

where $n$ is a non-negative integer.

Here $2\mu d = 2 \times 1.50 \times 0.5 \times 10^{-6}\text{ m} = 1.5 \times 10^{-6}\text{ m}$.

Putting $\lambda = 400$ nm in Eq. (i), we get

$$1.5 \times 10^{-6}\text{ m} = \left(n + \frac{1}{2}\right)(400 \times 10^{-9}\text{ m})$$

or $n = 3.25$

Putting $\lambda = 700$ nm in Eq. (i), we get

$$1.5 \times 10^{-6}\text{ m} = \left(n + \frac{1}{2}\right)(700 \times 10^{-9}\text{ m})$$

or $n = 1.66$

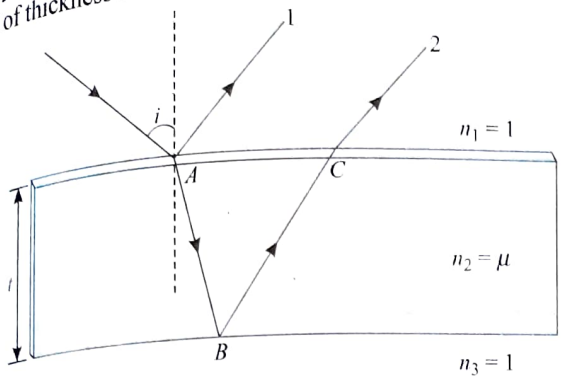
Thus, within 400 nm to 700 nm, the integer $n$ can take the values 2 and 3. Putting these values of $n$ in Eq. (i), the wavelengths become

$$\lambda = \frac{4\mu d}{2n + 1} = 600\text{ nm and } 429\text{ nm}$$

Thus, light of wavelengths 429 nm and 600 nm are strongly reflected.

**ILLUSTRATION 3.18**

A ray (wavelength $\lambda$) is incident at an angle $i$ at $A$ of a glass slab of thickness $t$ and refractive index $= \mu$.




Find the

(a) phase difference between the rays 1 and 2

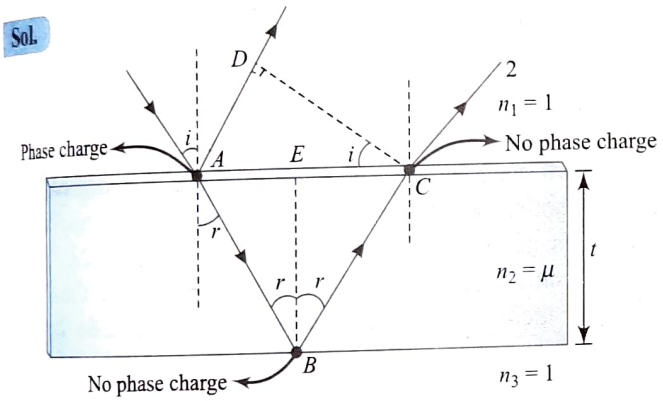
(b) value of $t$ for normal incidence $i = 0$ so as to have a constructive superposition of the rays 1 and 2

(c) if 50 % of light gets reflected at each interface, find the intensity of light after superposition of the rays 1 and 2, assume $I_0$ = intensity of incident ray.

(d) minimum intensity after superposition of the rays 1 and 2 (put $i = 0$)

(e) value of $t$ for minima for normal incidence of the light $i = 0$.

**Sol.**




(a) Optical path difference $\Delta x_{optical} = \mu(AB + BC) - AD$

In $\Delta ABC$, $AB = BC = \dfrac{t}{\cos r}$ ...(i)

Also $AE = EC = t\tan r$

Hence $AC = 2AE = 2t\tan r$ ...(ii)

In $\Delta ACD$: $AD = AC\sin i = 2t\tan r\sin i$ ...(iii)

Snell law at $A$: $1\sin i = \mu\sin r$ ...(iv)

from (iii) and (iv), $AD = 2t\tan r(\mu\sin r) = \dfrac{2\mu t\sin^2 r}{\cos r}$

Hence required path difference

$$\Delta x_{optical} = \mu\left[\frac{2t}{\cos r}\right] - \left[\frac{2\mu t\sin^2 r}{\cos r}\right]$$

$$= \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right] = \frac{2\mu t}{\cos r}\cos^2 r$$

$\Rightarrow \Delta x_{optical} = 2\mu t\cos r$

When a ray is reflected at the boundary of a rarer to denser medium, a phase difference of ($\pi$) or path difference $\dfrac{\pi}{2}$ occur for the ray '1'. There is no path difference due to transmission at $C$ including the change in path difference due to reflection, the true path difference

$$\Delta x_t = 2\mu t\cos r - \frac{\lambda}{2} \quad \text{...(v)}$$

Net phase difference between the ray '1' and '2'

$$\phi_t = \frac{2\pi}{\lambda}\cdot\Delta x_t = \frac{2\pi}{\lambda}\left(2\mu t\cos r - \frac{\lambda}{2}\right) \quad \text{...(vi)}$$

**(b)** For normal incidence $i \approx 0$, the value of $r$ should also near to zero. Hence for constructive interference of the rays 1 and 2, we have

$$2\mu t\cos\theta - \frac{\lambda}{2} = n\lambda \quad \text{or} \quad t = \frac{(2n+1)\lambda}{4\mu},\ n = 0,1,2,\ldots$$

**(c)** As 50% energy get reflected at $A$, $B$ and $C$. Hence the intensities of rays '1'and '2' will be $I_1 = \dfrac{I_0}{2}$ and $I_2 = \dfrac{I_0}{8}$

The resultant intensity of ray '1' and '2' will be

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_t = \frac{I_0}{2} + \frac{I_0}{8} = 2\sqrt{\frac{I_0}{2}\times\frac{I_0}{8}}\cos\phi_t$$

$$\Rightarrow I_R = \frac{I_0}{2}\left(\frac{5}{4} + \cos\phi_t\right) \quad \text{...(vii)}$$

On substituting the value of $\phi_t$ from Eq. (vi) and substituting in Eq. (vii) we can get the resultant intensity of ray '1' and '2'.

**(d)** The minimum intensity can be obtained by putting

$$\cos\phi_t = -1 \text{ and } I_{min} = \frac{I_0}{8}$$

**(e)** In case of normal incident ($i = 0$) for minima

$$2\mu t - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad 2\mu t = (n+1)\lambda$$

The phase relationship of the interfering waves does not change if one full wave is added or subtracted from any of the interfering waves.

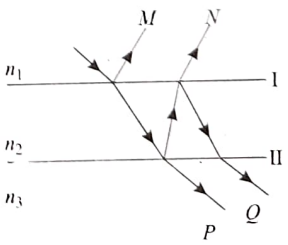
Hence above equation can be written as

$$2\mu t = m\lambda \quad \text{or} \quad t = \frac{m\lambda}{2\mu}$$

where $m = 1, 2, 3\ldots$

## CONCEPT APPLICATION EXERCISE 3.2

**1.** A ray of light is incident on a thin film. As shown in figure $M$, $N$ are two reflected rays and $P$, $Q$ are two transmitted rays. Rays $N$ and $Q$ undergo a phase change of $\pi$. Correct ordering of the refracting indices is




(a) $n_2 > n_3 > n_1$

(b) $n_3 > n_2 > n_1$

(c) $n_3 > n_1 > n_2$

(d) none of these, the specified changes cannot occur

**2.** Refractive index of a thin soap film of a uniform thickness is 1.34. Find the smallest thickness of the film that gives an interference maximum in the reflected light when light of wavelength 5360 Å falls at normal incidence.

**3.** A thin transparent film of thickness 3000 Å and refractive index 1.5 is deposited on a sheet made of a metal. Assuming normal incidence of light and also that the film is a plane

parallel one, which wavelength will be strongly reflected from this sheet when observed in white light?

4. By an anodizing process, a transparent film of aluminium oxide of thickness $t = 250$ nm and index of refraction $n_2 = 1.80$ is deposited on a sheet of polished aluminium. Which wavelength will be strongly reflected if utensils made from this sheet with observer in white light? Assume normal incidence of the light.
5. An oil film ($n = 1.45$) floating on water is illuminated by white light at normal incidence. The film is 280 nm thick. Find: **(a)** the wavelength of the light in the visible spectrum most strongly reflected and **(b)** the color of the light in the spectrum most strongly transmitted. Explain your reasoning.
6. A material having an index of refraction of 1.30 is used as an anti-reflective coating on a piece of glass ($n = 1.50$). What should be the minimum thickness of this film in order to minimize reflection of 500 nm light?
7. A glass plate of refractive index 1.5 is coated with a thin layer of thickness $t$ and refractive index 1.8. Light of wavelength $\lambda$ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and two reflected rays interfere. Write the condition for their constructive interference. If $\lambda = 648$ nm, obtain the least value of $t$ for which the rays interfere constructively.

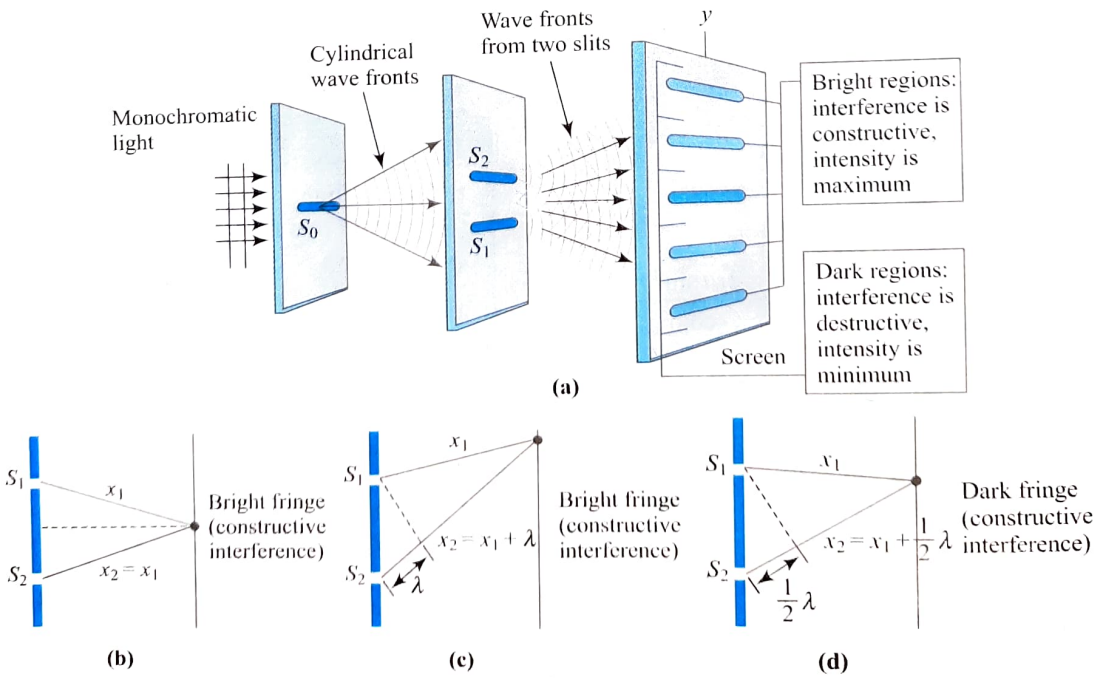
**ANSWERS**

**1.** (b) **2.** 0.1 μm **3.** 6000 Å
**4.** 450 nm **5.** (a) 541 nm, green (b) 406 nm, violet
**6.** 0.500 cm **7.** 90 nm

## YOUNG'S DOUBLE-SLIT EXPERIMENT

In 1801, the English scientist Thomas Young (1773–1829) performed a historic experiment that demonstrated the wave nature of light by showing that two overlapping light waves interfered with each other. His experiment was particularly important because he was also able to determine the wavelength of the light from his measurements, the first such determination of this important property. Figure (a) shows one arrangement of Young's experiment, in which light of a single wavelength (monochromatic light) passes through a single narrow slit and falls on two closely spaced narrow slits $S_1$ and $S_2$. These two slits act as coherent sources of light waves that interfere constructively and destructively at different points on the screen to produce a pattern of alternating bright and dark fringes. The purpose of the single slit is to ensure that only light from one direction falls on the double slit from different points on the light source would strike the double slit from different directions and cause the pattern on the screen to be washed out. The slits $S_1$ and $S_2$ act as coherent sources of light waves because the light from each originates from the same primary source, namely, the single slit.

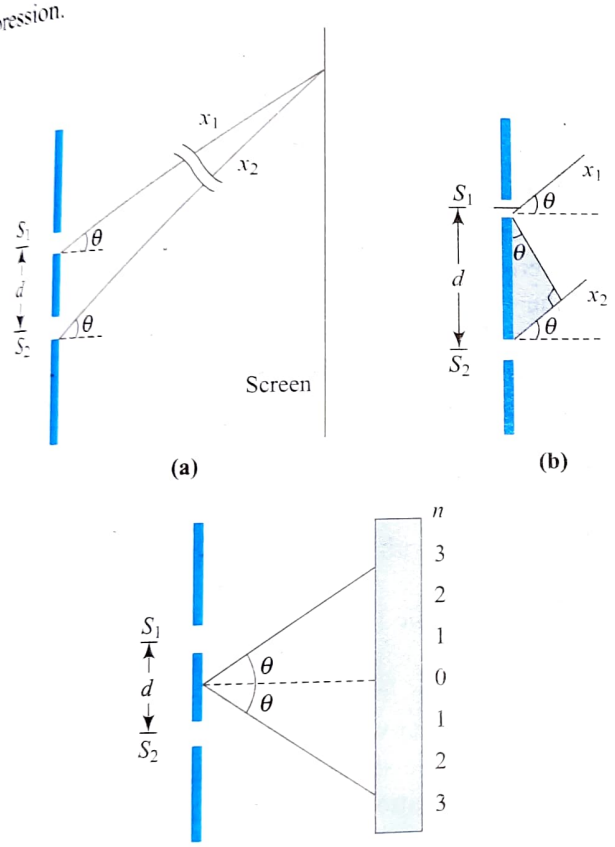
To explain the origin of the bright and dark fringes, the figure presents three top views of the double slit and the screen. Figure (b) illustrates how a bright fringe arises directly opposite the midpoint between the two slits. In this part of the figure, the waves (identical) from each slit travel to the midpoint on the screen. At this location, the distances $x_1$ and $x_2$ to the slits are equal, each containing the same number of wavelengths. Therefore, constructive interference results, leading to the bright fringe. Figure (c) indicates that constructive interference produces another bright fringe on one side of the midpoint when the distance $x_2$ is larger than $x_1$ by exactly one wavelength. A bright fringe also occurs symmetrically on the other side of the midpoint when the distance $x_1$ and $x_2$ by one wavelength; for clarity, however, this bright fringe is not shown. Constructive interference produces additional bright fringes on both sides of the middle wherever the difference between $x_1$ and $x_2$ is an integer number of wavelengths: $\lambda$, $2\lambda$, $3\lambda$, etc. Figure (d) shows how the first dark fringe arises. Here, the distance $x_2$ is larger than $x_1$ by exactly one-half a wavelength, so the waves




**The waves slits $S_1$ and $S_2$ interfere constructively (Parts (b) and (c)) or destructively (Part (d)) on the screen, depending on the difference in distance between the slits and the screen. The slit widths and the distance between the slits have been exaggerated for clarity.**

interfere destructively, giving rise to the dark fringe. Destructive interference creates additional dark fringes on both sides of the center whenever the difference between $x_1$ and $x_2$ equals an odd integer number of half-wavelengths: $1\left(\frac{\lambda}{2}\right), 3\left(\frac{\lambda}{2}\right)$, etc.

The position of the fringes observed on the screen in Young's experiment can be calculated with the aid of following figure. If the screen is located far away compared with the separation $d$ of the slits, then the lines labeled $x_1$ and $x_2$ in Fig. (a) are nearly parallel. Being nearly parallel, these lines make approximately equal angles $\theta$ with the horizontal. The distances $x_1$ and $x_2$ differ by an amount $\Delta x$, which is the length of the short side of the shaded triangle in Fig. (b). Since the triangle is a right angled triangle, it follows that $\Delta x = d \sin \theta$. Constructive interference occurs when the distances differ by an integer number $n$ of wavelengths $\lambda$, or $\Delta x = d \sin \theta = n\lambda$. Therefore, the angle $\theta$ for the interference maxima can be determined from the following expression.







**(a) Rays from slits $S_1$ and $S_2$, which make approxi-mately the same angle $\theta$ with the horizontal, strike a distant screen at the same spot. (b) The difference in the path lengths of the two rays is $\Delta l = d \sin \theta$. (c) The angle $\theta$ is the angle at which a bright fringe ($m = 2$, here) occurs on either side of the central bright fringe ($m = 0$).**

**Bright fringes of a double slit:**

$$\sin \theta = n\frac{\lambda}{d}, n = 0, 1, 2, 3, \ldots \quad \ldots(i)$$

The value of $m$ specifies the order of the fringe. Thus, $n = 2$ identifies the 'second-order' bright fringe. Figure (c) stresses that the angle $\theta$ given by Eq. (i) locates bright fringes on either side of the midpoint between the slits. A similar line of reasoning leads to the conclusion that the dark fringes, which lie between the bright fringes, are located according to the following expression.

**Dark fringes of a double slit:**

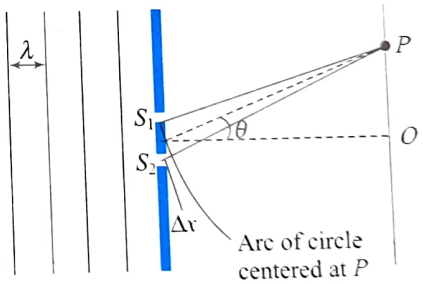
$$\sin \theta = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}, n = 0, 1, 2, 3, \ldots \quad \ldots(ii)$$

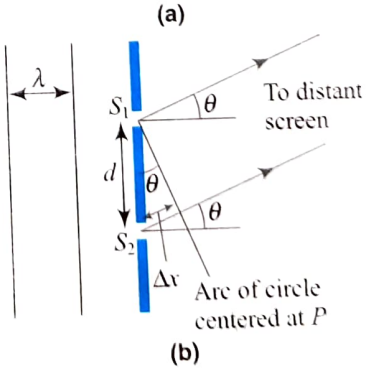
**Conditions for observing sustained interference:** Following are the conditions for observing sustained interference with good contrast:

1. The initial phase difference between the interfering waves must remain constant otherwise the interference will not be sustained.
2. The frequencies and wavelength of the two waves should be equal. If not, the phase difference will not remain constant and so the interference will not be sustained.
3. The light must be monochromatic. This eliminates overlapping of pattern as each wavelength corresponds to one interference pattern.
4. The amplitudes of the interfering waves must be equal. This improves constant with $I_{max} = 4I_0$ and $I_{min} = 0$.
5. The sources must be close to each other, otherwise due to small fringe, width fringes may be so close to each other that the eye cannot resolve them, resulting in uniform illumination.
6. The sources must be narrow. A board source will be equal to a large number of narrow sources and each set of two sources will give its own pattern and overlapping of patterns will result in uniform illumination.

## POSITION OF BRIGHT AND DARK FRINGES IN YDSE

Let us consider point $P$ on the distant screen, at a distance $D$ from the slits width $D >> d$. The small arc of the circle from $P$ is almost a straight line. From figure, path difference $\Delta x = d \sin \theta$.




The condition for constructive interference is

$\Delta x = S_2P - S_1P = \pm n\lambda$, where $n = 0, 1, 2, \ldots$

$d \sin \theta = \pm n\lambda$ [Condition for maxima] ...(i)

The condition for destructive interference is

$$\Delta x = S_2P - S_1P = \pm\left(n - \frac{1}{2}\right)\lambda, \quad \text{where} \quad n = 1, 2, \ldots$$

$$d \sin \theta = \pm\left(n - \frac{1}{2}\right)\lambda \quad \text{[Condition for minima]} \quad \ldots(ii)$$

If the separation between screen and slits is large ($D >> d$), then we have

$$\sin\theta \approx \tan\theta = \theta = \frac{y}{D}$$

where $y$ is the vertical distance from the center of the pattern. Position of $n$th bright and dark fringes are, respectively,

$$\frac{y_n d}{D} = n\lambda \quad \text{or} \quad y_n = n\lambda\left(\frac{D}{d}\right)$$

and $$\frac{y_n d}{D} = \left(n-\frac{1}{2}\right)\lambda \quad \text{or} \quad y_n = \left(n-\frac{1}{2}\right)\lambda\frac{D}{d}$$

Each value of $n$ corresponds to particular bright or dark fringe. The absolute value of $n$ is called the order of interference. The $n$th and $(n+1)$th maxima are given by

$$y_n = \frac{n\lambda D}{d} \quad \text{or} \quad y_{n+1} = \frac{(n+1)\lambda D}{d}$$

## FRINGE WIDTH

The distance between two successive maxima (or minima) is called fringe width given as

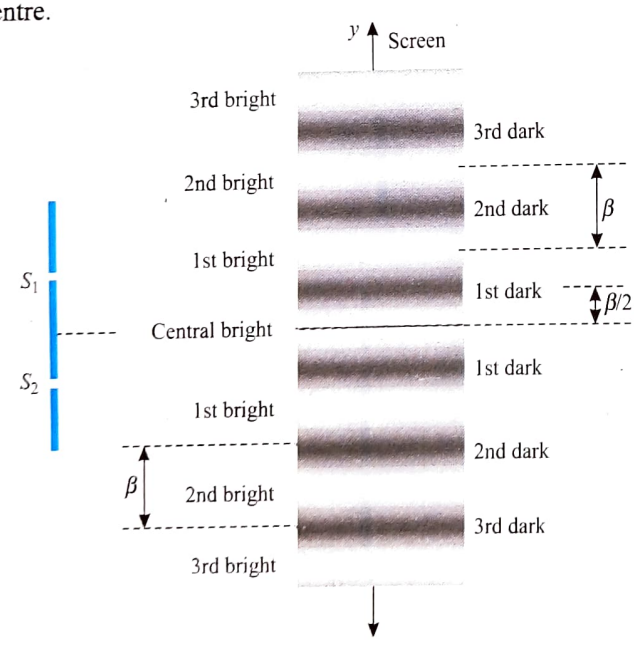
$$\beta = y_{n+1} - y_n$$

Distance between $(n+1)^{th}$ maxima and $n^{th}$ maxima will also equal to fringe width

$$\beta = (n+1)\frac{\lambda D}{d} - n\frac{\lambda D}{d}$$

or $$\beta = \frac{\lambda D}{d}$$

The fringe width is measured from centre of one fringe to centre of next fringe. The bright fringe has maximum intensity at its centre similarly dark fringe has minimum intensity at its centre.

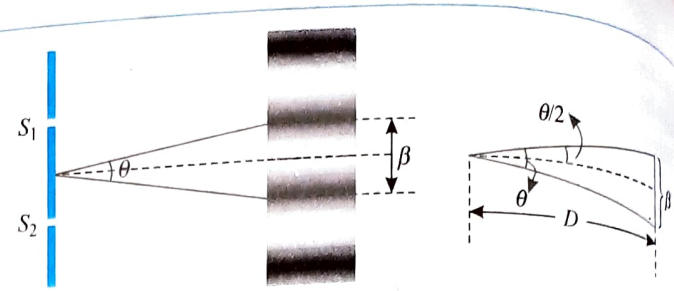



**Fringe pattern in YDSE**

## ANGULAR FRINGE WIDTH

The angular fringe width is $\theta = \frac{\beta}{D} = \frac{(\lambda D/d)}{D}$

or $$\theta = \frac{\lambda}{d}$$




The angular fringe width is defined as the ratio of wavelength of light and the distance between the slits $S_1$ and $S_2$ (sources) of coherent light.

**Important Points:**

- Fringe width is independent of $n$, i.e., all the interference fringes have same width in experiments where there is a division of wavefront of the incoming waves.
- Fringe width is directly proportional to the wavelength of light used, i.e., $\beta \propto \lambda$. So, fringes for red light are wider than those for blue light.
- Fringe width is inversely proportional to the separation between the slits, i.e., $\beta \propto (1/d)$. Thus, with increase in separation between the sources, fringe width decreases.
- With increase in distance between screen and plane of slits, fringe width $\beta$ increases linearly with $D$. However, with increase in $D$, intensity of light sources and hence of interfering waves is adversely affected.
- If the interference experiment is performed in a medium of refractive index $\mu$ (say water) instead of air, the wavelength of light will change from $\lambda$ to $(\lambda/\mu)$ and so

$$\beta' = \frac{D}{d}\left[\frac{\lambda}{\mu}\right] = \frac{\beta}{\mu} \quad \text{...(i)}$$

  That is, fringe width reduces and becomes $(1/\mu)$ times of its value in air.
- If monochromatic light is replaced by white light, due to overlapping of patterns (each corresponding to a single wavelength with fringe width $\beta \propto \lambda$), central band, i.e., principal maxima will be white with red edges on either side of it also we shall get a few colored bands and then uniform illumination.

### ILLUSTRATION 3.19

In Young's experiment for interference of light the slits 0.2 cm apart are illuminated by yellow light ($\lambda$ = 6000 Å). What would be the fringe width on a screen placed 1 m from the plane of slits ? What will be the fringe width if the system is immersed in water. (Refractive index = 4/3)

**Sol.** Fringe width, $\beta = \frac{\lambda D}{d} = \frac{6000\times10^{-10}\times1}{0.2\times10^{-2}}$

$= 3.0\times10^{-4}$ mm $= 0.3$ mm

If system is immersed in water ($\mu$ = 4/3), then the fringe width becomes

$$\beta' = \frac{\beta}{\mu} = \frac{0.3}{4/3}\text{mm} = 0.225 \text{ mm}$$

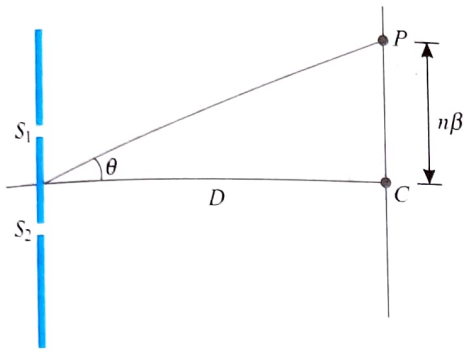
**ILLUSTRATION 3.20**

The slits of the double slit is illuminated by light of wave length 5000Å. The slit are 1mm apart and the screen is placed 1 m away. Calculate :
(i) the angular position of the 10th maxima in radian and
(ii) separation of the two adjacent minima.

**Sol.**

(i) Angular position of the $n^{th}$ maximum

$$\theta_n = \frac{n\beta}{D} = \frac{n\lambda D/d}{D} = \frac{n\lambda}{d}.$$




Hence the angular position of the 10th maxima

$\lambda = 5000\,\text{Å} = 5000\times10^{-10}$ m, $d = 1\times10^{-3}$ m and $n = 10$

$$\theta_n = \frac{n\lambda}{d} = \frac{10\times5000\times10^{-10}}{10^{-3}} = 5\times10^{-3}\text{ rad}$$

(ii) Separation between two adjacent minima = fringe width $\beta$

$$\beta = \frac{\lambda D}{d} = \frac{5000\times10^{-10}\times1}{1\times10^{-3}} = 5\times10^{-4}\text{ m} = 0.5\text{ mm}$$

**ILLUSTRATION 3.21**

Young's double slit experiment is performed inside water ($\mu = 4/3$) with light of frequency $6 \times 10^{14}$ Hz. If the slits are separated by 0.2 mm and the screen kept 1 m from the slits, find the fringe width. Using the same set-up, what will the fringe width be if the experiment is performed in air?

**Sol.** Wavelength of given light in air can be obtained from the relation

$$c = f\lambda_{air} \Rightarrow \lambda_{air} = \frac{c}{f} = \frac{3\times10^8}{6\times10^{14}} = 5\times10^{-7}\text{ m}$$

Wavelength of light of same frequency ($6 \times 10^{14}$ Hz) in water will be different and

$$\lambda_{water} = \frac{\lambda_{air}}{\mu_{water}} = \frac{5\times10^{-7}}{4/3} = 3.75\times10^{-7}\text{ m}$$

As the experiment is performed in water, so fringe width,

$$\beta = \frac{\lambda_{water} D}{d}$$

Here, $D = 1$ m and $d = 0.2$ mm $= 0.2 \times 10^{-3}$ m.

$$\therefore \quad \beta = \frac{3.75\times10^{-7}\times1}{0.2\times10^{-3}}\text{m}$$

or $\beta = 1.87$ mm

If the experiment is performed in air, then fringe width,

$$\beta = \frac{\lambda_{air} D}{d} = \frac{5\times10^{-7}\times1}{0.2\times10^{-3}} = 2.5\text{ mm}$$

**ILLUSTRATION 3.22**

In Young's double slit interference experiment, with monochromatic light, fringes are obtained on a screen placed at some distance from the slits. If the screen is moved by $5 \times 10^{-2}$ m towards the slits, the change in fringe width is $3 \times 10^{-5}$ m. If the distance between the slits is $10^{-3}$ m, calculate the wavelength of the light used.

**Sol.** Initial fringe width $\beta_1 = \frac{\lambda D}{d}$

The fringe width when the screen is moved by $5 \times 10^{-2}$ m towards the slits

$$\beta_2 = \frac{\lambda(D - 5\times10^{-2})}{d}$$

The fringe width will decrease in second case.

It is given $|\beta_2 - \beta_1| = \left|\frac{\lambda(D-5\times10^{-2})}{d} - \frac{\lambda D}{d}\right| = 3\times10^{-5}$ m

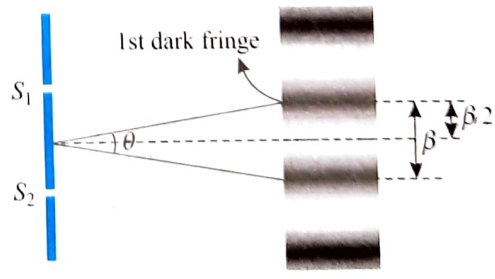
$$\therefore \quad 3\times10^{-5} = \frac{\lambda\times5\times10^{-2}}{d}$$

$$\therefore \quad \lambda = \frac{3\times10^{-5}\times10^{-3}}{5\times10^{-2}} = 6000\,\text{Å}$$

**ILLUSTRATION 3.23**

In Young's double slit interference experiment, the distance between two sources is $\frac{9}{100\pi}$ mm. The distance of the screen from the source is 1.0 m. Wavelength of light used is 5000 Å. Then what is the angular position of the first dark fringe?

**Sol.** Angular fringe width, $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$




The first dark fringe will be at half the angular fringe width from the mid point of central maximum. Thus the angular position of first dark fringe will be

$$\frac{\theta}{2} = \frac{1}{2}\left[\frac{\lambda}{d}\right] = \frac{1}{2}\left[\frac{5000\times100\pi}{9\times10^{-3}}\times10^{-10}\right]\frac{180}{\pi} = 0.5^\circ$$

## Maximum Order of Interference Fringes

The position of $n$th order maxima on the screen, $y = \frac{n\lambda D}{d}$, $n = 0, \pm1, \pm2$ [for interference maxima, but $n$ cannot take infinitely large values, as that would violate the approximation (II)]

i.e., $\theta$ is small or $y << D \Rightarrow \frac{y}{D} = \frac{n\lambda}{d} \ll 1$

Hence, the above formula for interference maxima is applicable when $n << d/\lambda$.

When $n$ becomes comparable to $d/\lambda$, path difference can no longer be given by $dy/D$. Hence, for maxima,

$$\Delta x = n\lambda \Rightarrow d\sin\theta = n\lambda \Rightarrow n = \frac{d\sin\theta}{\lambda}$$

Hence, the highest order of interference maxima,

$$n_{max} = \left[\frac{d}{\lambda}\right]$$

where [ ] represents the greatest integer function.

Similarly, the highest order of interference minima,

$$n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right]$$

### Illustration 3.24

In a YDSE, $D = 1$ m, $d = 1$ mm, and $\lambda = 1/2$ mm.

**(a)** Find the distance between the first and central maxima on the screen.

**(b)** Find the number of maxima and minima obtained on the screen.

**Sol.**

**(a)** $D >> d$

Hence, path difference at any angular position $\theta$ on the screen, $\Delta x = d\sin\theta$

The path difference for first maxima

$$\Delta x = d\sin\theta = \lambda \Rightarrow \sin\theta = \frac{\lambda}{d} = \frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ$$

Hence, distance between central maxima and first maxima

$$y = D\tan\theta = \frac{1}{\sqrt{3}}\text{ m}$$

**(b)** Maximum path difference, $\Delta x_{max} = d = 1$ mm

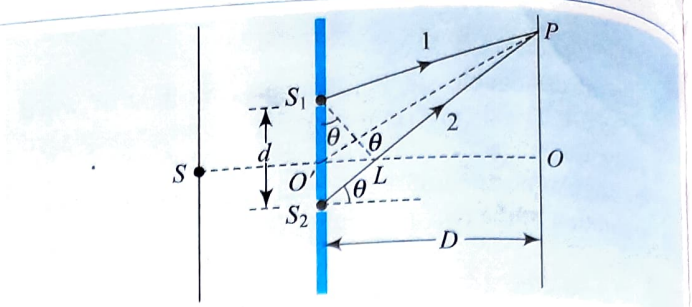
$\Rightarrow$ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$ and highest order minima $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total number of maxima $= 2n_{max} + 1 = 5$

Total number of minima $= 2n_{min} = 4$

### Illustration 3.25

In Young's experiment performed with light of wavelength 5500 Å, what should be the separation between the two slits so that the highest order of maximum intensity in the interference pattern is 2? You may assume $D << d$.




**Sol.** $S_2L = d\sin\theta$ or $\Delta x = d\sin\theta$

For intensity at $P$ to be maximum, $\Delta x = d\sin\theta = n\lambda$

$$\Rightarrow \sin\theta = \frac{n\lambda}{d}$$

Now, $\sin\theta \le 1 \Rightarrow \frac{n\lambda}{d} \le 1$

or $n \le \frac{d}{\lambda} \Rightarrow n_{max} = \frac{d}{\lambda}$

In this problem, $n_{max} = 2$ and $\lambda = 5500$ Å $= 5500 \times 10^{-10}$ m.

$\therefore$ $d = n_{max}\lambda = 2 \times 5500 \times 10^{-10} = 1.1 \times 10^{-6}$ m

### Illustration 3.26

In Young's experiment, light of wavelength 600 nm falls on the double slits separated by 0.1 mm. What is the highest order of maximum intensity in the interference pattern obtained on a screen kept 3 m from the slits? How does the highest order change if the distance of screen from the slits is changed?

**Sol.** As we know, $n_{max} = \frac{d}{\lambda}$

Given, $d = 0.1$ mm $= 0.1 \times 10^{-3}$ m

and $\lambda = 600$ nm $= 600 \times 10^{-9}$ m.

$\therefore$ $n_{max} = \frac{0.1 \times 10^{-3}}{600 \times 10^{-9}} = 166.67$

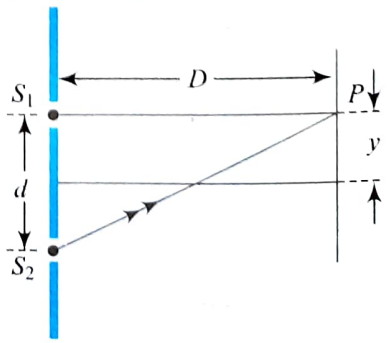
$\Rightarrow$ $n_{max} = 166$

Obviously, $n_{max} = d/\lambda$ does not depend on the distance of screen from the slits.

Hence, it remains 166 if the distance between the slits and the screen is changed.

### Illustration 3.27

At a point on the screen directly in front of one of the slits, find the missing wavelengths.



**Sol.** According to theory of interference, position $y$ of a point on the screen is given by

$$y = \frac{D}{d}(\Delta x)$$

As for missing wavelengths, intensity will be minimum (= 0), so

$$\Delta x = (2n-1)\frac{\lambda}{2}$$

$$\therefore \quad y = D\frac{(2n-1)\lambda}{2d}$$

However, here $d = b$ and $y = (b/2)$.

Hence, $\lambda = \dfrac{b^2}{(2n-1)D}$ with $n = 1, 2, 3, \ldots$

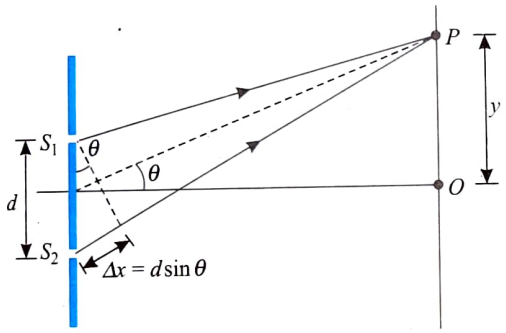
i.e., wavelengths $(b^2/D)$, $(b^2/3D)$, $(b^2/5D)$, etc., will be absent (or missing) at point $P$.

## INTENSITY DISTRIBUTION ON SCREEN IN YDSE

If $I_1$ and $I_2$ be the intensity of the light from slits $S_1$ and $S_2$ respectively reaching at point $P$ on the screen. Then resultant intensity at $P$ is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \qquad \ldots(i)$$

where $\phi$ is the phase difference between the waves reaching at $P$.




If intensities of the light coming out from slits are equal i.e., $I_1 = I_2 = I_0$, then from Eq. (i), $I_R = 2I_0 + 2I_0\cos\phi = 2I_0(1+\cos\phi)$

or $\quad I_R = 4I_0\cos^2\dfrac{\phi}{2}$

As phase difference $\phi = \dfrac{2\pi}{\lambda}\cdot\Delta x$ and in Young's double slit experiment (YDSE), $\Delta x = d\sin\theta$ then $\phi = \dfrac{2\pi}{\lambda}\cdot(d\sin\theta)$

It mean the resultant intensity at $P$ can be expressed as

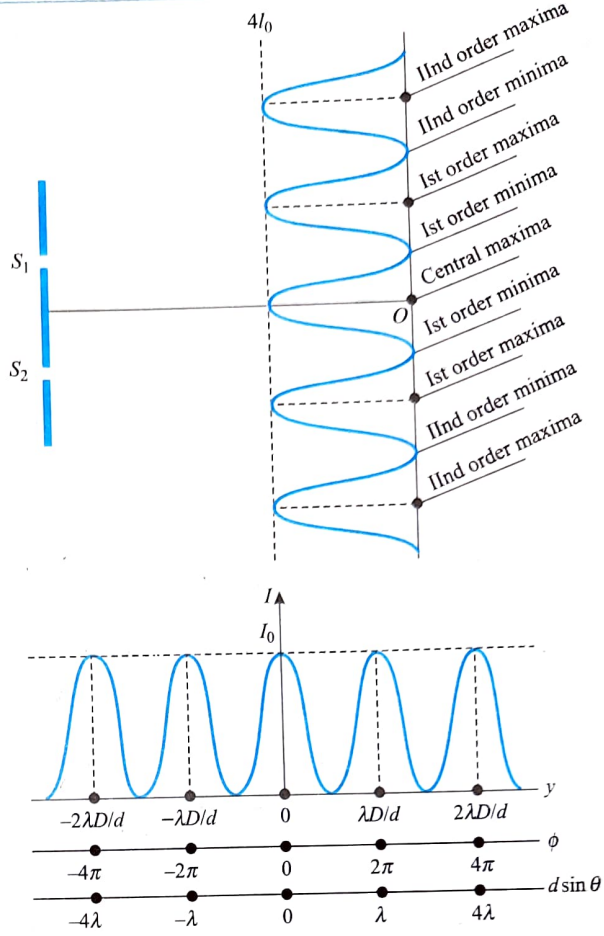
$$I_R = 4I_0\cos^2\left(\frac{\pi}{\lambda}d\sin\theta\right)$$

If we are observing near centre of screen we can express path difference as, $\Delta x = \dfrac{yd}{D}$

The phase difference, $\phi = \dfrac{2\pi}{\lambda}\cdot\left(\dfrac{yd}{D}\right) = \dfrac{2\pi yd}{\lambda D}$

And the resultant intensity at $P$ can be expressed as

$$I_R = 4I_0\cos^2\left(\frac{\pi yd}{\lambda D}\right)$$




**Intensity maxima occur where $\phi$ is an integral multiple of $2\pi$ and $d$ sin $\theta$ is an integral multiple of $\lambda$**

**Phase difference, path difference and intensity variation within one fringe:**

The phase difference, $\phi = \dfrac{2\pi}{\lambda}\cdot\left(\dfrac{yd}{D}\right) = \dfrac{2\pi yd}{\lambda D}$ ...(i)

We express the fringe width as, $\beta = \dfrac{\lambda D}{d}$ ...(ii)

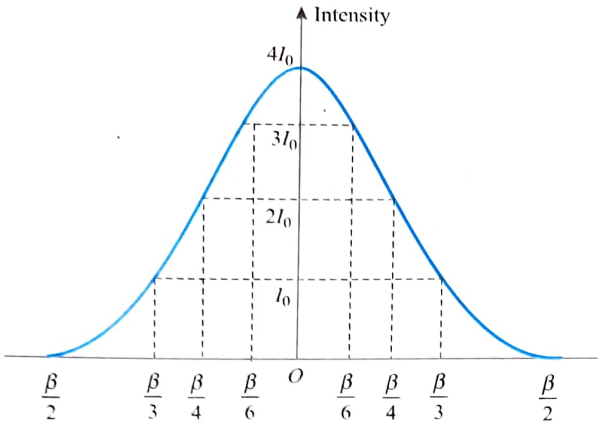
From (i) and (ii), $\phi = \dfrac{2\pi y}{\beta}$

The resultant intensity on screen in function of $y$ can be expressed as, $I_R = 4I_0\cos^2\left(\dfrac{\pi yd}{\lambda D}\right)$ ...(iii)

Then Eq. (iii) in terms of fringe width can be expressed as

$$I_R = 4I_0\cos^2\left(\frac{\pi y}{\beta}\right) \qquad \ldots(iv)$$

| Value of y | Phase difference ($\phi$) | Path difference $\Delta x = \dfrac{\lambda}{2\pi}\phi$ | Intensity |
|---|---|---|---|
| $\dfrac{\beta}{6}$ | $\dfrac{\pi}{3}$ or 60° | $\dfrac{\lambda}{6}$ | $3I_0$ |
| $\dfrac{\beta}{4}$ | $\dfrac{\pi}{2}$ or 90° | $\dfrac{\lambda}{4}$ | $2I_0$ |
| $\dfrac{\beta}{3}$ | $\dfrac{2\pi}{3}$ or 120° | $\dfrac{\lambda}{3}$ | $I_0$ |
| $\dfrac{\beta}{2}$ | $\pi$ or 180° | $\dfrac{\lambda}{2}$ | 0 |




**Important Point:**

The intensity of light coming through the slits is directly proportional to the opening of the slit (or width of the slit) i.e. Intensity $\propto$ width of slit $\propto$ (amplitude)$^2$

$$\Rightarrow \quad I \propto w \propto a^2 \Rightarrow \frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2}$$

## ILLUSTRATION 3.28

In YDSE interference pattern, if the slit widths are in the ratio 1:4. Then find out the ratio of minimum and maximum intensity.

**Sol.** As we know Intensity $\propto$ width of slit $\propto$ (amplitude)$^2$

Hence the slit width ratio

$$\because \quad \frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{1}{2} \Rightarrow 2a_1 = a_2$$

$$\therefore \quad \frac{I_{min}}{I_{max}} = \frac{(a_1 - a_2)^2}{(a_1 + a_2)^2} = \frac{(a_1 - 2a_1)^2}{(a_1 + 2a_1)^2} = \frac{1}{9} = 1:9$$

## ILLUSTRATION 3.29

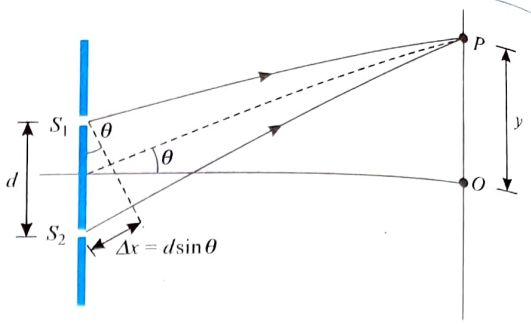
In YDSE one slit is wider than other slit. It is found that the amplitude of light reaching the central point of the screen from one of the slit, acting alone is twice that from the other slit when acting alone. Find the ratio resulting intensity at a direction $\theta$ on the screen and that at the centre of the screen.

**Sol.** The intensity at any point $P$ on screen is given by

$$I_P = K\left\{a^2 + (2a)^2 + 2(a)(2a)\cos\phi\right\}$$

At centre of screen, $\phi = 0$

or $\quad I_C = 9Ka^2 \quad \ldots(i)$




At any angular position $\theta$, at $P$,

$$I_\theta = K\left\{a^2 + (2a)^2 + 2(a)(2a)\cos\phi\right\}$$

$$= Ka^2\left\{5 + 4\cos\phi\right\}$$

$$= Ka^2\left\{1 + 8\cos^2\frac{\phi}{2}\right\}$$

where $\quad \phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} d\sin\theta$

or $\quad I_\theta = Ka^2\left(1 + 8\cos^2\frac{\pi d\sin\theta}{\lambda}\right) \quad \ldots(ii)$

From Eqs. (i) and (ii), $\frac{I_\theta}{I_C} = \frac{1}{9}\left\{1 + 8\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)\right\}$

## ILLUSTRATION 3.30

In Young's experiment, the slits, separated by $d = 0.8$ mm, are illuminated with light of wavelength 7200 Å. Interference pattern is obtained on a screen $D = 2$ m from the slits. Find the minimum distance from central maximum at which the average intensity is 50% of the maximum?

**Sol.** Resultant intensity at a point due to superposition of coherent waves and phase difference $\phi$ can be expressed as

$$I_R = 4I_0\cos^2\frac{\phi}{2}$$

At the position of interference maxima, $\phi = 2n\pi$

$$\cos^2(\phi/2) = \cos^2 n\pi = 1 \qquad [n \text{ is integer}]$$

$$I_{max} = 4I_0 \quad \Rightarrow \quad I_R = I_{max}\cos^2(\phi/2)$$

Here, $\quad I_R = \frac{I_{max}}{2}$ $\qquad$ [50% of maximal]

$$\frac{I_{max}}{2} = I_{max}\cos^2\frac{\phi}{2} \quad \Rightarrow \quad \cos^2\frac{\phi}{2} = \frac{1}{2}$$

$$\cos\frac{\phi}{2} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \frac{\phi}{2} = \frac{\pi}{4} \quad \text{or} \quad \phi = \pi/2$$

Therefore, phase difference at a point at which resultant intensity is 50% of maximum is $\pi/2$.

Now, phase difference, $\phi = (2\pi/\lambda)$ (path difference)

$$\therefore \quad \text{Path difference} = \frac{\lambda}{2\pi} \times \frac{\pi}{2} = \frac{\lambda}{4} \quad \ldots(i)$$

But $\quad \Delta x = d\frac{y}{D}$

So, from (i), $\frac{dy}{D}=\frac{\lambda}{4} \Rightarrow y=\frac{\lambda D}{4d}$

Given $\lambda = 7200$ Å and $D = 2$ m, $d = 0.8$ mm.

$\therefore \quad y=\frac{7200\times10^{-10}\times2}{4\times0.8\times10^{-3}}=4.5\times10^{-4}$ m $= 45$ mm

## SHAPE OF INTERFERENCE FRINGES IN YDSE

Here, we discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.

Fringes are locus of points which move in such a way that its path difference from the two slits remains constant.

$S_2P - S_1P = \Delta x =$ constant ...(i)

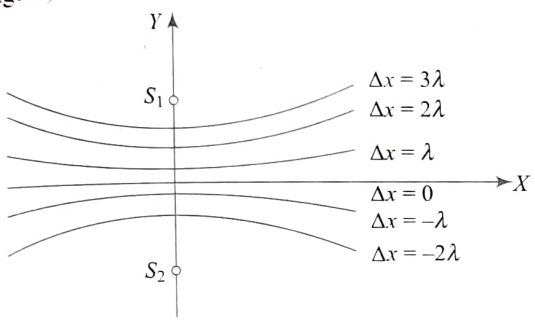
If $\Delta x = \pm\lambda/2$, the fringe represents 1st minima.

If $\Delta x = \pm3\lambda/2$, it represents 2nd minima.

If $\Delta x = 0$, it represents central maxima.

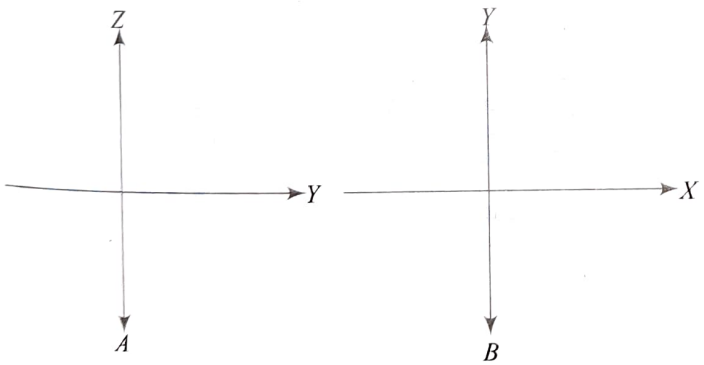
If $\Delta x = \pm\lambda$, it represents 1st maxima, etc.

Equation (i) represents a hyperbola with its two foci at $S_1$ and $S_2$ (see figure).




The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis $S_1S_2$.

If the screen is perpendicular to the $X$-axis, i.e., in the $YZ$ plane, as is generally the case, fringes are hyperbolic with a straight central section [see Fig. (a)].




(a) (b)

If the screen is in the $XY$ plane, again fringes are hyperbolic [see Fig. (b)].

## YOUNG'S DOUBLE-SLIT EXPERIMENT WITH WHITE LIGHT

The central maxima will be white because all wavelengths will constructively interference here. However, slightly below (or above) the position of central maxima fringes will be colored. For example, if $P$ is a point on the screen such that

$$S_2P - S_1P = \frac{\lambda_{violet}}{2} = 190 \text{ nm}$$

then completely destructive interference will occur for violet light. Hence, we will have a line devoid of violet color that will appear reddish. And if

$$S_2P - S_1P = \frac{\lambda_{red}}{2} = 350 \text{ nm}$$

then completely destructive interference for red light results and the line at this position will be violet. The colored fringes disappear at points far away from the central white fringe; for these points, there are so many wavelengths which interfere constructively that we obtain a uniform white illumination. For example, if $S_2P - S_1P = 3000$ nm, then constructive interference will occur for wavelengths $\lambda = 3000/n$ nm. In the visible region, these wavelengths are 750 nm (red), 600 nm (yellow), 500 nm (greenish-yellow), and 430 nm (violet). Clearly, such a light will appear white to the unaided eye.

Thus, with white light we get a white central fringe at the point of zero path difference, followed by a few colored fringes on its both sides, the color soon fading off to a uniform white.

In the usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of central maxima in such cases.

### ILLUSTRATION 3.31

White coherent light (4000–7000 Å) is sent through the slits of a YDSE. The separation between the slits is 0.5 mm and screen is 50 cm away from the slits. There is a hole in the screen at a point 1.0 mm away (along the width of the fringe) from the central line.

**(a)** Which wavelength(s) will be absent in the light coming from the hole?

**(b)** Which wavelength(s) will have a strong intensity?

**Sol.**

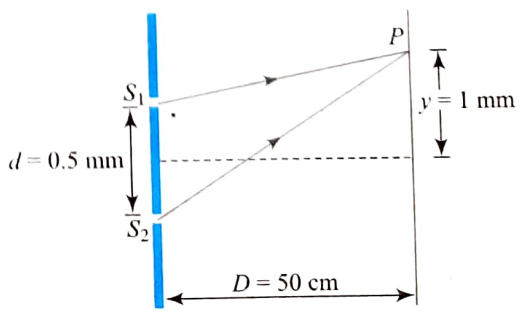
**(a)** The absent wavelength will correspond to minima at this position order of minima corresponding to 4000 Å.

$$y_n=\frac{(2n-1)D\lambda}{2d} \Rightarrow (2n-1)=\frac{y_n 2d}{D\lambda}$$

$$\Rightarrow \quad n=\frac{1}{2}\left[\frac{2d\,y_n}{D\lambda}+1\right]$$

$$n_1=\frac{1}{2}\left[\frac{0.5\times10^{-3}\times1\times10^{-3}\times2}{50\times10^{-2}\times4000\times10^{-10}}+1\right]$$

$$=\frac{1}{2}\left[\frac{2\times10^4}{4000}+1\right]=3$$



Order of minima corresponding to 7000 Å,

$$n_2 = \frac{1}{2}\left[\frac{2\times10^4}{7000}+1\right] = 1.9$$

Number of integers between 1.9 and 3.0 are 2 and 3.
Wavelength corresponding to $n = 2$ is

$$\lambda_{(2)} = \frac{y_n 2d}{(2n-1)D} = \frac{1\times10^{-3}\times2\times0.5\times10^{-3}}{(2\times2-1)\times50\times10^{-2}}$$

$$= \frac{10^{-4}}{(2\times2-1)\times50} = \frac{2\times10^{-6}}{3} = 0.6667\times10^{-6}\text{ m}$$

$$= 667\text{ nm}$$

Now, wavelength corresponding to $n = 3$ is

$$\lambda_{(3)} = \frac{2\times10^{-6}}{(2\times3-1)} = 400\text{ nm}$$

Hence the wavelengths 400 nm and 667 nm will be absent at the hole.

**(b)** The wavelengths corresponding to strong intensity will correspond to maxima at the position. Order of maxima corresponding to $\lambda$,

$$y_n = \frac{nD\lambda}{d} \Rightarrow n = \frac{y_n d}{D\lambda} = \frac{1\times10^{-3}\times0.5\times10^{-3}}{50\times10^{-2}\times\lambda}$$

$$n_{4000} = \frac{10^{-6}}{4000\times10^{-10}} = \frac{10000}{4000} = 2.5$$

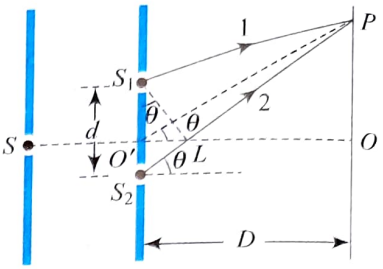
and $n_{7000} = \dfrac{10000}{7000} = 1.4$

Integer between 1.4 and 2.5 is 2.

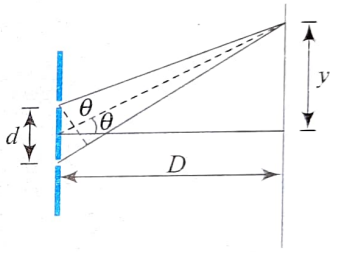
$$\therefore\ \lambda_{(2)} = \frac{y_n d}{nD} = \frac{10^{-6}}{2} = 500\text{ nm}$$

## CONCEPT APPLICATION EXERCISE 3.3

1. In Young's double slit experiment, one of the slits is so painted that intensity of light emitted from it is half of that of the light emitted from other slit. Then
   (a) fringe system will disappear
   (b) bright fringes will become brighter and dark fringes will be darker
   (c) both bright and dark fringes will become darker
   (d) dark fringes will become less dark and bright fringes will become less bright.
2. In YDSE performed with wavelength $\lambda$ = 5890 Å, the angular fringe width is 0.40°. What is the angular fringe width if the entire set-up is immersed in water?
3. In YDSE for wavelength $\lambda$ = 589 nm, the interference fringes have angular separation of $3.50\times10^{-3}$ rad. For what wavelength would the angular separation be 10.0% greater?
4. In Young's double slit experiment with wavelength 5890 Å, there are 60 fringes in the field of vision. How many fringes will be observed in the same field of vision if wavelength used is 5460 Å?
5. A beam of light consisting of wavelengths 6000 Å and 4500 Å is used in a YDSE with $D$ = 1 m and $d$ = 1 mm. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.
6. In Young's experiment, find the distance between two slits that results in the third minimum for 420 nm violet light at an angle 30°.




7. In an interference arrangement similar to double-slit experiment, $S_1$ and $S_2$ are illuminated with coherent source microwave source each of frequency 1 MHz. The sources are synchronized to have zero phase difference. The slits are separated by distance $d$ = 150.0 m. The intensity $I(\theta)$ is measured as a function of $\theta$, where $\theta$ is defined as shown in figure. If $I_0$ is maximum intensity, calculate $I(\theta)$ for:




**(a)** $\theta = 0$ **(b)** $\theta = 30°$ **(c)** $\theta = 90°$

8. Young's interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm, and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the center of the pattern. What is the wavelength?
9. Young's double-slit experiment is performed with a 600 nm light and a distance of 2.00 m between the slits and the screen. The tenth interference minimum is observed 7.5 mm from the central maximum. Determine the spacing of the slits.
10. Two slits are separated by 0.320 mm. A beam of 500 nm light strikes the slits, producing an interference pattern. Determine the number of maxima observed in the angular range $-30.0° < \theta < 30.0°$.
11. The intensity on the screen at a certain point in a double-slit interference pattern is 25.0% of the maximum value. **(a)** What minimum phase difference (in radians) between the sources produces this result? **(b)** Express this phase difference as a path difference for 486.1 nm light.
12. The intensity of the light coming from one of the slits in Young's double slit experiment is double the intensity from the other slit. Find the ratio of the maximum intensity to the minimum intensity in the interference fringe pattern observed.

13. The width of one of the two slits in Young's double slit experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportional to the slit width, find the ratio of the maximum to minimum intensity in the interference pattern.

14. Suppose that one of the slits in Young's experiment is wider than the other, so the amplitude of the light reaching the central part of the screen from one slit, acting alone, is twice that from the other slit, acting alone. Obtain an expression for the light intensity $I$ at the screen as a function of $\theta$, the angular position.

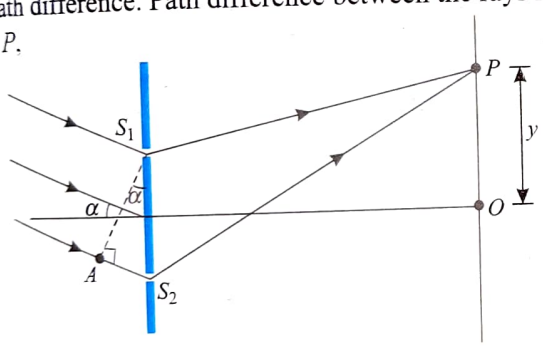
**ANSWERS**

1. (d) **2.** 0.30° **3.** 648 nm **4.** 65 **5.** 1.8 mm
6. $2.1 \times 10^{-6}$ m **7.** (a) $I_0$ (b) $I_0/2$ (c) 0 **8.** 515 nm
9. 1.52 mm **10.** 641 **11.** (a) $\frac{2\pi}{3}$ (b) 162 nm **12.** 34
13. 9

## DIFFERENT CASES IN YOUNG'S DOUBLE-SLIT EXPERIMENT

### Rays Not Parallel to Principal Axis

**Case 1. When source is at infinite distance and oblique incidence above the axis:** Here, rays reaching at $S_1$ and $S_2$ have initial path difference. Path difference between the rays reaching at point $P$,




Path difference between the rays reaching point $P$ i.e.,

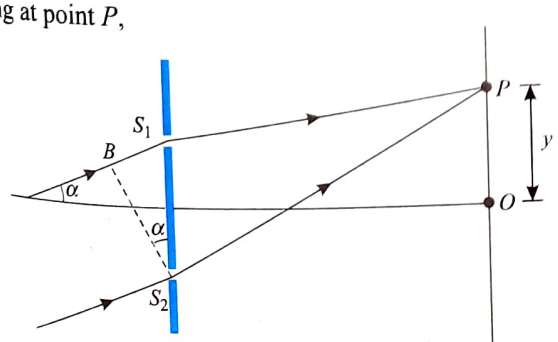
$$\Delta x = (AS_2 + S_2P) - S_1P$$

$$\Delta x = AS_2 + (S_2P - S_1P)$$

$$\Delta x = d\sin\alpha + \frac{yd}{D} \qquad \left[\therefore S_2P - S_1P = \frac{yd}{D}\right]$$

And for, maxima $\Delta x = \pm n\lambda$ for, minima $\Delta x = \pm(2n-1)\frac{\lambda}{2}$

**Case 2. When source is at infinite distance and oblique incidence below the axis:** Path difference between the rays reaching at point $P$,




$$\Delta x = (S_2P) - (BS_1 + S_1P)$$

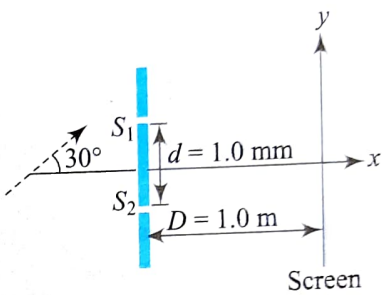
$$\Delta x = (S_2P - S_1P) - BS_1$$

$$\Delta x = \frac{yd}{D} - d\sin\alpha \qquad \left[\therefore S_2P - S_1P = \frac{yd}{D}\right]$$

And for maxima $\Delta x = \pm n\lambda$, for minima $\Delta x = \pm(2n-1)\frac{\lambda}{2}$

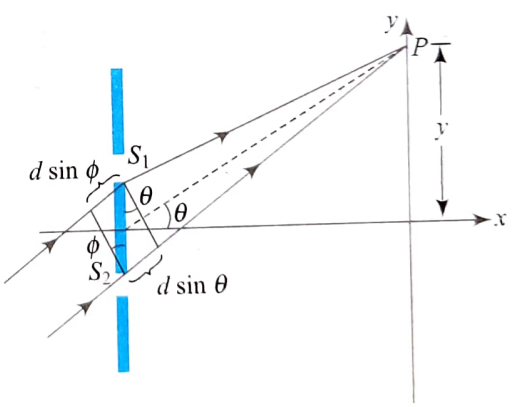
**ILLUSTRATION 3.32**

A coherent parallel beam of microwaves of wavelength $\lambda = 0.5$ mm falls on a Young's double-slit apparatus. The separation between the slits is 1.0 mm. The intensity of microwaves is measured on a screen placed parallel to the plane of the slits at a distance of 1.0 m from it as shown in figure.




If the incident beam makes an angle of 30° with the $x$-axis (as in the dotted arrow shown in the figure), find the $y$-coordinates of the first minima on either side of the central maximum.

**Sol.** Path difference before slits = $d\sin\phi$




Path difference after slits = $d\sin\theta$

As path of rays before slits is longer at $S_1$ and $S_2P > S_1P$ after slits, so net path difference for first minima is

$$\Delta x = d\sin\theta - d\sin\phi = \pm\frac{\lambda}{2}$$

$$\Rightarrow \quad \sin\theta = \sin\phi \pm \frac{\lambda}{2d} = \sin 30° \pm \frac{0.5}{2\times 1} = \frac{3}{4} \text{ or } \frac{1}{4}$$

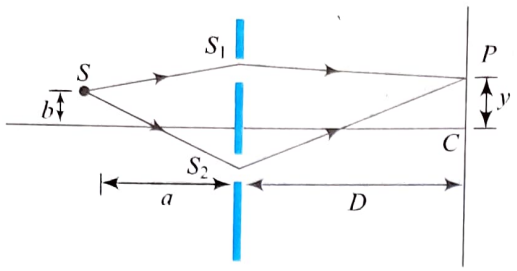
$$\Rightarrow \quad \tan\theta = \frac{3}{\sqrt{7}} \text{ and } \frac{1}{\sqrt{15}}$$

So, the position of first minima on either side of the central maxima is

$$y = D\tan\theta = \frac{3}{\sqrt{7}} \text{ and } \frac{1}{\sqrt{15}} \text{ m}$$

### Source Placed Beyond the Central Line

Here, the source is placed a little beyond the center line. In this case, the waves interaction with $S_1$ and $S_2$ has initial phase difference as shown in figure.




Path difference at point $P$ on the screen,

$$\Delta x = (SS_2 + S_2P) - (SS_1 + S_1P)$$
$$= (-SS_1 + SS_2) + (S_2P - S_1P)$$
$$= \frac{bd}{a} + \frac{yd}{D} \quad \ldots(i)$$

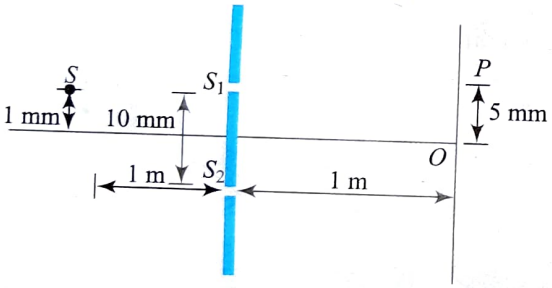
For maxima: $\Delta x = n\lambda$

For minima: $\Delta x = (2n-1)\frac{\lambda}{2}$

If we know the value of $\Delta x$ from Eq. (i), we can calculate different positions of maxima and minima.

### ILLUSTRATION 3.33

In Young's double-slit experiment, the point source is placed slightly off the central axis as shown in figure.




**(a)** Find the nature and order of the interference at point $P$.
**(b)** Find the nature and order of the interference at point $O$.

**Sol.**

**(a)** As discussed in previous case, here $b = 1$ mm, $d = 10$ mm, $a = 1$ m, and $D = 1$ m.

Using Eq. (i) for path difference between the waves reaching at point $P$,

$$\Delta x = \frac{1.10}{10^3} + \frac{5\times10}{2\times10^3} = 35\times10^{-3} \text{ mm}$$

If it is maximum at point $P$, then $\Delta x/\lambda$ will be an integer, i.e.,

$$\frac{35\times10^{-3}}{500\times10^{-9}\times10^3} = 70$$

A maximum of the order 70 occurs at $P$.

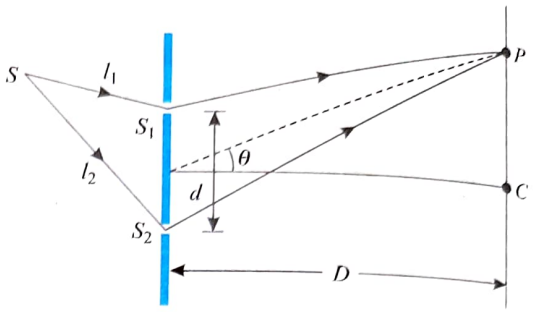
**(b)** At $O$, $\Delta x_0' = \frac{1\times10}{10^3} = 10\times10^{-3}$ mm

$$\Rightarrow \quad \frac{\Delta x}{\lambda} = 20$$

Hence, maximum of the order 20 occurs at $O$.

### Source Placed Beyond the Central Line at Certain Finite Distance

In this case also the rays reaching at $S_1$ and $S_2$ have initial path difference.



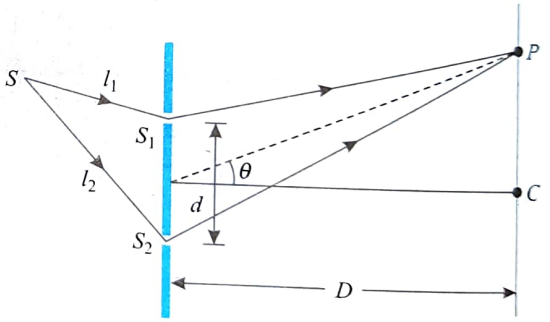

Path difference at point $P$,

$$\Delta x = [(SS_2 + S_2P) - (SS_1 + S_1P)]$$
$$\Delta x = (SS_2 - SS_1) - (S_2P - S_1P)$$
$$\Delta x = (l_2 - l_1) + d\sin\theta = (l_2 - l_1) + \frac{yd}{D}$$

Here, we assume that interference pattern formed at point $P$ is above the point '$C$'.

### ILLUSTRATION 3.34

In a Young's experiment the light source is at distance $l_1 = 20\ \mu$m and $l_2 = 40\ \mu$m from the slits. The light of wavelength $\lambda = 500$ nm is incident on slits separated at a distance 10 μm. A screen is placed at a distance $D = 2$ m away from the slits as shown in figure.




**(a)** Find the values of $\theta$ relative to the central line where maxima appear on the screen.
**(b)** How many maxima will appear on the screen?

**Sol.**

**(a)** The optical path difference between the beams arriving at $P$, $\Delta x = (l_2 - l_1) + d\sin\theta$

The condition for maximum intensity is, $\Delta x = n\lambda$ where, $n = 0, \pm1, \pm2, \ldots$

Thus, $\sin\theta = \frac{1}{d}[\Delta x - (l_2 - l_1)] = \frac{1}{d}[n\lambda - (l_2 - l_1)]$

$$= \frac{1}{10\times10^{-6}}[n\times500\times10^{-9} - 20\times10^{-6}] = 2\left[\frac{n}{40} - 1\right]$$

hence, $\theta = \sin^{-1}\left[2\left(\frac{n}{40} - 1\right)\right]$

(b) $|\sin\theta| \le 1$

$\therefore \quad -1 \le 2\left[\frac{n}{40} - 1\right] \le 1$

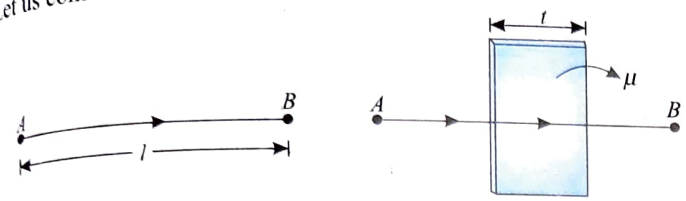
or $\quad -20 \le (n-40) \le 20$

or $\quad 20 \le n \le 60$

Hence, number of maxima = 60 – 20 = 40

## Change in Optical Path in a Transparent Slab

Let us consider two cases:




**Case I** **Case II**

**Case I:** A ray of light is travelling in air from $A$ to $B$. The optical path covered by the ray is '$l$'.

$$(L_1)_{\text{optical}} = l$$

**Case II:** Now a transparent slab of thickness '$t$' and refractive index $\mu$ is introduced in the path of the ray. In this case the ray travelled a distance $(l - t)$ in air and distance '$t$' in the slab.

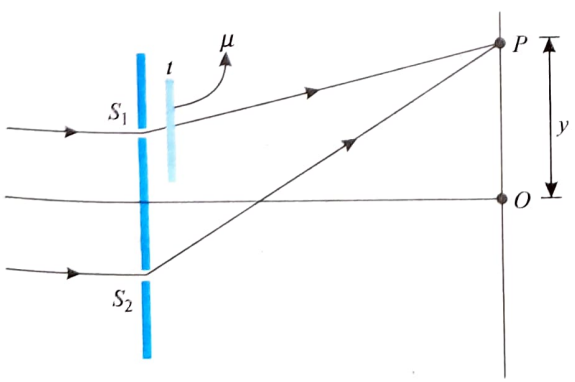
The length of the path '$t$' covered in slab is equal to the length of the path '$\mu t$' covered in vacuum in air. Hence the optical path in this case,

$$(L_2)_{\text{optical}} = (l-t) + \mu t = l + t(\mu - 1)$$

It means if we introduce a transparent slab, the optical path increases by $t(\mu - 1)$, where $\mu$ is the refractive index of slab and $t$ is the thickness and optical path difference in case II and case I is equal to $t(\mu - 1)$.

## Fringe Shift Due to Thin Plates

If we introduce a thin transparent slab of thickness $t$ and refractive index $\mu$ is placed in front of the upper slit in YDSE. Due to the presence of the slab the optical path of upper ray $(S_1P)$ gets increase by $t(\mu - 1)$. The optical path difference between the rays reaching point $P$




$$\Delta x = (S_2P) - \left(S_1P + t(\mu - 1)\right)$$

$$\Delta x = (S_2P - S_1P) - t(\mu - 1)$$

$$\Delta x = \frac{yd}{D} - t(\mu - 1) \qquad \ldots(i) \qquad \left[\because S_2P - S_1P = \frac{yd}{D}\right]$$

For $n$th maxima on upper side, $\Delta x = n\lambda$ ...(ii)

From (i) and (ii), $\frac{yd}{D} - t(\mu - 1) = n\lambda$

The position of $n^{\text{th}}$ maxima $y = \frac{n\lambda D}{d} + \frac{(\mu - 1)tD}{d}$

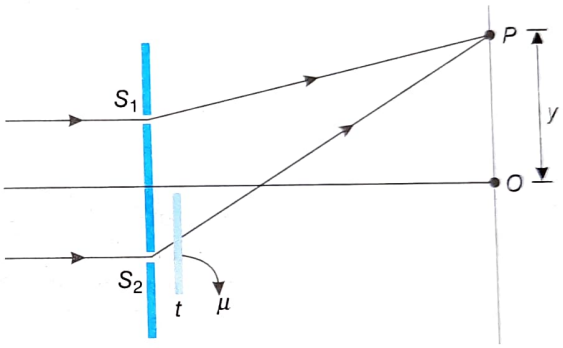
And we know before introduction of the slab it was at a distance $\frac{n\lambda D}{d}$ from $O$, it means on introduction of a transparent slab the fringe pattern will be shifted in upward direction. Hence the shift ($S$) of fringe pattern

$$S = \frac{(\mu - 1)tD}{d}$$

The positions of all the maxima and minima will be shifted to new position

For, maxima $\Delta x = \pm n\lambda$ and for, minima $\Delta x = \pm(2n-1)\frac{\lambda}{2}$

Similarly if we introduce a thin transparent slab in front of lower slit in YDSE as shown in figure. Here due to the presence of the slab the optical path of upper ray $(S_2P)$ gets increase by $t(\mu - 1)$. The optical path difference between the rays reaching point $P$




$$\Delta x = \left(S_2P + t(\mu - 1)\right) - (S_1P) = (S_2P - S_1P) + t(\mu - 1)$$

$$\Delta x = \frac{yd}{D} + t(\mu - 1) \qquad \ldots(i) \qquad \left[\therefore S_2P - S_1P = \frac{yd}{D}\right]$$

For $n^{\text{th}}$ maxima on upper side,

$$\Delta x = n\lambda \qquad \ldots(ii)$$

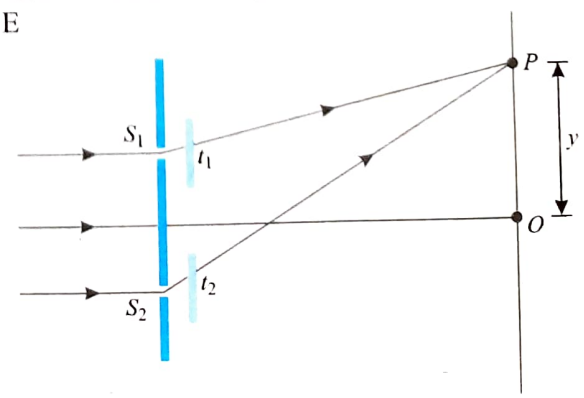
The position of $n^{\text{th}}$ maxima, $= \frac{n\lambda D}{d} - \frac{(\mu - 1)tD}{d}$

It means on introduction of a transparent slab the fringe pattern will be shifted in upward direction. The magnitude of shift will be same as in previous case but in downward direction

Here also for, maxima $\Delta x = \pm n\lambda$

And for, minima $\Delta x = \pm(2n-1)\frac{\lambda}{2}$

If we introduce a thin transparent slabs in front of both the slit in YDSE



Path difference between the rays reaching point $P$,

$$\Delta x = \left(S_2P + t_2(\mu_2 - 1)\right) - \left(S_1P + t_1(\mu_1 - 1)\right)$$

$$\Delta x = (S_2P - S_1P) + (t_1 - t_2) + (\mu_2 t_2 - \mu_1 t_1)$$

$$\Delta x = \frac{yd}{D} + (t_1 - t_2) + (\mu_2 t_2 - \mu_1 t_1) \quad \left[\because S_2P - S_1P = \frac{yd}{D}\right]$$

And for maxima $\Delta x = \pm n\lambda$, for minima $\Delta x = \pm(2n-1)\frac{\lambda}{2}$

**Important Points:**

- This fringe shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upward and if the glass slab is placed before the lower slit, the fringe pattern gets shifted downward.
- The number of fringes shifted

$$N = \frac{S}{\beta} = \frac{(\mu-1)tD/d}{D\lambda/d} = \frac{(\mu-1)t}{\lambda}$$

- Shift is independent of $n$, (the order of the fringe), i.e., shift of central maxima = shift of $10^{th}$ order maxima.
- The shift of fringe pattern is independent of $\lambda$.
  Shift of red colour fringe = shift of violet colour fringe.

## ILLUSTRATION 3.35

Monochromatic light of wavelength of 600 nm used in YDSE. If one of the slit is covered with a transparent sheet of thickness $1.8 \times 10^{-5}$ m made of a material of refractive index 1.6, how many fringes will be shifted?

**Sol.** The total fringe shift $= \frac{D\lambda}{d}\frac{(\mu-1)t}{\lambda} = \frac{D(\mu-1)t}{d}$, where $\frac{D\lambda}{d}$ = fringe width (FW) = $\beta$ or the number of fringes that will shift $= \frac{\text{Total shift}}{\text{FW}} = \frac{D(\mu-1)t/d}{D\lambda/d}$

$$= \frac{(\mu-1)t}{\lambda} = \frac{(1.6-1)\times 1.8\times 10^{-5}}{600\times 10^{-9}} = 18$$

## ILLUSTRATION 3.36

A Young's double slit apparatus is immersed in a liquid of refractive index $\mu_1$. The slit plane touches the liquid surface. A parallel beam of monochromatic light of wavelength $\lambda$ (in air) is incident normally on the slits.

**(a)** Find the fringe width.

**(b)** If one of the slits (say $S_2$) is covered by a transparent slab of refractive index $\mu_2$ and thickness $t$ as shown, find the new position of central maxima.

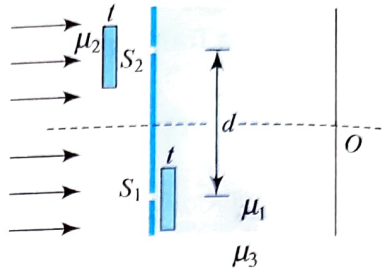
**(c)** Now, the other slit $S_1$ is also covered by a slab of same thickness and refractive index $\mu_3$ as shown in figure due to which the central maxima recovers its position, find the value of $\mu_3$.

**(d)** Find the ratio of intensities at $O$ in the three conditions (a), (b), and (c).

**Sol.**

**(a)** Fringe width, $\beta = \frac{\lambda_{\mu 1} D}{d} = \frac{\lambda D}{\mu_1 d}$

**(b)** Position of central maximum is shifted upward by distance $\frac{(\mu_2 - 1)tD}{d}$.




**(c)** $\frac{(\mu_2-1)tD}{d} = \frac{\left(\frac{\mu_3}{\mu_1} - 1\right)tD}{d} \Rightarrow \frac{\mu_3}{\mu_1} = \mu_2$ or $\mu_3 = \mu_1\mu_2$

**(d)** $I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$, where $\phi = \left(\frac{2\pi}{\lambda}\right)\Delta x$ or $\frac{\phi}{2} = \left(\frac{\pi}{\lambda}\right)\Delta x$

$$\Rightarrow \quad I \propto \cos^2\left(\frac{\phi}{2}\right)$$

In the first and third case, $\Delta x = 0$ while in the second case $\Delta x = (\mu_2 - 1)t$. Therefore, the desired ratio is

$$I_1 : I_2 : I_3 = 1 : \cos^2\left\{\frac{\pi(\mu_2-1)t}{\lambda}\right\} : 1$$

## ILLUSTRATION 3.37

A thin paper of thickness 0.02 mm having a refractive index 1.45 is pasted across one of the slits in a YDSE. The paper transmits 4/9 of the light energy falling on it.

**(a)** Find the ratio of maximum intensity to the minimum intensity in interference pattern.

**(b)** How many fringes will cross through the centre if an identical paper piece is pasted on the other slit also? The wavelength of the light used is 600 nm.

**Sol.**

**(a)** $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{(1+2/3)^2}{(1-2/3)^2} = \frac{5^2}{1} = 25$

**(b)** Fringe width, $\beta = \frac{D\lambda}{d}$

Shifting of fringe pattern due to paper sheet

$$s = \frac{t(\mu-1)D}{d}$$

If we paste paper sheet on the other slit, the fringe pattern will shift by same amount on that side.

Hence, number of fringes crossing the centre is

$$n = \frac{s}{\beta} = \frac{t(\mu-1)}{\lambda} = \frac{0.02(1.45-1)}{600\times 10^{-9}} \times 10^{-3}$$

$$= \frac{0.02\times 0.45\times 10^6}{600} = \frac{90}{6} = 15$$

### ILLUSTRATION 3.38

A double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and the screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å.

(a) Calculate the fringe width.

(b) One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minima on the axis.

**Sol.** As fringe width is given by $\beta = D\lambda/d$ and by presence of medium the wavelength becomes $\lambda/\mu$, so the fringe width in liquid will be

$$\beta' = \frac{D\lambda}{\mu_M d} = \frac{1.33 \times 6300 \times 10^{-10}}{1.33 \times 1 \times 10^{-3}} = 0.63 \text{ mm}$$

Now, as the distance of a minima from adjacent maxima is $\beta'/2$, so according to given problem, shift

$$y_0 = \frac{D}{d}(\mu - 1)t = \frac{\beta'}{2}$$

$$\Rightarrow \quad \frac{D(\mu-1)t}{d} = \frac{1}{2}\left(\frac{D\lambda}{\mu_M d}\right)$$

$$\Rightarrow \quad t = \frac{\lambda}{2\mu_M(\mu-1)} = \frac{6300}{2 \times 1.33 \times (1.53-1)}$$

$$= 4468.7 \text{ Å} = 446.87 \text{ μm}$$

### ILLUSTRATION 3.39

A monochromatic light of $\lambda$ = 500 Å is incident on two identical slits separated by a distance of $5 \times 10^{-4}$m. The interference pattern is seen on a screen placed at a distance of 1 m from the plane of slits. A thin glass plate of thickness $1.5 \times 10^{-6}$ m and refractive index $\mu = 1.5$ is placed between one of the slits and the screen. Find the intensity at the center of the screen.

**Sol.** In case of interference, $I_R = I_1 + I_2 + 2(\sqrt{I_1 I_2})\cos\phi$

Now, as for identical slits $I_1 = I_2 = I$,

so $I_R = 2(1 + \cos\phi) = 4I\cos^2(\phi/2)$

But for central maxima, $\phi = 0°$, and here $I_R = I_0$ (given)

$$I_0 = 4I\cos(0°) = 4I$$

Hence, $I_R = I_0 \cos^2\left(\frac{\phi}{2}\right)$ ...(i)

Now, when the glass plate is introduced, path difference between the waves at the position of central maxima will become

$$\Delta x = (\mu - 1)t = (1.5 - 1)1.5 \times 10^{-6} = 7.5 \times 10^{-7} \text{ m} \quad ...(ii)$$

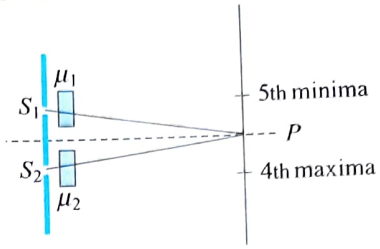
$$\therefore \quad \phi = \frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{5 \times 10^{-7}} \times 7.5 \times 10^{-7} = 3\pi$$

So, intensity at central maxima will now be

$$I_R = I_0 \cos^2\left(\frac{3\pi}{2}\right) = I_0 \times 0 = 0$$

### ILLUSTRATION 3.40

In Young's experiment, the upper slit is covered by a thin glass plate of refractive index 1.4 while the lower slit is covered by another glass plate, having the same thickness as the first one but having refractive index 1.7. Interference pattern is observed using light of wavelength 5400 Å.

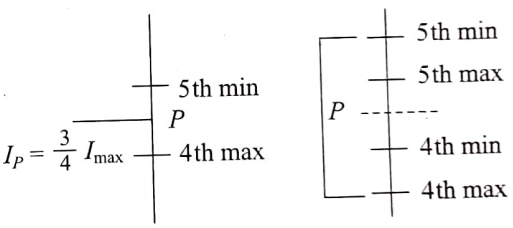



It is found that point $P$ on the screen where the central maximum ($n$ = 0) fell before the glass plates were inserted now has 3/4 the original intensity. It is further observed that what used to be the fourth maximum earlier, lies below point $P$ while the fifth minimum lies above $P$. Calculate the thickness of the glass plate. (Absorption of light by glass plate may be neglected.)

**Sol.** At point $P$ (central maxima), originally it has maximum intensity. If the intensity coming out of each slit is $I_0$, the intensity at point $P$ should be $4I_0$.

$$I = 2I_0(1 + \cos\phi) \quad \Rightarrow \quad \frac{3}{4}(4I_0) = 2I_0(1 + \cos\phi)$$

$$\cos\phi = \frac{1}{2} \quad \Rightarrow \quad \phi = 2n\pi \pm \frac{\pi}{3} \quad (n \text{ is integer})$$




Path difference at $P$,

$$4\lambda \le \Delta x \le \left(\frac{2 \times 5 - 1}{2}\right)\lambda$$

$$\Rightarrow \quad 4\lambda \le \Delta x \le \frac{9}{2}\lambda \quad \Rightarrow \quad 4\lambda \le \Delta x \le 4.5\lambda \quad ...(i)$$

The relation between phase difference and path difference is

$$\phi = \frac{2\pi}{\lambda}\Delta x \quad \Rightarrow \quad \Delta x = n\lambda \pm \frac{\lambda}{6} \quad ...(ii)$$

$$\Delta x = \lambda\left(4 \pm \frac{1}{6}\right) \quad ...(iii)$$

From Eqs. (ii) and (iii), $n$ = 4. Hence,

$$\Delta x = \lambda\left(4 + \frac{1}{6}\right) = \frac{25}{6}\lambda \Rightarrow \Delta x = t(\mu - 1)$$

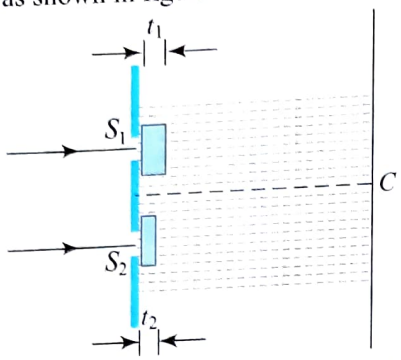
$$\Rightarrow \quad t = \frac{25\lambda}{6(\mu_2 - \mu_1)} = \frac{25 \times 5400 \times 10}{6 \times 0.3 \times 10^{-1}} = 7500 \times 10^{-9} \text{ m}$$

$$= 7.5 \text{ μm}$$

## ILLUSTRATION 3.41

A screen is at a distance $D = 80$ cm from a diaphragm having two narrow slits $S_1$ and $S_2$ which are $d = 2$ mm apart.

Slit $S_1$ is covered by a transparent sheet of thickness $t_1 = 2.5$ μm and slit $S_2$ is covered by another sheet of thickness $t_2 = 1.25$ μm as shown in figure.




Both sheets are made of same material having refractive index $\mu = 1.40$.

Water is filled in the space between diaphragm and screen. A monodichromatic light beam of wavelength $\lambda = 5000$ Å is incident normally on the diaphragm.

Assuming intensity of beam to be uniform, calculate ratio of intensity of $C$ to maximum intensity of interference pattern obtained on the screen ($\mu_w = 4/3$).

**Sol.** Path difference at $C$,

$$\Delta x = t_1(\mu - 1) - t_2(\mu - 1)$$
$$= \mu(t_1 - t_2) - (t_1 - t_2)$$
$$= (t_1 - t_2)(\mu - 1)$$
$$= (2.5 - 1.25)\left(\frac{1.4 \times 3}{4 \times 10} - 1\right) = 1.25 \times \frac{2}{40} = \frac{2.5}{400}$$

$$\Rightarrow \quad \Delta x = \frac{1}{16}\ \mu\text{m}$$

Corresponding phase difference,

$$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi \times 4}{5000 \times 3 \times 10^{-10}} \times \frac{1}{16} \times 10^{-6} = \frac{\pi}{3}$$

$$\Rightarrow \quad I_{max} = 4I_0$$

Intensity at $C$, $I_C = 2I_0(1 + \cos\frac{\pi}{3}) = 3I_0$

$$\therefore \quad \text{Required ratio} = \frac{I_C}{I_{max}} = \frac{3}{4}$$

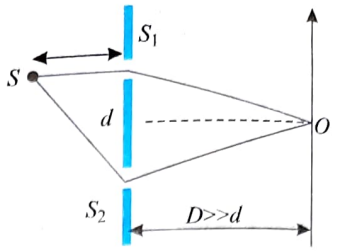
## CONCEPT APPLICATION EXERCISE 3.4

**1.** In a YDSE with two identical slits, when the upper slit is covered with a thin, perfectly transparent sheet of mica, the intensity at the centre of screen reduces to 75% of the initial value. Second minima is observed to be above this point and third maxima below it. Which of the following cannot be a possible value of phase difference caused by the mica sheet?

(a) $\frac{\pi}{3}$ (b) $\frac{13\pi}{3}$

(c) $\frac{17\pi}{3}$ (d) $\frac{11\pi}{3}$

**2.** To make the central fringe at the centre $O$, a mica sheet of refractive index 1.5 is introduced. Choose the correct statement(s).




(a) The thickness of sheet is $2(\sqrt{2}-1)d$ infront of $S_1$

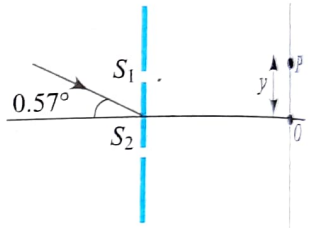
(b) The thickness of sheet is $(\sqrt{2}-1)d$ infront of $S_2$

(c) The thickness of sheet is $2\sqrt{2}d$ infront of $S_1$

(d) The thickness of sheet is $(2\sqrt{2}-1)d$ infront of $S_1$

**3.** White light is used in a YDSE with $D = 1$ m and $d = 0.9$ mm. Light reaching the screen at position $y = 1$ mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.

**4.** In YDSE with $D = 1$ m, $d = 1$ mm, light of wavelength 500 nm is incident at an angle of 0.57° w.r.t. the axis of symmetry of the experimental set-up. If center of symmetry of screen is $O$ as shown (figure),

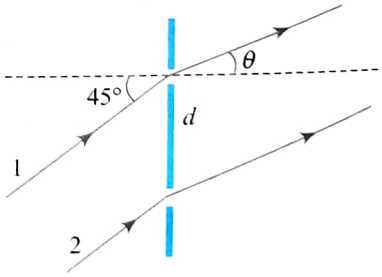



(a) Find the position of central maxima.

(b) Gind intensity at point $O$ in terms of intensity of central maxima $I_0$.

(c) Find number of maxima lying between $O$ and the central maxima.

**5.** Distance between the slits shown in figure is $d = 20\lambda$, where $\lambda$ is the wavelength of light used. Find the angle $\theta$ where




(a) central maxima (where path difference is zero) is obtained, and

(b) third-order maxima is obtained.

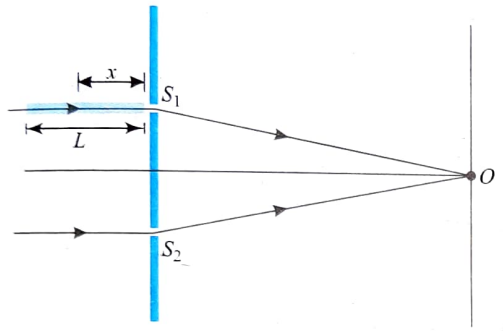
**6.** Monochromatic light of wavelength of 600 nm is used in YDSE. One of the slits is covered by a transparent sheet of thickness $1.8 \times 10^{-5}$ m made of a material of refractive index 1.6. How many fringes will shift due to the introduction of the sheet?

**7.** Interference fringes are produced by a double slit arrangement and a piece of plane parallel glass of refractive index 1.5 is interposed in one of the interfering beams. If the fringes are displaced through 30 fringe widths for light of wavelength $6 \times 10^{-5}$ cm, find the thickness of the plate.

8. In YDSE with $d = 1$ mm and $D = 1$ m, slabs of ($t = 1$ μm, $\mu = 3$) and ($t = 0.5$ μm, $\mu = 2$) are introduced in front of upper and lower slits, respectively. Find the shift in the fringe pattern.

9. In Young's double slit experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen, when a mica sheet of refractive index 1.6 and thickness 2 micron is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment.

10. Light incident on the slit $S_1$ passes through a medium of variable refractive index $\mu = 1 + ax$ (where '$x$' is the distance from the plane of slits as shown, up to a distance '$L$' before falling on $S_1$). If at '$O$' a minima is formed, find the minimum value of '$a$'




11. Find the minimum thickness of a mica sheet having $\mu = \frac{3}{2}$ which should be placed in front of one of the slits in YDSE is required to reduce the intensity at the centre of screen to half of maximum intensity .

## ANSWERS

1. (a) **2.** (a) **3.** 600 nm and 360 nm

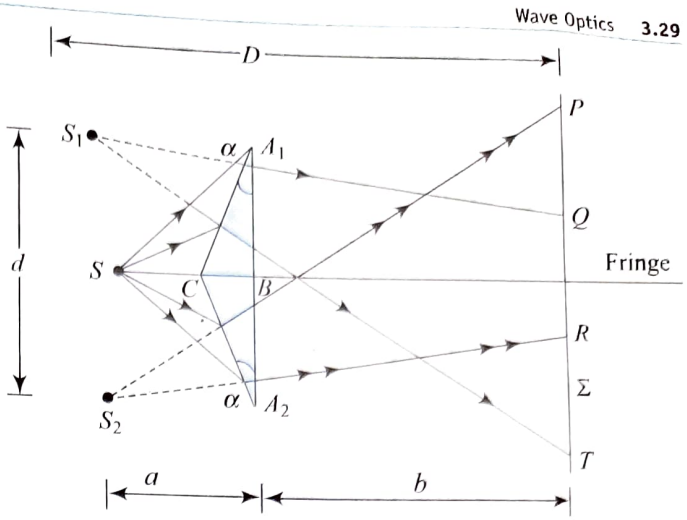
4. (a) −1 cm (b) $I_0$ (c) 19 **5.** (a) 45° (b) $\sin^{-1}\left(\frac{1}{\sqrt{2}} + \frac{3}{20}\right)$

6. 18 **7.** $3.6 \times 10^{-3}$ cm **8.** 1.5 mm upward

9. 6000 Å **10.** $\frac{\lambda}{L^2}$ **11.** $5\lambda/6$

# FRESNEL'S BIPRISM

Figure shows a schematic diagram of Fresnel's biprism and interference of light using it. Two thin prisms $A_1BC$ and $A_2BC$ are joined at the base to form a biprism. The refracting angles $A_1$ and $A_2$ (denoted by $\alpha$ in the figure) are of the order of half a degree each. In fact, it is a simple prism whose base angles are extremely small. A narrow slit $S$, allowing monochromatic light, is placed parallel to the refracting edge $C$, the light going through the prisms $A_1BC$ and $A_2BC$. A screen $\Sigma$ is placed to intercept the transmitted light. Interference fringes are formed on the portion $QR$ of the screen where the two cones overlap.

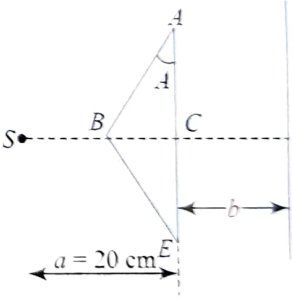



One can treat the points $S_1$ and $S_2$ as two coherent sources sending light to the screen. The arrangement is then equivalent to Young's double-slit experiment with $S_1$ and $S_2$ acting as the two slits. Suppose the separation between $S_1$ and $S_2$ is $d$ and the separation between the planes of $S_1S_2$ and $\Sigma$ is $D$ then, the fringe width obtained on the screen is

$$\beta = \frac{D\lambda}{d}$$

### ILLUSTRATION 3.42

Two small angled transparent prisms (each of refracting angle $A = 1°$) are so placed that their bases coincide, so that common base is $BC$. This device is called Fresnel's biprism and is used to obtain coherent sources of a point source $S$ illuminated by monochromatic light of wavelength 6000 Å placed at a distance $a$ = 20 cm. Calculate the separation between coherent sources. If a screen is placed at a distance $b$ = 80 cm from the device, what is the fringe width of fringes obtained (Refractive index of material of each prism = 1.5).




**Sol.** The virtual images of slit $S$ are formed at $S_1$ and $S_2$ by means of prism $ABC$ and $BCE$, respectively. These images act as coherent sources. The deviation produced by small-angled prism is

$$\delta = (\mu - 1)A$$

Also $\tan\delta \approx \delta \approx \frac{SS_1}{AS_1} = \frac{d/2}{a}$

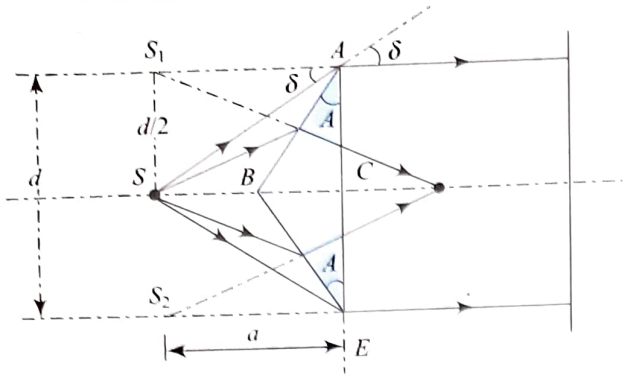
Therefore, separation between coherent sources is

$$d = 2a\delta = 2a(\mu - 1)A$$

Here, $a = 20$ cm $= 0.20$ m

$$A = 1° = \frac{\pi}{180} = \frac{3.14}{180} \text{ rad}$$

$$\therefore \quad d = 2 \times 0.20 \times (1.5 - 1)\frac{3.14}{180} \text{ m} = 3.84 \times 10^{-3} \text{ m}$$




Separation between coherent sources and screen is

$D = a + b = 20 + 80 = 100 \text{ cm} = 1 \text{ m}$

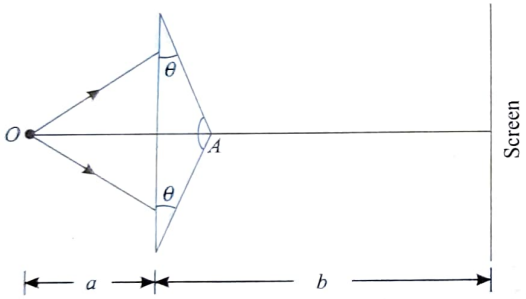
$\lambda = 6000 \text{ Å} = 6000 \times 10^{-10} \text{ cm}$

Therefore, fringe width

$$\beta = \frac{D\lambda}{d} = \frac{1 \times 6000 \times 10^{-10}}{3.48 \times 10^{-3}} = 1.724 \times 10^{-4} \text{ m} = 0.1724 \text{ mm}$$

### ILLUSTRATION 3.43

A point source of light $O$ (behaving as a point object) is placed infront of a prism at a distance '$a$' from it. A screen is placed at a distance $b$ from the source is prism. If the angle $\theta$ is very small compared to the angle $A$ of prism, find the value of $A$ and assuming $n$ = refractive index of prism and $\beta$ = fringe width of the interference pattern formed on the screen.

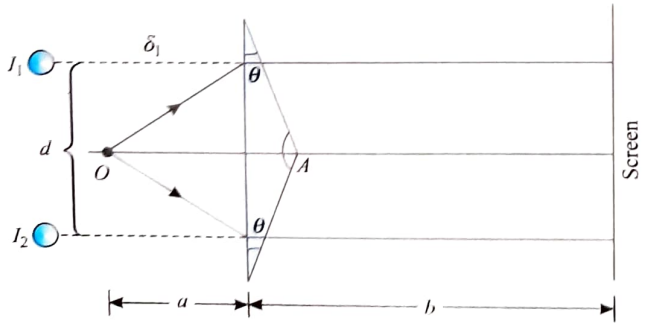



**Sol** The object $O$ has images $I_1$ and $I_2$ due to the refraction at two slant surface of the prism. The deviation δ of the ray is

$$\delta = (n-1)\theta$$

Then, $d = a\delta + a\delta$

or $d = 2a(n-1)\theta$




Hence, the fringe width is $\beta = \frac{\lambda D}{d}$

$$\beta = \lambda \frac{(a+b)}{2a(n-1)\theta}$$

or $$\theta = \frac{\lambda(a+b)}{2a(n-1)\beta}$$

Hence, the angle of prism is $A = \pi - 2\theta = \pi - \frac{\lambda(a-b)}{a(n-1)\beta}$

## LLOYD'S MIRROR EXPERIMENT

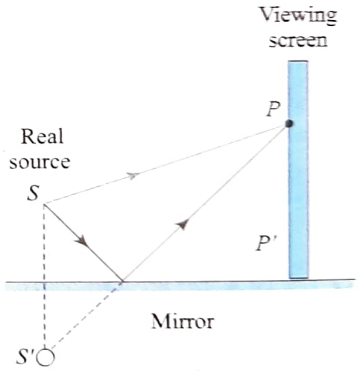
### CHANGE OF PHASE DUE TO REFLECTION

Young's method of producing two coherent light sources involves illuminating a pair of slits with a single source. Another simple, although ingenious, arrangement for producing an interference pattern with a single light source is known as Lloyd's mirror. A light source is placed at point $S$, close to a mirror, as illustrated in figure. Waves can reach the viewing point $P$ either by the direct path $SP$ or by the path involving reflection from the mirror. The reflected ray can be treated as a ray originating at the source $S'$ behind the mirror. Source $S'$, which is the image of $S$, can be considered a virtual source.

At points far from the source, we would expect an interference pattern due to waves from $S$ and $S'$, just as is observed for two real coherent sources. An interference pattern is indeed observed. However, the positions of the dark and bright fringes are reversed relative to the pattern of two real coherent sources (Young's experiment). This is because the coherent sources $S$ and $S'$ differ in phase by 180°. This 180° phase change is produced by reflection.

An interference pattern is produced on a screen at $P$ as a result of the combination of the direct ray and the reflected ray.

The reflected ray undergoes a phase change of 180°.




(a)

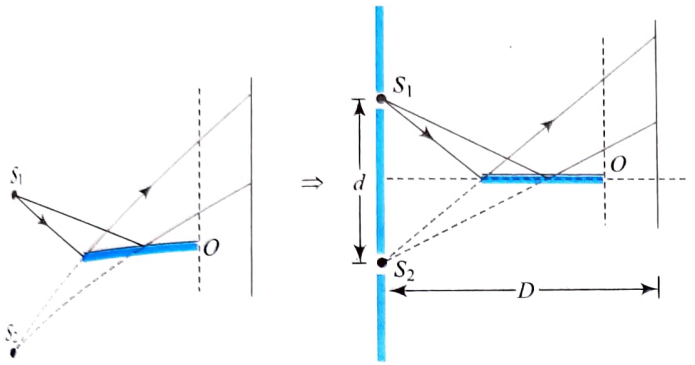
To illustrate this further, consider point $P'$, at which the mirror meets the screen [see Fig. (a)]. This point is equidistant from $S$ and $S'$. If path difference alone were responsible for the phase difference, we would expect to see a bright fringe at $P'$ (because the path difference is zero for this point), corresponding to the central fringe of the two-slit interference pattern. Instead, we observe a 'dark fringe at $P'$ because of the 180° phase change produced by reflection. In general, an electromagnetic wave undergoes a phase change of 180° upon reflection from a medium of higher index of refraction than the one in which it was travelling.

In this arrangement, the light reflected from a long mirror and the light coming directly from the source without reflection produce interference on the screen [see Fig. (b)]. An important feature of this experiment lies in the fact that when the screen is placed in contact with the end of the mirror, the edge $O$ of the reflecting surface comes at the center of a dark fringe instead of a bright

fringe. The direct beam does not suffer any phase change of $\pi$ radian. Hence, at point $P$ on the screen the conditions for minima and maxima are

$S_2P - S_1P = n\lambda,\ n = 0, 1, 2, 3, \ldots$ [minima]

$S_2P - S_1P = \left(n + \frac{1}{2}\right)\lambda$ [maxima]




(b)

## ILLUSTRATION 3.44

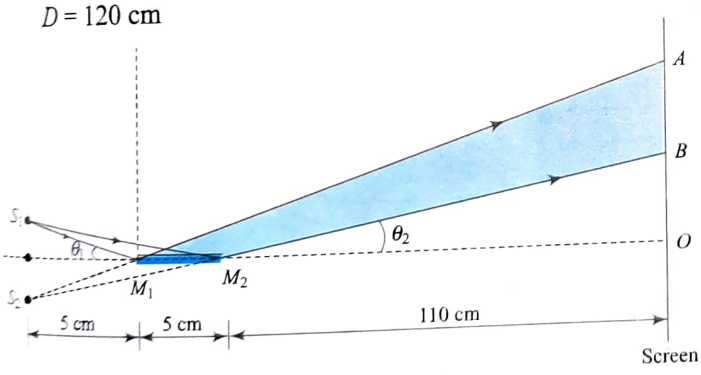
A Lloyd's mirror of length 5 cm is illuminated with monochromatic light of wavelength $\lambda$ (= 6000 Å) from a narrow 1 mm slit in its plane and 5 cm plane from its near edge. Find the fringe width on a screen 120 cm from the slit and width of interference pattern on the screen.

**Sol.** In a plane mirror, image is formed behind it as the object is in front of it. So,

$d = 2$ mm $= 0.2$ cm

$\lambda = 6000$ Å $= 6000 \times 10^{-8}$ cm

$D = 120$ cm




Therefore, fringe width,

$$\beta = \frac{\lambda D}{d} = \frac{6 \times 10^{-5} \times 120}{0.2} = 0.036 \text{ cm}$$

The width of fringe pattern is $AB$. From figure,

$$\tan\theta_1 = \frac{0.1}{5} \quad \text{and} \quad \tan\theta_2 = \frac{0.1}{10}$$

In right-angled triangles $AM_1O$ and $BM_2O$,

$$\tan\theta_1 = \frac{0.1}{5} = \frac{OA}{M_1O} \Rightarrow OA = 115 \times \frac{0.1}{5} \text{ cm},$$

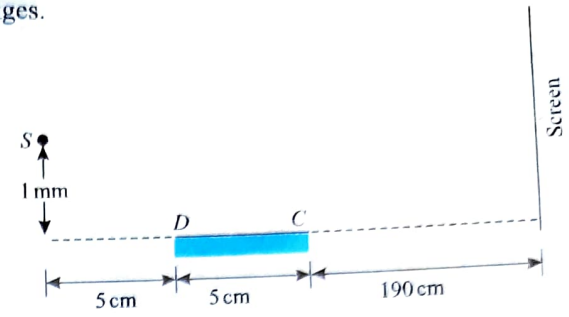
$$\tan\theta_2 = \frac{0.1}{10} = \frac{OB}{OM_2} \Rightarrow OB = 110 \times \frac{0.1}{10} \text{ cm}$$

Hence, width of fringe pattern is

$$OA - OB = \frac{115 \times 0.1}{5} - \frac{110 \times 0.1}{10} = 1.2 \text{ cm}$$

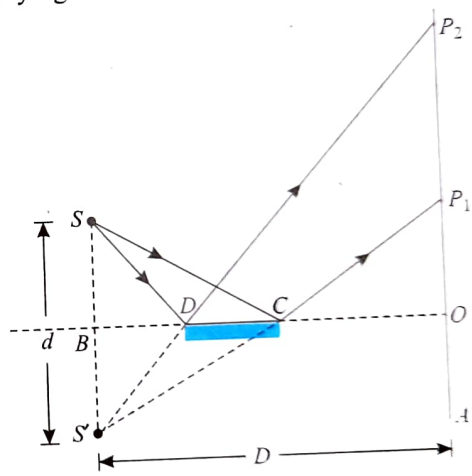
## ILLUSTRATION 3.45

A source of light of wavelength $\lambda = 5000$ Å is kept infront of a screen as shown in the figure. Taking the interference of the direct waves and reflected waves at $C$ and $D$ of the mirror, find **(a)** the region where the fringe will be visible **(b)** number of fringes.




**Sol.**

**(a)** The reflected lights at the ends $C$ and $D$ of the mirror will hit the screen at the points $P_1$ and $P_2$ respectively. The superposition of the reflected rays and direct rays from $S$ will take place between $P_1$ and $P_2$. Then, the number of fringes lying in this region is




$$N = \frac{P_1P_2}{\beta}$$

where $\beta$ = fringe width $= \frac{\lambda D}{d}$

Hence, $N = \frac{(P_1P_2)d}{\lambda D}$ ...(i)

$$P_1P_2 = AP_2 - AP_1 = \frac{(AS')}{BD} \cdot BS' - \frac{(AS')BS'}{BC}$$

$$= \frac{(190 + 5 - 5)(0.1)}{5} - \frac{(190 + 5 - 5)(0.1)}{10}$$

$$= 4 \text{ cm} - 2 \text{ cm} = 2 \text{ cm}$$

**(b)** Putting $P_1P_2 = 2 \times 10^{-2}$ m,

$d = 2$ mm $= 2 \times 10^{-3}$ m,

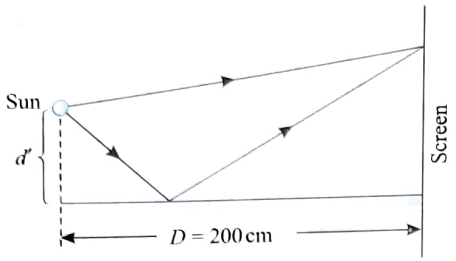
$D = (190 + 5 + 5)$ cm $= 200$ cm $= 2$ m

$\lambda = 5000 \times 10^{-10}$ m in Eq. (i), we have

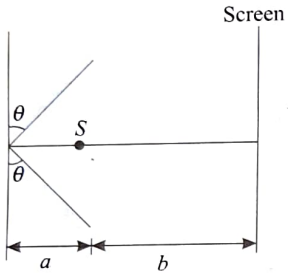
$$N = \frac{(2 \times 10^{-2}\text{ m})(2 \times 10^{-3}\text{ m})}{(5000 \times 10^{-10}\text{ m}) \times 2} = 40$$

## CONCEPT APPLICATION EXERCISE 3.5

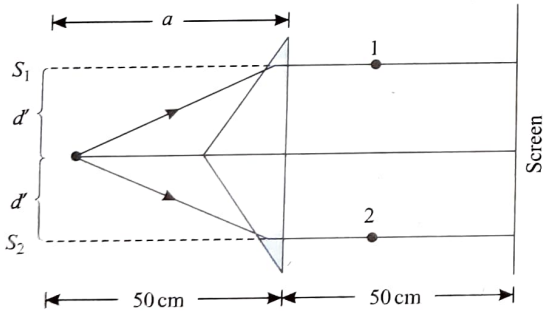
**1.** A source of light of wavelength 5000 Å is placed at one end of a table of 200 cm long and 5 mm above its reflecting surface. Find the fringe width of the interference pattern formed on the screen.



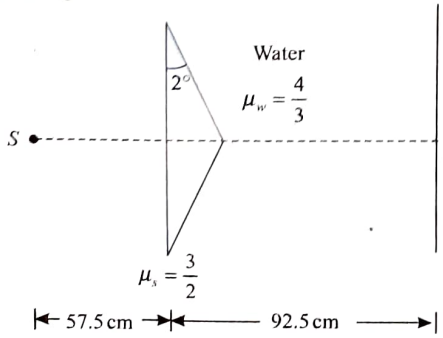

**2.** Two plane mirrors, a source $S$ of light, emitting monochromatic rays of wavelength $\lambda$ and a screen are arranged as shown in figure. If angle $\theta$ is very small, calculate fringe width of the interference pattern formed by reflected rays.



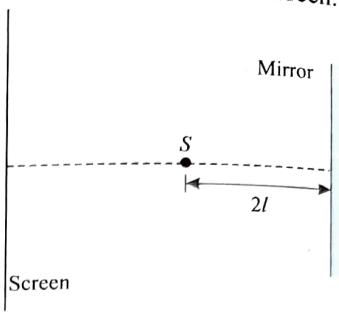

**3.** The distance between the slit and bi-prism and between the prism and screen are 50 cm each. The angle of prism is 179° and its refractive index is 1.5. If the distance between successive fringe is $\beta$, find the wavelength of light.




**4.** The figure shows the modified Fresnel biprism experiment with monochromatic source of wavelength 500 nm. The refractive angle of the glass prism is 2°. Find the number of bright fringes formed.




**5.** Consider interference of direct and reflected light in the situation as shown in the figure. Find the number of minima that will be formed on the screen.



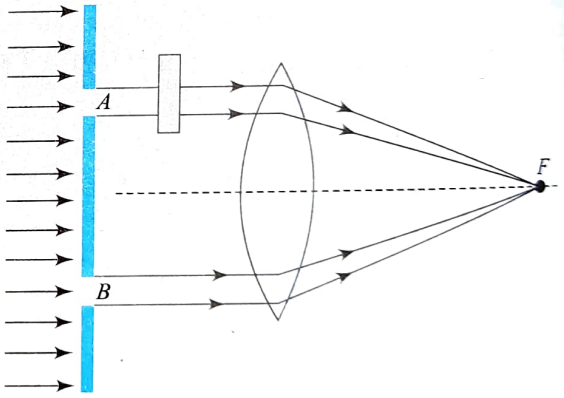

### ANSWERS

**1.** 0.01 cm **2.** $\frac{\lambda(b+2a)}{4a\theta}$ **3.** 5000 Å **4.** 71 **5.** 7

## Solved Examples

### EXAMPLE 3.1

In a modified Young's double-slit experiment, a monochromatic uniform and parallel beam of light of wavelength 6000 Å and intensity $(10/\pi)$ W m$^{-2}$ is incident normally on two circular apertures $A$ and $B$ of radii 0.001 m and 0.002 m, respectively.




A perfectly transparent film of thickness 2000 Å and refractive index 1.5 for the wavelength of 6000 Å is placed in front of aperture $A$ (see figure). Calculate the power (in watt) received at the focal spot $F$ of the lens. The lens is symmetrically placed with respect to the aperture. Assume that 10% of the power received by each aperture goes in the original direction and is brought to the focal spot.

**Sol.** The intensities of sources $A$ and $B$ are

$$I_A = \left(\frac{10}{\pi}\right) \times \pi r_A^2 = 10 \times (0.001)^2 = 10^{-5} \text{ W}$$

$$I_B = \left(\frac{10}{\pi}\right) \times \pi r_B^2 = 10 \times (0.002)^2 = 4 \times 10^{-5} \text{ W}$$

Intensities of sources $A$ and $B$ received along $F$ are

$$I_A = \frac{10}{100}(1 \times 10^{-5}) = 10^{-6} \text{ W}$$

$$I_B = \frac{10}{100}(4\times10^{-5}) = 4\times10^{-6}\ \text{W}$$

Path difference,

$$\Delta = (\mu - 1)t = (1.5 - 1)\times 2000\ \text{Å} = 1000\ \text{Å}$$

Phase difference,

$$\delta = \frac{2\pi}{\lambda}\times\Delta = \frac{2\pi}{6000}\times 1000 = \frac{\pi}{3}$$

Intensity at point $F$ is given by

$$I_F = a_A^2 + a_B^2 + 2a_A a_B\cos\delta$$

$$= I_A + I_B + 2\sqrt{I_A I_B}\cos\delta$$

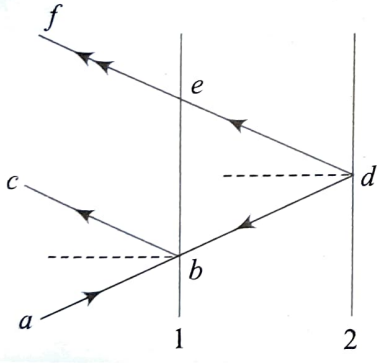
$$= 10^{-6} + 4\times10^{-6} + 2\sqrt{10^{-6}\times4\times10^{-6}}\cos\frac{\pi}{3}$$

$$= 10^{-6} + 4\times10^{-6} + 2\times2\times10^{-6}\times\frac{1}{2}$$

$$= 10^{-6} + 4\times10^{-6} + 2\times10^{-6} = 7\times10^{-6}\ \text{W}$$

## EXAMPLE 3.2

A narrow monochromatic beam of light of intensity $I$ is incident on a glass plate as shown in figure. Another identical glass plate is kept close to the first one and parallel to it. Each glass plate reflects 25% of the light incident on it and transmits the remaining. Find the ratio of minimum and maximum intensities in the interference pattern formed by two beams obtained after one reflection at each plate.




**Sol.** $I$ is the intensity of incident beam $ab$. The interfering waves are $bc$ and $ef$, reflected from surface of plates 1 and 2, respectively.

Reflection coefficient of intensity, $r = 25\% = 0.25$

Transmission coefficient of intensity, $t = 75\% = 0.75$

The intensity of beam $bc$, $I_1 = 0.25\,I = \frac{1}{4}I$

The intensity of beam $bd = 0.75I$

The intensity of beam $de = 0.25\times0.75I$

The intensity of beam $ef$, $I_2 = 0.75\times0.25\times0.75I = \frac{9}{64}I$

Ratio of maximum and minimum intensities,

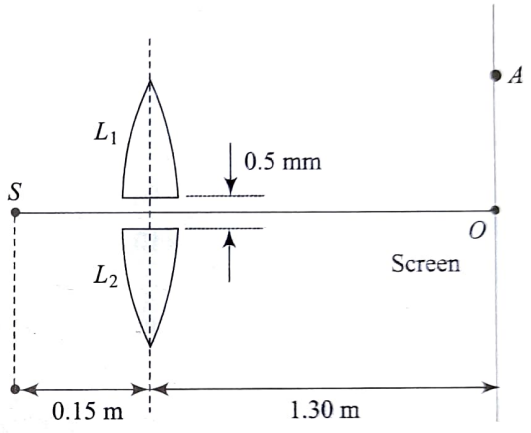
$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{a_1^2 + a_2^2 + 2a_1a_2}{a_1^2 + a_2^2 - 2a_1a_2}$$

$$= \frac{I_1 + I_2 + 2\sqrt{(I_1I_2)}}{I_1 + I_2 - 2\sqrt{(I_1I_2)}} = \frac{\frac{1}{4}I + \frac{9}{64}I + 2\left(\frac{1}{4}I\times\frac{9}{64}I\right)}{\frac{1}{4}I + \frac{9}{64}I - 2\sqrt{\left(\frac{1}{4}I\times\frac{9}{64}I\right)}}$$

$$= \frac{(49/64)I}{(1/64)I} = \frac{49}{1}$$

## EXAMPLE 3.3

In figure, $S$ is a monochromatic point source emitting light of wavelength $\lambda = 500$ nm. A thin lens of circular shape and focal length 0.01 m is cut into two identical halves $L_1$ and $L_2$ by a plane passing through a diameter. The two halves are placed symmetrically about the central axis $SO$ with a gap of 0.5 mm. The distance along the axis from $S$ to $L_1$ and $L_2$ is 0.15 m, while that from $L_1$ and $L_2$ to $O$ is 1.30 m. The screen at $O$ is normal to $SO$.




**(a)** If the third intensity maximum occurs at point $A$ on the screen, find the distance $OA$.

**(b)** If the gap between $L_1$ and $L_2$ is reduced from its original value of 0.5 mm, will the distance $OA$ increase, decrease, or remain the same?

**Sol.**

**(a)** The images of $S$ due to both parts of lenses are formed at $S_1$ and $S_2$ whose distance $v$ from the lens is given by

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Here, $u = -0.15$ m, $f = 0.10$ m.

$$\therefore\quad \frac{1}{0.10} = \frac{1}{v} - \frac{1}{(-0.15)}$$

$$\Rightarrow\quad \frac{1}{v} = \frac{1}{0.10} - \frac{1}{0.15} = \frac{3-2}{0.30} = \frac{1}{0.30}$$

$$\therefore\quad v = 0.30\ \text{m}$$

As the ray passing through optical center passes undeviated, therefore, angle $\theta$ subtended by lens gap and images $S_1$ and $S_2$ must be same. We have,

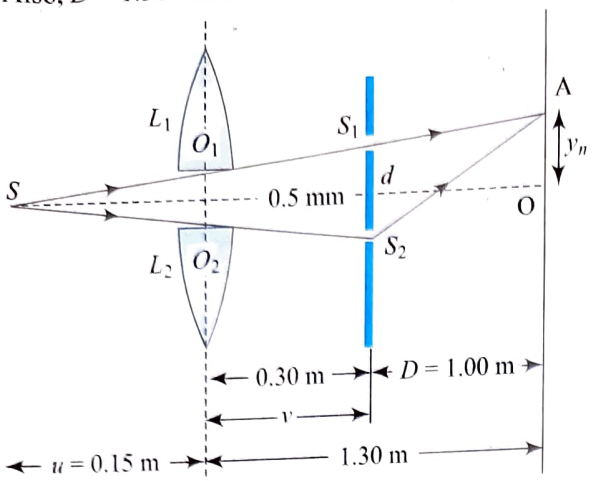
$$\theta = \frac{O_1O_2}{u} = \frac{S_1S_2}{u+v}$$

$$\therefore\quad S_1S_2 = \frac{u+v}{u}O_1O_2 = \frac{0.30 + 0.15}{0.15}\times0.5\ \text{mm}$$

$$= 1.5\ \text{mm}$$

i.e., $S_1S_2 = d = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Also, $D = 1.30 - 0.30 = 1.00 \text{ m}$.




$S_1$ and $S_2$ act as coherent sources and produce interference. The positions of maxima are given by

$$y_n = \frac{nD\lambda}{d}$$

As point $A$ is third maxima,

$$\therefore \quad y_n = OA = \frac{3D\lambda}{d} = \frac{3 \times 1.00 \times 500 \times 10^{-9}}{1.5 \times 10^{-3}}$$

$$= 1000 \times 10^{-6} \text{ m} = 1.0 \text{ mm}$$

**(b)** If gap between $L_1$ and $L_2$ is reduced, then $S_1S_2$ or $d$ decreases and hence $OA$ increases.

## EXAMPLE 3.4

A double-slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å.

**(a)** Calculate the fringe width.

**(b)** One of the slits of the apparatus is covered by a thin glass sheet of refractive index 1.53. Find the smallest thickness of the sheet to bring the adjacent minimum on the axis.

**Sol.** Given $\mu_l = 1.33$, $d = 1 \text{ mm} = 10^{-3} \text{ m}$, $D = 1.33 \text{ m}$, $\lambda = 6300 \text{ Å} = 6.3 \times 10^{-7} \text{ m}$.

When the experiment is performed in liquid, $\lambda$ changes to

$$\lambda' = \frac{\lambda}{\mu_l}$$

**(a)** Fringe width,

$$\beta = \frac{D\lambda'}{d} = \frac{D(\lambda/\mu_l)}{d} = \frac{D\lambda}{\mu_l d}$$

$$= \frac{1.33 \times 6.3 \times 10^{-7}}{1.33 \times 10^{-3}} = 6.3 \times 10^{-4} \text{m} = 0.63 \text{ mm}$$

**(b)** Displacement of fringes,

$$\Delta y = \frac{\beta}{\lambda'}({}^{l}\mu_g - 1)t = \frac{\beta}{\lambda/\mu_l}\left(\frac{\mu_g}{\mu_l} - 1\right)t$$

$$= \frac{\beta}{\lambda}(\mu_g - \mu_l)t \quad \Rightarrow \quad t = \frac{\Delta y \lambda}{\beta.(\mu_g - \mu_l)}$$

But $\Delta y$ = separation between bright and adjacent dark fringe $= \frac{\beta}{2}$

$$\therefore \quad t = \frac{\left(\frac{\beta}{2}\right)\lambda}{\beta(\mu_g - \mu_l)} = \frac{\lambda}{2(\mu_g - \mu_l)}$$

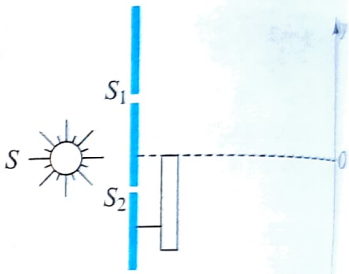
Given $\mu_g = 1.53$, $\mu_l = 1.33$

$$\therefore \quad t = \frac{6.3 \times 10^{-7}}{2(1.53 - 1.33)}$$

$$= \frac{6.3 \times 10^{-7}}{2 \times 0.20} = 1.575 \times 10^{-6} \text{ m} = 1.575 \text{ μm}$$

## EXAMPLE 3.5

The Young's double-slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit $S_2$ is covered by a thin glass sheet of refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in figure.




**(a)** Find the location of central maximum (bright fringe with zero path difference) on the $y$-axis.

**(b)** Find the light intensity of point $O$ relative to the maximum fringe intensity.

**(c)** Now, if 600 nm light is replaced by white light of range 400–700 nm, find the wavelengths of the light that form maxima exactly at point $O$.

(All wavelengths in the problem are for the given medium of refractive index 4/3. Ignore dispersion.)

**Sol.** Given $\lambda = 600 \text{ nm} = 6 \times 10^{-7} \text{ m}$, $d = 0.45 \text{ mm} = 0.45 \times 10^{-3} \text{ m}$

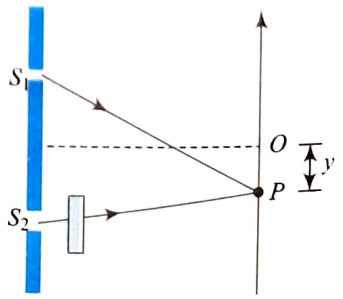
$D = 1.5 \text{ m}$

Thickness of glass sheet, $t = 10.4 \text{ μm} = 10.4 \times 10^{-6} \text{ m}$.

Refractive index of glass sheet, $\mu_g = 1.5$.

**(a)** Let central maximum is obtained at a distance $y$ below point $O$. Then, $\Delta x_1 = S_1P - S_2P = \frac{yd}{D}$

Path difference due to glass sheet, $\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$



Net path difference will be zero when $\Delta x_1 = \Delta x_2$

$$\Rightarrow \frac{yd}{D} = \left(\frac{\mu_g}{\mu_m} - 1\right)t \Rightarrow y = \left(\frac{\mu_g}{\mu_m} - 1\right)t\frac{D}{d}$$

Substituting the values, we have $y = \left(\frac{\mu_g}{\mu_m} - 1\right)t\frac{D}{d}$

or we can say $y = 4.33$ mm.

(b) At $O$, $\Delta x_1 = 0$ and $\Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$

$\therefore$ Net path difference $\Delta x = \Delta x_2$

Corresponding phase difference, $\Delta\phi$ or simply $\phi = \frac{2\pi}{\lambda}\Delta x$.

Substituting the values, we have

$$\phi = \frac{2\pi}{6\times10^{-7}}\left(\frac{1.5}{4/3} - 1\right)(10.4\times10^{-6}) = \left(\frac{13}{3}\right)\pi$$

Now, $I(\phi) = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

$$I = I_{max}\cos^2\left(\frac{13\pi}{6}\right) = \frac{3}{4}I_{max}$$

(c) At $O$, path difference is $\Delta x = \Delta x_2 = \left(\frac{\mu_g}{\mu_m} - 1\right)t$

For maximum intensity at $O$, $\Delta x = n\lambda$ (here $n = 1, 2, 3, ...$)

$\lambda = \frac{\Delta x}{1}, \frac{\Delta x}{2}, \frac{\Delta x}{3}$, ... and so on

$$\Delta x = \left(\frac{1.5}{4/3} - 1\right)(10.4\times10^{-6}\text{ m}) = \left(\frac{1.5}{4/3} - 1\right)(10.4\times10^{3}\text{ nm})$$

$= 1300$ nm

Maximum intensity will be corresponding to

$$\lambda = 1300\text{ nm}, \frac{1300}{2}\text{nm}, \frac{1300}{3}\text{nm}, \frac{1300}{4}\text{nm}, ...$$

$= 1300$ nm, 650 nm, 433.33 nm, 325 nm, ...

The wavelengths in the range 400 to 700 nm are 650 nm and 433.33 nm.

## EXAMPLE 3.6

A glass of refractive index 1.5 is coated with a thin layer of thickness $t$ and refractive index 1.8. Light of wavelength $\lambda$ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and two reflected rays interfere. Write the condition for their constructive interference. If $\lambda = 648$ nm, obtain the least value of $t$ for which the rays interfere constructively.

**Sol.** Path difference between rays reflected from upper and lower faces of layer $= 2\mu t$ (for normal incidence)

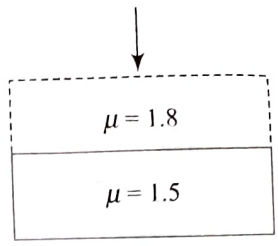
Path difference due to reflection from denser (upper) surface is $\lambda/2$.

Condition of constructive interference (or maxima) is

$$2\mu t + \frac{\lambda}{2} = n\lambda$$

$$t = \frac{1}{2\mu}(2n-1)\frac{\lambda}{2}$$

$$= \frac{(2n-1)\lambda}{4\mu},\ n = 1, 2, 3, ...$$

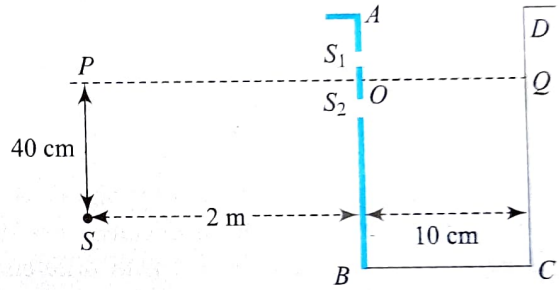



For least thickness, $n = 1$.

$$\therefore \quad t_{min} = \frac{\lambda}{4\mu} = \frac{648}{4\times1.8}\text{ nm} = 90\text{ nm}$$

## EXAMPLE 3.7

A vessel $ABCD$ of 10 cm width has two small slits $S_1$ and $S_2$ sealed with identical glass plates of equal thickness. The distance between the slits is 0.8 mm. $POQ$ is the line perpendicular to the plane $AB$ and passing through $O$, the middle point of $S_1$ and $S_2$. A monochromatic light source is kept at $S$, 40 cm below $P$ and 2 m from the vessel, to illuminate the slits as shown in figure. Calculate the position of the central bright fringe on the other wall $CD$ with respect to line $OQ$. Now, a liquid is poured into the vessel and filled up to $OQ$. The central bright fringe is found to be at $Q$. Calculate the refractive index of the liquid.

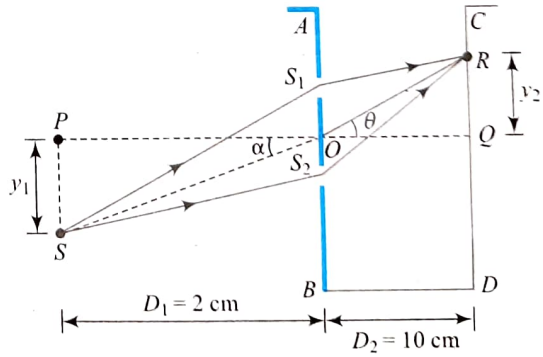



**Sol.** Given $y_1 = 40$ cm, $D_1 = 2$ m $= 200$ cm, $D_2 = 10$ cm.

$$\tan\alpha = \frac{y_1}{D_1} = \frac{40}{200} = \frac{1}{5}$$

$$\alpha = \tan^{-1}\left(\frac{1}{5}\right)$$

$$\sin\alpha = \frac{1}{\sqrt{26}} \approx \frac{1}{5} = \tan\alpha \text{ (see figure)}$$




Path difference between $SS_1$ and $SS_2$ is

$$\Delta x_1 = d\sin\alpha = (0.8\text{ mm})\left(\frac{1}{5}\right) = 0.16\text{ mm} \quad ...(i$$

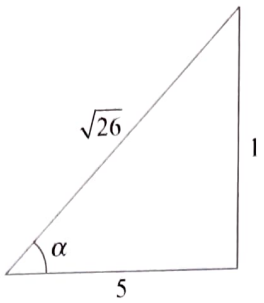
Now, let at point $R$ on the screen, central bright fringe is observe
(i.e., net path difference = 0).

Path difference between $S_2R$ and $S_1R$ would be

$$\Delta x_2 = S_2R - S_1R = d\sin\theta \quad ...(ii$$

Central bright fringe will be observed when net path difference is zero.

$$\Delta x_2 - \Delta x_1 = 0$$
$$\Delta x_2 = \Delta x_1$$
$$d \sin\theta = 0.16$$
$$(0.8)\sin\theta = 0.16$$
$$\sin\theta = \frac{0.16}{0.8} = \frac{1}{5}$$
$$\tan\theta = \frac{1}{\sqrt{24}} \approx \sin\theta = \frac{1}{5}$$




Hence, $\tan\theta = \frac{y_2}{D_2} = \frac{1}{5}$

$$y_2 = \frac{D_2}{5} = \frac{10}{5} = 2 \text{ cm}$$

Therefore, central bright fringe is observed at 2 cm above point $O$ on side $CD$.

**Alternative solution:**

$\Delta x$ at $R$ will be zero if $\Delta x_1 = \Delta x_2$

$$d\sin\alpha = d\sin\theta$$
$$\alpha = \theta$$
$$\tan\alpha = \tan\theta$$
$$\frac{y_1}{D_1} = \frac{y_2}{D_2}$$
$$y_2 = \frac{D_2}{D_1} y_1 = \left(\frac{10}{200}\right)(40) \text{ cm}$$
$$y_2 = 2 \text{ cm}$$

The central bright fringe will be observed at point $Q$. If the path difference created by the liquid slab of thickness $t$ = 10 cm or 100 mm is equal to $\Delta x$, so that the net path difference at $Q$ becomes zero, then $(\mu - 1)t = \Delta x_1$

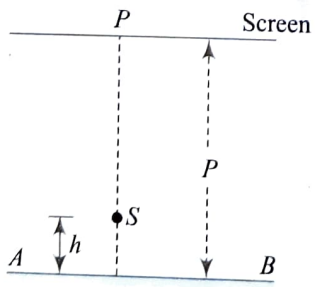
$\Rightarrow$ $(\mu - 1)(100) = 0.16$

$\Rightarrow$ $\mu - 1 = 0.0016$

$\Rightarrow$ $\mu = 1.0016$

## EXAMPLE 3.8

A point source $S$ emitting light of wavelength 600 nm is placed at a very small height $h$ above a flat reflecting surface $AB$ (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance $D$ from it.




**(a)** What is the shape of the interference fringes on the screen?

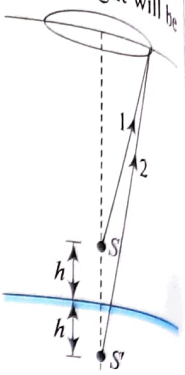
**(b)** Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near point $P$ (shown in the figure).

**(c)** If the intensity at point $P$ corresponds to a maximum, calculate the minimum distance through which the reflecting surface $AB$ should be shifted so that the intensity at $P$ again becomes maximum.

**Sol.**

**(a)** The path difference between the rays 1 and 2 coming from $S$ and $S'$ will be equal on the circular path on the screen, hence fringes will be circular.

**(b)** Let the intensity of light coming from S be $I_1 = I$, then as per the problem, the velocity of the reflected light will be $I_2 = 0.36\, I$.




As we know $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

Hence, $\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

$$= \frac{(\sqrt{I} + \sqrt{0.36\, I})^2}{(\sqrt{I} - \sqrt{0.36\, I})^2} = \frac{1}{16}$$

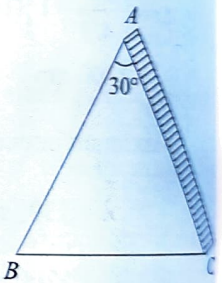
**(c)** To have the next maximum at $P$, the path difference between the interfering waves must change by $\lambda$. If $AB$ is moved by a distance $x$, it will cause an additional path difference $2x$.

$$2x = \lambda \text{ (For minimum value of } x)$$

$$\Rightarrow \quad x = \frac{\lambda}{2} = 300 \text{ mm}$$

## EXAMPLE 3.9

A prism of refracting angle 30° is coated with a thin film of transparent material of refractive index 2.2 on face $AC$ of the prism. A light ray of wavelength 6600 Å is incident on face $AB$ such that angle of incidence is 60°.




**(a)** Find the angle of emergence.

**(b)** Find the minimum value of thickness of the coated film on the face $AC$ for which the light emerging from the face has maximum intensity. (Given refractive index of the material of the prism is $\sqrt{3}$.)

**Sol.**

**(a)** Given, $i_1 = 60°$, $A = 30°$, $\mu_{prism} = \sqrt{3}$.

From Snell's law, $\sin i_1 = \mu \sin r_1$

$$\Rightarrow \quad \sin r_1 = \frac{1}{\mu}\sin i_1 = \frac{1}{\sqrt{3}} \times \sin 60° = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{2}$$

$$\Rightarrow \quad r_1 = 30°$$

Also, $r_1 + r_2 = A \quad \Rightarrow \quad r_2 = A - r_1 = 30° - 30° = 0°$

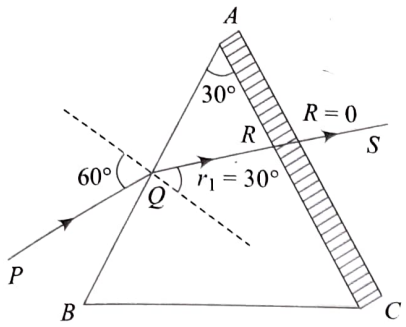
That is, the ray is incident normally on face $AC$ and angle of emergence $i_2 = 0°$.

**(b)** For maximum intensity, the thickness of coated film must satisfy the condition of maxima in transmitted light, i.e.,

$$2\mu t \cos r = (2n - 1)\,\lambda/2 \Rightarrow t = \frac{(2n-1)\lambda}{4\mu \cos r}$$

For normal incidence, $r = 0$. So, $\cos r = 1$ and for minimum thickness, $n = 1$.

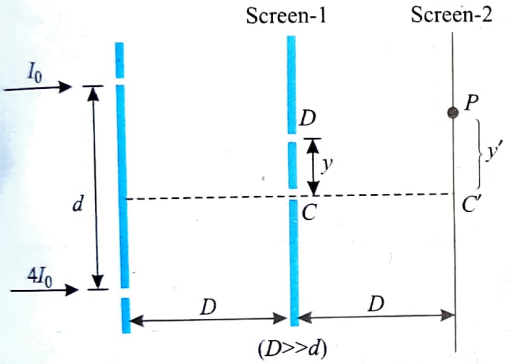
$$\therefore \quad t_{min} = \frac{\lambda}{4\mu} = \frac{6600\text{Å}}{4 \times 2.2} = 750\text{ Å}$$




## EXAMPLE 3.10

Two light sources of intensity $I_0$ and $4I_0$ produce interference pattern on the screen. Two screens are placed parallel to each other as shown in the figure. In screen-1 two small holes are made at a distance $y$ ($y = \beta/2$), where $\beta$ = fringe width = $\frac{D\lambda}{d}$.

Find the **(a)** resulting intensity at $D$ **(b)** ratio of the maximum to minimum intensities on the screen-2.




**Sol.**

(a) The path difference between the rays at $C = \Delta x = 0$

Then, the phase difference $= \phi = \frac{2\pi}{\lambda}\Delta x = 0$

Hence, the resulting intensity at $C$ is

$$I_C = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$I_C = I_0 + 4I_0 + 2\sqrt{I_0(4I_0)}\cos 0° = 9I_0$$

Similarly, at $D$, the path difference between two waves is

$$\Delta x' = d\sin\theta \simeq d\tan\theta = \frac{dy}{D} = \frac{d}{D}\left(\frac{D\lambda}{2d}\right) = \frac{\lambda}{2}$$

Then, the phase difference at $D$, $\phi = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$

Hence, the resulting intensity at $D$ is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos = I_0 + 4I_0 + 2\sqrt{I_0(4I_0)}\cos\pi = I_0$$

(b) Now, the slits $C$ and $D$ acts as two different monochromatic sources of light of intensities $I_C = 9I_0$ and $I_D = I_0$ respectively. The light emitted from the slits will get superimposed on the screen. The maximum intensity of light on the screen is

$$I_{max} = (\sqrt{I_C} + \sqrt{I_D})^2 = (\sqrt{9I_0} + \sqrt{I_0})^2 = 16I_0$$

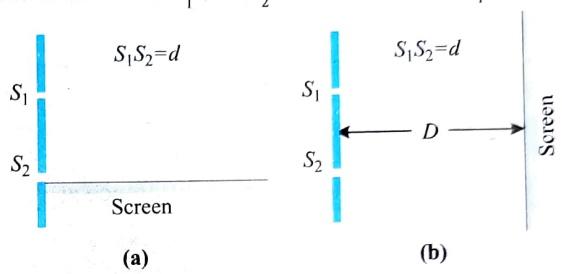
The maximum intensity of light on the screen is

$$I_{min} = (\sqrt{I_C} - \sqrt{I_D})^2 = (\sqrt{9I_0} - \sqrt{I_0})^2 = 4I_0$$

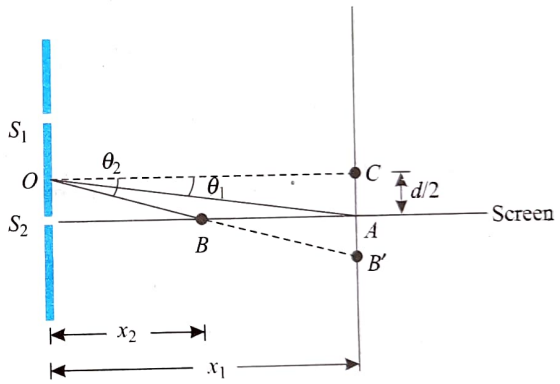
Hence, the ratio of $I_{max}$ and $I_{min}$ is, $\frac{I_{max}}{I_{min}} = \frac{16I_0}{4I_0} = 4:1$

## EXAMPLE 3.11

The interference pattern of a Young's double slit experiment is observed in two ways by placing the screen as shown in Figs. (a) and (b). The distance between two consecutive right most minima on the screen of Fig. (a) using light of wavelength $\lambda_1 = 3000$ Å is observed to be 600 times the fringe width in the screen of Fig. (b) using the wavelength $\lambda_2 = 8000$ Å. If $D$ (as shown in figure) is 2 m then find the separation between the coherent sources $S_1$ and $S_2$. Given that $d > 3\lambda_1/2$.




**Sol.** $A$ and $B'$ would have been the positions of first and second minima (last two), had the screen be perpendicular to $S_2A$.




Since, the angular positions of minima do not depend on the position of the screen. Therefore, second minima is formed at $B$ on the screen.

For right most minima at $A$, $d\sin\theta_1 = \frac{\lambda_1}{2}$ ...(i)

For small angles, $\sin\theta_1 \approx \tan\theta_1 = \frac{d/2}{x_1}$

Substituting in Eq.(i), we get $x_1 = \frac{d^2}{\lambda_1}$ ...(ii)

For next minima at $B$, $d\sin\theta_2 = \frac{3}{2}\lambda_1$ ...(iii)

For small angles, $\sin\theta_2 \approx \tan\theta_2 = \frac{d/2}{x_2}$

Substituting in Eq. (iii), we have $x_2 = \frac{d^2}{3\lambda_1}$ ...(iv)

$$\therefore \quad AB = x_1 - x_2 = \frac{2d^2}{3\lambda_1} \quad \text{...(v)}$$

In the second case, fringe width $\beta = \frac{\lambda_2 D}{d}$

Given that, $AB = 300\beta$

$$\therefore \quad \frac{2d^2}{3\lambda_1} = 300\frac{\lambda_2 D}{d}$$

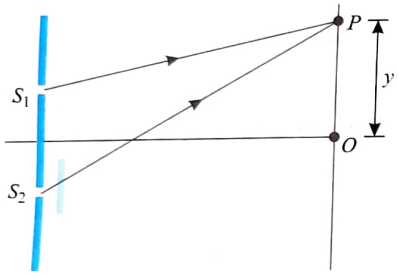
$$\therefore \quad d^3 = 450\lambda_1\lambda_2 D = 450\times3000\times8000\times10^{-20}\times2 = 216\times10^{-12}$$

$$\therefore \quad d = 6\times10^{-4}\text{ m} = 0.6\text{ mm}$$

## EXAMPLE 3.12

In Young's double slit experiment set-up with light of wavelength $\lambda = 6000$ Å, distance between the two slits is 2 mm and distance between the plane of slits and the screen is 2 m. The slits are of equal intensity. When a sheet of glass of refractive index 1.5 (which permits only a fraction $\eta$ of the incident light to pass through) and thickness 8000 Å is placed infront of the lower slit, it is observed that the intensity at a point $P$, 0.15, mm above the central maxima does not change. Find the value of $\eta$.

**Sol.** Without inserting the slab, path difference at $P$,




$$\Delta = \frac{yd}{} = \frac{0.15\times10^{-3}\times2\times10^{-3}}{} = 1.5\times10^{-7}\text{ m}$$

Corresponding phase difference at $P$, $\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x)$

$$= \left(\frac{2\pi}{6000\times10^{-10}}\right)(1.5\times10^{-7}) = \frac{\pi}{2}$$

$$\therefore \quad \frac{\phi}{2} = \frac{\pi}{4}$$

Intensity at $P$, $I = 4I_0\cos^2\frac{\phi}{2} = 2I_0$

Phase difference after placing the glass sheet,

$$\phi' = \phi + \frac{2\pi}{\lambda}(\mu - 1)t$$

$$= \frac{\pi}{2} + \frac{2\pi}{6000+10^{-10}}(1.5-1)(8000\times10^{-10}) = \frac{11\pi}{6}$$

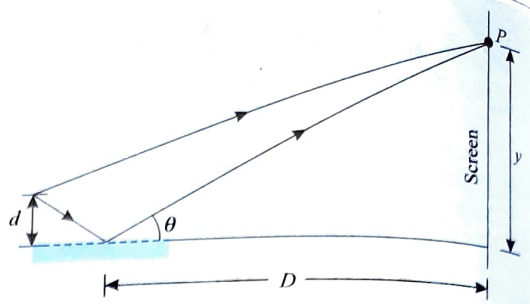
The intensity at $P$ is now,

$$I' = I_0 + \eta I_0 + 2\sqrt{\eta I_0^2}\cos\frac{11\pi}{6} = 2I_0 \text{ (given)}$$

Solving this equation, we get $\eta = 0.21$.

## EXAMPLE 3.13

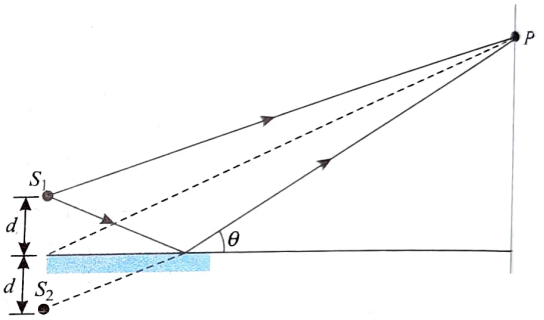
A point monochromatic source of light of wavelength $\lambda$ is put above a plane mirror at a distance $d$. The intensity of light is $I_0$. If half of the light is reflected, find the:




**(a)** intensity of light at $P$. ($y << d$ and $D >> d$); $I = f(\theta)$
**(b)** intensity $I$ at $P$ as the function of $y$
**(c)** position of $n$th maxima
**(d)** fringe width
**(e)** intensity at the center $C$.

**Sol.**

**(a)** Once direct wave goes with intensity $I_0$. The reflected wave has intensity $I_0/2$. Both the waves superimpose at $P$. The resulting intensity is

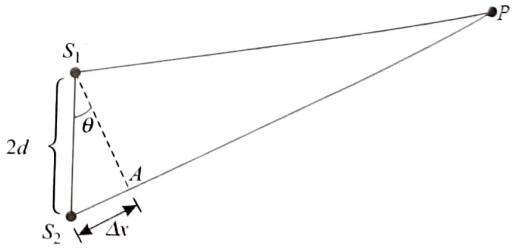



$$I = I_1 + I_2 + 2I_1I_2\cos\phi = I_0 + \frac{I_0}{2} + 2\cdot I_0\cdot\frac{I_0}{2}\cos\phi$$

$$I = I_0\left(\frac{3}{2} + I_0\cos\phi\right) \quad \text{...(i)}$$

The phase difference $\phi$ is given as $\phi = \frac{2\pi}{\lambda}\Delta x + \pi$

when $\Delta x$ = path difference = extra phase difference $\pi$ arises due to reflection of light and at the mirror.




Then $\phi = \frac{4\pi d}{\lambda}\sin\theta + \pi$ ...(ii)

By using Eqs. (i) and (ii),

$$I = I_0\left[\frac{3}{2} + I_0\cos\left(\frac{4\pi d}{\lambda}\sin\theta\right)\right]$$

**(b)** Putting $\sin\theta \simeq \tan\theta = \frac{y}{D}$

$$I = I_0\left[\frac{3}{2} - I_0\cos\left(\frac{4\pi dy}{\lambda D}\right)\right]$$

(c) For maxima, $\cos\frac{4\pi dy}{\lambda D} = -1$

or $\frac{4\pi dy}{\lambda D} = (2n+1)\pi$

or $y = \left(\frac{2n+1}{4}\right)\frac{\lambda D}{d}$

(d) The fringe width is $\beta = y_n - y_{n-1}$

$$= \frac{\lambda D}{4d}[(2n+1) - \{2(n-1)+1\}] = \frac{\lambda D}{2d}$$

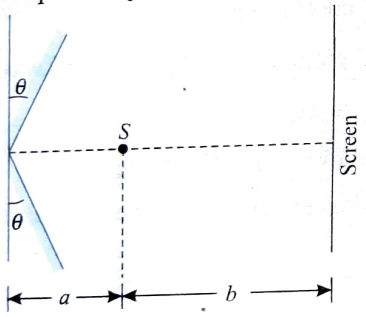
(e) At the center, put $\theta = 0$ to obtain $\phi = \pi$

Then, the minima takes place at the center.

Hence, $I_{\text{center}} = \frac{I_0}{2}$

## EXAMPLE 3.14

Two plane mirrors are inclined with each other in front of a points sources of light '$S$', emitting wavelengths of $\lambda_1 = 2000$ Å and $\lambda_2 = 6000$ Å and a screen are arranged as shown in figure. The angle $\theta$ shown is 0.05 radian and distance $a$ and $b$ are 1 cm and 48 cm respectively.

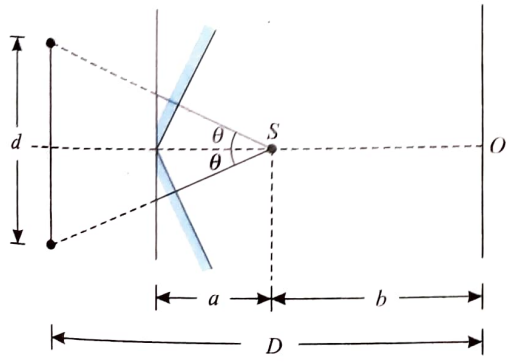



(a) Find the fringe width of interference pattern formed on screen by the blue light.

(b) Calculate the distance of first black line from central bright fringe.

(c) Find the separation between two black lines which are nearest to the central bright fringe.

**Sol.** Interference is due to reflected waves, therefore, images $S_1$ and $S_2$ of source $S$ behave like two coherent sources.




Distance of $S$ from each mirror $= a\cos\theta$

$\therefore \quad SS_1 = SS_2 = 2a\cos\theta$

$\therefore \quad S_1S_2 = SS_1\sin\theta + SS_2\sin\theta = 4a\cos\theta\sin\theta$

Since $\theta$ is small

$\cos\theta = 1$ and $\sin\theta \simeq \theta$

$d = 4a\theta$

Distance $RS = SS_1\cos\theta = 2a\cos^2\theta \simeq 2a$

$\therefore \quad D = RO = RS + SO = (2a + b)$

$\therefore$ Fridge width $\beta = \frac{D\lambda}{d} = \frac{(2a+b)\lambda}{4a\theta}$

**(a)** For wavelength $\lambda_1 = 4000$ Å, $a = 1$ cm, $b = 48$ cm

$$\therefore \quad \beta_1 = \frac{2\times1+48}{4\times1\times0.05}\times2000\times10^{-10}\text{ m} = 50\,\mu\text{m}$$

**(b)** Fringe width for light of wavelength $\lambda_2 = 6000$ Å is

$$\beta_2 = \frac{2\times1+48}{4\times1\times0.05}\times6000\times10^{-10}\text{ m} = 150\,\mu\text{m}$$

A black line is formed at the position where dark fringes of both the wavelengths will coincide.

Let the first black line be at a distance $y$ from the central fringe. Let this be the position of $m^{\text{th}}$ dark fringe of 2000 Å and $n^{\text{th}}$ dark fringe of 6000 Å light.

$$\therefore \quad y = \left(m - \frac{1}{2}\right)\beta_1 = \left(n - \frac{1}{2}\right)\beta_2$$

$$\Rightarrow \quad \frac{2m-1}{2n-1} = \frac{\beta_2}{\beta_1} = \frac{150}{50} = \frac{3}{1}$$

For first black line, $y$ should be minimum possible which corresponds to least possible integral values of $m$ and $n$.

$$\therefore \quad \frac{2m-1}{2n-1} = \frac{3}{1}$$

or $\quad m = 2, n = 1$

$\therefore$ Position of first black line $y = \left(m - \frac{1}{2}\right)\beta_1 = 25\,\mu\text{m}$

**(c)** Since the interference pattern is symmetric about the central fringe, there will be one dark line at a distance $y$ above it and the other will be $y$ below it.

Hence the separation between two black lines which are nearest to the central bright fringe, $2y = 560\ \mu\text{m}$.

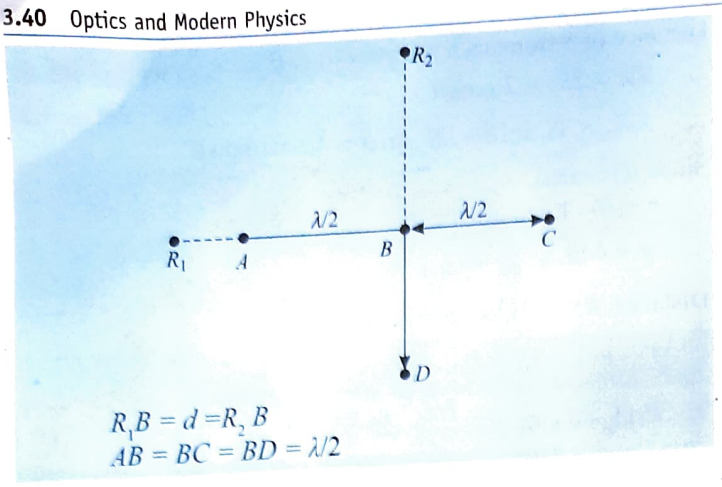
## EXAMPLE 3.15

Four identical monochromatic sources $A$, $B$, $C$, $D$ as shown in the (figure) produce waves of the same wavelength $\lambda$ and are coherent. Two receiver $R_1$ and $R_2$ are at great but equal distances from $B$.

**(a)** Which of the two receivers picks up the larger signal?

**(b)** Which of the two receivers picks up the larger signal when $B$ is turned off?

**(c)** Which of the two receivers picks up the larger signal when $D$ is turned off?




**Sol.**

**(a)** Let us consider the disturbances at the receiver $R_1$, which is at a distance $d$ from $B$.

Let the equation of wave at $R_1$, because of $A$ be

$$y_A = a\cos\omega t \quad \ldots(i)$$

The path difference of the signal from $A$ with that from $B$ is $\lambda/2$ and hence, the phase difference

$$\Delta\phi = \frac{2\pi}{\lambda}\times(\text{path difference}) = \frac{2\pi}{\lambda}\times\frac{\lambda}{2} = \pi$$

Thus, the wave equation at $R_1$, because of $B$ is

$$y_B = a\cos(\omega t - \pi) = -a\cos\omega t \quad \ldots(ii)$$

The path difference of the signal from $C$ with that from $A$ is $\lambda$ and hence the phase difference

$$\Delta\phi = \frac{2\pi}{\lambda}\times(\text{path difference}) = \frac{2\pi}{\lambda}\times\lambda = 2\pi$$

Thus, the wave equation at $R_1$, because of $C$ is

$$y_C = a\cos\omega t = a\cos(\omega t - 2\pi) = a\cos\omega t \quad \ldots(iii)$$

The path difference between the signal from D with that of $A$ is

$$\Delta x_{R_1} = \sqrt{d^2 + \left(\frac{\lambda}{2}\right)^2} - \left(d - \frac{\lambda}{2}\right) = d\left(1 + \frac{\lambda^2}{4d^2}\right)^{1/2} - d + \frac{\lambda}{2}$$

$$= d\left(1 + \frac{\lambda^2}{8d^2}\right) - d + \frac{\lambda}{2} \approx \frac{\lambda}{2} \quad (\because d >> \lambda)$$

Therefore, phase difference is $\pi$.

$$\therefore \quad y_D = a\cos(\omega t - \pi) = -a\cos\omega t \quad \ldots(iii)$$

The resultant signal picked up at $R_1$, from all the four sources is the summation of all four waves,

$$y_{R_1} = y_A + y_B + y_C + y_D$$

$$y_{R_1} = a\cos\omega t - a\cos\omega t + a\cos\omega t - a\cos\omega t = 0$$

Thus, the signal picked up at $R_1$ is zero.

Now let us consider the resultant signal received at $R_2$.

Let the equation of wave at $R_2$, because of $B$ be

$$y_B = a_1\cos\omega t \quad \ldots(iv)$$

The path difference of the signal from $D$ with that from $B$ is $\lambda/2$ and hence, the phase difference

$$\Delta\phi = \frac{2\pi}{\lambda}\times(\text{path difference}) = \frac{2\pi}{\lambda}\times\frac{\lambda}{2} = \pi$$

Thus, the wave equation at $R_2$, because of $D$ is

$$y_B = a_1\cos(\omega t - \pi) = -a_1\cos\omega t \quad \ldots(v)$$

The path difference between signal at $A$ and that at $B$ is

$$\Delta x_{R_2} = \sqrt{(d)^2 + \left(\frac{\lambda}{2}\right)^2} - d = d\left(1 + \frac{\lambda^2}{4d^2}\right)^{1/2} - d \simeq \frac{\lambda^2}{8d^2}$$

As $d >> \lambda$, therefore this path differences $\Delta x_{R_2} \to 0$ and phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\times(\text{path difference}) = \frac{2\pi}{\lambda}\times 0 \approx 0 \text{ (very small)}$$

$$= \phi(\text{say})$$

Hence, $y_B = a_1\cos(\omega t - \phi)$

The resultant signal picked up at $R_2$, from all the four sources is the summation of all four waves, $y_{R_2} = y_A + y_B + y_C + y_D$

$$y_{R_2} = a_1\cos\omega t - a_1\cos\omega t + a_1\cos(\omega t - \phi) + a_1\cos(\omega t - \phi)$$

$$= 2a_1\cos(\omega t - \phi)$$

$\therefore$ Signal picked up by $R_2$ is $y_{R_2} = 2a_1\cos(\omega t - \phi)$

$$\therefore \quad |y_{R_2}|^2 = 4a_1^2\cos^2(\omega t - \phi) \Rightarrow \langle I_{R_2}\rangle = 2a_1^2$$

Thus, $R_2$ picks up the larger signal.

**(b)** If $B$ is switched off, $R_1$, picks up, $y = a\cos\omega t$

$$\therefore \quad \langle I_{R_1}\rangle = a^2 < \cos^2\omega t >= \frac{a^2}{2}$$

$R_2$ picks up, $y = a\cos\omega t$

$$\therefore \quad \langle I_{R_2}\rangle = a^2 < \cos^2\omega t >= \frac{a^2}{2}$$

Thus, $R_1$, and $R_2$ pick up the same signal

**(c)** If $D$ is switched off.

$R_1$ picks up $y = a\cos\omega t$

$$\therefore \quad \langle I_{R_1}\rangle = a^2 < \cos^2\omega t >= \frac{a^2}{2}$$

$R_2$ picks up, $y = 3a\cos\omega t$

$$\therefore \quad \langle I_{R_2}\rangle = 9a^2 < \cos^2\omega t >= \frac{9a^2}{2}$$

Thus, $R_2$ picks up larger signal compared to $R_1$.

# Exercises

## Single Correct Answer Type

1. In Young's double-slit experiment, the slits are illuminated by monochromatic light. The entire set-up is immersed in pure water. Which of the following act cannot restore the original fringe width?
   (1) Bringing the slits close together.
   (2) Moving the screen away from the slit plane.
   (3) Replacing the incident light by that of longer wavelength.
   (4) Introducing a thin transparent slab in front of one of the slits.

2. The maximum intensity in Young's double-slit experiment is $I_0$. Distance between the slits is $d = 5\lambda$, where $\lambda$ is the wavelength of monochromatic light used in the experiment. What will be the intensity of light in front of one of the slits on a screen at a distance $D = 10d$?
   (1) $\frac{I_0}{2}$ (2) $\frac{3}{4}I_0$
   (3) $I_0$ (4) $\frac{I_0}{4}$

3. A light ray of frequency $\nu$ and wavelength $\lambda$ enters a liquid of refractive index 3/2. The ray travels in the liquid with
   (1) frequency $\nu$ and wavelength $\left(\frac{2}{3}\right)\lambda$
   (2) frequency $\nu$ and wavelength $\left(\frac{3}{2}\right)\lambda$
   (3) frequency $\nu$ and wavelength $\lambda$
   (4) frequency $\left(\frac{3}{2}\right)\nu$ and wavelength $\lambda$

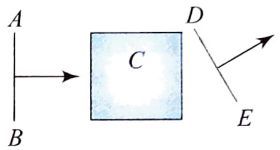
4. In a double-slit experiment, instead of taking slits of equal width, one slit is made twice as wide as the other. Then in the interference pattern
   (1) the intensities of both the maxima and the minima increase
   (2) the intensity of the maxima increases and the minima has zero intensity
   (3) the intensity of the maxima decreases and that of the minima increases
   (4) the intensity of the maxima decreases and the minima has zero intensity

5. In Young's double-slit interference experiment, if the slit separation is made threefold, the fringe width becomes
   (1) sixfold (2) threefold
   (3) 3/6-fold (4) 1/3-fold

6. Light of wavelength $\lambda = 5890$ Å falls on a double-slit arrangement having separation $d = 0.2$ mm. A thin lens of focal length $f = 1$ m is placed near the slits. The linear separation of fringes on a screen placed in the focal plane of the lens is
   (1) 3 mm (2) 4 mm
   (3) 2 mm (4) 1 mm

7. One of the two slits in YDSE is painted over, so that it transmits only light waves having intensity half of the intensity of the light waves through the other slit. As a result of this
   (1) fringe pattern disappears
   (2) bright fringes become brighten and dark ones become darker
   (3) dark and bright fringes get fainter
   (4) dark fringes get brighter and bright fringes get darker

8. A wave front $AB$ passing through a system $C$ emerges as $DE$. The system $C$ could be




   (1) a slit (2) a biprism
   (3) a prism (4) a glass slab

9. In Young's interference experiment, if the slits are of unequal width, then
   (1) no fringes will be formed
   (2) the positions of minimum intensity will not be completely dark
   (3) bright fringe as displaced from the original central position
   (4) distance between two consecutive dark fringes will not be equal to the distance between two consecutive bright fringes

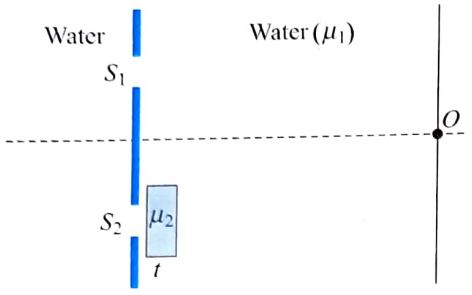
10. In Young's interference experiment, the central bright fringe can be identified due to the fact that it
    (1) has greater intensity than other fringes which are bright
    (2) is wider than the other bright fringes
    (3) is narrower than the other bright fringes
    (4) can be obtained by using white light instead of monochromatic light

11. In a double-slit experiment, instead of taking slits of equal width, one slit is made twice as wide as the other. Then, in the interference pattern
    (1) the intensities of both the maxima and the minima increases
    (2) the intensity of the maximum increase and minima has zero intensity
    (3) the intensity of the maxima decreases and that of minima increases
    (4) the intensity of the maxima decreases and the minima has zero intensity

12. In a two-slit experiment with white light, a white fringe is observed on a screen kept behind the slits. When the screen in moved away by 0.05 m, this white fringe

(1) does not move at all
(2) gets displaced from its earlier position
(3) becomes colored
(4) disappears

**13.** Young's double-slit experiment is conducted in water ($\mu_1$) as shown in figure, and a glass plate of thickness $t$ and refractive index $\mu_2$ is placed in the path of $S_2$. Find the magnitude of the optical path difference at '$O$'.

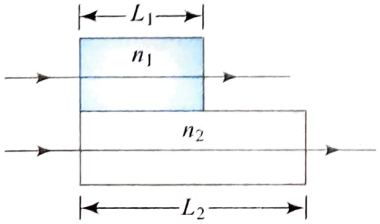



(1) $\left|\left(\frac{\mu_2}{\mu_1}-1\right)t\right|$ (2) $\left|\left(\frac{\mu_1}{\mu_2}-1\right)t\right|$
(3) $|(\mu_2-\mu_1)t|$ (4) $|(\mu_2-1)t|$

**14.** In Young's double-slit experiment, the wavelength of light was changed from 7000 Å to 3500 Å. While doubling the separation between the slits, which of the following is not true for this experiment?
(1) The width of fringes changes.
(2) The color of bright fringes changes.
(3) The separation between successive bright fringes changes.
(4) The separation between successive dark fringes remains unchanged.

**15.** Two beams of light having intensities $I$ and $4I$ interfere to produce a fringe pattern on a screen. The phase between the beams is $\pi/2$ at point $A$ and $\pi$ at point $B$. Then, the difference between the resultant intensities at $A$ and $B$ is
(1) $2I$ (2) $4I$
(3) $5I$ (4) $7I$

**16.** Two waves of light in air have the same wavelength and are initially in phase. They then travel through plastic layers with thicknesses of $L_1 = 3.5$ mm and $L_2 = 5.0$ mm and indices of refraction $n_1 = 1.7$ and $n_2 = 1.25$ as shown in figure. The rays later arrive at a common point. The longest wavelength of light for which constructive interference occurs at the point is

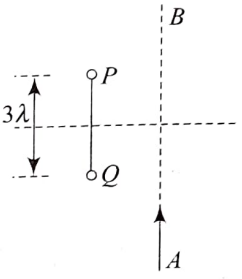



(1) 0.8 μm (2) 1.2 μm
(3) 1.7 μm (4) 2.9 μm

**17.** In YDSE, $D = 1$ m, $d = 1$ mm, and $\lambda = 5000$ nm. The distance of 100th maxima from the central maxima is
(1) $\frac{1}{2}$m (2) $\frac{\sqrt{3}}{2}$m
(3) $\frac{1}{\sqrt{3}}$m (4) does not exist

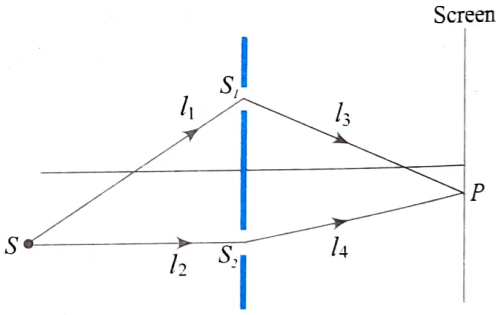
**18.** The path difference between two interfering waves at a point on the screen is $\lambda/6$. The ratio of intensity at this point and that at the central bright fringe will be (assume that intensity due to each slit is same)
(1) 0.853 (2) 8.53
(3) 0.75 (4) 7.5

**19.** Two coherent light sources, each of wavelength $\lambda$, are separated by a distance $3\lambda$. The maximum number of minima formed on line $AB$, which runs from $-\infty$ to $+\infty$, is




(1) 2 (2) 4
(3) 6 (4) 8

**20.** The YDSE apparatus is as shown in figure. The condition for point $P$ to be a dark fringe is



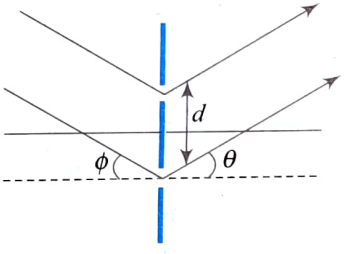

($\lambda$ = wavelength of light waves)
(1) $(l_1 - l_3) + (l_2 - l_4) = n\lambda$
(2) $(l_1 - l_2) + (l_3 - l_4) = \frac{(2n-1)\lambda}{2}$
(3) $(l_1 + l_3) + (l_2 + l_4) = \frac{(2n-1)\lambda}{2}$
(4) $(l_1 - l_2) + (l_4 - l_3) = \frac{(2n-1)\lambda}{2}$

**21.** Two wavelengths of light $\lambda_1$ and $\lambda_2$ are sent through Young's double-slit apparatus simultaneously. If the third-order bright fringe coincides with the fourth-order bright fringe, then
(1) $\frac{\lambda_1}{\lambda_2} = \frac{4}{3}$ (2) $\frac{\lambda_1}{\lambda_2} = \frac{3}{4}$
(3) $\frac{\lambda_1}{\lambda_2} = \frac{5}{4}$ (4) $\frac{\lambda_1}{\lambda_2} = \frac{4}{5}$

22. Microwaves from a transmitter are directed normally toward a plane reflector. A detector moves along the normal to the reflector. Between positions of 14 successive maxima the detector travels a distance 0.14 m. The frequency of the transmitter is ($c = 3 \times 10^8$ m s$^{-1}$)
(1) $1.5 \times 10^{10}$ Hz (2) $10^{10}$ Hz
(3) $3 \times 10^{10}$ Hz (4) $6 \times 10^{10}$ Hz

23. In a double-slit experiment, two parallel slits are illuminated first by light of wavelength 400 nm and then by light of unknown wavelength. The fourth-order dark fringe resulting from the known wavelength of light falls in the same place on the screen as the second-order bright fringe from the unknown wavelength. The value of unknown wavelength of the light is
(1) 900 nm (2) 700 nm
(3) 300 nm (4) none of these

24. Light is incident at an angle $\phi$ with the normal to a plane containing two slits of separation $d$. Select the expression that correctly describes the positions of the interference maxima in terms of the incoming angle $\phi$ and outgoing angle $\theta$.
(1) $\sin\phi + \sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{d}$
(2) $d \sin\theta = m\lambda$
(3) $\sin\phi - \sin\theta = (m+1)\frac{\lambda}{d}$
(4) $\sin\phi + \sin\theta = m\frac{\lambda}{d}$




25. Blue light of wavelength 480 nm is most strongly reflected off a thin film of oil on a glass slab when viewed near normal incidence. Assuming that the index of refraction of the oil is 1.2 and that of the glass is 1.6, what is the minimum thickness of the oil film (other than zero)?
(1) 100 nm (2) 200 nm
(3) 300 nm (4) none

26. In a YDSE with identical slits, the intensity of the central bright fringe is $I_0$. If one of the slits is covered, the intensity at the same point is
(1) $2I_0$ (2) $I_0$
(3) $I_0/2$ (4) $I_0/4$

27. Two identical coherent sources are placed on a diameter of a circle of radius $R$ at separation $x$ ($<< R$) symmetrical about the center of the circle. The sources emit identical wavelength $\lambda$ each. The number of points on the circle of maximum intensity is ($x = 5\lambda$)
(1) 20 (2) 22
(3) 24 (4) 26

28. In Young's double-slit experiment $d/D = 10^{-4}$ ($d$ = distance between slits, $D$ = distance of screen from the slits). At point $P$ on the screen, resulting intensity is equal to the intensity due to the individual slit $I_0$. Then, the distance of point $P$ from the central maximum is ($\lambda = 6000$ Å)
(1) 2 mm (2) 1 mm
(3) 0.5 mm (4) 4 mm

29. A thin film of refractive index 1.5 and thickness $4 \times 10^{-5}$ cm is illuminated by light normal to the surface. What wavelength within the visible spectrum will be intensified in the reflected beam?
(1) 4800 Å (2) 5800 Å
(3) 6000 Å (4) 6800 Å

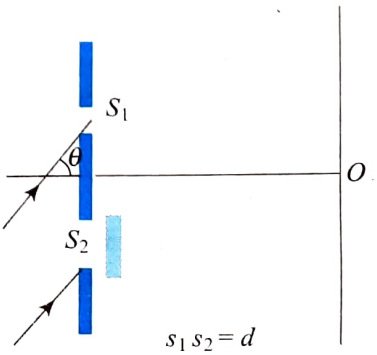
30. A plane wave of monochromatic light falls normally on a uniform thin film of oil which covers a glass plate. The wavelength of source can be varied continuously. Complete destructive interference is observed for $\lambda = 5000$ Å and $\lambda = 1000$ Å and for no other wavelength in between. If $\mu$ of oil is 1.3 and that of glass is 1.5, the thickness of the film will be
(1) $6.738 \times 10^{-5}$ cm (2) $5.7 \times 10^{-5}$ cm
(3) $4 \times 10^{-5}$ cm (4) $2.8 \times 10^{-5}$ cm

31. Two light waves having the same wavelength $\lambda$ in vacuum are in phase initially. Then the first ray travels a path of length $L_1$ through a medium of refractive index $\mu_1$. The second ray travels a path of length $L_2$ through a medium of refractive index $\mu_2$. The two waves are then combined to observe interference effects. The phase difference between the two, when they interfere, is
(1) $\frac{2\pi}{\lambda}(L_1 - L_2)$ (2) $\frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$
(3) $\frac{2\pi}{\lambda}(\mu_2 L_1 - \mu_1 L_2)$ (4) $\frac{2\pi}{\lambda}\left[\frac{L_1}{\mu_1} - \frac{L_2}{\mu_2}\right]$

32. In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude $A$ and of wavelength $\lambda$. In another experiment with the same setup, the two slits are sources of equal amplitude $A$ and wavelength $\lambda$, but are incoherent. The ratio of intensity of light at the mid-point of the screen in the first case to that in the second case is
(1) 1:1 (2) 1:2
(3) 2:1 (4) 4:1

33. In Young's double-slit experiment, the $y$-coordinates of central maxima and 10th maxima are 2 cm and 5 cm, respectively. When the YDSE apparatus is immersed in a liquid of refractive index 1.5, the corresponding $y$-coordinates will be
(1) 2 cm, 7.5 cm (2) 3 cm, 6 cm
(3) 2 cm, 4 cm (4) 4/3 cm, 103 cm

34. A monochromatic beam of light falls on YDSE apparatus at some angle (say $\theta$) as shown in figure. A thin sheet of glass is inserted in front of the lower slit $S_2$. The central bright fringe (path difference = 0) will be obtained
(1) at $O$
(2) above $O$
(3) below $O$
(4) anywhere depending on angle $\theta$, thickness of plate $t$, and refractive index of glass $\mu$



**35.** A plate of thickness $t$ made of a material of refractive index $\mu$ is placed in front of one of the slits in a double-slit experiment. What should be the minimum thickness $t$ which will make the intensity at the center of the fringe pattern zero?

(1) $(\mu-1)\frac{\lambda}{2}$ (2) $(\mu-1)\lambda$

(3) $\frac{\lambda}{2(\mu-1)}$ (4) $\frac{\lambda}{(\mu-1)}$

**36.** In Young's double-slit experiment, how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringe ($\lambda = 2000$ Å and $d = 7000$ Å)?

(1) 12 (2) 7

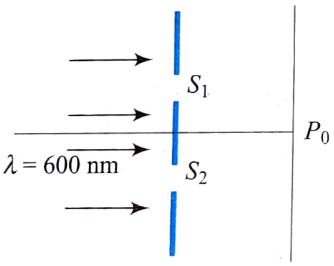
(3) 18 (4) 4

**37.** Young's double-slit experiment is made in a liquid. The 10th bright fringe in liquid lies where 6th dark fringe lies in vacuum. The refractive index of the liquid is approximately

(1) 1.8 (2) 1.54

(3) 1.67 (4) 1.2

**38.** In a standard Young's double-slit experiment with coherent light of wavelength 600 nm, the fringe width of the fringes in the central region (near the central fringe, $P_0$) is observed to be 3 mm. An extremely thin glass plate is introduced in front of the first slit, and the fringes are observed to be displaced by 11 mm. Another thin plate is placed before the second slit and it is observed that the fringes are now displaced by an additional 12 mm. If the additional optical path lengths introduced are $\Delta_1$ and $\Delta_2$, then




(1) $11\Delta_1 = 12\Delta_2$ (2) $12\Delta_1 = 11\Delta_2$

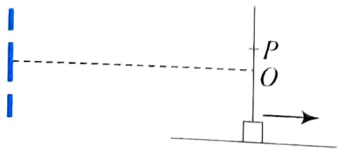
(3) $11\Delta_1 > 12\Delta_2$ (4) none of these

**39.** In Young's double-slit experiment, the separation between two coherent sources $S_1$ and $S_2$ is $d$ and the distance between the source and screen is $D$. In the interference pattern, it is found that exactly in front of one slit, there occurs a minimum. Then the possible wavelengths used in the experiment are

(1) $\lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D}$ (2) $\lambda = \frac{d^2}{D}, \frac{d^2}{5D}, \frac{d^2}{9D}$

(3) $\lambda = \frac{d^2}{D}, \frac{d^2}{2D}, \frac{d^2}{3D}$ (4) $\lambda = \frac{d^2}{3D}, \frac{d^2}{7D}, \frac{d^2}{11D}$

**40.** In Young's double-slit experiment, the first maxima is observed at a fixed point $P$ on the screen. Now, the screen is continuously moved away from the plane of slits. The ratio of intensity at point $P$ to the intensity at point $O$ (centre of the screen)




(1) remains constant

(2) keeps on decreasing

(3) first decreases and then increases

(4) first decreases and then becomes constant

**41.** A parallel beam of white light is incident on a thin film of air of uniform thickness. Wavelengths 7200 Å and 5400 Å are observed to be missing from the spectrum of reflected light viewed normally. The other wavelength in the visible region missing in the reflected spectrum is

(1) 6000 Å (2) 4320 Å

(3) 5500 Å (4) 6500 Å

**42.** In Young's double-slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, the number of fringes observed in the same segment of the screen is given by

(1) 12 (2) 18

(3) 24 (4) 30

**43.** In the ideal double-slit experiment, when a glass plate (refractive index 1.5) of thickness $t$ is introduced in the path of one of the interfering beams (wavelength $\lambda$), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass plate is

(1) $2\lambda$ (2) $2\lambda/3$

(3) $\lambda/3$ (4) $\lambda$

**44.** A double-slit arrangement produces interference fringes for sodium light ($\lambda = 589$ nm) that have an angular separation of $3.50 \times 0^{-3}$ rad. For what wavelength would the angular separation be 10% greater?

(1) 527 nm (2) 648 nm

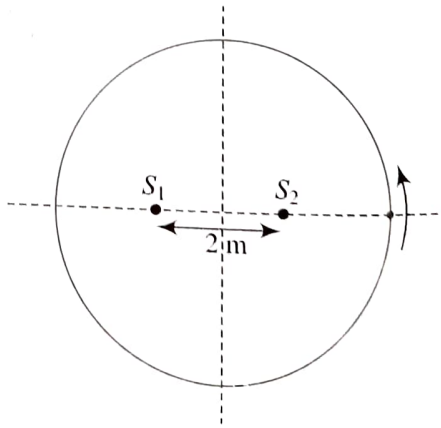
(3) 722 nm (4) 449 nm

**45.** In Young's double-slit experiment, the intensity of light at a point on the screen, where the path difference is $\lambda$, is $I$. The intensity of light at a point where the path difference becomes $\lambda/3$ is

(1) $\frac{I}{4}$ (2) $\frac{I}{3}$

(3) $\frac{I}{2}$ (4) $I$

**46.** Two point sources separated by 2.0 m are radiating in phase with $\lambda = 0.50$ m. A detector moves in a circular path around the two sources in a plane containing them. How many maxima are detected?

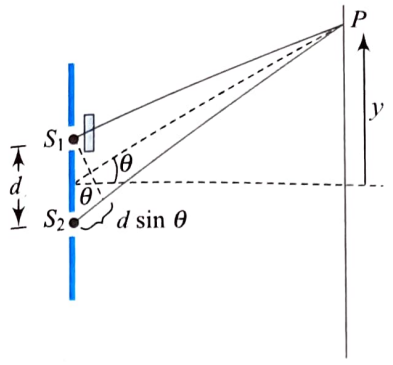



(1) 16 (2) 20

(3) 24 (4) 32

**47.** In YDSE, light of wavelength $\lambda = 5000$ Å is used, which emerges in phase from two slits a distance $d = 3 \times 10^{-7}$ m

apart. A transparent sheet of thickness $t = 1.5 \times 10^{-7}$ m, refractive index $n = 1.17$, is placed over one of the slits. Where does the central maxima of the interference now appear from the center of the screen? (Find the value of $y$.)




(1) $\frac{D(\mu-1)t}{2d}$ (2) $\frac{2D(\mu-1)t}{d}$

(3) $\frac{D(\mu+1)t}{d}$ (4) $\frac{D(\mu-1)t}{d}$

48. A double-slit experiment is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6830 Å. Then the fringe width is

(1) $6.3 \times 10^{-4}$ m (2) $8.3 \times 10^{-4}$ m

(3) $6.3 \times 10^{-2}$ m (4) $6.3 \times 10^{-5}$ m

49. In Young's double-slit experiment, the slit separation is 0.5 mm and the screen is 0.5 m away from the slit. For a monochromatic light of wavelength 500 nm, the distance of 3rd maxima from the 2nd minima on the other side of central maxima is

(1) 2.75 mm (2) 2.5 mm

(3) 22.5 mm (4) 2.25 mm

50. A light of wavelength 6000 Å shines on two narrow slits separated by a distance 1.0 mm and illuminates a screen at a distance 1.5 m away. When one slit is covered by a thin glass plate of refractive index 1.8 and other slit by a thin glass plate of refractive index $\mu$, the central maxima shifts by 0.1 rad. Both plates have the same thickness of 0.5 mm. The value of refractive index $\mu$ of the glass is

(1) 1.4 (2) 1.5

(3) 1.6 (4) none of these

51. Calculate the wavelength of light used in an interference experiment from the following data: Fringe width = 0.03 cm. Distance between slits and eyepiece through which the interference pattern is observed is 1 m. Distance between the images of the virtual source when a convex lens of focal length 16 cm is used at a distance of 80 cm from the eyepiece is 0.8 cm.

(1) 6000 Å (2) 0.00006 Å

(3) 6000 cm (4) 0.00006 m

52. In Young's double-slit experiment, the angular width of a fringe formed on a distant screen is 1°. The wavelength of light used is 6000 Å. What is the spacing between the slits?

(1) 344 mm (2) 0.1344 mm

(3) 0.0344 mm (4) 0.034 mm

53. In a double-slit experiment, the distance between the slits is $d$. The screen is at a distance $D$ from the slits. If a bright fringe is formed opposite to a slit on the screen, the order of the fringe is

(1) $\frac{d^2}{2\lambda D}$ (2) $\frac{d}{2\lambda D}$

(3) $\frac{d^2}{4\lambda D}$ (4) 0

54. In Young's double-slit experiment using monochromatic light, the light pattern shifts by a certain distance on the screen when a mica sheet of refractive index $\mu$ and thickness $t$ microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of light?

(1) $(1/2)t\,(\mu-1)$ (2) $t\,(\mu-1)$

(3) $\mu t$ (4) $3\mu t$

55. In YDSE, find the thickness of a glass slab ($\mu = 1.5$) which should be placed before the upper slit $S_1$ so that the central maximum now lies at a point where 5th bright fringe was lying earlier (before inserting the slab). Wavelength of light used is 5000 Å.

(1) $5 \times 10^{-6}$ m (2) $3 \times 10^{-6}$ m

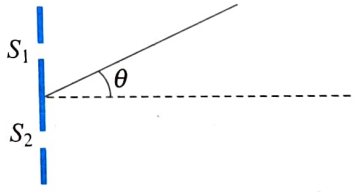
(3) $10 \times 10^{-6}$ m (4) $5 \times 10^{-5}$ m

56. In Young's double-slit experiment, the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slits. It is found that the ninth bright fringe is at a distance of 7.5 mm from the second dark fringe from the center of the fringe pattern. The wavelength of the light used is

(1) 5000 Å (2) $\frac{5000}{7}$ Å

(3) 2500 Å (4) $\frac{2500}{7}$ Å

57. Figure shows two coherent sources $S_1$ and $S_2$ emitting wavelength $\lambda$. The separation $S_1S_2 = 1.5\lambda$ and $S_1$ is ahead in phase by $\pi/2$ relative to $S_2$. Then the maxima occur in direction $\theta$ given by $\sin^{-1}$ of




(i) 0 (ii) 1/2

(iii) −1/6 (iv) −5/6

Correct options are

(1) (ii), (iii), and (iv) (2) (i), (ii), and (iii)

(3) (i), (iii), and (iv) (4) All the above

**58.** Light from a source emitting two wavelengths $\lambda_1$ and $\lambda_2$ is allowed to fall on Young's double-slit apparatus after filtering one of the wavelengths. The position of interference maxima is noted. When the filter is removed both the wavelengths are incident and it is found that maximum intensity is produced where the fourth maxima occured previously. If the other wavelength is filtered, at the same location the third maxima is found. What is the ratio of wavelengths?

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$

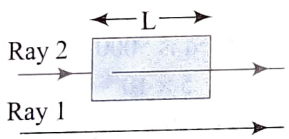
(3) $\frac{3}{4}$ (4) $\frac{4}{3}$

**59.** The wavefront of a light beam is given by the equation $x + 2y + 3x = c$ (where $c$ is arbitrary constant), then the angle made by the direction of light with the $y$-axis is

(1) $\cos^{-1}\frac{1}{\sqrt{14}}$ (2) $\sin^{-1}\frac{2}{\sqrt{14}}$

(3) $\cos^{-1}\frac{2}{\sqrt{14}}$ (4) $\sin^{-1}\frac{3}{\sqrt{14}}$

**60.** As shown in figure, waves with identical wavelengths and amplitudes and which are initially in phase travel through different media. Ray 1 travels through air and Ray 2 through a transparent medium for equal length $L$, in four different situations. In each situation, the two rays reach a common point on the screen. The number of wavelengths in length $L$ is $N_2$ for Ray 2 and $N_1$ for Ray 1. In the following table, values of $N_1$ and $N_2$ are given for all four situations.




| Situations | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $N_1$ | 2.25 | 1.80 | 3.00 | 3.25 |
| $N_2$ | 2.75 | 2.80 | 3.25 | 4.00 |

The order of the situations according to the intensity of the light at the common point in descending order is

(1) $I_3 = I_4 > I_2 > I_1$ (2) $I_1 > I_3 = I_4 > I_2$

(3) $I_1 > I_2 > I_3 > I_4$ (4) $I_2 > I_3 = I_4 > I_1$

**61.** If the distance between the first maxima and fifth minima of a double-slit pattern is 7 mm and the slits are separated by 0.15 mm with the screen 50 cm from the slits, then wavelength of the light used is

(1) 600 nm (2) 525 nm

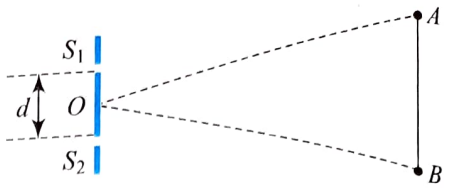
(3) 467 nm (4) 420 nm

**62.** Let $S_1$ and $S_2$ be the two slits in Young's double-slit experiment. If central maxima is observed at $P$ and angle $\angle S_1PS_2 = \theta$, then the fringe width for the light of wavelength $\lambda$ will be (assume $\theta$ to be a small angle)

(1) $\lambda/\theta$ (2) $\lambda\theta$

(3) $2\lambda/\theta$ (4) $\lambda/2\theta$

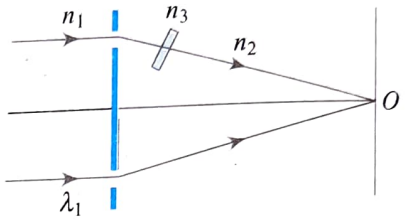
**63.** Figure shows two coherent sources $S_1$ and $S_2$ vibrating in same phase. $AB$ is an irregular wire lying at a far distance from the sources $S_1$ and $S_2$. Let $\frac{\lambda}{d} = 10^{-3}$, $\angle BOA = 0.12°$. How many bright spots will be seen on the wire, including points $A$ and $B$?




(1) 2 (2) 3

(3) 4 (4) more than 4

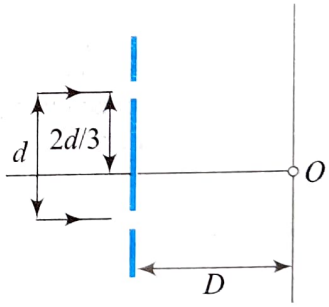
**64.** In YDSE shown in figure, a parallel beam of light is incident on the slits from a medium of refractive index $n_1$. The wavelength of light in this medium is $\lambda_1$. A transparent slab of thickness $t$ and refractive index $n_3$ is put in front of one slit. The medium between the screen and the plane of the slits is $n_2$. The phase difference between the light waves reaching point $O$ (symmetrical, relative to the slits) is




(1) $\frac{2\pi}{n_1\lambda_1}(n_3 - n_2)t$ (2) $\frac{2\pi}{\lambda_1}(n_3 - n_2)t$

(3) $\frac{2\pi n_1}{n_2\lambda_1}\left(\frac{n_3}{n_2} - 1\right)t$ (4) $\frac{2\pi n_1}{\lambda_1}(n_3 - n_2)t$

**65.** In figure, if a parallel beam of white light is incident on the plane of the slits, then the distance of the nearest white spot on the screen from $O$ is [assume $d << D$, $\lambda << d$]




(1) 0 (2) $d/2$

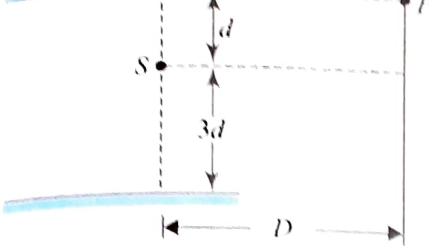
(3) $d/3$ (4) $d/6$

**66.** In YDSE, coherent monochromatic light having wavelength 600 nm falls on the slits. First-order bright fringe is at 4.84 mm from central maxima. Determine the wavelength for which the first-order dark fringe will be observed at the same location on screen. (Take $D = 3$ m.)

(1) 600 nm (2) 1200 nm

(3) 300 nm (4) 900 nm

**67.** Consider the optical system shown in figure. The point source of light $S$ is having wavelength equal to $\lambda$. The light is reaching screen only after reflection. For point $P$ to be second maxima, the value of $\lambda$ would be ($D >> d$ and $d >> \lambda$)




(1) $\frac{12d^2}{D}$ (2) $\frac{6d^2}{D}$

(3) $\frac{3d^2}{D}$ (4) $\frac{24d^2}{D}$

68. In YDSE, if a bichromatic light having wavelengths $\lambda_1$ and $\lambda_2$ is used, then the maxima due to both lights will overlap at a certain distance $y$ from the central maxima. Take separation between slits as $d$ and distance between screen and slits as $D$. Then the value of $y$ will be

(1) $\left(\frac{\lambda_1+\lambda_2}{2D}\right)d$ (2) $\left(\frac{\lambda_1-\lambda_2}{D}\right)2d$

(3) LCM of $\frac{\lambda_1 D}{d}$ and $\frac{\lambda_2 D}{d}$ (4) HCF of $\frac{\lambda_1 D}{d}$ and $\frac{\lambda_2 D}{d}$

69. In YDSE, $d = 2$ mm, $D = 2$ m, and $\lambda = 500$ nm. If intensities of two slits are $I_0$ and $9I_0$, then find intensity at $y = \frac{1}{6}$ mm.

(1) $7I_0$ (2) $10I_0$

(3) $16I_0$ (4) $4I_0$

70. In YDSE, let $A$ and $B$ be two slits. Films of thicknesses $t_A$ and $t_B$ and refractive indices $\mu_A$ and $\mu_B$ are placed in front of $A$ and $B$, respectively. If $\mu_A t_A = \mu_A t_B$, then the central maxima will

(1) not shift

(2) shift toward $A$

(3) shift toward $B$

(4) (2) if $t_B < t_A$ and (3) if $t_B < t_A$

71. In YDSE, having slits of equal width, let $\beta$ be the fringe width and $I_0$ be the maximum intensity. At a distance $x$ from the central bright fringe, the intensity will be

(1) $I_0 \cos\left(\frac{x}{\beta}\right)$ (2) $I_0 \cos^2 \frac{2\pi x}{\beta}$

(3) $I_0 \cos^2 \frac{\pi x}{\beta}$ (4) $\frac{I_0}{4}\cos^2 \frac{\pi x}{\beta}$

72. In YDSE of equal width slits, if intensity at the center of screen is $I_0$, then intensity at a distance of $\beta/4$ from the central maxima is

(1) $I_0$ (2) $\frac{I_0}{2}$

(3) $\frac{I_0}{4}$ (4) $\frac{I_0}{3}$

73. In Young's double-slit experiment, the slits are 2 mm apart and are illuminated with a mixture of two wavelengths $\lambda_0 = 750$ nm and $\lambda = 900$ nm. The minimum distance from the common central bright fringe on a screen 2 m from the slits, where a bright fringe from one interference pattern coincides with a bright fringe from the other, is

(1) 1.5 mm (2) 3 mm

(3) 4.5 mm (4) 6 mm

74. A flake of glass (refractive index 1.5) is placed over one of the openings of a double-slit apparatus. The interference pattern displaced itself through seven successive maxima toward the side where the flake is placed. If wavelength of the light is $\lambda = 600$ nm, then the thickness of the flake is

(1) 2100 nm (2) 4200 nm

(3) 8400 nm (4) none of above

75. Two identical sources each of intensity $I_0$ have a separation $d = \lambda/8$, where $\lambda$ is the wavelength of the waves emitted by either source. The phase difference of the sources is $\pi/4$. The intensity distribution $I(\theta)$ in the radiation field as a function of $\theta$, which specifies the direction from the sources to the distant observation point $P$ is given by

(1) $I(\theta) = I_0 \cos^2 \theta$

(2) $I(\theta) = \frac{I_0}{4}\cos^2\left(\frac{\pi\theta}{8}\right)$

(3) $I(\theta) = 4I_0 \cos^2\left[\frac{\pi}{8}(\sin\theta + 1)\right]$

(4) $I(\theta) = I_0 \sin^2 \theta$

76. A certain region of a soap bubble reflects red light of vacuum wavelength $\lambda = 650$ nm. What is the minimum thickness that this region of the soap bubble could have? Take the index of reflection of the soap film to be 1.41.

(1) $1.2 \times 10^{-7}$ m (2) $650 \times 10^{-9}$ m

(3) $120 \times 10^{7}$ m (4) $650 \times 10^{5}$ m

77. In Young's double-slit experiment using monochromatic light of wavelength $\lambda$, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 2.0 μm is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the plane of slits and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the light.

(1) 4500 Å (2) 5700 Å

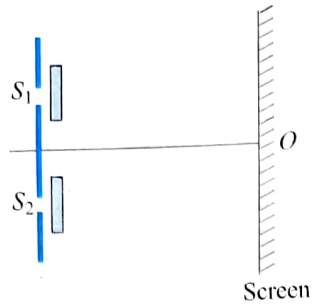
(3) 6000 Å (4) 4000 Å

78. In YDSE, when a glass plate of refractive index 1.5 and thickness $t$ is placed in the path of one of the interfering beams (wavelength $\lambda$), intensity at the position where central maximum occured previously remains unchanged. The minimum thickness of the glass plate is

(1) $2\lambda$ (2) $(2/3)\lambda$

(3) $\lambda/3$ (4) $\lambda$

79. Young's double-slit experiment is carried with two thin sheets of thickness 10.4 μm each and refractive index $\mu_1 = 1.52$ and $\mu_2 = 1.40$ covering the slits $S_1$ and $S_2$, respectively. If white light of range 400 nm to 780 nm is used, then which wavelength will form maxima exactly at point $O$, the center of the screen?




(1) 416 nm only (2) 624 nm only
(3) 416 nm and 624 nm only (4) None of these

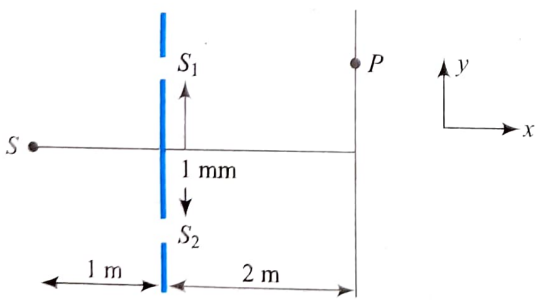
**80.** High-quality camera lenses are often coated to prevent reflection. A lens has an optical index of refraction of 1.72 and a coating with an optical index of refraction 1.31. For near normal incidence, the minimum thickness of the coating to prevent reflection for wavelength of $5.3 \times 10^{-7}$ m is

(1) 0.75 μm (2) 0.2 mm
(3) 0.1 μm (4) 1.75 mm

**81.** A long horizontal slit is placed 1 mm above a horizontal plane mirror. The interference between the light coming directly from the slit and that after reflection is seen on a screen 1 m away from the slit. If the mirror reflects only 64% of the light falling on it, the ratio of the maximum to the minimum intensity in the interference pattern observed on the screen is

(1) 8:1 (2) 3:1
(3) 81:1 (4) 9:1

**82.** In Young's double-slit experiment set up, source $S$ of wavelength 50 nm illuminates two slits $S_1$ and $S_2$ which act as two coherent sources. The source $S$ oscillates about its own position according to the equation $y = 0.5 \sin \pi t$, where $y$ is in nm and $t$ in seconds. The minimum value of time $t$ for which the intensity at point $P$ on the screen exactly in front of the upper slit becomes minimum is




(1) 1 s (2) 2 s
(3) 3 s (4) 1.5 s

**83.** Intensity observed in an interference pattern is $I = I_0 \sin^2 \theta$. At $\theta = 30°$, intensity $I = 5 \pm 0.002$. The percentage error in angle is

(1) $4\sqrt{3} \times 10^{-2}$ % (2) $\frac{4}{\pi} \times 10^{-2}$ %
(3) $\frac{4\sqrt{3}}{\pi} \times 10^{-2}$ % (4) $\sqrt{3} \times 10^{-2}$ %

**84.** A thin uniform film of refractive index 1.75 is placed on a sheet of glass of refractive index 1.5. At room temperature (20°C), this film is just thick enough for light with wavelength 600 nm reflected off the top of the film to be canceled by light reflected from the top of the glass. After the glass is placed in an oven and slowly heated to 170°C, the film conceals reflected light with wavelength 606 nm. The coefficient of linear expansion of the film is (ignore any changes in the refractive index of the film due to the temperature change)

(1) $3.3 \times 10^{-5}$ °C$^{-1}$ (2) $6.6 \times 10^{-5}$ °C$^{-1}$
(3) $9.9 \times 10^{-5}$ °C$^{-1}$ (4) $2.2 \times 10^{-5}$ °C$^{-1}$

**85.** Two slits spaced 0.25 mm apart are placed 0.75 m from a screen and illuminated by coherent light with a wavelength of 650 nm. The intensity at the center of the central maximum ($\theta = 0°$) is $I_0$. The distance on the screen from the center of the central maximum to the point where the intensity has fallen to $I_0/2$ is nearly

(1) 0.1 mm (2) 0.25 mm
(3) 0.4 mm (4) 0.5 mm

**86.** Two thin parallel slits that are 0.012 mm apart are illuminated by a laser beam of wavelength 650 nm. On a very large distant screen, the total number of bright fringes including the central fringe and those on both sides of it is

(1) 38 (2) 37
(3) 40 (4) 39

**87.** The index of refraction of a glass plate is 1.48 at $\theta_1 = 30°$C and varies linearly with temperature with a coefficient of $2.5 \times 10^{-5}$ °C$^{-1}$. The coefficient of linear expansion of the glass is $5 \times 10^{-5}$ °C$^{-1}$. At 30°C, the length of the glass plate is 3 cm. This plate is placed in front of one of the slits in Young's double-slit experiment. If the plate is being heated so that its temperature increases at a rate of 5°C$^{-1}$ min, the light source has wavelength $\lambda = 589$ nm and the glass plate initially is at $\theta = 30$°C. The number of fringes that shift on the screen in each minute is nearly (use approximation)

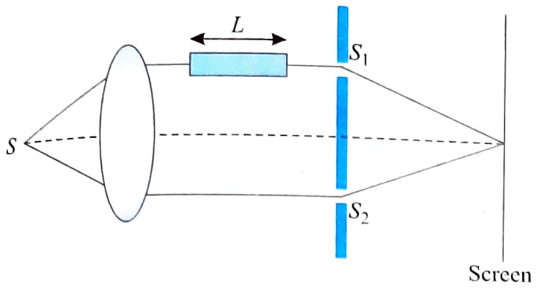
(1) 1 (2) 11
(3) 110 (4) $1.1 \times 10^3$

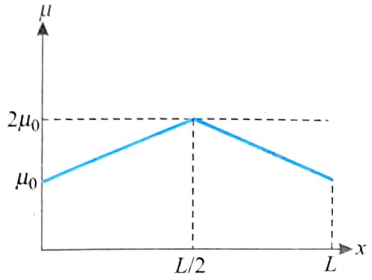
**88.** Two thin parallel slits are made in an opaque sheet of film when a monochromatic beam of light is shone through them at normal incidence. The first bright fringes in the transmitted light occur at ±45° with the original direction of the light beam on a distant screen when the apparatus is in air. When the apparatus is immersed in a liquid, the same bright fringes now occur at ±30°. The index of refraction of the liquid is

(1) $\sqrt{2}$ (2) $\sqrt{3}$
(3) $\frac{4}{3}$ (4) $\frac{3}{2}$

**89.** Figure shows a narrow-slit $S$ illuminated by a monochromatic light of wavelength $\lambda$ in a double-slit experiment. In the path of the rays reaching the upper slit $S_1$, a tube of length $L$ is interposed in which the index of refraction of the medium varies linearly as shown in figure. The position of the central

maximum in the interference pattern on the screen was displaced by $N$ fringes. Find the value of $N$ in terms of $\mu_0$, $L$ and $\lambda$.




(1) $\frac{(1.5\mu_0 - 1)L}{\lambda}$ (2) $\frac{(\mu_0 - 1)L}{\lambda}$

(3) $\frac{3(\mu_0 - 1)L}{2\lambda}$ (4) $\frac{2(\mu_0 - 1)L}{3\lambda}$

## Multiple Correct Answers Type

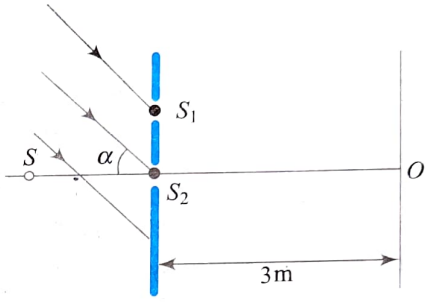
1. Two monochromatic coherent point sources $S_1$ and $S_2$ are separated by a distance $L$. Each source emits light of wavelength $\lambda$; where $L >> \lambda$. The line $S_1S_2$ when extended meets a screen perpendicular to it at point $A$. Then
   (1) the interference fringes on the screen are circular in shape
   (2) the interference fringes on the screen are straight lines perpendicular to the line $S_1S_2A$
   (3) point $A$ is an intensity maxima if $L = n\lambda$
   (4) point $A$ is always an intensity maxima for any separation $L$

2. In Young's double-slit experiment, let $A$ and $B$ be the two slits. A thin film of thickness $t$ and refractive index $\mu$ is placed in front of $A$. Let $\beta$ = fringe width. Then the central maxima will shift
   (1) toward $A$ (2) toward $B$
   (3) by $t(\mu - 1)\frac{\beta}{\lambda}$ (4) by $\mu t \frac{\beta}{\lambda}$

3. If the first minima in Young's double-slit experiment occurs directly in front of one of the slits (distance between slit and screen $D = 12$ cm and distance between slits $d = 5$ cm), then the wavelength of the radiation used can be
   (1) 2 cm (2) 4 cm
   (3) $\frac{2}{3}$ cm (4) $\frac{4}{3}$ cm

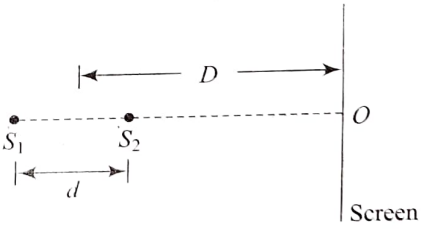
4. If one of the slits of a standard Young's double-slit experiment is covered by a thin parallel sided glass slab so that it transmits only one-half the light intensity of the other, then
   (1) the fringe pattern will get shifted toward the covered slit
   (2) the fringe pattern will get shifted away from the covered slit
   (3) the bright fringes will become less bright and the dark ones will become more bright
   (4) the fringe width will remain unchanged

5. A parallel beam of light ($\lambda = 500$ nm) is incident at an angle $\alpha = 30°$ with the normal to the slit plane in Young's double-slit experiment. Assume that the intensity due to each slit at any point on the screen is $I_0$. Point $O$ is equidistant from $S_1$ and $S_2$. The distance between slits is 1 mm. Then



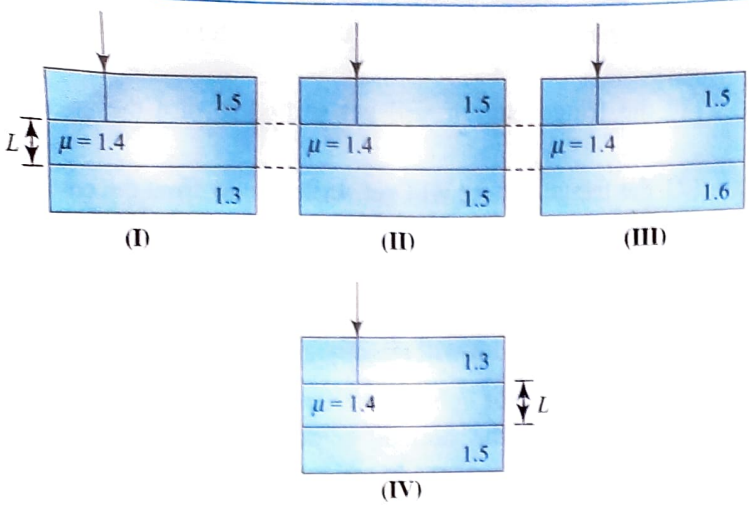

   (1) the intensity at $O$ is $4I_0$
   (2) the intensity at $O$ is zero
   (3) the intensity at a point on the screen 1 m below $O$ is $4I_0$
   (4) the intensity at a point on the screen 1 m below $O$ is zero

6. Two points monochromatic and coherent sources of light of wavelength $\lambda$ each are placed as shown in figure. The initial phase difference between the sources is zero $O$. ($D >> d$). Mark the correct statement(s).




   (1) If $d = \frac{7\lambda}{2}$, $O$ will be a minima.
   (2) If $d = \lambda$, only one maxima can be observed on the screen.
   (3) If $d = 4.8\ \lambda$, then total 10 minima would be there on the screen
   (4) If $d = \frac{5\lambda}{2}$, the intensity at $O$ would be minimum.

7. Consider a film of thickness $L$ as shown in four different cases below. Notice the observation of film with perpendicularly falling light. Mark the correct statement(s).




(Take $L = 1.5\ \lambda$)

(1) For (I) and (II), the reflection at film interfaces causes zero phase difference for two reflected rays.

(2) For (II) and (III), the reflection at film interfaces causes a phase difference of $\pi$ for two reflected rays.

(3) For (I), the film will appear dark, if it is observed through reflected rays from film interfaces.

(4) For (III), the film will appear dark, if it is observed through reflected rays from film interfaces.

**8.** In YDSE, the source is placed symmetrical to the slits. If a transparent slab is placed in front of the upper slit, then (slab can absorb a fraction of light)

(1) intensity of central maxima may change

(2) intensity of central maxima may not change

(3) central maxima will be shifted up

(4) intensity of dark fringes will be always zero

**9.** A transparent slab of thickness $t$ and refractive index $\mu$ is inserted in front of upper slit of YDSE apparatus. The wavelength of light used is $\lambda$. Assume that there is no absorption of light by the slab. Mark the correct statement(s).

(1) The intensity of dark fringes will be 0, if slits are identical.

(2) The change in optical path due to insertion of plate is $\mu t$.

(3) The change in optical path due to insertion of plate is $(\mu - 1)t$.

(4) For making intensity zero at center of screen, the thickness can be $\dfrac{5\lambda}{2(\mu - 1)}$.

**10.** A light wave of wavelength $\lambda_0$ propagates from point $A$ to point $B$. We introduce in its path a glass plate of refractive index $n$ and thickness $l$. The introduction of the plate alters the phase of the plate at $B$ by an angle $\phi$. If $\lambda$ is the wavelength of light on emerging from the plate, then

(1) $\Delta\phi = 0$ (2) $\Delta\phi = \dfrac{2\pi l}{\lambda_0}$

(3) $\Delta\phi = 2\pi l\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda_0}\right)$ (4) $\Delta\phi = \dfrac{2\pi l}{\lambda_0}(n-1)$

**11.** A ray of light of wavelength $\lambda_0$ and frequency $\nu_0$ enters a glass slab of refractive index $\mu$ from air. Then

(1) its wavelength increases, frequency decreases

(2) its wavelength decreases, frequency remains same

(3) its wavelength increases, frequency remains same

(4) $\Delta\lambda = \lambda_0\left(\dfrac{1}{\mu} - 1\right)$ and $\Delta\nu = 0$

**12.** In Young's double-slit experiment, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. This implies that

(1) the intensities at the screen due to the two slits are 5 units and 4 units, respectively

(2) the intensities at the screen due to the two slits are 4 units and 1 units, respectively

(3) the amplitude ratio is 3

(4) the amplitude ratio is 2

**13.** In Young's double-slit experiment, two wavelengths of light are used simultaneously where $\lambda_2 = 2\lambda_1$. In the fringe pattern observed on the screen,

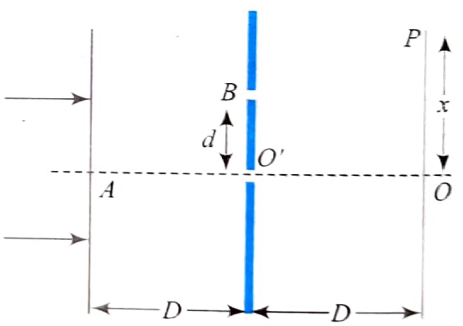
(1) maxima of wavelength $\lambda_2$ can coincide with minima of wavelength $\lambda_1$.

(2) fringe width of $\lambda_2$ will be double that of fringe width of $\lambda_1$ and $n$th order maxima of $\lambda_2$ will coincide with 2nd order maxima of $\lambda_1$

(3) $n$th order minima of $\lambda_2$ will coincide with $2n$th order minima of $\lambda_1$

(4) none of these

**14.** The minimum value of $d$ so that there is a dark fringe at $O$ is $d_{min}$. For the value of $d_{min}$, the distance at which the next bright fringe is formed is $x$. Then




(1) $d_{min} = \sqrt{\lambda D}$ (2) $d_{min} = \sqrt{\dfrac{\lambda D}{2}}$

(3) $x = \dfrac{d_{min}}{2}$ (4) $x = d_{min}$

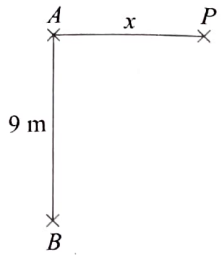
**15.** Slit 1 of Young's double-slit experiment is wider than slit 2, so that the light from slits are given as $A_1 = A_0 \sin\omega t$ and $A_2 = 3A_0 \sin\left(\omega t + \dfrac{\pi}{3}\right)$. The resultant amplitude and intensity at a point where the path difference between them is zero, are $A$ and $I$, respectively. Then

(1) $A = \sqrt{13}\,A_0$ (2) $A = 4A_0$

(3) $I \propto 16A_0^2$ (4) $I \propto 13A_0^2$

**16.** A radio transmitting station operating at a frequency of 120 MHz has two identical antennas that radiate in phase. Antenna $B$ is 9 m to the right of antenna $A$. Consider point $P$ at a horizontal distance $x$ to the right of antenna $A$ as shown

in figure. The value of $x$ and order for which the constructive interference will occur at point $P$ are




(1) $x = 14.95$ m, $n = 1$ (2) $x = 5.6$ m, $n = 2$

(3) $x = 1.65$ m, $n = 3$ (4) $x = 0$, $n = 3.6$

17. A two-slit interference experiment uses coherent light of wavelength $5 \times 10^{-7}$ m. Intensity in the interference pattern for the following points are $I_1$, $I_2$, $I_3$, and $I_4$, respectively.

1. A point that is close to one slit than the other by $5 \times 10^{-7}$ m.
2. A point where the light waves received from the two slits are out of phase by $\frac{4\pi}{3}$ rad.
3. A point that is closer to one slit than the other by $7.5 \times 10^{-7}$ m.
4. A point where the light waves received by the two slits are out of phase by $\frac{\pi}{3}$ rad.

Then which of following statements is/are correct?

(1) $I_1 > I_4 > I_2 > I_3$ (2) $I_1 > I_4 > I_2 > I_3$

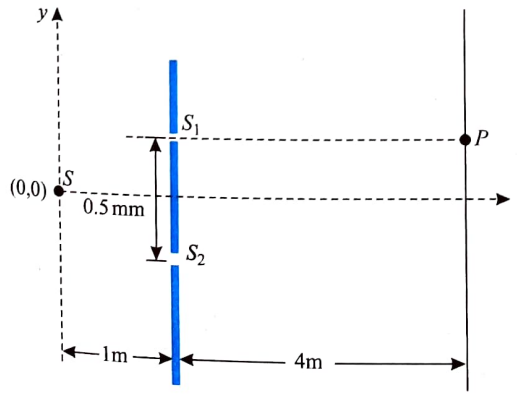
(3) $3I_2 = I_4$ (4) $I_3 = 0$

18. White light is used to illuminate the two slits in Young's double-slit experiment. The separation between the slits is $b$ and the screen is at a distance $D$ ($>b$) from the slits. At a point on the screen directly in front of one of the slits, certain wavelengths are missing. Some of these missing wavelengths are

(1) $\lambda = \frac{b^2}{D}$ (2) $\lambda = \frac{2b^2}{D}$

(3) $\lambda = \frac{b^2}{3D}$ (4) $\lambda = \frac{2b^2}{3D}$

19. Figure shows a Young's double slit experiment setup. The source $S$ of wavelength 4000 Å oscillates along $y$-axis according to the equation $y = \sin \pi t$, where $y$ is in millimeters and $t$ is in seconds. The distance between two slits $S_1$ and $S_2$ is 0.5 mm.




(1) The position of the central maxima as a function of time is $4 \sin \pi t$

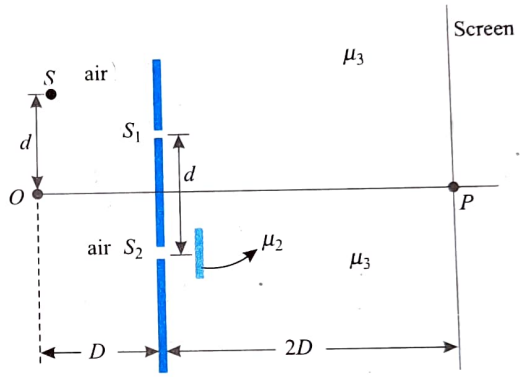
(2) The position of the central maxima as a function of time is $4 \sin \pi t$

(3) The instant at which maximum intensity occurs at $P$ for first time is $\frac{1}{\pi} \sin^{-1}\left(\frac{54}{80}\right)$

(4) The instant at which maximum intensity occurs at $P$ for first time is $\frac{1}{\pi} \sin^{-1}\left(\frac{27}{80}\right)$

20. Consider the situation shown in the figure. Two slits $S_1$ and $S_2$ are placed symmetrically about the line $OP$ which is perpendicular to screen. The space between screen and slits is filled with a liquid of refractive index $\mu_3$. A plate of thickness $t$ and refractive index $\mu_2$ is placed in front of one of the slit. A source $S$ is placed above $OP$ at a distance $d$. Choose the correct alternatives.

[(Given that $D = 1$ m, $d = 2$ mm, $t = 6 \times 10^{-6}$ m, $\mu_2 = 1.2$, $\mu_3 = 1.8$]




(1) Position of central maxima from point $P$ is 2 mm

(2) Position of central maxima from point $P$ is $\frac{2}{9}$ mm

(3) Thickness of glass slab so that central maxima forms at point $P$ is $\frac{20}{3} \times 10^{-6}$ m

(4) If glass slab is removed, the central maxima shifts by a distance of 2 mm

21. If the source of light used in a Young's double slit experiment is changed from red to blue, then

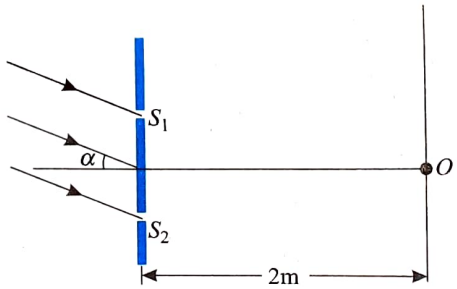
(1) the fringes will become brighter

(2) consecutive fringes will come closer

(3) the number of maxima formed on the screen increases

(4) the central bright fringe will become a dark fringe

22. A parallel beam of light ($\lambda = 5000$ Å) is incident at an angle $\alpha = 30°$ with the normal to the slit plane in a Young's double slit experiment. The intensity due to each slit is $I_0$. Point $O$ is equidistant from $S_1$ and $S_2$. The distance between slits is 1 mm.



(1) The intensity at $O$ is $I_0$

(2) The intensity at $O$ is zero

(3) The intensity at a point on the screen 4 mm above $O$ is $4I_0$

(4) The intensity at a point on the screen 4 mm above $O$ is zero

**23.** Bichromatic light of wavelengths $\lambda_1 = 500$ Å and $\lambda_2 = 7000$ Å are used in YDSE. Then,

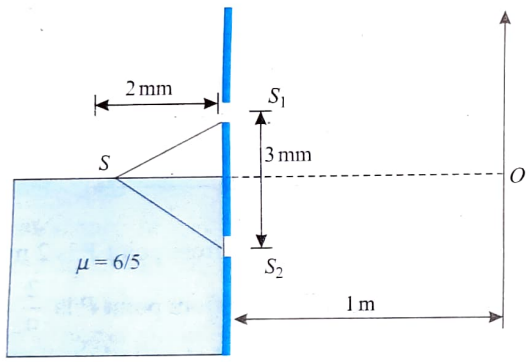
(1) $14^{th}$ order maxima of $\lambda_1$ will coincide with $10^{th}$ order maxima of $\lambda_2$

(2) $21^{st}$ order maxima of $\lambda_2$ will coincide with $15^{th}$ order maxima of $\lambda_1$

(3) $11^{th}$ order maxima of $\lambda_1$ will coincide with $8^{th}$ order maxima of $\lambda_2$

(4) $3^{rd}$ order maxima of $\lambda_1$ will coincide with $4^{th}$ order maxima of $\lambda_2$

**24.** In Young's double-slit experiment, a point source is placed on a solid slab of refractive index 6/5 at a distance of 2 mm from two slits spaced 3 mm apart as shown and at equal distance from both the slits. The screen is at a distance of 1 m from the slits. Wavelength of light used is 500 nm.




(1) The central maxima is at $\frac{1}{6}$ m below $O$.

(2) The central maxima is at $\frac{1}{3}$ m below $O$.

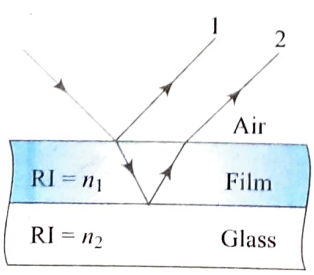
(3) The order of the fringe formed at $O$ is 750.

(4) If a film of refractive index 1.8 is to be placed in front of $S_1$ so that central maxima is formed where 200th maxima was formed. Then the thickness of film is 1.25 mm.

## Linked Comprehension Type

### For Problems 1–3

A thin film of a specific material can be used to decrease the intensity of reflected light. There is destructive interference of waves reflected from upper and lower surfaces of the film. These films are called non-reflecting or anti-reflection coatings. The process of coating the lens or surface with non-reflecting film is called blooming as shown in figure. The refracting index of coating ($n_1$) is less than that of the glass ($n_2$).




**1.** If a light of wavelength $\lambda$ is incident normally and the thickness of film is $t$, then optical path difference between waves reflected from upper and lower surface of the film is

(1) $2n_1t$ (2) $2n_1t - \frac{\lambda}{2}$

(3) $2n_1t + \frac{\lambda}{2}$ (4) $2t$

**2.** Magnesium fluoride ($MgF_2$) is generally used as anti-reflection coating. If refractive index of $MgF_2$ is 1.25, then minimum thickness of film required is (Take $\lambda = 500$ nm)

(1) 125 nm (2) 75 nm

(3) 100 nm (4) 225 nm

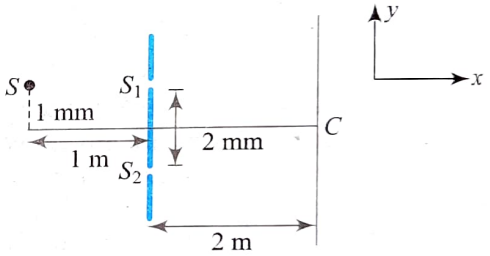
**3.** If the thickness of film in above question is not technologically possible to manufacture, then next thickness of film required is (approximately)

(1) 300 nm (2) 125 nm

(3) 750 nm (4) 550 nm

### For Problems 4–6

In a Young's double-slit experiment set up, source $S$ of wavelength 6000 Å illuminates two slits $S_1$ and $S_2$ which act two coherent sources. The source $S$ oscillates about its shown position according to the equation $y = 1 + \cos \pi t$, where $y$ is in millimeter and $t$ in second.




**4.** At $t = 0$, fringe width is $\beta_1$, and at $t = 2$ s, fringe width of figure is $\beta_2$. Then

(1) $\beta_1 > \beta_2$ (2) $\beta_2 > \beta_1$

(3) $\beta_1 = \beta_2$ (4) data is insufficient

**5.** At $t = 2$ s, the position of central maxima is

(1) 2 mm above $C$ (2) 2 mm below $C$

(3) 4 mm above $C$ (4) 4 mm below $C$

**6.** At $t = 1$ s, a slab of thickness $2 \times 10^{-3}$ mm and refractive index 1.5 is placed just in front of $S_1$. The central maximum is formed at

(1) 1 mm above $C$ (2) 1 mm below $C$

(3) 2 mm above $C$ (4) 2 mm below $C$

### For Problems 7–9

In YDSE, the source is red light of wavelength $7 \times 10^{-7}$ m. When a thin glass plate of refractive index 1.5 is put in the path of one of the interfering beams, the central bright fringe shifts by $10^{-3}$ m to the position previously occupied by the 5th bright fringe.

**7.** What is the thickness of the plate?

(1) 5 μm (2) 0.005 μm

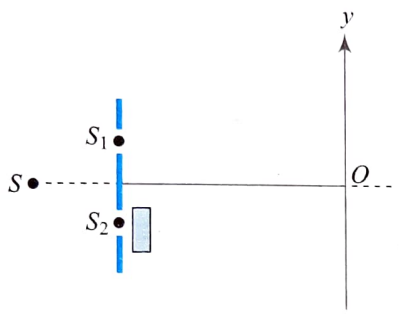
(3) 7 μm (4) 0.007 μm

8. If the source is now changed to green light of wavelength $10^{-7}$ m, the central fringe shifts to a position initially occupied by the sixth bright fringe due to red light. What will be refractive index of glass plate for the second light for changed source of light?
(1) 2.6 μm (2) 1.6 μm
(3) 1.2 μm (4) 2.2 μm

9. Change in fringe width produced due to change in wavelength is
(1) $-0.57 \times 10^{-4}$ m (2) $-0.47 \times 10^{-4}$ m
(3) $-0.37 \times 10^{-4}$ m (4) $-0.27 \times 10^{-4}$ m

**For Problems 10–12**

A YDSE is performed in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit $S_2$ is covered by a thin glass plate of thickness 10.4 mm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown in figure. (All the wavelengths in this problem are for the given medium of refractive index 4/3, ignore absorption.)




10. The location of the central maximum (bright fringe with zero path difference) on the $y$-axis will be
(1) 2.33 mm (2) 4.33 mm
(3) 6.33 mm (4) 4.43 mm

11. Find the light intensity at point $O$ relative to maximum fringe intensity.
(1) $\frac{1}{4}I_{max}$ (2) $\frac{5}{4}I_{max}$
(3) $\frac{1}{2}I_{max}$ (4) $\frac{3}{4}I_{max}$

12. Now, if 600 nm light is replaced by white light of range 400 to 800 nm, find the wavelength of the light that forms maxima exactly at point $O$.
(1) 650 nm, 433.3 nm
(2) 550 nm, 750 nm
(3) 450 nm, 645.3 nm
(4) 375 nm, 525.3 nm

**For Problems 13–15**

In a YDSE performed with light of wavelength 600 Å, the screen is placed 1 m from the slits. Fringes formed on the screen are observed by a student sitting close to the slits. The student's eye can distinguish two neighboring fringes. If they subtend an angle more than 1 minute of arc, then

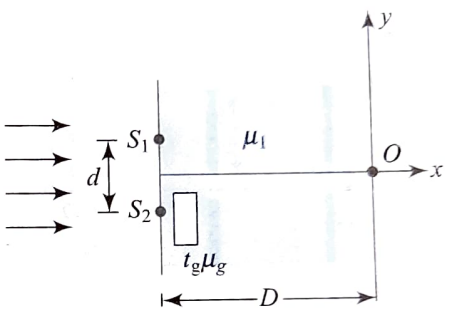
13. In order to have the clear visibility of the fringe, the maximum distance that can be maintained between the slits is
(1) 3.06 mm (2) 2.06 mm
(3) 1.31 mm (4) 3.31 mm

14. Find the location of third bright fringe from center of the screen.
(1) $8.74 \times 10^{-4}$ mm
(2) $6.74 \times 10^{-4}$ mm
(3) $5 \times 10^{-4}$ mm
(4) $8.74 \times 10^{-7}$ mm

15. Under similar conditions or arrangement, what will be the position of fifth dark fringe from the center of screen?
(1) 4.26 μm (2) 2.93 μm
(3) 1.31 μm (4) 3.14 μm

**For Problems 16–17**

In a modified YDSE, the region between the screen and slits is immersed in a liquid whose refractive index varies with time as $\mu_1 = (5/2) - (T/4)$ until it reaches a steady state value of 5/4. A glass plate of thickness 36 μm and refractive index 3/2 is introduced in front of one of the slits.



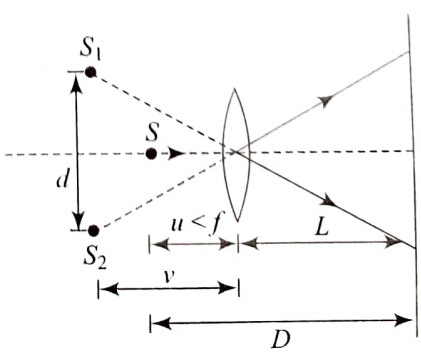

$D = 1$ m
$d = 2$ mm

16. Find the time when central maxima is at point $O$, located symmetrically on the $x$-axis.
(1) 2 s (2) 4 s
(3) 6 s (4) 8 s

17. What is the speed of the central maxima when it is at $O$?
(1) $3 \times 10^{-3}$ m s$^{-1}$
(2) $4 \times 10^{-3}$ m s$^{-1}$
(3) $6 \times 10^{-3}$ m s$^{-1}$
(4) $1 \times 10^{-3}$ m s$^{-1}$

**For Problems 18–19**

A lens of focal length $f$ is cut along the diameter into two identical halves. In this process, a layer of the lens $t$ in thickness is lost, then the halves are put together to form a composite lens. In between the focal plane and the composite lens, a narrow slit is placed near the focal plane. The slit is emitting monochromatic light with wavelength $\lambda$. Behind the lens, a screen is located at a distance $L$ from it.



**18.** Find the fringe width for the pattern obtained under given arrangement on the screen.

(1) $\frac{\lambda f}{2t}$ (2) $\frac{\lambda f}{t}$

(3) $\frac{tf}{\lambda}$ (4) $\frac{tf}{2\lambda}$

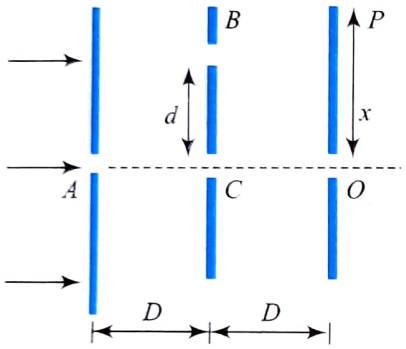
**19.** The expression for the number of visible maxima which are obtained through above said arrangement will turn out to be

(1) $\frac{Lt^2}{\lambda f^2}$ (2) $\frac{2Lt^2}{\lambda f^2}$

(3) $\frac{Lt}{2\lambda f^2}$ (4) $\frac{Lt^2}{2\lambda f^2}$

**For Problems 20–22**

In the arrangement shown in figure, $D >> d$.




**20.** For what minimum value of $d$ is there a dark band at point $O$ on the screen?

(1) $\sqrt{\frac{D\lambda}{4}}$ (2) $\sqrt{\frac{3D\lambda}{4}}$

(3) $\sqrt{\frac{D\lambda}{8}}$ (4) $\sqrt{\frac{2D\lambda}{3}}$

**21.** Find the distance $x$ at which the next bright fringe is formed.

(1) $\frac{3\lambda}{2}$ (2) $\frac{\lambda}{4}$

(3) $\frac{\lambda}{2}$ (4) $\frac{5\lambda}{2}$

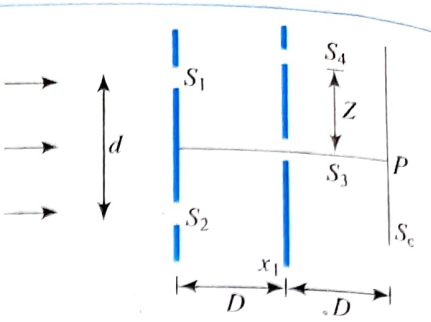
**22.** Find the fringe width.

(1) $d$ (2) $2d$

(3) $4d$ (4) $3d$

**For Problems 23–25**

Consider the situation shown in figure. The two slits $S_1$ and $S_2$ placed symmetrically around the central line are illuminated by monochromatic light of wavelength $\lambda$. The separation between the slits is $d$. The light transmitted by the slits falls on a screen $S_0$ placed at a distance $D$ from the slits. The slit $S_3$ is at the central line and the slit $S_4$ is at a distance $Z$ from $S_3$. Another screen $S_c$ is placed a further distance $D$ away from $S_0$. Find the ratio of the maximum to minimum intensity observed on $S_c$.




**23.** If $z = \frac{\lambda D}{2d}$

(1) 1 (2) 1 / 2

(3) 3 / 2 (4) 2

**24.** If $z = \frac{\lambda D}{d}$

(1) 4 (2) 2

(3) $\infty$ (4) 1

**25.** If $z = \frac{\lambda D}{4d}$

(1) $\left[3-2\sqrt{2}\right]^2$ (2) $\left[3+\sqrt{2}\right]^2$

(3) $\left[3-\sqrt{2}\right]^2$ (4) $\left[3+2\sqrt{2}\right]^2$

**For Problems 26–28**

When two coherent sources interact with each other, there will be production of alternate bright and dark fringes on the screen. Young's double-slit experiment demonstrates the idea of making two coherent sources. For better visibility, one has to choose proper amplitude for the sources. The phenomena is good enough to satisfy the conservation of energy principle. The pattern formed in YDSE is of uniform thickness and is usually placed on a long distance screen.

**26.** Law of conservation of energy is satisfied because

(1) equal loss and gain in intensity is observed

(2) all bright fringes are equally bright

(3) all dark fringes are of zero brightness

(4) the average intensity on screen is equal to the sum of intensities of the two sources

**27.** For better visibility of fringe pattern

(1) amplitudes of the sources are equal

(2) the width of the slits should not be equal

(3) dark should be the darkest and bright should be the brightest

(4) the widths should be same

**28.** The best combination of independent sources to produce sustained pattern among the following is

$Y_1 = a \sin \omega t$ $Y_2 = a \cos \omega t$

$Y_3 = a \sin\left(\omega t + \frac{\pi}{4}\right)$ $Y_4 = 2a \sin(\omega t + \pi)$

(1) $Y_1$, $Y_2$ only (2) $Y_2$, $Y_3$ only

(3) $Y_3$, $Y_4$ only (4) none of these

**For Problems 29–31**

When a light wave passes from a rarer medium to a denser medium, there will be a phase change of $\pi$ radians. This difference brings change in the conditions for constructive and destructive interference. This phenomena also reasons the formation of interference pattern in thin films like, oily layer, soap film, etc., but has no reason on the shifting of fringes from the central portion outward. The shift is dependent on the refractive index of the material as per the relation, $\Delta y = (\mu - 1)t$.

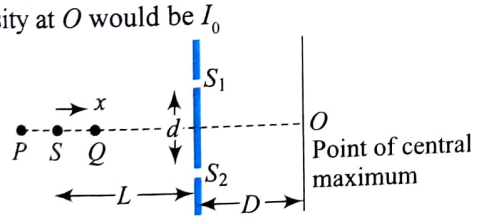
29. Thin film interference happens with
(1) point or spherical source
(2) broad source
(3) film thickness of the order of 10,000 Å
(4) very thick transparent slabs

30. On introducing a transparent slab ($\mu$), the central fringe shifts to the point originally occupied by the fifth bright fringe. The thickness of the slab is
(1) $\frac{5\lambda}{\mu - 1}$ (2) $\frac{4\lambda}{\mu - 1}$
(3) $\frac{\mu - 1}{4\lambda}$ (4) $\frac{\mu - 1}{5\lambda}$

31. The condition for constructive interference in Lloyd's single mirror experiment is the path difference which is equal to
(1) $N\lambda$ (2) $(2N - 1)\frac{\lambda}{2}$
(3) $(N - 1)\frac{\lambda}{2}$ (4) $\frac{\lambda}{2(2N - 1)}$

**For Problems 32–34**

In YDSE set up (see figure), the light source executes SHM between $P$ and $Q$ according to the equation $x = A \sin \omega t$, $S$ being the mean position. Assume $d \to 0$ and $A << L$. $\omega$ is small enough to neglect Doppler effect. If the source were stationary at $S$, intensity at $O$ would be $I_0$




Read the paragraph carefully and answer the following questions.

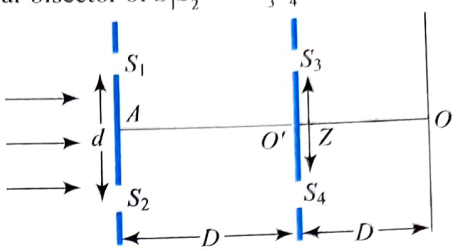
32. The fractional change in intensity of the central maximum as function of time is
(1) $\frac{A \sin \omega t}{L}$ (2) $\frac{2A \sin \omega t}{L}$
(3) $\frac{3A \sin \omega t}{L}$ (4) $\frac{4A \sin \omega t}{L}$

33. When the source comes toward the point $Q$,
(1) the bright fringes will be less bright.
(2) the dark fringes will no longer remain dark
(3) the fringe width will increase
(4) none of these

34. The fringe width $\beta$ can be expressed as
(1) $\beta = \beta_0 \sin \omega t$ (2) $\beta = \beta_0 \cos \omega t$
(3) $\beta = \beta_0 \sin 2\omega t$ (4) none of these

**For Problems 35–37**

In the arrangement shown in figure, slits $S_1$ and $S_4$ are having a variable separation $Z$. Point $O$ on the screen is at the common perpendicular bisector of $S_1S_2$ and $S_3S_4$.



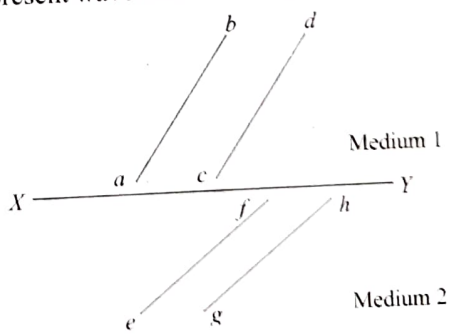

35. When $Z = \frac{\lambda D}{2d}$, the intensity measured at $O$ is $I_0$. The intensity at $O$ when $Z = \frac{2\lambda D}{d}$ is
(1) $I_0$ (2) $2I_0$
(3) $3I_0$ (4) $4I_0$

36. The minimum value of $Z$ for which the intensity at $O$ is zero is
(1) $\frac{2\lambda D}{d}$ (2) $\frac{\lambda D}{2d}$
(3) $\frac{\lambda D}{3d}$ (4) $\frac{\lambda D}{d}$

37. If a hole is made at $O'$ on $AO'O$ and the slit $S_3$ is closed, then the ratio of the maximum to minimum observed on screen at $O$, if $O'S_3$ is equal to $\frac{\lambda D}{4d}$, is
(1) 1 (2) infinity
(3) 34 (4) 4

**For Problems 38–40**

Figure shows a surface $XY$ separating two transparent media, medium 1 and medium 2. Lines $ab$ and $cd$ represent wavefronts of a light wave traveling in medium 1 and incident on $XY$. Lines $ef$ and $gh$ represent wavefront of the light wave in medium 2 after refraction.




38. Light travels as a
(1) parallel beam in each medium
(2) convergent beam in each medium
(3) divergent beam in each medium
(4) divergent beam in one medium and convergent beam in the other medium

**39.** The phases of the light waves at $c$, $d$, $e$, and $f$ are $\phi_c$, $\phi_d$, $\phi_e$, and $\phi_f$, respectively. It is given that $\phi_c \neq \phi_f$. Then

(1) $\phi_c$ cannot be equal to $\phi_d$

(2) $\phi_d$ can be equal to $\phi_e$

(3) $\phi_d - \phi_f$ is equal to $\phi_c - \phi_e$

(4) $\phi_d - \phi_c$ is not equal to $\phi_f - \phi_e$

**40.** Speed of light is

(1) the same in medium 1 and medium 2

(2) larger in medium 1 than in medium 2

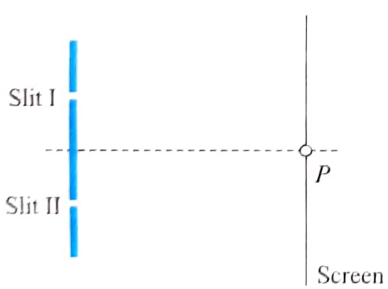
(3) larger in medium 2 than in medium 1

(4) different at $b$ and $d$

## Matrix Match Type

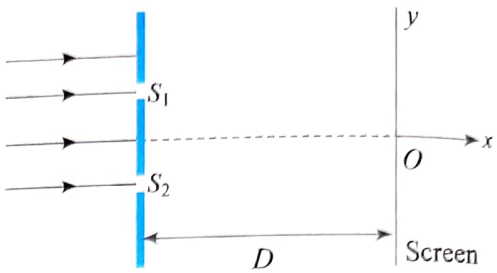
**1.** A double-slit interference pattern is produced on a screen, as shown in figure, using monochromatic light of wavelength 500 nm. Point $P$ is the location of the central bright fringe, that is produced when light waves arrive in phase without any path difference. A choice of three strips $A$, $B$, and $C$ of transparent materials with different thicknesses and refractive indices is available, as shown in the table. These are placed over one or both of the slits, singularly or in conjunction, causing the interference pattern to be shifted across the screen from the original pattern. In Column I, how the strips have been placed, is mentioned whereas in Column II, order of the fringe at point $P$ on the screen that will be produced due to the placement of the strip(s), is shown. Correctly match both the columns.

| Film | A | B | C |
|---|---|---|---|
| Thickness (in μm) | 5 | 1.5 | 0.25 |
| Refractive index | 1.5 | 2.5 | 2 |



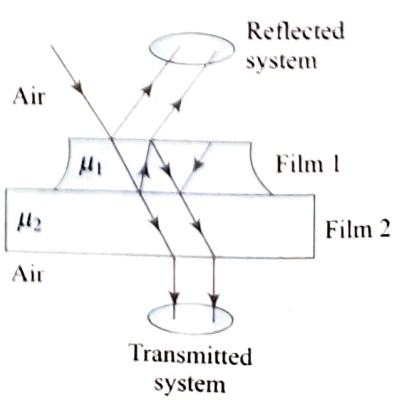

| Column I | Column II |
|---|---|
| i. Only strip $B$ is placed over slit I. | a. First bright |
| ii. Strip $A$ is placed over slit I and strip $C$ is placed over slit II. | b. Fourth dark |
| iii. Strip $A$ is placed over slit I and strip $B$ and strip $C$ are placed over slit II in conjunction. | c. Fifth dark |
| iv. Strip $A$ and strip $C$ are placed over slit I (in conjunction) and strip $B$ is placed over slit II. | d. Central bright |

**2.** A monochromatic parallel beam of light of wavelength $\lambda$ is incident normally on the plane containing slits $S_1$ and $S_2$. The slits are of unequal width such that intensity only due to one slit on screen is four times that only due to the other slit. The screen is placed along $y$-axis as shown in figure. The distance between slits is $d$ and that between the screen and slits is $D$. Match the statements in Column I with results in Column II.




| Column I | Column II |
|---|---|
| i. The distance between two points on the screen having equal intensities, such that intensity at those points is $\frac{1}{9}$th of maximum intensity. | a. $\frac{D\lambda}{3d}$ |
| ii. The distance between two points on the screen having equal intensities, such that intensity at those points is $\frac{3}{9}$th of maximum intensity. | b. $\frac{D\lambda}{d}$ |
| iii. The distance between two points on the screen having equal intensities, such that intensity at those points is $\frac{5}{9}$th of maximum intensity. | c. $\frac{2D\lambda}{d}$ |
| iv. The distance between two points on the screen having equal intensities, such that intensity at those points is $\frac{7}{9}$th of maximum intensity. | d. $\frac{3D\lambda}{d}$ |

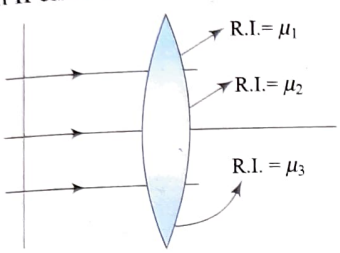
**3.** For the situation shown in figure, match the entries in Column I with Column II.



Assume thickness of films to be very small compared to wavelength of incident light.

| Column I | Column II |
|---|---|
| i. $\mu_1 = \mu_2$ | a. Film 1 appears shiny from the reflected system |
| ii. $\mu_1 > \mu_2$ | b. Film 1 appears dark from the reflected system |
| iii. $\mu_1 < \mu_2$ | c. Film 1 appears shiny from the transmitted system |
| iv. $\mu_1 \neq \mu_2$ | d. Film 1 appears dark from the transmitted system |

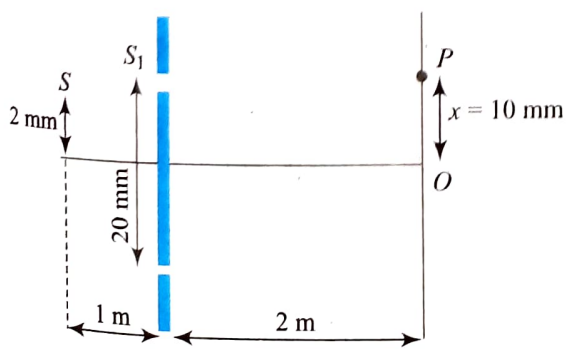
4. For the situation shown in figure, Column I relates the values of refractive index (RI) $\mu_1$, $\mu_2$ and $\mu_3$ and Column II shows the possible shapes of wavefronts. Match Column I entries with Column II entries.




Plane wavefront

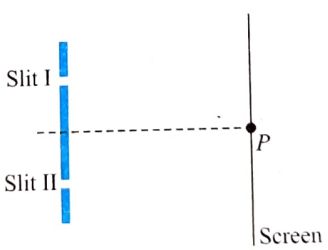
| Column I | Column II |
|---|---|
| i. $\mu_1 = 1.4, \mu_2 = 1.5, \mu_3 = 1.6$ | a. |
| ii. $\mu_1 = 1.6, \mu_2 = 1.5, \mu_3 = 1.4$ | b. |
| iii. $\mu_1 = 1.7, \mu_2 = 1.5, \mu_3 = 1.7$ | c. |
| iv. $\mu_1 = 1.3, \mu_2 = 1.5, \mu_3 = 1.3$ | d. |

5. In Young's double-slit experiment, the point source $S$ is placed slightly off the central axis as shown in figure. If $\lambda = 500$ nm, then match the following.




| Column I | Column II |
|---|---|
| i. Nature and order of interference at point $P$, $OP = 10$ mm | a. Bright fringe of order 80 |
| ii. Nature and order of interference at point $O$ | b. Bright fringe of order 262 |
| iii. If a transparent paper (refractive index $\mu = 1.45$) of thickness $t = 0.02$ mm is pasted on $S_1$, i.e., one of the slits, the nature and order of the interference at $P$ | c. Bright fringe of order 62 |
| iv. After inserting the transparent paper in front of slit $S_1$, the nature and order of interference at $O$ | d. Bright fringe of order 280 |

6. A double slit interference pattern is produced on a screen as shown in the figure, using monochromatic light of wavelength 500 nm. Point $P$ is the location of the central bright fringe, that is produced when light waves arrive in phase without any path difference. A choice of three strips $A$, $B$ and $C$ of transparent; materials with different thicknesses and refractive indices is available, as shown in the table, these are placed over one or both of the slits, singularly or in conjunction, causing the interference pattern to be, shifted across the screen from the original pattern. In the column-I, how the strips have been placed, is mentioned whereas in the column-II, order of the fringe at point $P$ on the screen that will be produced due to the placement of the strip(s), is shown. Correctly match both the column.




| Film | $A$ | $B$ | $C$ |
|---|---|---|---|
| Thickness (in mm) | 5 | 1.5 | 0.25 |
| Refractive index | 1.5 | 2.5 | 2 |

| Column I | Column II |
|---|---|
| i. Only strip $B$ is placed over slit-I | a. First bright |
| ii. Strip $A$ is placed over slit-I and strip $C$ is placed over slit-II | b. Fourth dark |
| iii. Strip $X$ is placed over the slit-I and strip $B$ and strip $C$ are placed over the slit-II in conjunction. | c. Fifth dark |
| iv. Strip $A$ and strip $C$ are placed over slit-I (in conjuction) and strip $B$ is placed over slit-II. | d. Central bright |
| | e. First dark |

(1) i.–c.; ii.–c.; iii.–d.; iv.–a.

(2) i.–e.; ii.–c.; iii.–d.; iv.–d.

(3) i.–a.; ii.–b.; iii.–c.; iv.–e.

(4) i.–c.,d.; ii.–e.; iii.–d.; iv.–b.

**7.** Two waves from coherent sources meet at a point in a path difference of $\Delta x$. Both the waves have same intensities. Match the following two columns.

| Column I | Column II |
|---|---|
| i. If $\Delta x = \frac{\lambda}{3}$ | a. Resultant intensity will become three times |
| ii. If $\Delta x = \frac{\lambda}{6}$ | b. Resultant intensity will remain same |
| iii. If $\Delta x = \frac{\lambda}{4}$ | c. Resultant intensity will become two times |
| iv. If $\Delta x = \frac{\lambda}{2}$ | d. Resultant intensity will become zero |

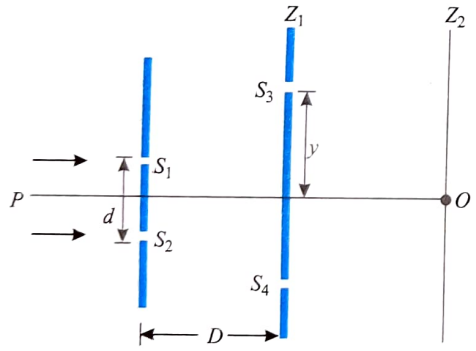
(1) i.–d.; ii.–a.; iii.–c.; iv.–b.

(2) i.–b.; ii.–a.; iii.–c.; iv.–d.

(3) i.–a.; ii.–b.; iii.–d.; iv.–c.

(4) i.–d.; ii.–c.; iii.–a.; iv.–b.

**8.** In the figures shown, $Z_1$ and $Z_2$ are two screens. Lines $PO$ is the bisector line of $S_1S_2$ and $S_3S_4$. When $Z_1$ is removed, resultant intensity at $O$ due to slits $S_1$ and $S_2$ is $I$. Now, $Z_1$ is placed. For different values of $y$ given in column I, match the resultant intensity at $O$ given in column II.




| Column I | Column II |
|---|---|
| i. $y = \frac{\lambda D}{2d}$ | a. $3I$ |
| ii. $y = \frac{\lambda D}{6d}$ | b. Zero |
| iii. $y = \frac{\lambda D}{4d}$ | c. $I$ |
| iv. $y = \frac{\lambda D}{3d}$ | d. None of these |

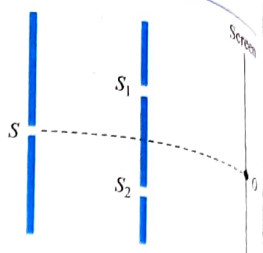
(1) i.–c.; ii.–a.; iii.–d.; iv.–b.

(2) i.–d.; ii.–c.; iii.–a.; iv.–b.

(3) i.–a.; ii.–b.; iii.–c.; iv.–d.

(4) i.–b.; ii.–a.; iii.–d.; iv.–c.

**9.** Figure shows a set up to perform Young's double slit experiment. A monochromatic source of light is placed at $S$, $S_1$ and $S_2$ and act as coherent sources and interference pattern is obtained on the screen.




| Column I | Column II |
|---|---|
| i. A thin transparent plate is placed in front of $S_1$ | a. Interference fringes disappear |
| ii. $S_1$ is closed | b. There is a uniform illumination on a large part of the screen |
| iii. A thin transparent plate is placed in front of $S_2$ | c. The zero order fringe will not form at $O$ |
| iv. $S$ is removed and two different sources emitting light of same wavelength are placed at $S_1$ and $S_2$ | d. Central maxima is formed below $O$ |

(1) i.–a., b.; ii.–c., d.; iii.–b., c.; iv.–b., d.

(2) i.–a.; ii.–c.; iii.–d.; iv.–b.

(3) i.–c.; ii.–a., b.; iii.–c., d.; iv.–a., b.

(4) i.–b., c.; ii.–a., b.; iii.–c.; iv.–a.

## Numerical Value Type

**1.** In Young's double slit experiment $\lambda = 500$ nm, $d = 1$ mm, and $D = 4$ m. The minimum distance from the central maximum for which the intensity is half of the maximum intensity is '*' $\times 10^{-4}$ m. What is the value of '*'?

**2.** A monochromatic light of $\lambda = 500$ Å is incident on two identical slits separated by a distance of $5 \times 10^{-4}$ m. The interference pattern is seen on a screen placed at a distance of 1 m from the plane of slits. A thin glass plate of thickness $1.5 \times 10^{-6}$ m and refractive index $\mu = 1.5$ is placed between one of the slits and the screen. Find the intensity at the centre of the slit now.

**3.** A glass of refractive index 1.5 is coated with a thin layer of thickness of $t$ of refractive index 1.8. Light of wavelength $\lambda$ travelling in air is incident normally on the layer. It is partly reflected at the upper and the lower surfaces of the layer and the two reflected rays interfere. If $\lambda = 648$ nm, obtain the least value of $t$ (in $10^{-8}$ m) for which the rays interfere constructively.

**4.** In YDSE, find the thickness (in μm) of a glass slab ($\mu = 1.5$) which should be placed before the upper slit $S_1$ so that the central maximum now lies at a point where 5th bright fringe was lying earlier (before inserting the slab). Wavelength of light used is 5000 Å.

**5.** A monochromatic beam of light of wavelength 5000 Å is used in Young's double slit experiment. If one of the slits is covered by a transparent sheet of thickness $1.4 \times 10^{-5}$ m,

having refractive index of its medium 1.25. Then the number of fringes shifted is

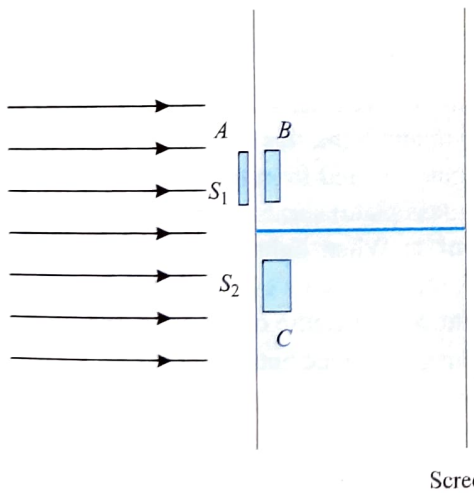
6. In Young's double-slit experiment, the two slits act as coherent sources of equal amplitude $A$ and of wavelength $\lambda$. In another experiment with the same setup, the two slits are sources of equal amplitude $A$ and wavelength $\lambda$, but are incoherent. Find the ratio of intensity of light at the midpoint of the screen in the first case to that in the second case.

7. Interference fringes are produced by a double-slit experiment and a piece of plane parallel glass of refractive index 1.5 is interposed in one of the interfering beams. If the fringes are displaced through 30 fringe widths for light of wavelength $6 \times 10^{-5}$ cm, find the thickness of the plate (in μm).

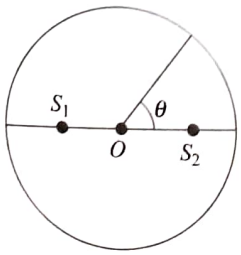
8. In YDSE, find the thickness of a glass slab (in μm) which should be placed before the upper slit so that the central maximum now lies at a point where 5th bright fringe was lying earlier (before inserting the slab). Wavelength of light used is 5000 Å. The refractive index of material of glass plate is $\mu = 1.5$.

9. Bichromatic light is used in YDSE having wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 700$ nm. Find minimum order of $\lambda_1$ which overlaps with $\lambda_2$.

10. Three slabs $A$, $B$ and $C$ are placed in front of slits in YDSE as shown in figure. Refractive index of slabs $A$, $B$ and $C$ are 4/3, 5/3 and 8/3 respectively and thickness of $A$, $B$ and $C$ are 2 mm, 1 mm and 1.5 mm respectively. A liquid of refractive index $2\mu/3$ is filled between slits and screen. A monochromatic light of wavelength 5000 Å is incident on the slits. If the central bright fringe is obtained at $C$ then find the value of 'μ'.

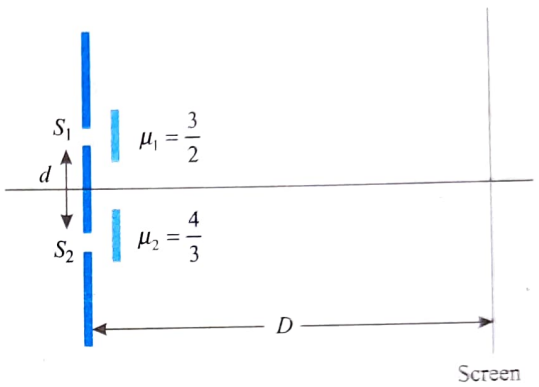



11. Two coherent sources $S_1$ and $S_2$ separated by distance $2\lambda$ emit light of wavelength $\lambda$ in phase as shown in the figure. A circular wire of radius $100\lambda$ is placed in such a way that $S_1S_2$ lies in its plane and the mid-point of $S_1S_2$ is at the centre of wire. The angular positions on the wire for which intensity reduces to half of its maximum value for the first time is given as θ. Find the value of cosθ.




12. One slit of a double slit experiment is covered by a thin glass plate of refractive index 1.4 and the other by a thin glass plate of refractive index 1.7. The point on the screen, where central bright fringe was formed before the introduction of the glass sheets, is now occupied by the 5th bright fringe. Assuming that both the glass plates have the same thickness and wavelength of light used is 4800 Å, find their thickness (in μm).

13. A monochromatic beam of light of 6000Å is used in YDSE set-up. The two slits are covered with two thin films of equal thickness $t$ but of different refractive indices as shown in figure. Considering the intensity of the incident beam on the slits to be $I_0$, find the point on the screen at which intensity is $I_0$ and is just above the central maxima. (Assume that there is no change in intensity of the light after passing through the films.)




Consider $t = 6$ μm, $d = 1$ mm, and $D = 1$ m, where $d$ and $D$ have their usual meaning. Give your answer in mm.

14. A double slit of separation 1.5 mm is illuminated by white light (between 4000 and 8000 Å). On a screen 120 cm away colored interference pattern is formed. If a pinhole is made on this screen at a distance of 3.0 mm from the central white fringe, some wavelengths will be absent in the transmitted light. Find the second longest wavelength (in Å) which will be absent in the transmitted light.

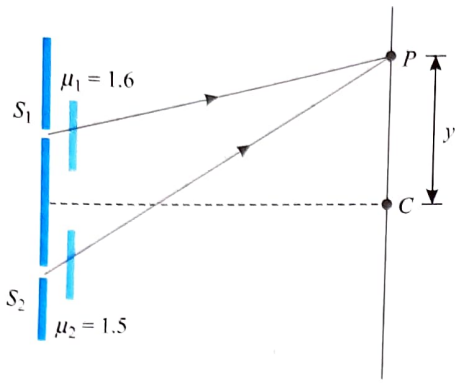
15. A monochromatic light of $\lambda = 5000$ Å is incident on two slits separated by a distance of $5 \times 10^{-4}$ m. The interference pattern is seen on a screen placed at a distance of 1 m from the slits. A thin glass plate of thickness $1.5 \times 10^{-6}$ m and refractive index $\mu = 1.5$ is placed between one of the slits and the screen. Find the intensity at the centre of the screen, if the intensity there is $I_0$ in the absence of the plate. Also find the lateral shift of the central maximum (in mm).

16. In a Biprism experiment with sodium light, bands of width 0.0195 cm are observed at 100 cm from slit. On introducing a convex lens 30 cm away from the slit between biprism and screen, two images of the slit are seen 0.7 cm apart at 100 cm distance from the slit. Calculate the wavelength of sodium light (in Å).

17. A lens ($\mu = 1.5$) is coated with a thin film of refractive index 1.2 in order to reduce the reflection from its surface at $\lambda = 4800$ Å. Find the minimum thickness of the film (in μm) which will minimize the intensity of the reflected light.

18. Consider two cases in Young's double-slit experiment. **In case (a):** the amplitude of source $S_1$ is three times the

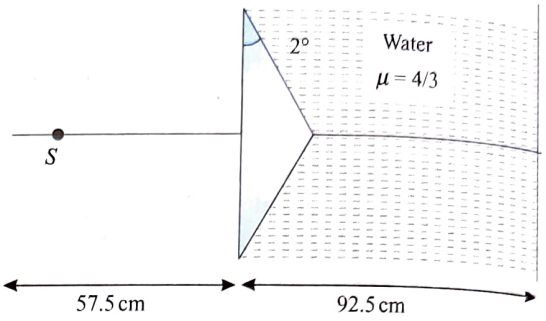
amplitude of the source $S_2$. These sources are covered by perfectly transparent thin plates of same thickness but different refractive indices 1.6 and 1.5, respectively. The central maxima is found to formed at point $P$ on the screen, the intensity at $P$ is found to be $I_1$.




**In case (b):** The plates are interchanged and the amplitude of source $S_1$ is made same as that of $S_2$ the intensity at $P$ is found to be $I_2$.

Then find the ratio of intensity at $P$, i.e., $\frac{I_1}{I_2}$. Take thickness of the plate equal to $(110/3)\lambda$, where $\lambda$ is the wavelength of light used.

**19.** Figure shows a modified Fresnel biprism experiment with monochromatic source of wavelength 500 nm. The refracting angle of glass prism is 2°. Find the fringe width (in mm).




# Archives

## JEE Main

### Single Correct Answer Type

**1.** A mixture of lights, consisting of wavelength 590 nm and an unknown wavelength illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of the known light coincides with the fourth bright fringe of the unknown light. From this data, the wavelength of the unknown light is

(1) 393.4 nm (2) 885.0 nm
(3) 442.5 nm (4) 776.8 nm

**(AIEEE 2009)**

**Statement for Problems 2–4**

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where $\mu_0$ and $\mu_2$ are positive constants and $I$ is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius. **(AIEEE 2010)**

**2.** As the beam enters the medium, it will

(1) diverge
(2) converge
(3) diverge near the axis and converge near the periphery
(4) travel as a cylindrical beam

**3.** The initial shape of the wave front of the beam is

(1) convex
(2) concave
(3) convex near the axis and concave near the periphery
(4) planar

**4.** The speed of light in the medium is

(1) minimum on the axis of the beam
(2) the same everywhere in the beam
(3) directly proportional to the intensity $I$
(4) maximum on the axis of the beam

**5. Direction:** The question has a paragraph followed by two statements, Statement 1 and Statement 2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

**Statement 1:** When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of $\pi$.

**Statement 2:** The centre of the interference pattern is dark.

(1) Statement 1 is true but statement 2 is false.
(2) Statement 1 is true, statement 2 is true, statement 2 is the correct explanation of statement 1.
(3) Statement 1 is true, statement 2 is true, statement 2 is not the correct explanation of statement 1.
(4) Statement 1 is false but statement 2 is true.

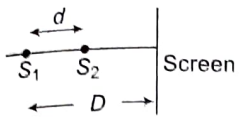
**(AIEEE 2011)**

**6.** In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If $I_m$ be the maximum intensity, the resultant intensity $I$ when they interfere at phase difference $\phi$ is given by

(1) $\frac{I_m}{9}(4 + 5\cos\phi)$ (2) $\frac{I_m}{3}\left(1 + 2\cos^2\frac{\phi}{2}\right)$
(3) $\frac{I_m}{5}\left(1 + 4\cos^2\frac{\phi}{2}\right)$ (4) $\frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$

**(AIEEE 2012)**

7. Two coherent point sources $S_1$ and $S_2$ are separated by a small distance $d$ as shown. The fringes obtained on the screen will be




(1) straight lines (2) semi-circles
(3) concentric circles (4) points

**(JEE Main 2013)**

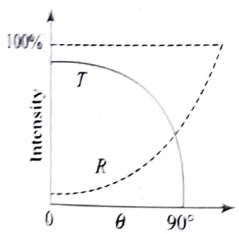
8. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygen's principle leads us to conclude that as it travels, the light beam
(1) becomes narrower
(2) goes horizontally without any deflection
(3) bends downwards
(4) bends upwards **(JEE Main 2015)**

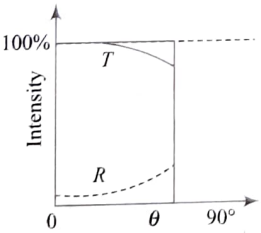
9. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is
(1) 9.75 mm
(2) 15.6 mm
(3) 1.56 mm
(4) 7.8 mm **(JEE Main 2017)**

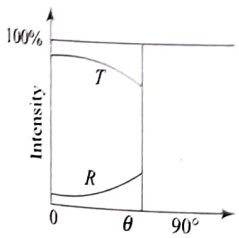
## JEE ADVANCED

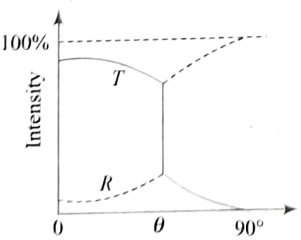
### Single Correct Answer Type

1. A light ray traveling in glass medium is incident on glass-air interface at an angle of incidence $\theta$. The reflected ($R$) and transmitted ($T$) intensities, both as function of $\theta$, are plotted. The correct sketch is

(1)



(2)



(3)



(4)



**(IIT-JEE 2011)**

2. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are $\beta_G$, $\beta_R$ and $\beta_B$, respectively. Then,
(1) $\beta_G > \beta_B > \beta_R$ (2) $\beta_B > \beta_G > \beta_R$
(3) $\beta_R > \beta_B > \beta_G$ (4) $\beta_R > \beta_G > \beta_B$.

**(IIT-JEE 2012)**

3. In Young's double-slit experiment using a monochromatic light of wavelength $\lambda$, the path difference (in terms of an integer $n$) corresponding to any point having half the peak intensity is
(1) $(2n+1)\frac{\lambda}{2}$ (2) $(2n+1)\frac{\lambda}{4}$
(3) $(2n+1)\frac{\lambda}{8}$ (4) $(2n+1)\frac{\lambda}{16}$

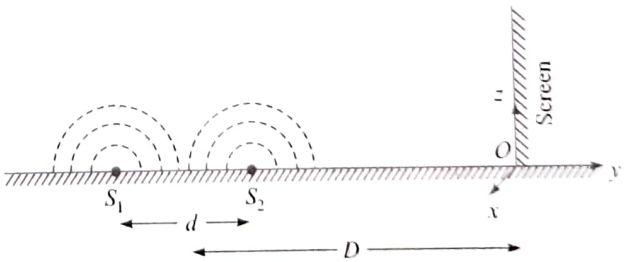
**(JEE Advanced 2013)**

### Multiple Correct Answers Type

1. A light source, which emits two wavelengths $\lambda_1 = 400$ nm and $\lambda_2 = 600$ nm, is used in a Young's double slit experiment. If recorded fringe widths for $\lambda_1$ and $\lambda_2$ are $\beta_1$ and $\beta_2$ and the number of fringes for them within a distance $y$ on one side of the central maximum are $m_1$ and $m_2$, respectively, then
(1) $\beta_2 > \beta_1$
(2) $m_1 > m_2$
(3) From the central maximum, 3rd maximum of $\lambda_2$ overlaps with 5th minimum of $\lambda_1$
(4) The angular separation of fringes for $\lambda_1$ is greater than $\lambda_2$

**(JEE Advanced 2014)**

2. While conducting the Young's double slit experiment, a student replaced to two slits with a large opaque plate in the $x$-$y$ plane containing two small holes that act as two coherent point sources ($S_1$, $S_2$) emitting light of wavelength 600 nm. The student mistakenly placed the screen parallel to the $x$-$z$ plane (for $z > 0$) at a distance $D = 3$ m from the mid-point of $S_1S_2$, as shown schematically in the figure. The distance between the sources $d = 0.6003$ mm. The origin $O$ is at the intersection of the screen and the line joining $S_1S_2$. Which of the following is(are) true of the intensity pattern on the screen?

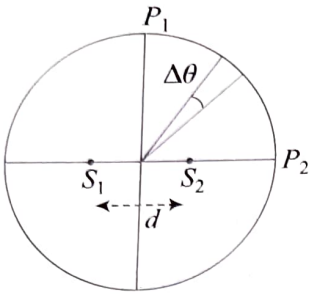



(1) Semicircular bright and dark bands centered at point $O$
(2) Hyperbolic bright and dark bands with foci symmetrically placed about $O$ in the $x$-direction
(3) The region very close to the point $O$ will be dark
(4) Straight bright and dark bands parallel to the $x$-axis

**(JEE Advanced 2016)**

3. Two coherent monochromatic point sources $S_1$ and $S_2$ of wavelength $\lambda = 600$ nm are placed symmetrically on either

side of the centre of the circle as shown. The sources are separated by a distance $d = 1.8$ mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?




(1) A dark spot will be formed at the point $P_2$

(2) The angular separation between two consecutive bright spots decreases as we move from $P_1$ to $P_2$ along the first quadrant

(3) At $P_2$ the order of the fringe will be maximum

(4) The total number of fringes produced between $P_1$ and $P_2$ in the first quadrant is close to 3000

**(JEE Advanced 2017)**

## Linked Comprehension Type

### For Problems 1–2

Most material have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $\left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, $v$ its speed in the medium, $\varepsilon_r$ and $\mu_r$ are the relative permittivity and permeability of the medium respectively. In normal materials, both $\varepsilon_r$ and $\mu_r$ are positive, implying positive $n$ for the medium. When both $\varepsilon_r$ and $\mu_r$ are negative, one must choose the negative root of $n$. Such negative refractive index material can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since $n$ is negative, it result in a change in the direction of proportion of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refractive even in meta-materials. **(IIT-JEE 2012)**

**1.** Choose the correct statement.

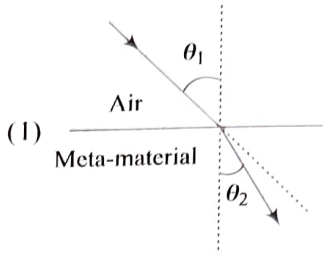
(1) The speed of light in the meta-material is $v = c\,|n|$

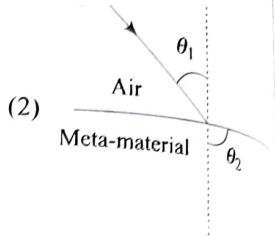
(2) The speed of light in the meta-material is $v = \frac{c}{|n|}$

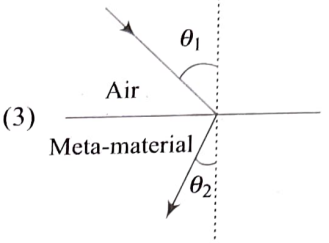
(3) The speed of light in the meta-material is $v = c$

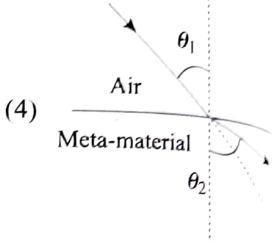
(4) The wavelength of the light in the meta-material ($\lambda_m$) is given by $\lambda_m = \lambda_{air}\,|n|$, where $\lambda_{air}$ is the wavelength of the light in air.

**2.** For light incident from air on a meta-material, the appropriate ray diagram is

(1)



(2)


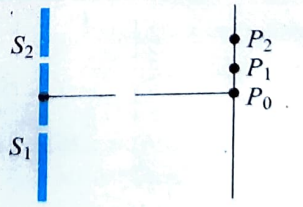
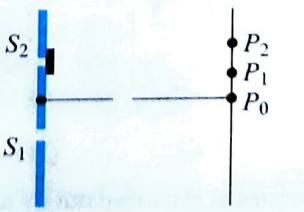

(3)



(4)


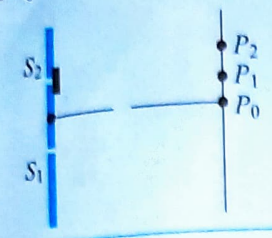

## Matrix Match Type

The question given in this section contains statements given in two columns, which have to be matched.

**1.** Column I shows four situations of standard Young's double-slit arrangement with the screen placed far away from the slits $S_1$ and $S_2$. In each of these cases, $S_1P_0 = S_2P_0$, $S_1P_1 - S_2P_1 = \lambda/4$, and $S_1P_2 - S_2P_2 = \lambda/3$, where $\lambda$ is the wavelength of the light used. In the cases *b*, *c*, and *d*, a transparent sheet of refractive index $\mu$ and thickness $t$ is pasted on slit $S_2$. The thicknesses of the sheets are different in the three cases. The phase difference between the light waves reaching point $P$ on the screen from the two slits is denoted by $\delta(P)$ and the intensity by $I(P)$. Match each situation given in Column I with the statement(s) in Column II valid for that situation.

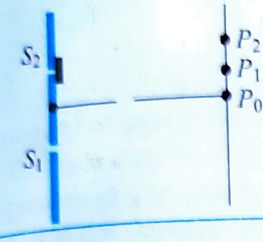
| Column I | Column II |
|---|---|
| i.   | a. $\delta(P_0) = 0$ |
| ii. $(\mu - 1)t = \lambda/4$   | b. $\delta(P_1) = 0$ |

iii. $(\mu - 1)t = \lambda/2$




c. $I(P_1) = 0$

iv. $(\mu - 1)t = 3\lambda/4$



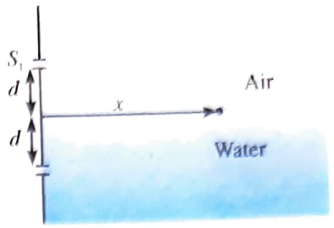

d. $I(P_0) > I(P_1)$

e. $I(P_2) > I(P_1)$

**(IIT-JEE 2009)**

## Numerical Value Type

1. A Young's double slit interference arrangement with slits $S_1$ and $S_2$ is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where $\lambda$ is the wavelength of light in air (refractive index = 1). $2d$ is the separation between the slits and $m$ is an integer. The value of $p$ is _____.




**(JEE Advanced 2015)**

# Answers Key

## EXERCISES

### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| 1. (4) | 2. (1) | 3. (1) | 4. (1) | 5. (4) |
| 6. (1) | 7. (4) | 8. (3) | 9. (2) | 10. (4) |
| 11. (1) | 12. (1) | 13. (1) | 14. (4) | 15. (2) |
| 16. (2) | 17. (3) | 18. (3) | 19. (3) | 20. (2) |
| 21. (1) | 22. (1) | 23. (2) | 24. (4) | 25. (2) |
| 26. (4) | 27. (1) | 28. (1) | 29. (1) | 30. (1) |
| 31. (2) | 32. (3) | 33. (3) | 34. (4) | 35. (3) |
| 36. (2) | 37. (1) | 38. (2) | 39. (1) | 40. (3) |
| 41. (2) | 42. (2) | 43. (1) | 44. (2) | 45. (1) |
| 46. (1) | 47. (4) | 48. (1) | 49. (4) | 50. (3) |
| 51. (1) | 52. (3) | 53. (1) | 54. (1) | 55. (1) |
| 56. (1) | 57. (1) | 58. (3) | 59. (3) | 60. (4) |
| 61. (1) | 62. (1) | 63. (2) | 64. (1) | 65. (4) |
| 66. (2) | 67. (1) | 68. (3) | 69. (1) | 70. (4) |
| 71. (3) | 72. (2) | 73. (3) | 74. (3) | 75. (3) |
| 76. (1) | 77. (3) | 78. (1) | 79. (3) | 80. (3) |
| 81. (3) | 82. (1) | 83. (3) | 84. (2) | 85. (4) |
| 86. (2) | 87. (2) | 88. (1) | 89. (1) | |

### Multiple Correct Answers Type

| | | |
|---|---|---|
| 1. (1),(3) | 2. (1),(3) | 3. (1),(3) |
| 4. (1),(3),(4) | 5. (1),(3) | 6. (1),(2),(3),(4) |
| 7. (2),(4) | 8. (1),(2),(3) | 9. (1),(3),(4) |
| 10. (3),(4) | 11. (2),(4) | 12. (2),(4) |
| 13. (2),(3) | 14. (2),(4) | 15. (1),(4) |
| 16. (1),(2),(3) | 17. (2),(3),(4) | 18. (1),(3) |
| 19. (2),(3) | 20. (2),(3) | 21. (2),(3) |
| 22. (1),(3) | 23. (1),(3) | 24. (1),(3),(4) |

### Linked Comprehension Type

| | | | | |
|---|---|---|---|---|
| 1. (1) | 2. (3) | 3. (1) | 4. (3) | 5. (4) |
| 6. (1) | 7. (3) | 8. (2) | 9. (1) | 10. (2) |
| 11. (4) | 12. (1) | 13. (2) | 14. (1) | 15. (3) |
| 16. (2) | 17. (1) | 18. (2) | 19. (4) | 20. (1) |
| 21. (3) | 22. (2) | 23. (1) | 24. (3) | 25. (4) |
| 26. (4) | 27. (1),(3),(4) | | 28. (4) | 29. (2),(3) |
| 30. (1) | 31. (2) | 32. (2) | 33. (4) | 34. (4) |
| 35. (2) | 36. (4) | 37. (3) | 38. (1) | 39. (3) |
| 40. (2) | | | | |

### Matrix Match Type

1. i. → c.; ii. → c.; iii. → d.; iv. → a.
2. i. → b., c., d.; ii. → a., b., c., d.; iii. → b., c., d.; iv. → a.,b., c., d.
3. i. → b., c.; ii. → b., d.; iii. → a., d.; iv. → a., b., c., d.
4. i. → a.; ii. → b.; iii. → c.; iv. → d.
5. i. → d.; ii. → a.; iii. → b.; iv. → c.
6. (1) 7. (2) 8. (4) 9. (3)

### Numerical Value Type

| | | | | |
|---|---|---|---|---|
| 1. (5) | 2. (0) | 3. (9) | 4. (5) | 5. (7) |
| 6. (2.00) | 7. (36.00) | 8. (5.00) | 9. (0.700) | 10. (5.00) |
| 11. (0.125) | 12. (8.00) | 13. (30.00) | 14. (5769) | 15. (1.50) |
| 16. (5850) | 17. (0.100) | 18. (16) | 19. (0.1125) | |

## ARCHIVES

### JEE Main

#### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| 1. (3) | 2. (2) | 3. (4) | 4. (1) | 5. (2) |
| 6. (4) | 7. (3) | 8. (4) | 9. (4) | |

### JEE Advanced

#### Single Correct Answer Type

1. (3) 2. (4) 3. (2)

#### Multiple Correct Answers Type

1. (1),(2),(3) 2. (1),(3) 3. (3),(4)

#### Linked Comprehension Type

1. (2) 2. (3)

#### Matrix Match Type

1. i. → a., d.; ii. → b.; iii. → e.; iv. → c., d., e.

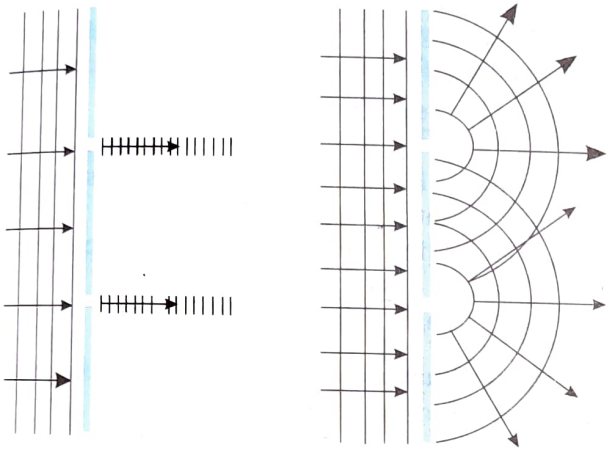
#### Numerical Value Type

1. (3)

# 4 Diffraction and Polarization

## INTRODUCTION

The basic property of light is its travelling in a straight line. Because of this, light casts the shadow of the object coming in its path. In previous chapter, we learned if light passes through a pair of narrow slits it shows interference phenomenon. In fact, the light waves from the two slits overlap as they spread out, filling what we expect to be shadowed regions with light and producing interference fringes. If light waves did not spread out after passing through the slits, no interference would occur.

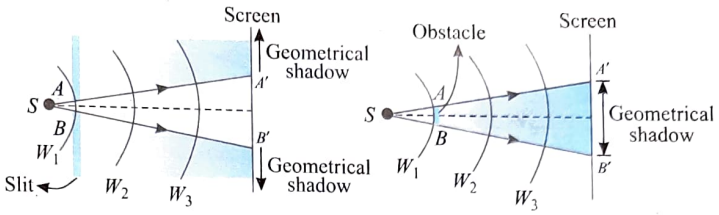


(a) (b)

Also close to a sharp edge of an opaque obstacle or through a narrow slit, we do observe some deviation of light from its straight line path.

(i) Let us consider a source of light $S$ placed at a short distance from a narrow (circular) aperture $AB$. Let a screen be placed at some distance from the aperture [Fig. (a)], only the part $A'B'$ of the screen should get illuminated and no light should enter the regions $A'X$ and $B'Y$ of the screen as shown in Fig. (a). But we do not find complete darkness in these regions. $A'B'$ is the portion on the screen which is illuminated by light passing through the aperture. Beyond $A'$ and B′ on both the sides is the region of the geometrical shadow. If $W_1$ represents a primary wavefront falling on the aperture and if secondary wavefronts $W_2$, $W_3$, ... are drawn according to Huygens' principle, it is found that some portions of these wavefronts would go into the geometrical shadow beyond $A'$ and $B'$. The shadow is, therefore, not sharp.

(ii) Similarly, if an **opaque (circular) obstacle** $AB$ is placed between the source $S$ and the screen [Fig. (b)], some portions of the secondary wavefronts $W_2$ and $W_3$ etc. will enter the region of geometrical shadow $A'B'$ on the screen and as the shadow is again not sharp. But this illumination of the geometrical shadow will be observed only if the aperture is very narrow or if the size of the obstacle is very small.




(a) (b)

The phenomenon of bending of light round the sharp corners and spreading into the regions of the geometrical shadow is called **diffraction**.

## FRESNEL AND FRAUNHOFER DIFFRACTIONS

Before discussing how diffraction patterns are formed, we must briefly explain the distinction between Fresnel and Fraunhofer diffractions.

1. **Fresnel diffraction:** The source and the screen are placed at finite distances from the obstacle (or slit) and no lenses are used. Since the source of light is close to the slit, the wavefront is either spherical (in case of point source) or cylindrical (in case of a line source) in nature. The screen is also at a finite distance from the slit. This is also called **near-field diffraction**.

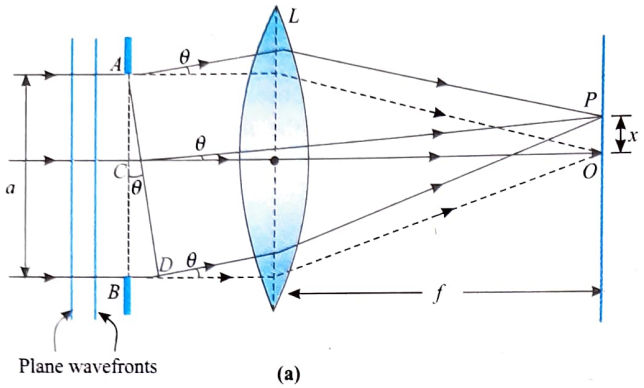
   Fresnel gave the exact explanation of the phenomenon of diffraction. According to him, diffraction is due to the interference of secondary wavelets originating from the portion of the primary wavefront, which is not blocked by the slit or the obstacle.

2. **Fraunhofer diffraction:** In Fraunhofer diffraction, the incident waves as well as the diffracted waves can be approximated to have plane wavefronts. This can be achieved by placing the source and the screen far away from the obstacle or by using lenses so that the source and the screen are at focal planes. Fraunhofer diffraction is also called **far-field diffraction**. Owing to the very nature of the wavefronts, Fraunhofer diffraction (planar wavefronts) is much simpler to analyse than Fresnel diffraction (spherical wavefronts). Therefore, we shall limit ourselves to Fraunhofer diffraction.

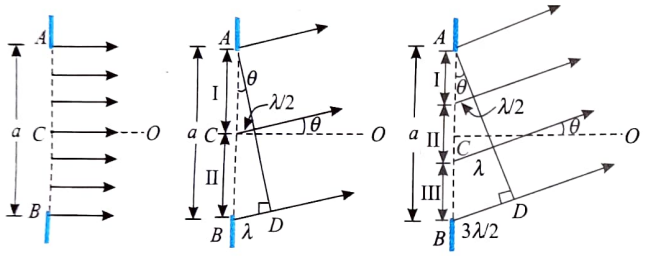
### FRAUNHOFER DIFFRACTION AT A SINGLE SLIT

Let a plane wavefront falling on a single narrow slit $AB$ of width '$a$' [Fig. (a)]. According to our knowledge of that light travels in straight path, we should expect a sharp image of the slit on

the screen. But this is not the case because light travels only approximately in straight lines and is diffracted (i.e., spreads out) into the regions above $A$ and below $B$ where we do not expect it. The simple laws of geometrical optics do not apply and as such instead of a sharp image of the slit, we get the central image [Fig. (a)] flanked by a series of dark and bright fringes (or bands). The intensity of the bright bands goes on decreasing as we proceed outwards from the central band either way. This has been explained as follows.




(a)




(b)

According to Huygens' hypothesis, each point on the plane wavefront falling on $AB$ gives rise to secondary wavelets which are initially in the same phase. These wavelets spread out in all directions.

(a) **Position of central maximum:** According to Huygens' principle, when light falls on the slit, it becomes a source of secondary wavelets. These wavelets are initially in phase and spread out in all directions. Let $C$ be the centre of the slit $AB$. The wavelets which fall on the lens in a direction parallel to $CO$ meet at point $O$ in the same phase. This is because, the wavelets are initially in phase and their optical paths between the slit and the point $O$ are also equal. Therefore, the wavelets reinforce each other and give rise to the central maximum at point $O$.

(b) **Positions of minima:** Consider a point $P$ on the screen at which wavelets travelling in a direction making angle $\theta$ with $CO$ are brought to focus by the lens. The wavelets from different parts of the slit do not reach point $P$ in phase, although they are initially in phase. It is because, they cover unequal distances in reaching the point $P$. The wavelets from points $A$ and $B$ will have a path difference equal to $BD$.

$$BD = a\sin\theta \qquad \left(\text{as } \sin\theta = \frac{BD}{AB} = \frac{BD}{a}\right)$$

(i) Let the point $P$ be so situated on the screen that $a\sin\theta = \lambda$, where $\lambda$ is the wavelength of the incident monochromatic light. In this case, the path difference for the wavelets from $A$ and $C$ and similarly for those from $C$ and $B$ will be $\lambda/2$, where $C$ is the midpoint of the slit.

(ii) If we imagine the slit $AB$ to be divided into two halves $AC$ and $CB$ (denoted by I and II) as shown in Fig. [(b) (ii)], then for every point in the upper half $AC$ (i.e., Part I), there is a corresponding point in the lower half $CB$ (i.e., Part II), for which the path difference is $\lambda/2$. The wavelets from each pair of corresponding points in the two halves will reach $P$ in opposite phases and obviously will interfere destructively with each other and produce the **first minimum or a dark band** there. Thus, for the first minimum,

$$a\sin\theta = \lambda$$

or $$\sin\theta = \frac{\lambda}{a} \qquad \ldots(i)$$

Since $\theta$ is small, $\sin\theta \approx \theta$ and as such

$$\theta = \frac{\lambda}{a} \qquad \ldots(ii)$$

Proceeding on the same lines, it can be shown that the condition for the $n^{th}$ minimum is

$$\theta_n = \frac{n\lambda}{a} \qquad \ldots(iii)$$

where $n = 1, 2, 3, \ldots$, but not zero (as that is the condition for the central maximum)

Thus, the first, second third,...minima occur when

$$\theta_1 = \frac{\lambda}{a}, \theta_2 = \frac{2\lambda}{a}, \theta_3 = \frac{3\lambda}{a} \ldots \qquad \ldots(iv)$$

(c) **Positions of maxima:** Let point $P$ on the screen be situated such that the path difference

$$a\sin\theta = \frac{3\lambda}{2} \qquad \ldots(v)$$

Now consider the slit width to be divided into three equal parts as shown in Fig. [(b) (iii)] so that the path difference between the wavelets from the first two parts will again be $\lambda/2$. These wavelets will interfere destructively and cancel each other's effect. The wavelets from the third part will remain uncancelled. Since all of them are in the same phase, they reinforce each other and thus produce the first secondary maximum. Therefore, for the first secondary maximum, from Eq. (v),

$$\sin\theta = \frac{3\lambda}{2a}$$

or $$\theta = \frac{3\lambda}{2a} \quad (\text{as } \sin \approx \theta, \text{ where } \theta \text{ is small}) \qquad \ldots(vi)$$

Proceeding in the same way, it can be shown that the condition for the $n^{th}$ secondary maximum is

$$\theta_n' = (2n+1)\frac{\lambda}{2a} \qquad \ldots(vii)$$

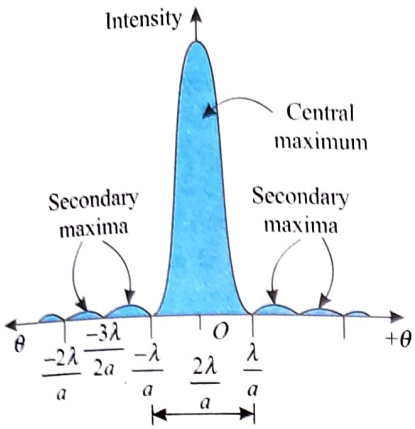
where $n = 1, 2, 3, \ldots$ and not zero.

Thus, the first, second, third...secondary maximum occur when

$$\theta_1 = \frac{3\lambda}{2a}, \theta_2 = \frac{5\lambda}{2a}, \theta_3 = \frac{7\lambda}{2a} \ldots$$

The relative intensities of the various maxima have been shown in Fig. (b).

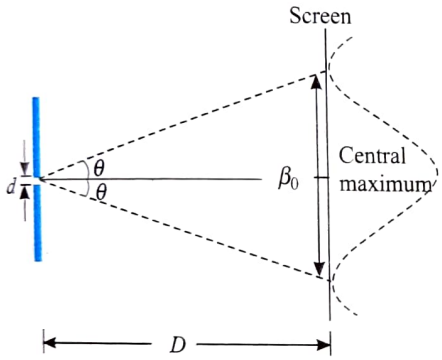
(a)




(b)

## Angular Width of Central Maximum

The angular width of the central maximum is the angular separation between the directions of the first minima on the two sides of the central maximum, as shown in figure.




The directions of first minima on either side of central maximum are given by $\theta = \lambda/a$. This angle is called half angular width of central maximum.

$\therefore$ Angular width of central maximum $= 2\theta = \dfrac{2\lambda}{a}$

## Linear Width of Central Maximum

If $D$ is the distance of the screen from the single slit, then the linear width of central maximum will be

$$\beta_0 = D \times 2\theta = \frac{2D\lambda}{a}$$

$$\left[2\theta(\text{rad}) = \frac{\text{Arc}}{\text{Radius}} = \frac{\beta_0}{D}\right]$$

## Linear Width of a Secondary Maximum

The angular width of $n^{th}$ secondary maximum is the angular separation between the directions of $n^{th}$ and $(n+1)^{th}$ minima.

Direction of $n^{th}$ minimum, $\theta_n = n\dfrac{\lambda}{a}$

Direction of $(n+1)^{th}$ minimum, $\theta_{n+1} = (n+1)\dfrac{\lambda}{a}$

$\therefore$ Angular width of $n^{th}$ secondary maximum

$$= \theta_{n+1} - \theta_n = (n+1)\frac{\lambda}{a} - n\frac{\lambda}{a} = \frac{\lambda}{a}$$

Hence, the linear width of $n^{th}$ secondary maximum = Angular width $\times D$

or $$\beta = \frac{D\lambda}{a}$$

Clearly, $\beta_0 = 2\beta$

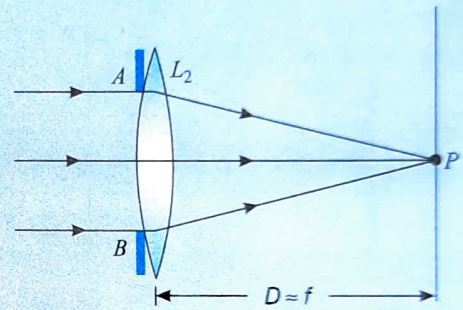
Thus the central maximum of a diffraction pattern is twice as wide as any secondary maximum.

Clearly, width of a secondary maximum $\propto \dfrac{1}{\text{slit width}}$

As the slit width is increased, the secondary maxima get narrower. If the slit is sufficiently wide, the secondary maxima disappear and only the central maximum is obtained which is the sharp image of the slit. Thus a distinct diffraction pattern is possible only if the slit is very narrow.

### Special Cases

Lens $L_2$ is shifted very near to slit $AB$. In this case distance between slit and screen will be nearly equal to the focal length of lens $L_2$ (i.e. $D \approx f$) $\theta_n = \dfrac{x_n}{f} = \dfrac{n\lambda}{a} \Rightarrow x_n = \dfrac{n\lambda f}{a}$




Width of central maxima $w_x = 2x = \dfrac{2\lambda f}{a}$ and angular width of central maxima $w_B = \dfrac{2x}{f} = \dfrac{2\lambda}{a}$

**Fringe width:** Distance between two consecutive maxima (bright fringe) or minima (dark fringe) is known as fringe width. Fringe width of central maxima is doubled then the width of other maxima i.e.,

$$\beta = x_{n+1} - x_n = (n+1)\frac{\lambda D}{a} - \frac{n\lambda D}{a} = \frac{\lambda D}{a}$$

### Important Points:

- Actually, there is no significant difference between an interference pattern and a diffraction pattern. Interference is usually referred to as the superposition of only a few (two, three or four) waves, whereas diffraction is concerned with the superposition of a very large number of waves.
- When $x = \dfrac{\lambda D}{d}$, a bright fringe is obtained in the interference pattern of two narrow slits distance $d$ apart whereas when $x = \dfrac{\lambda D}{d}$, first minimum (dark band) is obtained in the diffraction pattern of a single slit of width $a$.

### ILLUSTRATION 4.1

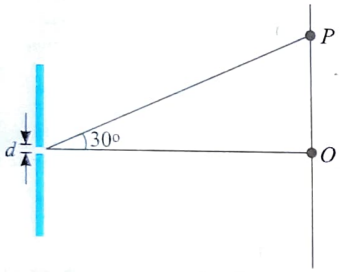
Why is the diffraction of sound waves more evident in daily experience than that of light wave?

**Sol.** The frequencies of sound waves lie between 20 Hz to 20 kHz , their wavelength ranges between 15 m to 15 mm. The diffraction occur if the wavelength of waves is nearly equal to slit width.

The wavelength of light waves is $7000 \times 10^{-10}$ m to $4000 \times 10^{-10}$ m. For observing diffraction of light we need very narrow slit width. In daily life experience we observe the slit width very near to the wavelength of sound waves as compared to light waves. Thus, the diffraction of sound waves is more evident in daily life than that of light waves.

### ILLUSTRATION 4.2

A slit of width '$d$' is illuminated by white light. For what value of '$d$' is the first minimum, for red light of $\lambda = 650$ nm, located at point $P$? For what value of the wavelength of light will the first diffraction maxima also fall at $P$?




**Sol.** For first minimum, $n = 1$

$$d \sin 30^\circ = \lambda$$

or $$d \times \frac{1}{2} = 650\,\text{nm} \quad \text{or} \quad d = 1300\,\text{nm}$$

For first maximum to lie at $P$, $d \sin\theta = \frac{3}{2}\lambda'$

or $$\lambda' = \frac{2d\sin\theta}{3} = \frac{2 \times 1300 \times \sin 30^\circ}{3} = 433.3\,\text{nm}$$

### ILLUSTRATION 4.3

A screen is placed 2 m away from the single narrow slit. Calculate the slit width if the first minimum lies 5 mm on either side of the central maximum. Incident plane waves have a wavelength of 5000 Å.

**Sol.** For the first minimum, $\theta = \frac{\lambda}{a}$ ...(i)

where $a$ is the slit width. Since the first minimum is 5 mm on either side of the central maximum on the screen and it subtends an angle $\theta$ at the slit 2 m away,

$$\theta = \frac{5\,\text{mm}}{2\,\text{m}} = \frac{5 \times 10^{-3}\,\text{m}}{2\,\text{m}} \qquad \left[\text{as } \theta = \frac{l}{r}\right]$$

or $$\theta = 2.5 \times 10^{-3}\,\text{rad} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), $\frac{\lambda}{a} = 2.5 \times 10^{-3}$

or $$a = \frac{\lambda}{2.5 \times 10^{-3}} = \frac{5000 \times 10^{-8}\,\text{m}}{2.5 \times 10^{-3}} = 2 \times 10^{-2}\,\text{m} = 0.02\,\text{m}$$

### ILLUSTRATION 4.4

A parallel beam of light of wavelength 600 nm is incident normally on a slit of width '$a$'. If the distance between the slits and the screen is 0.8 m and the distance of the second order maximum from the centre of the screen is 15 mm, calculate the width of the slit.

**Sol.** Here, $\lambda = 600\,\text{nm} = 6 \times 10^{-7}\,\text{m}$ , $D = 0.8$ m,
$n = 2$, $x_2 = 15$ mm $= 15 \times 10^{-3}\,\text{m}$

Distance of $n^{\text{th}}$ order secondary maximum from the centre of the screen, $x_n = \frac{(2n+1)\lambda D}{2a}$

Thus, $$x_2 = \frac{5\lambda D}{2a}$$

or $$a = \frac{5\lambda D}{2x_2} = \frac{5(6 \times 10^{-7}\,\text{m})(0.8\,\text{m})}{2(15 \times 10^{-3}\,\text{m})} = 8 \times 10^{-5}\,\text{m}$$

### ILLUSTRATION 4.5

Light of wavelength 550 nm is incident as parallel beam on a slit of width 0.1 mm. Find the angular width and the linear width of the principal maxima in the resulting diffraction pattern on a screen kept at a distance of 1.1 m from the slit. Which of these widths would not change if the screen were moved to a distance of 2.2 m from the slit?

**Sol.** Here $\lambda = 550\,\text{nm} = 550 \times 10^{-9}\,\text{m}$

$$a = 0.1\,\text{mm} = 0.1 \times 10^{-3}\,\text{m}, D = 1.1\,\text{m}$$

Angular width of principal maximum,

$$2\theta = \frac{2\lambda}{a} = \frac{2 \times 550 \times 10^{-9}}{0.1 \times 10^{-3}} = 1.1 \times 10^{-2}\,\text{rad}$$

Linear width of principal maximum,

$$\beta_0 = \frac{2D\lambda}{a} = 1.1 \times 0.011 = 0.0121\,\text{m} = 12.1\,\text{mm}$$

When the screen is moved to a distance of 2.2 m, the angular width would not change because it is independent of the distance $D$.

### ILLUSTRATION 4.6

Two wavelengths of sodium light 590 nm and 596 nm are used in turn to study the diffraction taking place at a single slit of aperture $2 \times 10^{-6}$ m. The distance between the slit and the screen is 1.5 m. Calculate the separation between the position of first maximum of the diffraction pattern obtained in the two cases.

**Sol.** For the first secondary maximum, $\theta = \frac{3\lambda}{2a}$

Denoting the wavelengths of the spectral lines $D_1$ and $D_2$ by $\lambda_1$ and $\lambda_2$,

$$\theta_1 = \frac{3\lambda_1}{2a} \text{ and } \theta_2 = \frac{3\lambda_2}{2a}$$

Clearly, $$\Delta\theta = (\theta_2 - \theta_1) = \frac{3\lambda_2}{2a} - \frac{3\lambda_1}{2a} = \frac{3(\lambda_2 - \lambda_1)}{2a}$$

or $$\Delta\theta = \frac{3(596 - 590) \times 10^{-9}}{2(2 \times 10^{-6})} = 4.5 \times 10^{-3}\,\text{rad}$$

Thus, spacing between the first maxima of two sodium lines
$= \Delta\theta \times (1.5) = (4.5\times10^{-3})(1.5) = 6.75\,\text{mm}$

## VALIDITY OF RAY OPTICS: FRESNEL'S DISTANCE

Geometrical optics is based upon the rectilinear propagation of light. Diffraction effects show that light does not travel in straight lines.

The distance of the screen from the slit, so that spreading of light due to diffraction from the centre of screen is just equal to size of the slit, is called **Fresnel distance**. It is denoted by $Z_F$.

The diffraction pattern of a slit consists of secondary maxima and minima on the two sides of the central maximum. Therefore, one can say that on diffraction from a slit, light spreads on the screen in the form of central maximum. The angular position of first secondary minimum is called **half angular width** of the central maximum and it is given by

$$\theta = \frac{\lambda}{d} \qquad \text{(provided } \theta \text{ is small)}$$

If the screen is placed at a distance $D$ from the slit, then the linear spread of the central maximum is given by

$$y_1 = D\theta_1 = \frac{D\lambda}{d}$$

It is, in fact, the distance of first secondary minimum from the centre of the screen. It follows that as the screen is moved away ($D$ is increased), the linear size of the central maximum i.e. spread of light on the screen goes on increasing.

From the definition of Fresnel distance,

when $D = D_F, y_1 = d$ (size of the slit)

Setting this condition in the above equation, we have

$$d = \frac{D_F\lambda}{d} \quad \text{or} \quad D_F = \frac{d^2}{\lambda}$$

It follows that if screen is placed at a distance beyond $D_F$, the spreading of light due to diffraction will be quite large as compared to the size of the slit.

If $D < D_F$, then there will not be too much broadening by diffraction i.e., the light will travel along straight lines and the concepts of ray optics will be valid.

As $D < D_F$ or $D < \frac{d^2}{\lambda}$ or $d > \sqrt{\lambda D}$

For a given value of $D$, the quantity $\sqrt{\lambda D}$ is called the size of Fresnel zone and is denoted by $d_F$.

i.e., $d_F = \sqrt{\lambda D}$

Hence the concepts of ray optics can be conveniently used without introducing any appreciable error if the size of the aperture is greater than the size of the Fresnel zone,

i.e., $d > d_F$

### ILLUSTRATION 4.7

Estimate the distance for which ray optics a good approximation for an aperture of 4 mm and the wavelength is 400 nm?

**Sol.** Fresnel distance (distance for which ray optics is a good approximation), $D_F = \frac{a^2}{\lambda}$

Here, $a = 4\,\text{mm} = 4\times10^{-3}\,\text{m}$

$\lambda = 400\,\text{nm} = 400\times10^{-9}\,\text{m}$

Thus, $D_F = \frac{(4\times10^{-3})^2}{400\times10^{-9}} = 40\,\text{m}$

### Difference between interference and diffraction

| Interference | Diffraction |
|---|---|
| (1) It is the phenomenon of superposition of two waves coming from two different coherent sources | (1) It is the phenomenon of superposition of two waves coming from two different parts of the same wavefront |
| (2) In interference pattern, all bright lines are equally bright and equally spaced | (2) All bright lines are not equally bright and equally wide. Brightness and width go on decreasing with the angle of diffraction |
| (3) All dark lines are totally dark | (3) Dark lines are not perfectly dark. Their contrast with bright lines and width goes on decreasing with angle of diffraction |
| (4) In interference bands are large in number | (4) In diffraction bands are a few in number |

### CONCEPT APPLICATION EXERCISE 4.1

1. A slit of width 0.15 cm is illuminated by light of wavelength $5\times10^{-7}$ m and a diffraction pattern is obtained on a screen 21 m away. Calculate the width of the central maxima.
2. Light of wavelength 5900 Å falls normally on a slit of width $11.8\times10^{-5}$ cm and the resulting Fraunhofer diffraction is received on a screen. Calculate the angular position of the first minimum. Also, find the angular width of the central maximum.
3. Microwaves of frequency 12,000 MHz are incident normally on a rectangular slit of width 10 cm. Calculate angular spread of the central maximum of the diffraction pattern of the slit.
4. Monochromatic light passes through a slit whose width is 0.050 mm. At what angle will the third minimum be seen and what is the width of the central maximum on a screen located 1.0 m from the slit for $\lambda = 400$ nm?
5. Parallel light of wavelength 5000 Å falls normally on a single slit. The central maximum spreads out to 30° on either side of the incident light. Find the width of the slit. For what width of the slit the central maximum would spread out to 90° from the direction of the incident light?
6. Calculate the distance that a beam of light of wavelength 500 nm can travel without significant broadening, if the diffracting aperture is 3 mm wide.

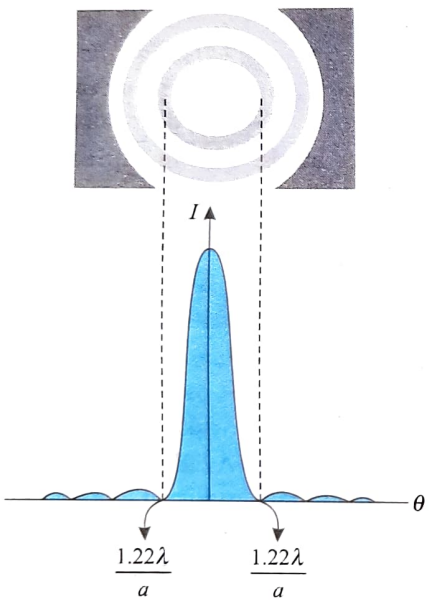
#### ANSWERS

**1.** 1.4 mm **2.** 30°, 60° **3.** 28.7° **4.** 1.4°, 1.6 cm
**5.** $10^{-6}$ m, $5\times10^{-7}$ m **6.** 18 m

# DIFFRACTION BY A CIRCULAR APERTURE

Here we consider diffraction by a circular aperture—that is, a circular opening, such as a circular lens, through which light can pass. In all the optical instruments, we use lenses or mirrors. The optical instruments are based on the laws of geometrical optics for defining magnification. Therefore, the image of a point object produced by an optical instrument must be a geometric point. In practice, the image of a bright geometric point produced by a lens is never a geometric point. It is always a circular patch of light with a bright disc at the centre surrounded by alternate dark and bright rings. It arises because of diffraction of light.

Suppose that light of wavelength $\lambda$ is incident on a lens of diameter $a$ (or a mirror with a circular aperture of diameter $a$). The light diffracted from this circular aperture will show a diffraction pattern consisting of circular rings as shown in figure.




If $\theta$ is the angle between the central maximum (called Airy disc) and the first minimum (the first dark fringe). The (complex) analysis of such patterns shows that the first minimum for the diffraction pattern of a circular aperture of diameter $a$ is located by

$$\sin\theta = 1.22\frac{\lambda}{a} \quad \text{(first minimum – circular aperture)} \quad \ldots(i)$$

Since $\theta$ is small, we may also write $\sin\theta \approx \theta$

Thus, $\theta = 1.22\frac{\lambda}{a}$

We write a similar equation for a narrow slit

$$\sin\theta = \frac{\lambda}{a} \quad \text{(first minimum – single slit)} \quad \ldots(ii)$$

which locates the first minimum for a long narrow slit of width $a$. The main difference is the factor 1.22, which enters because of the circular shape of the aperture.

# RESOLVING POWER OF OPTICAL INSTRUMENTS

## RAYLEIGH'S CRITERION

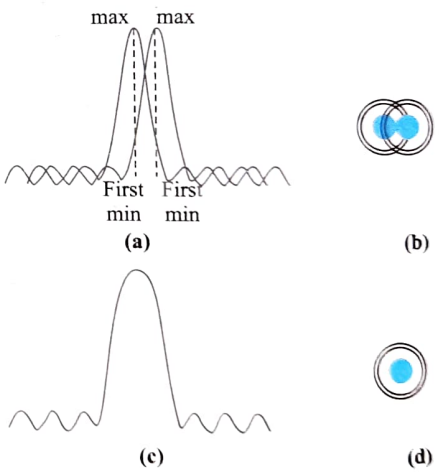
Two bright point objects lying very close to each other are seen with a naked eye or with the help of an optical instrument (a microscope or a telescope), then they may or may not be seen as two separate distinct objects due to the overlapping of their diffraction patterns. The ability to separate distinct objects depends upon limit of resolution and the resolving power of an optical instrument.

The smallest linear or angular separation between two point objects at which they can be just separately seen or resolved by an optical instrument is called the **limit of resolution of the instrument**.

The resolving power of an optical instrument is its ability to resolve or separate the images of two nearby point objects so that they can be distinctly seen. It is equal to the reciprocal of the limit of resolution of the optical instrument.

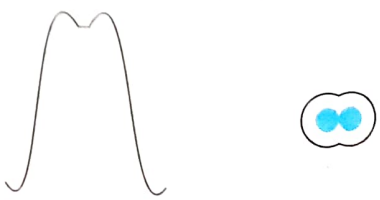
The smaller the limit of resolution of an optical instrument, greater is its resolving power.

**Rayleigh's criterion for resolution:** If we look through a telescope at two stars lying close to each other, their diffraction patterns overlap and the resultant pattern is little broader but otherwise similar to that of a single star, as shown in figure. So the two stars cannot be resolved or separately seen.




**Figures (a) and (b) show overlapping of diffraction patterns of two close point objects and Figs. (c) and (d) show their resultant diffraction patterns.**

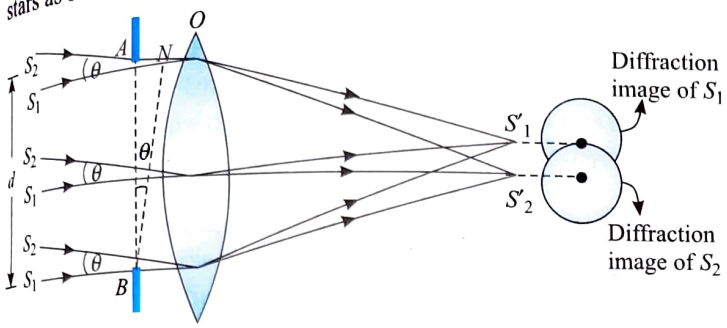
According to Rayleigh's criterion, the images of two point objects are just resolved when the central maximum of the diffraction pattern of one falls over the first minimum of the diffraction pattern of the other, as shown in figure. When seen through the telescope, the resultant diffraction has a well-marked depression at the top, showing that these are really two stars and not one. Thus the images of two stars have been just resolved.



**Observation from telescope at Rayleigh's criterion**

## Resolving Power of a Telescope

Let us consider that two distant stars $S_1$ and $S_2$ are observed through a telescope, whose objective ($O$) is of diameter '$a$'. Let $\theta$ be the angular separation between the two stars and $\lambda$, the wavelength of the light, in which the two stars are observed. The objective lens forms the diffraction images $S_1'$ and $S_2'$ of the two stars as shown in figure.




The path difference between the rays of light from the stars $S_1$ and $S_2$ is equal to $AN$. The two images $S_1'$ and $S_2'$ will be just resolved, when the path difference $AN$ is equal to the wavelength ($\lambda$) of the light. Since the aperture of the objective is circular, it corresponds to the smallest angular separation $\theta$ between the two stars, so that their images $S_1'$ and $S_2'$ are just resolved and is given by

$$\theta = \frac{1.22\lambda}{a} \qquad \ldots(i)$$

Here, $\theta$ is called **limit of resolution of a telescope**

**Resolving power of a telescope** is defined as the reciprocal of the smallest angular separation between two distant objects, so that they appear just separated, when seen through the telescope. Therefore, the resolving power of the telescope,

$$R = \frac{1}{\theta} = \frac{a}{1.22\lambda} \qquad \ldots(ii)$$

From the above expression, $R \propto a$, i.e., the larger the aperture of a telescope, higher is its resolving power. Therefore, telescopes with larger objectives form sharper images and are capable of resolving objects of smaller angular separation. The images are also brighter, not only because the light is concentrated in a **smaller Airy's disc**, but also due to the reason that a larger lens has more light gathering capacity.

Also, $R \propto \frac{1}{\lambda}$, it means the light of smaller wavelength is used to see the objects.

### Illustration 4.8

Calculate the separation of two points on the moon that can be resolved using 600 cm telescope. Given the distance of the moon from the earth is $3.8 \times 10^{10}$ cm. The wavelength most sensitive to the eye is $5.5 \times 10^{-5}$ cm.

**Sol.** Here $D = 600$ cm, $\lambda = 5.5\times10^{-5}$ cm

Limit of resolution, $\theta = \frac{1.22\lambda}{D} = \frac{1.22\times5.5\times10^{-5}}{600} = 1.1\times10^{-5}$ cm

Let $x$ be the distance between two points on the moon and $d$ be the distance between the moon and the objective of the telescope.

Then $\theta = \frac{x}{d}$

or $x = d\times\theta = 3.8\times10^{10}\times1.1\times10^{-7} = 4180$ cm

### Illustration 4.9

Two stars at a distance of 10 light years are viewed through a telescope having a lens of 20 cm diameter. What is the minimum separation of these stars for which they would be distinguishable as separate objects? $\lambda = 6000$ Å

**Sol.** Here, $a = 20$ cm, $\lambda = 6000$ Å $= 6\times10^{-5}$ cm

$D$ (distance of the star) = 10 ly

Thus, $\Delta\theta = \frac{1.22\lambda}{a} = \frac{1.22(6\times10^{-5}\text{ cm})}{20\text{ cm}} = 0.366\times10^{-5}$ rad

Minimum separation between the stars

$= D\Delta\theta = (10\text{ ly})(0.366\times10^{-5}\text{ rad})$

$= 3.66\times10^{-5}$ ly

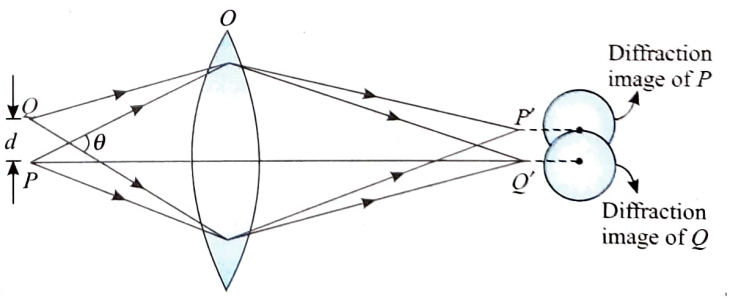
$= 3.66\times10^{-5}(9.461\times10^{12}\text{ km})$

$= 3.46\times10^{8}$ km

## Resolving Power of a Microscope

The resolving power of a microscope represents its ability to form distinctly separate images of two objects lying close to each other.

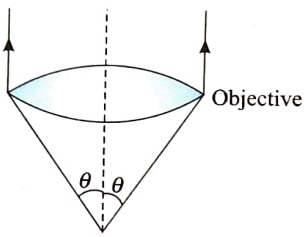
Consider that two objects $A$ and $B$ lying very close to each other are observed through a microscope. The objects are placed just beyond the focus of the objective ($O$) of the microscope. Let $\lambda$ be the wavelength of the light used to observe the objects. The objective forms the diffraction images $A'$ and $B'$ of the two objects as shown in figure.




Let $d$ be the least distance between the two objects so that their diffraction images $A'$ and $B'$ are just resolved. It can be proved that

$$d = \frac{1.22\lambda}{2\sin\theta} \qquad \ldots(i)$$

$\theta$ = half the angle of cone of light from each point object or the angle subtended by each point object on the radius of the objective (see figure).



The factor 1.22 appears due to circular aperture of the objective. If the space between the point objects and the objective lens is filled with medium (usually oil) of refractive index $\mu$, then the above relation modifies to

$$d = \frac{1.22\lambda}{2\mu \sin\theta} \quad \ldots(ii)$$

The least distance given by Eq. (ii) is called the **limit of resolution** of the microscope.

The resolving power of a microscope is defined as the reciprocal of the least separation between two close objects, so that they appear just separated, when seen through the microscope.

Therefore, resolving power of microscope, $R = \frac{1}{d} = \frac{2\mu \sin\theta}{1.22\lambda}$

From the above expression, the following conclusions can be drawn:

1. The resolving power of a microscope increases with increase in the value of the refractive index of the medium between its objective and the object. For this reason, **oil immersion objective microscopes** are used to achieve high resolving power.
2. The resolving power of a microscope increases with decrease in the value of the wavelength of the light used to illuminate the object. Since the wavelength of ultraviolet light is less than that of the visible light, the microscopes employing ultraviolet light for illuminating the objects are used to achieve high resolving power. Such microscopes are called **ultra microscopes**.

## Resolving Power of the Eye

Eye is an efficient natural optical instrument. The diameter ($a$) of the iris is about 2 mm and for green light ($\lambda$ = 500 nm = 5 × $10^{-5}$ cm), The limit of angular resolution of the eye,

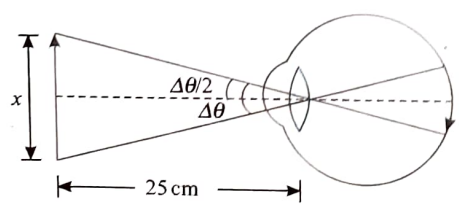
$$\Delta\theta = \frac{1.22\lambda}{a} = \frac{(1.22)(5\times10^{-5}\,\text{cm})}{2.00\times10^{-3}\,\text{m}} = 3.05\times10^{-4}\,\text{rad}$$

An average eye is not capable of resolving objects with an angular separation of less than 3 × $10^{-4}$ rad. The reason for this is the size of the cones, which are closely packed at the fovea. The diameter of each cone is 2 × $10^{-6}$ m. For two points to be clearly resolved, their images formed at the retina must be such that the two excited cones are separated by at least one cone.

### Smallest Size an Eye can Resolve

The limit of angular resolution of the eye, 3 × $10^{-4}$ rad, leads to the following smallest size that a normal eye can resolve. Suppose two point objects separated $x$ cm are placed at distance of distinct vision (25 cm) from the eye as shown in figure. Since $\Delta\theta$ is small,

$$\tan\left(\frac{\Delta\theta}{2}\right) \approx \frac{\Delta\theta}{2} = \frac{x/2}{25\,\text{cm}}$$




or $\Delta\theta = \frac{x}{25\,\text{cm}}$

or $x = (25\,\text{cm})\Delta\theta = (25\,\text{cm})(3\times10^{-4}\,\text{rad})$

$= 7.5\times10^{-3}\,\text{cm} = 0.075\,\text{mm}$ (taking $\Delta\theta = 3\times10^{-4}$ rad)

### ILLUSTRATION 4.10

Calculate the smallest angular separation resolved by the human eye. Given: Aperture $a$ = 2.5 mm and effective $\lambda$ = 5500 Å. If a scale with mm markings is viewed by the unaided eye, deduce the largest distance to which the markings will be visible.

**Sol.** Here, $\lambda = 5500\text{Å} = 5500\times10^{-10}\,\text{m}$,

$a = 2.5\,\text{mm} = 2.5\times10^{-3}\,\text{m}$

$$\Delta\theta = 1.22\left(\frac{\lambda}{a}\right) = 1.22\left(\frac{5500\times10^{-10}\,\text{m}}{2.5\times10^{-3}\,\text{m}}\right)$$

$= 2.7\times10^{-4}\,\text{rad}$ ...(i)

Angle subtended by 1mm markings from a distance $x$ m i.e.,

$$\theta = \frac{1\times10^{-3}\,\text{m}}{x\,\text{m}} = \frac{1}{x}\times10^{-3} \quad \ldots(ii)$$

From Eqs. (i) and (ii), $\frac{1}{x}\times10^{-3} = 2.7\times10^{-4}$

or $x = \frac{10^{-3}}{2.7\times10^{-4}} = 3.7\,\text{m}$

Thus, a normal eye can resolve objects that are separated by ~ 0.1 mm and placed at a distance of distinct vision. This is approximately equal to thickness of a human hair.

### CONCEPT APPLICATION EXERCISE 4.2

1. Two parallel pillars are 11 km away from an observer. Find the minimum distance between the pillars so that they can be seen separately.
2. A telescope is used to resolve two stars separated by 4.6 × $10^{-6}$ rad. If the wavelength of light used is 5460 Å, what should be the aperature of the objective of the telescope?
3. A microscope has objective of aperture 8 mm and focal length 2.5 cm. Estimate its resolving power. Given $\lambda$ = 5500 Å.
4. Calculate the least value of the angular separation of two stars which can be resolved by a telescope of 200 cm aperture. If the aperture of the human eye be 2 mm and if the focal length of the eyepiece be 1 inch, what must be the minimum focal of the objective if resolving power of the telescope is to be utilized. Take $\lambda$ = 5500 Å.
5. Diameter of human eye lens is 2 mm. What will be the minimum distance between two points to resolve them, which are situated at a distance of 50 m from eye? The wavelength of light is 5,000 Å.

**ANSWERS**

**1.** 3.2 m **2.** 0.1488 m **3.** 2 × $10^{-6}$ m
**4.** 3.35 × $10^{-7}$ rad **5.** 1.525 cm

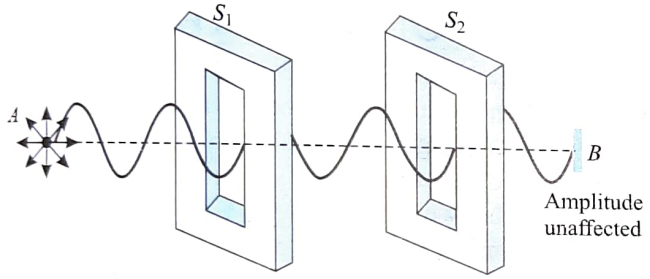
# POLARIZATION

Through the phenomena of interference and diffraction, we established without doubt the wave nature of light. As both these phenomena of interference and diffraction are exhibited by either type of wave, they do not tell us the exact nature of light. It was only the discovery of another phenomenon, known as 'polarization' which conclusively proved the transverse nature of light waves.

A wave exhibiting different properties in different directions is said to be **polarized** and the phenomenon is referred to as the **polarization**.

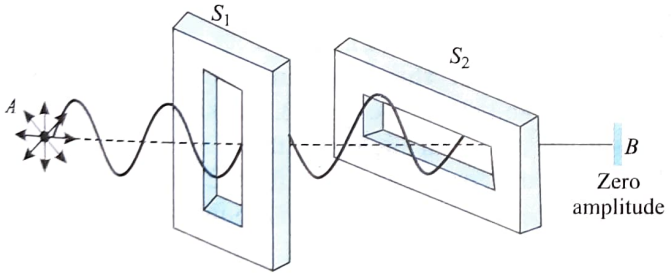
To understand the polarization of the waves, the following **mechanical analogy** is to be considered.

Two vertical slits $S_1$ and $S_2$ are placed parallel to each other and a string is passed through them. The end $B$ of the string is fixed and by continuously jerking the string at the end $A$ in all directions perpendicular to $AB$, transverse waves are set up in it in all possible perpendicular planes Then, transverse wave, whose vibrations are parallel to the length of the slit, will only pass through the slit $S_1$. If the slit $S_2$ is parallel to $S_1$, then amplitude of vibrations will remain unaffected [see Fig. (a)].




**(a)**

On the other hand, when the slit $S_2$ is at right angle to the slit $S_1$, no vibrations will pass through it i.e. amplitude of vibrations will become zero [see Fig. (b)].




**(b)**

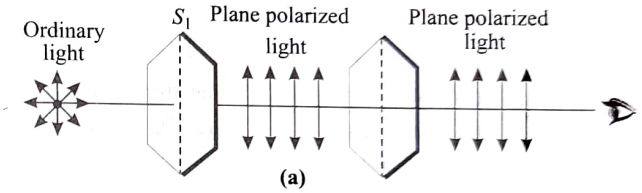
Thus, in the wave that emerges from $S_1$, all the vibrations are restricted to the vertical plane and as such the wave is said to be **plane-polarized**. The first slit is called the **polarizer** as it produces polarized vibrations. Since the slit $S_2$ is also vertical, this plane polarized wave will be able to pass through it and remain plane polarized. But if $S_2$ is gradually rotated about the string as the axis, the amplitude of vibrations passing through it goes on decreasing till it becomes zero when $S_2$ is at right angles to $S_1$. Thus, no wave emerges from $S_2$ as shown in Fig. (b). This confirms the polarized state of vibrations passing through $S_1$, and as $S_2$ detects this state, it is called the **analyzer**.

If the string is replaced by a spring in which longitudinal waves can be generated, these will pass through both the slits unaffected, irrespective of the two slits being parallel or perpendicular to each other.

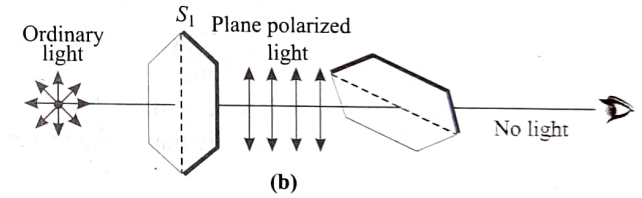
From the above mechanical experiment, it follows that if on rotating the slit $S_2$, the amplitude of vibration in the string decreases and becomes zero, when the two slits are at right angles, then the wave motion is transverse in nature. On the other hand, if the amplitude of vibration remains unaffected, the wave motion is a longitudinal one.

Thus, polarization is a characteristic of transverse waves only. The **polarization of light waves** can be demonstrated by an optical experiment similar to the mechanical experiment described earlier.

A Nicol prism or tourmaline crystal cut parallel to its crystallographic axis behaves in a similar manner for light waves as the slit does in case of the wave motion produced in the string. Suppose that the ordinary light is incident normally on a pair of such crystals $S_1$ and $S_2$. It is observed that the intensity of the emergent light is maximum, when the axis of the two crystals are parallel [Fig. (a)] and minimum, when the axis of the two crystals become at right angles to each other [Fig. (b)]. In view of the mechanical experiment discussed earlier, it follows that light is a transverse wave motion.



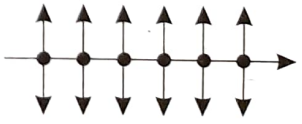

**(a)**




**(b)**

Light is electromagnetic in nature and an electromagnetic wave consists of varying electric and magnetic fields, such that the two fields are mutually perpendicular to each other and to the direction of propagation of waves. The optical phenomena i.e. the phenomena concerning light are described by the **electric field vector** in electromagnetic waves. Therefore, the transverse nature of light may be attributed to the vibrations of electric field vector in a direction perpendicular to the direction of propagation of light.

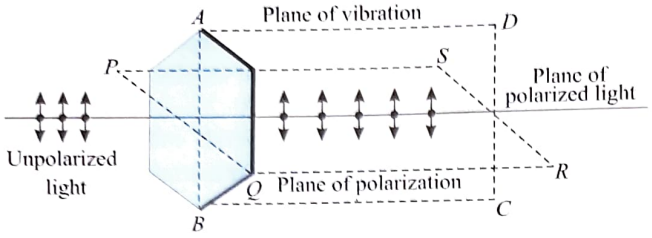
In an ordinary or unpolarized ray of light, the vibrations of electric field vector are symmetrically distributed in all the directions but perpendicular to the direction of propagation of the light. Such a ray of light is called a ray of **ordinary or unpolarized light**. It is schematically represented as shown in figure. The arrows represent vibrations in the plane of the paper, while the dots represent vibrations in a direction perpendicular to the plane of the paper.



## SOME USEFUL TERMS AND DEFINITIONS

**Polarization of light:** It is the phenomenon, due to which the vibrations of light are restricted in a particular plane, is called the polarization of light.

When ordinary light is passed through a tourmaline crystal, it gets polarized, i.e., its vibrations originally present in all directions perpendicular to the direction of propagation are now confined to only one of these directions which are parallel to its crystallographic axis *AB*. This plane contains the direction of vibration and the direction of propagation of light.




**Plane of vibration:** The plane within which the vibrations of the polarized light are confined is known as the **plane of vibration**. It is represented by the plane (*ABCD*) in figure.

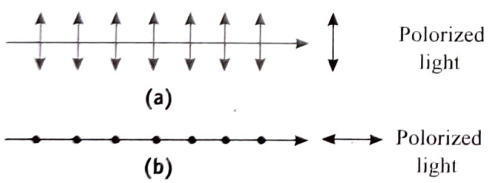
**Plane of polarization:** It is the plane at right angles to the plane of vibration and passing through the direction of propagation of light is known as the **plane of polarization**. The plane (*PQRS*) perpendicular to the plane of vibrations is called the plane of polarization.

**Plane polarized light:** It may be defined as the light, in which the vibrations of the light (vibrations of the electric vector) are restricted to a particular plane.

According to the electromagnetic theory of light, the electric vector acts as light vector. Therefore, in a plane polarized light, the electric vector vibrates along a fixed line in a plane perpendicular to the direction of propagation.

In Fig. (a), the plane polarized component whose vibrations are confined to the plane of paper but perpendicular to the direction of propagation, has been represented. The plane of polarization is therefore perpendicular to the plane of the paper.

Figure (b), on the other hand, is the representation of the other plane polarized component whose plane of polarization is in the plane of the paper and the vibrations which are perpendicular to the plane of the paper are represented by dots on the direction of propagation.

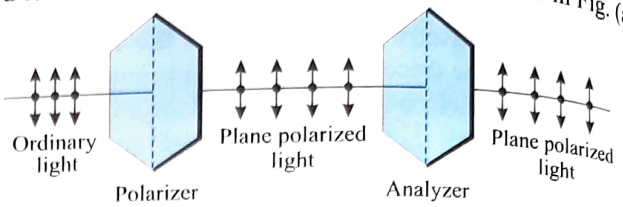



## DETECTING PLANE POLARIZED LIGHT

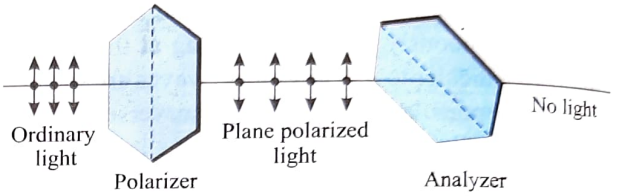
We cannot make distinction between the unpolarized light and the plane polarized light through the naked eye or the polarizer alone. Another such crystal required to analyze the nature of light is called **analyzer**.

A tourmaline crystal or a Nicol prism used to produce plane polarized light is called **polarizer**. If the polarizer is rotated in the path of the ordinary light, the intensity of the light transmitted from the polarizer remains unchanged. It is because, in each orientation of the polarizer, the plane polarized light is obtained, which has vibrations in a direction parallel to the axis of the crystal in that orientation.

If we rotate the analyzer in the path of the light transmitted from the polarizer, so that the axes of the polarizer and the analyzer are parallel to each other, then the intensity of light is found to remain unaffected (or maximum) as shown in Fig. (a).




But when the axis of the polarizer and the analyzer are perpendicular to each other, the intensity of light becomes minimum. In the positions shown in Fig.(b), the polarizer and the analyzer are said to be in **crossed positions**.

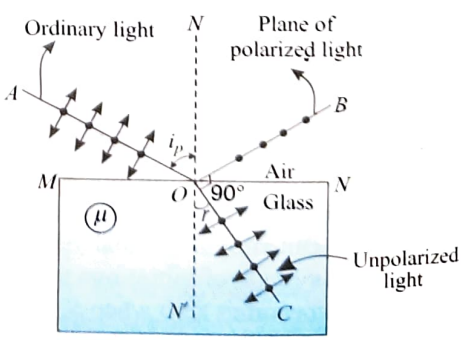



## POLARIZATION BY REFLECTION

When unpolarized light is reflected from a plane boundary between two transparent media, such as air and glass or air and water, the reflected light is partially polarized. The degree of polarization depends on the angle of incidence and the ratio of wave speeds in the two media.

One of the simplest method to produce plane polarized light is by reflection. When unpolarized light is reflected from a plane boundary between two transparent media, such as air and glass or air and water, the reflected light is partially polarized. The degree of polarization increases as the angle of incidence is increased. At a particular value of angle of incidence, the reflected beam becomes completely polarized. This angle of incidence is called the **polarizing angle** for that medium.

Let *MN* represent **the reflecting surface** separating two media, say air and glass (see figure). Consider a ray *AO* incident on the surface *MN* at an angle $i_p$ with the normal *NON'*. *OB* and *OC* represent the corresponding reflected and refracted rays. If the reflected beam is examined, it is found to be polarized, but not completely.




If the angle of incidence be gradually changed and the reflected beam examined in every case, it will be observed that for one particular value of the angle of incidence, this beam is completely polarized. This angle is called the **polarizing angle** ($i_p$) for that

medium. The plane polarized light has vibrations perpendicular to the plane of paper. The light travelling along the path $OC$ (refracted light) is unpolarized light.

## BREWSTER'S LAW

The explanation for obtaining the reflected beam as plane polarized at the polarizing angle was given by David Brewster. According to him,

'When a ray of light is incident at polarizing angle, the reflected ray is at right angle to the refracted ray.'

Thus, when $i = i_p$, $r = (90 - i_p)$.

As $$\mu = \frac{\sin i}{\sin r}, \mu = \frac{\sin i_p}{\sin(90 - i_p)} = \frac{\sin i_p}{\cos i_p}$$

or $$\mu = \tan i_p \quad \ldots(i)$$

This relation is known as the **Brewster's law**, it states that:

When light is incident at polarizing angle at the interface of a refracting medium, the refractive index of the medium is equal to the tangent of the polarizing angle.

The polarizing angle is sometimes called the **Brewster's angle**.

The value of the polarizing angle for glass is about 57.5° and that for water is about 37°.

**ILLUSTRATION 4.11**

The critical angle between a given transparent medium and air is denoted by $C$. A ray of light in air medium enters this transparent medium at an angle of incidence equal to polarizing angle $i_p$. Deduce a relation for the angle of refraction in terms of $C$.

**Sol.** According to Brewster's law, when a ray of light is incident at polarizing angle,

$$\mu = \tan i_p \quad \ldots(i)$$

If $C$ is critical angle for the transparent medium and air, then

$$\mu = \frac{1}{\sin C} \quad \ldots(ii)$$

From the Eqs. (i) and (ii), we have

$$\tan i_p = \frac{1}{\sin C} \quad \ldots(iii)$$

If $r$ is angle of refraction, then $i_p = 90° - r$

or $$\tan i_p = \tan(90° - r) = \cot r = \frac{1}{\tan r} \quad \ldots(iv)$$

From the Eqs. (iii) and (iv), we have $\tan r = \sin C$

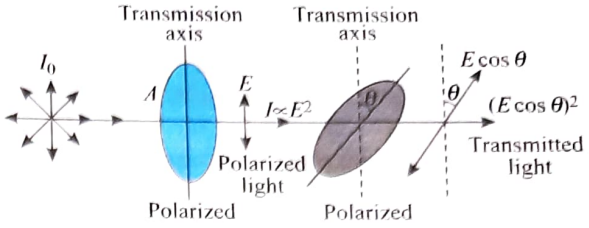
or $r = \tan^{-1}(\sin C)$

## MALUS'S LAW

The dependence of intensity of transmitted light on the angle between the analyzer and the polarizer was investigated by E.L. Malus.

It states that when a completely plane polarized light beam is incident on an analyzer, the intensity of the emergent light varies as the square of the cosine of the angle between the planes of transmission of the analyzer and the polarizer.

In Fig. (a), an unpolarized light beam incident on the polarizer where the transmission axis is oriented vertically. The light that is passing through this sheet is polarized vertically as shown, where the transmitted electric field vector is $E$. A second polarizing sheet, called the analyzer, intercepts this beam with its transmission axis at an angle $\theta$ to the axis of the polarizer. The component of $E$ perpendicular to the axis of the analyzer is completely absorbed, and the component of $E$ parallel to the axis of the analyzer is transmitted.



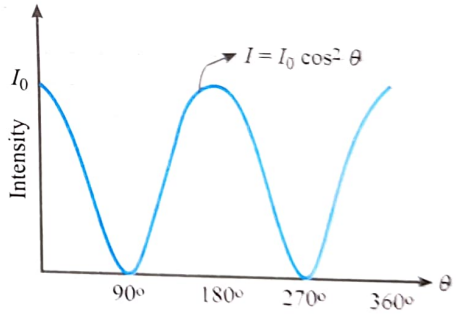

As the component of $E$ parallel to the axis of the analyzer is $E\cos\theta$ and as the transmitted intensity varies as the square of the transmitted amplitude, we conclude that the transmitted intensity ($I_\theta$) varies as

$$I_\theta = I\cos^2\theta \quad \ldots(i)$$

[as $I \propto E^2$ and $I_\theta \propto (E\cos\theta)^2$ ]

where $I$ is the intensity of polarized wave incident on the analyzer. This expression is known as **Malus's law** and is applicable to any two polarizing materials whose transmission axes are at an angle $\theta$ to each other.

**Intensity curve:** As the angle '$\theta$' between the transmission directions of polarizer and analyzer is varied, the intensity '$I$' of the light transmitted by the analyzer varies as a function of $\cos^2\theta$, as shown in figure.




**Graph of intensity $I$ through analyzer versus angle $\theta$ between polarizer and analyzer.**

**Important Points:**

- When $\theta = 0°$ or $180°$, $\cos\theta = \pm 1$, so that $I = I_0$
  So when the transmission directions of polarizer and analyzer are parallel or antiparallel to each other, the maximum intensity of plane polarized light is transmitted by the analyzer and is equal to the intensity emerging from the polarizer.
- When $\theta = 90°$, $\cos\theta = 0$, so that $I = 0$
  So when the transmission directions of polarizer and analyzer are perpendicular to each other, the intensity of light transmitted through the analyzer is zero.
- When a beam of unpolarized light is incident on the polarizer,

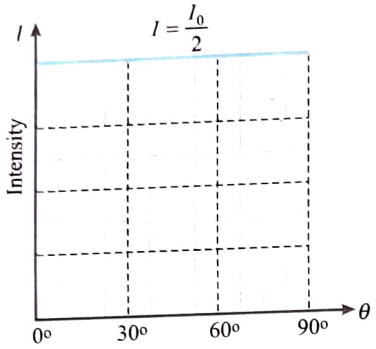
$$I = I_0\overline{\cos^2\theta} = I_0 \times \frac{1}{2}\left(\overline{1 + \cos 2\theta}\right)$$

$$= \frac{1}{2}I_0\left(1 + \overline{\cos 2\theta}\right) = \frac{1}{2}I_0(1 + 0) = \frac{1}{2}I_0$$

## ILLUSTRATION 4.12

Light from an ordinary source (say, a sodium lamp) is passed through a polarized sheet $P_1$. The transmitted light is then made to pass through a second polaroid sheet $P_2$ which can be rotated so that the angle ($\theta$) between the two polaroid sheets varies from 0° to 90°. Show graphically the variation of the intensity of light, transmitted by $P_1$ and $P_2$, as a function of the angle $\theta$. Take the incident beam intensity as $I_0$.

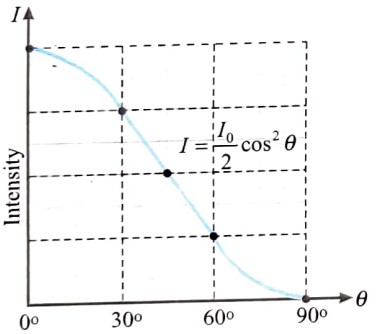
**Sol.** The intensity of light passing through $P_1$ remains constant equal to $I_0/2$. So $I$ vs. $\theta$ graph for $P_1$ is as shown below.




The intensity of light transmitted by analyzer $P_2$ varies as function of $\cos^2\theta$ as per relation:

$$I = \frac{I_0}{2}\cos^2\theta$$

So $I$ vs. $\theta$ graph for $P_2$ is as shown below.




## ILLUSTRATION 4.13

A polaroid examines two adjacent plane-polarized light beams $A$ and $B$ whose planes of polarization are mutually at right angles. In one position of the polaroid, the beam $B$ shows zero intensity. From this position a rotation of 30° shows the two beams of equal intensities. Find the intensity ratio $I_A/I_B$ of the two beams.

**Sol.** The planes of polarization of light beams $A$ and $B$ are mutually at right angles. Initially, the beam $B$ shows zero intensity. Therefore, $\theta = 90°$ for beam $B$ and $\theta = 0°$ for beam $A$. When the polaroid is rotated through 30°, we have $\theta = 60°$, for beam $A$ and $\theta = 30°$ for beam $B$. In this position, Intensity of emerging beam $A$ = Intensity of emerging beam $B$

$$\therefore \quad I_A \cos^2 30° = I_B \cos^2 60°$$

$$\text{or} \quad \frac{I_A}{I_B} = \frac{\cos^2 60°}{\cos^2 30°} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3} = 1:3$$

## ILLUSTRATION 4.14

Unpolarized light falls on two polarizing sheets placed one on top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of the transmitted light is: **(a)** one third the maximum intensity of the unpolarized light of the incident beam, **(b)** one third the maximum intensity of the incident beam? Assume that the sheet is ideal, i.e., it reduces the intensity of unpolarized light by exactly 50%.

**Sol.**

**(a)** If $I_0$ is the intensity of unpolarised light, the intensity of the polarized beam transmitted by the first sheet, i.e.,

$$I = \frac{1}{2}I_0 \qquad \ldots(i)$$

If $\theta$ is the angle between the characteristic directions of the two sheets, intensity of polarized light transmitted by second sheet, i.e.,

$$I_\theta = I\cos^2\theta \qquad \ldots(ii)$$

For $I_\theta = \frac{1}{3}I, \cos^2\theta = \frac{1}{3}$

or $\theta = \cos^{-1}\left[\pm(1/\sqrt{3})\right] = \cos^{-1}(\pm 0.577) = \pm 55°$

**(b)** From Eqs. (i) and (ii), $I_\theta = \frac{1}{2}I_0\cos^2\theta$

For $I_\theta = \frac{1}{3}I_0, \frac{1}{3} = \frac{1}{2}\cos^2\theta$

or $\theta = \cos^{-1}\left(\pm\sqrt{\frac{2}{3}}\right) = \cos^{-1}(\pm 0.8165) = \pm 35°$

## ILLUSTRATION 4.15

Two polaroids are crossed to each other. Now, one of them is rotated through 60°. What percentage of incident unpolarized light will pass through the system?

**Sol.** Let $I_0$ be the intensity of the incident unpolarized light on the system of the two polaroids. As the incident light is unpolarized, the intensity of light on being transmitted from the first polaroid will become

$$I = I_0\overline{\cos^2\theta} = \frac{I_0}{2} \qquad \left(\because \overline{\cos^2\theta} = \frac{1}{2}\right)$$

If $\theta$ is the angle between the transmission planes of the two polaroids, then the intensity of light on passing through the second polaroid is given by

$$I' = I\cos^2\theta = \frac{I_0}{2}\cos^2\theta$$

Since the two polaroids were initially crossed, the angle between the transmission planes of the two polaroids on rotating through 60° will become $\theta = 90° - 60° = 30°$.

$$\therefore \quad I' = \frac{I_0}{2} \times \cos^2 30° = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = 0.375\, I_0$$

Hence, the percentage of incident light that passes through the system,

$$\frac{I'}{I} \times 100 = 0.375 \times 100 = 37.5\%$$

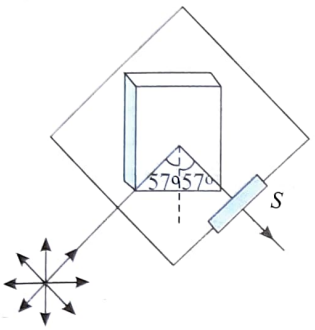
## CONCEPT APPLICATION EXERCISE 4.3

1. A ray of light is incident on the surface of a glass plate at an angle of incidence equal to Brewster's angle $\varphi$. If $\mu$ represents the refractive index of glass with respect to air, then the angle between reflected and refracted rays is

(a) $90+\varphi$ (b) $\sin^{-1}(\mu\cos\varphi)$

(c) $90^\circ$ (d) $90^\circ-\sin^{-1}(\sin\varphi/\mu)$

2. Figure represents a glass plate placed vertically on a horizontal table with a beam of unpolarized light falling on its surface at the polarizing angle of 57° with the normal. The electric vector in the reflected light on screen $S$ will vibrate with respect to the plane of incidence in a



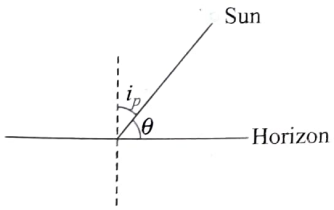

(a) Vertical plane (b) Horizontal plane

(c) Plane making an angle of 45° with the vertical

(d) Plane making an angle of 57° with the horizontal

3. When sunlight is incident on water at an angle of 53°, the reflected light is found to be completely plane-polarized. Determine: **(a)** angle of refraction of light and **(b)** refractive index of water.

4. Sunlight is reflected from the smooth surface of a pond. What is the Sun's altitude when the polarization of the reflected light is greatest?




[Refractive index of water $\mu_{water}=\frac{4}{3}$]

5. Two polaroids are used to study polarization. One of them (the polarizer) is kept fixed and the other (the analyzer) is initially kept with its axis parallel to the polarizer axis. The analyzer is then rotated through angles of 45°, 90° and 180° in turn. How would the intensity of light coming out of the analyzer be affected for these angles of rotation, as compared to the initial intensity and why?

6. Two polarizing sheets have their polarizing directions parallel so that the intensity of the transmitted light is maximum. Through what angle must the either sheet be turned if the intensity is to drop by one-half?

### ANSWERS

1. (c) **2.** (a) **3.** (a) 37° (b) 1.327
4. 37° **6.** ±45° or ±135°

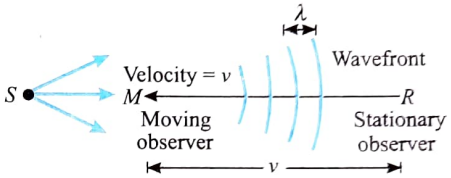
## DOPPLER EFFECT FOR LIGHT

You have already learnt Doppler effect for sound waves. When a source of sound travels towards an observer, the apparent frequency is higher than the frequency actually emitted by the source. When the source moves away, the apparent frequency is lower than the actual frequency. Doppler effect is a basic property of all waves and so occurs in case of light also.

Whenever there is a relative motion between source of light and observer, the frequency of light received by the observer is different from the frequency actually emitted by the source. This phenomenon of the apparent change in the frequency of light is called **Doppler effect** for light.

### EXPRESSION FOR THE APPARENT FREQUENCY OF LIGHT

Suppose a source of light emits waves of frequency $f$ and wavelength $\lambda$. If $c$ is the speed of light, then $\lambda=\frac{c}{f}$




Suppose an observer moves towards the source with velocity $v$. In one second, the source and observer come closer by a distance $v$.

∴ Apparent frequency = Number of light waves emitted per second by the source + Number of light waves contained in distance $v$

or $$f'=f+\frac{v}{\lambda}=f+\frac{v}{c/f}=f+v\cdot\frac{f}{c}$$

or $$f'=f\left(1+\frac{v}{c}\right) \quad ...(i)$$

Clearly, $f'>f$ i.e., the apparent frequency increases when source and observer approach each other.

When source and observer move away from each other, the apparent frequency can be obtained by replacing $v$ by $-v$ in the above equation. Then

$$f'=f\left(1-\frac{v}{c}\right) \quad ...(ii)$$

Clearly, $f'<f$ i.e., the apparent frequency decreases when source and observer move away from each other.

**Blue shift and red shift:** Equations (i) and (ii) can be combined together as

$$f'=f\left(1\pm\frac{v}{c}\right)$$

or $$f'-f=\pm\frac{v}{c}\cdot f$$

The frequency change $\Delta f=f'-f$ is called Doppler shift.

Putting $f=\frac{c}{\lambda}$ and $f'=\frac{c}{\lambda'}$, we get $\frac{c}{\lambda'}-\frac{c}{\lambda}=\pm\frac{v}{c}\cdot\frac{c}{\lambda}$

or $$\frac{\lambda-\lambda'}{\lambda'}=\pm\frac{v}{c}$$

But $\frac{\lambda-\lambda'}{\lambda'} \simeq \frac{\lambda-\lambda'}{\lambda}$

$\therefore \quad \frac{\lambda-\lambda'}{\lambda} = \pm\frac{v}{c}$

or $\lambda-\lambda' = \pm\frac{v}{c}\lambda$

(i) When source and observer approach each other, positive sign is taken. Then $\lambda - \lambda'$ is positive or $\lambda' < \lambda$, i.e., the wavelengths in the middle part of the visible spectrum shift towards the blue region. This is called **blue shift**.

(ii) When source and observer move away from each other, negative sign is taken. Then $\lambda - \lambda'$ is negative or $\lambda' > \lambda$, i.e., the wavelengths in the middle part of the visible spectrum shift towards the red region. This is called **red shift**.

**Applications of Doppler Effect**:

1. Light received from stars and galaxies shows a red shift which indicates that the universe is expanding.
2. By measuring Doppler shift in the e.m. wave reflected from an automobile, the speed of the automobile can be determined.
3. Doppler shift of light received from Saturn rings shows that the rings consist of a number of discontinuous satellites.
4. By measuring Doppler shift in the light received from eastern and western edges of the Sun, the speed of rotation of the Sun has been determined to be 2 km $s^{-1}$ from east to west relative to the earth.

## Concept Application Exercise 4.4

1. If the shift of wavelength of light emitted by a star is towards violet, then this shows that star is
   (a) stationary
   (b) moving towards earth
   (c) moving away from earth
   (d) information is incomplete
2. A star emits light of wavelength 5500 Å. It appears blue to an observer on the earth, it means
   (a) star is going away from the earth
   (b) star is stationary
   (c) star is coming towards earth
   (d) none of the above
3. A star is moving towards the earth with a speed of $9.0 \times 10^6$ $ms^{-1}$. If the wavelength of a particular spectral line emitted by it is 6000 Å, then find the apparent wavelength.
4. A spaceship is moving with a velocity of $10^8$ m/ s towards a star that emits radiations of wavelength $6 \times 10^{11}$ m. Calculate the wavelength of the radiation as received by the crew in the ship.
5. What speed should a galaxy move with respect to us that the sodium line at 589.0 nm is observed at 589.6 nm?

### Answers

**1.** (b) **2.** (c) **3.** 5820 Å
**4.** $4.5 \times 10^{-7}$ m **5.** $3.1 \times 10^5$ m/s

# Solved Examples

## Example 4.1

A laser light beam of power 20 mW is focused on a target by a lens of focal length 0.05 m. If the aperture of the laser be 1 mm and the wavelength of its light 7000 Å, calculate the angular spread of the laser, the area of the target hit by it, and the intensity of the impact on the target.

**Sol.** Here $P = 20\,\text{mW} = 20\times10^{-3}\,\text{W}$,

$f = 0.05\,\text{m}$, $d = 1\,\text{mm} = 10^{-3}\,\text{m}$,

$\lambda = 7000\,\text{Å} = 7000\times10^{-10}\,\text{m}$

Angular spread of the laser beam,

$$\theta = \frac{1.22\lambda}{d} = \frac{1.22\times7000\times10^{-10}}{1\times10^{-3}} = 8.54\times10^{-4}\ \text{radian}$$

Linear spread of the laser $= f\cdot\theta = 5\times10^{-2}\times8.54\times10^{-4}\,\text{m}$

$\therefore$ Areal spread of the laser, i.e., area of the target hit by it

$$= \left(5\times8.54\times10^{-6}\right)^2 = 1.823\times10^{-15}\,\text{m}^2$$

Intensity of impact of the laser on the target

$$= \frac{\text{Power of laser}}{\text{Area hit}} = \frac{20\times10^{-3}}{1.823\times10^{-15}} = 10.97\times10^{12}\,\text{Wm}^{-2}$$

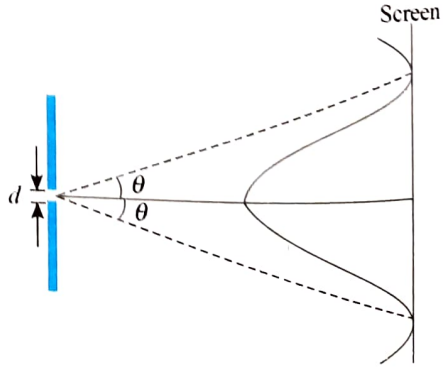
## Example 4.2

Angular width of a central maximum in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in the angular-width of central maximum is obtained when the original apparatus is immersed in a liquid. Find refractive index of the liquid.

**Sol.** In single slit diffraction, first minimum occurs at

$$d\sin\theta = \lambda \quad \text{or} \quad \sin\theta = \frac{\lambda}{d}$$

As $\lambda << d$, so $\theta \simeq \sin\theta = \frac{\lambda}{d}$



Angular width of central maximum is $\phi = 2\theta = \frac{2\lambda}{d}$

$\therefore \quad \frac{\phi_2}{\phi_1} = \frac{\lambda_2}{\lambda_1}$

or $\lambda_2 = \frac{\phi_2}{\phi_1} \cdot \lambda_1 = \frac{70}{100} \times 6000 = 4200$ Å

$[\because \phi_2 = 70\%$ of $\phi_1]$

When the apparatus is immersed in the liquid, the decrease in angular width is same. This indicates that the wavelength of light in the liquid is also 4200 Å.

$$\mu = \frac{\lambda}{\lambda_1} = \frac{6000}{4200} = 1.43$$

## EXAMPLE 4.3

A telescope has an objective of diameter 60 cm. The focal lengths of the objective and eye-piece are 2.0 m and 1.0 cm respectively. The telescope is directed to view two distant almost point sources of light (e.g. two stars of a binary). The sources are at roughly the same distance (= $10^4$ light years) along the line of sight but are separated transverse to the line of sight by a distance of $10^{10}$ m. Will the telescope resolve the two objects i.e. will it see two distinct stars ?

**Sol.** Separation between the two stars is $y = 10^{10}$ m

Distance of the stars from the earth is

$$x = 10^4 \text{ light years} = 10^4 \times 9.47 \times 10^{15} \text{ m}$$

Therefore, angle subtended by the line joining the two stars on the objective lens (or on eye) is

$$d\theta = \frac{y}{x} = \frac{10^{10}}{10^4 \times 9.47 \times 10^{15}} = 0.106 \times 10^{-9} \text{ rad}$$

Now diameter of objective, $D = 60\,\text{cm} = 0.60\,\text{m}$

For mean yellow colour, $\lambda = 600\,\text{nm} = 6 \times 10^{-7}$ m

According to Rayleigh's criterion, the limit of resolution of the telescope is

$$d\theta' = \frac{1.22\lambda}{D} = \frac{1.22 \times 6 \times 10^{-7}}{0.60} = 1.22 \times 10^{-6} \text{ rad}$$

As the angle $d\theta$ subtended by the transverse separation of the two stars is much too small compared to the limit of resolution $d\theta'$, hence the two stars of the binary cannot be resolved by the given telescope.

## EXAMPLE 4.4

The objective of a microscope has a focal length of 1 cm and diameter 0.5 cm. Using light of 6000 Å, is it possible to resolve the two point objects if they are separated by a distance of $10^{-6}$m?

**Sol.** Suppose the two objects are distance $\Delta x$ apart. Since these objects are almost at the focal length of the objective, the angle subtended by $\Delta x$, i.e., $\Delta\phi \approx \frac{\Delta x}{f_0}$

For the two objects to be clearly resolved, $\Delta\phi > \Delta\theta$

or $\frac{\Delta x}{f_0} > \frac{1.22\lambda}{a}$

or $\Delta x > \frac{1.22 f_0 \lambda}{a}$

or $\Delta x > \frac{(1.22)(1 \times 10^{-2}\,\text{m})(6000 \times 10^{-10}\,\text{m})}{(0.5 \times 10^{-2}\,\text{m})} = 1.46 \times 10^{-6}\,\text{m}$

Because $10^{-6}$ m is smaller than $1.46 \times 10^{-6}$ m, the two objects will not be resolved.

## EXAMPLE 4.5

Two polaroids are placed 90° to each other. What happens when $N-1$ more polaroids are inserted between two crossed polaroids (at 90° to each other). Their axes are equally spaced. How does the transmitted intensity behave for large $N$?

**Sol.** Transmitted intensity through first polaroid is

$$I_1 = I_0 \cos^2\theta$$

where $I_0$ is the original intensity. Similarly, the transmitted intensity through second polaroid will be

$$I_2 = I_1 \cos^2\theta = I_0 \cos^4\theta$$

If $N$ polaroids are used, then $I_N = I_0 (\cos\theta)^{2N}$

As the optic axes of the polaroids are equally inclined, so angle of rotation $\theta$ is same for each polaroid.

Thus $\frac{I_N}{I_0} = (\cos\theta)^{2N}$

But angle between successive polaroids is $\theta = \frac{90°}{N} = \frac{\pi}{2N}$ radian

$$\therefore \quad \left(\cos\frac{\pi}{2N}\right)^{2N} = \left(1 - \frac{\pi^2}{8N^2} + \cdots\right)^{2N} \simeq \left[1 - \frac{2N\pi^2}{8N^2} + \cdots\right]$$

which approaches 1 for large $N$. Hence fractional intensity,

$$\frac{I_N}{I_0} = 1 \qquad \text{or} \quad I_N = I_0$$

## EXAMPLE 4.6

Sodium light has two wavelengths, $\lambda_1 = 589$ nm and $\lambda_2 = 589.6$ nm. As the path difference increases, when is the visibility of the fringes a minimum?

**Sol.** The visibility of fringes would be minimum when the point where the $n^{\text{th}}$ dark fringe is formed by the light of wavelength $\lambda_1$ is the same where the $n^{\text{th}}$ bright fringe is formed by the light of wavelength $\lambda_2$.

For the $n^{\text{th}}$ dark fringe formed by $\lambda_1$, the path difference, $\Delta x_1 = (2n+1)\frac{\lambda_1}{2}$ and for the $n^{\text{th}}$ bright fringe formed by $\lambda_2$, the path difference, $\Delta x_2 = n\lambda_2$

Since the $n^{\text{th}}$ dark fringe due to $\lambda_1$ is formed at the same point where the $n^{\text{th}}$ bright fringe due to $\lambda_2$ is formed, $\Delta x_1 = \Delta x_2$

or $(2n+1)\frac{\lambda_1}{2} = n\lambda_2$

or $\left(n+\frac{1}{2}\right)\lambda_1 = n\lambda_2$

or $n\lambda_2 - n\lambda_1 = \frac{\lambda_1}{2}$

or $n\left[\frac{\lambda_2}{\lambda_1} - 1\right] = \frac{1}{2}$

or $n\left[\frac{589.6}{589.0} - 1\right] = \frac{1}{2}$

or $n(1.0010 - 1) = \frac{1}{2}$

or $n = \frac{1}{2 \times 0.0010} = 500$

Thus, $\Delta x_1 = \Delta x_2 = n\lambda_2 = 500 \times 589.6\,\text{nm} = 294800\,\text{nm} = 0.29\,\text{mm}$

## EXAMPLE 4.7

Unpolarized light of intensity 32 W $m^{-2}$ passes through three polarizers such that the transmission axis of the last polarizer is crossed with the first. If the intensity of the emerging light is 3 W $m^{-2}$, what is the angle between the transmission axes of the first two polarizers ? At what angle will the transmitted intensity be maximum ?

**Sol.** Let $P_1, P_2, P_3$ be the three polarizers and $\theta$ be the angle between the transmission axes of $P_1$ and $P_2$. As $P_1$ and $P_3$ are crossed, the angle between $P_2$ and $P_3$ is $90° - \theta$.

Let $I_0$ be the intensity of the unpolarized light falling on $P_1$. Then the intensity of light emerging from $P_1$ will be

$$I_1 = \frac{1}{2} I_0$$

By Malus law, the intensity of light emerging from $P_2$ is

$$I_2 = I_1 \cos^2\theta = \frac{1}{2} I_0 \cos^2\theta$$

The intensity of light emerging from $P_3$ is

$$I_3 = I_2 \cos^2(90° - \theta) = \frac{1}{2} I_0 \cos^2\theta \sin^2\theta$$

$$= \frac{1}{8} I_0 \sin^2 2\theta$$

$\therefore$ $\sin^2 2\theta = \frac{8I_3}{I_0} = \frac{8 \times 3}{32} = \frac{3}{4}$

or $\sin 2\theta = \sqrt{3}/2$

or $2\theta = 60°$

or $\theta = 30°$

As $I_3 = \frac{1}{8} I_0 \sin^2 2\theta$

$\therefore$ $I_3$ will be maximum when

$\sin^2 2\theta = 1$(maximum) or $\sin 2\theta = 1 = \sin 90°$

or $\theta = 45°$

# Exercises

## Single Correct Answer Type

1. In case of linearly polarized light, the magnitude of the electric field vector
(1) does not change with time
(2) varies periodically with time
(3) increases and decreases linearly with time
(4) is parallel to the direction of propagation

2. A diffraction is obtained by using a beam of red light. What will happen if the red light is replaced by the blue light?
(1) Bands will narrower and crowded together
(2) Bands become broader and move further apart
(3) No change will take place
(4) Bands disappear

3. When an unpolarized light of intensity $I_0$ is incident on a polarizing sheet, the intensity of the light which does not get transmitted is
(1) $\frac{1}{2}I_0$ (2) $\frac{1}{4}I_0$
(3) zero (4) $I_0$

4. In a diffraction pattern by a wire, on increasing diameter of wire, fringe width
(1) decreases
(2) increases
(3) remains unchanged
(4) increasing or decreasing will depend on wavelength

5. What will be the angular width of central maxima in Fraunhofer diffraction when light of wavelength 6000 Å is used and slit width is $12\times10^{-5}$ cm?
(1) 2 rad (2) 3 rad
(3) 1 rad (4) 8 rad

6. If we observe the single slit Fraunhofer diffraction with wavelength $\lambda$ and slit width $d$, the width of the central maxima is $2\theta$. On decreasing the slit width for the same $\lambda$
(1) $\theta$ increases
(2) $\theta$ remains unchanged
(3) $\theta$ decreases
(4) $\theta$ increases or decreases depending on the intensity of light

7. In order to see diffraction, the thickness of the film is
(1) 100 Å (2) 10,000 Å
(3) 1 mm (4) 1 cm

8. A spectral line $\lambda = 5000$ Å in the light coming from a distant star is observed as a 5200 Å. What will be recession velocity of the star?
(1) $1.15\times10^7$ cm/s (2) $1.15\times10^7$ m/s
(3) $1.15\times10^7$ km/s (4) 1.15 km/s

9. The apparent wavelength of the light from a star moving away from the earth is 0.01% more than its real wavelength. Then the velocity of star is
(1) 60 km/s (2) 15 km/s
(3) 150 km/s (4) 30 km/s

10. A star emitting light of wavelength 5896 Å is moving away from the earth with a speed of 3600 km/s. The wavelength of light observed on earth will
(1) decrease by 5825.25 Å
(2) increase by 5966.75 Å
(3) decrease by 70.75 Å
(4) increase by 70.75 Å
($c = 3\times10^8$ m/s is the speed of light)

11. A star moves away from earth at speed 0.8 $c$ while emitting light of frequency $6\times10^{14}$ Hz. What frequency will be observed on the earth (in units of $10^{14}$ Hz) ($c$ = speed of light)?
(1) 0.24 (2) 1.2
(3) 30 (4) 3.3

12. A light source approaches the observer with velocity 0.8 $c$. The doppler shift for the light of wavelength 5500 Å is
(1) 4400 Å (2) 1833 Å
(3) 3167 Å (4) 7333 Å

13. In the spectrum of light of a luminous heavenly body the wavelength of a spectral line is measured to be 4747 Å while actual wavelength of the line is 4700 Å. The relative velocity of the heavenly body with respect to earth will be (velocity of light is $3\times10^8$ m/s)
(1) $3\times10^5$ m/s moving towards the earth
(2) $3\times10^5$ m/s moving away from the earth
(3) $3\times10^6$ m/s moving towards the earth
(4) $3\times10^6$ m/s moving away from the earth

14. 6563 Å line emitted by hydrogen atom in a star is found to be red shifted by 5 Å. The speed with which the star is receding from the earth is
(1) $17.29\times10^9$ m/s (2) $4.29\times10^7$ m/s
(3) $3.39\times10^5$ m/s (4) $2.29\times10^5$ m/s

15. A slit of width $a$ is illuminated by white light. For red light ($\lambda = 6500$ Å), the first minima is obtained at $\theta = 30°$. Then the value of $a$ will be
(1) 3250 Å (2) $6.5\times10^{-4}$ mm
(3) 1.24 microns (4) $2.6\times10^{-4}$ cm

16. The light of wavelength 6328 Å is incident on a slit of width 0.2 mm perpendicularly, the angular width of central maxima will be
(1) 0.36° (2) 0.18°
(3) 0.72° (4) 0.90°

17. A slit of size 0.15 cm is placed at 2.1 m from a screen. On illuminated it by a light of wavelength $5\times10^{-5}$ cm. The width of central maxima will be
(1) 70 mm (2) 0.14 mm
(3) 1.4 mm (4) 0.14 cm

**18.** What will be the angle of diffraction for the first minimum due to Fraunhofer diffraction with sources of light of wave length 550 nm and slit of width 0.55 mm?
(1) 0.001 rad (2) 0.01 rad
(3) 1 rad (4) 0.1 rad

**19.** Angular width ($\beta$) of central maximum of a diffraction pattern on a single slit does not depend upon
(1) distance between slit and source
(2) wavelength of light used
(3) width of the slit
(4) frequency of light used

**20.** A single slit of width 0.20 mm is illuminated with light of wavelength 500 nm. The observing screen is placed 80 cm from the slit. The width of the central bright fringe will be
(1) 1 mm (2) 2 mm
(3) 4 mm (4) 5 mm

**21.** The observed wavelength of light coming from a distant galaxy is found to be increased by 0.5% as compared with that coming from a terrestrial source. The galaxy is
(1) stationary with respect to the earth
(2) approaching the earth with velocity of light
(3) receding from the earth with the velocity of light
(4) receding from the earth with a velocity equal to $1.5\times10^6$ m/s

**22.** A rocket is moving away from the earth at a speed of $6\times10^7$ m/s . The rocket has blue light in it. What will be the wavelength of light recorded by an observer on the earth? (Wavelength of blue light = 4600 Å)
(1) 4600 Å (2) 5520 Å
(3) 3680 Å (4) 3920 Å

**23.** Yellow light is used in single slit diffraction experiment with slit width 0.6 mm. If yellow light is replaced by X-rays then the pattern will reveal
(1) that the central maxima is narrower
(2) no diffraction pattern
(3) more number of fringes
(4) less number of fringes

**24.** A plane wavefront ($\lambda = 6\times10^{-7}$ m) falls on a slit 0.4mm wide. A convex lens of focal length 0.8m placed behind the slit focusses the light on a screen. What is the linear diameter of second maximum?
(1) 6 mm (2) 12 mm
(3) 3 mm (4) 9 mm

**25.** Red light is generally used to observe diffraction pattern from single slit. If blue light is used instead of red light, then diffraction pattern
(1) will be more clear (2) will contract
(3) will expand (4) will not be visualized

**26.** Light of wavelength 589.3 nm is incident normally on the slit of width 0.1 mm. What will be the angular width of the central diffraction maximum at a distance of 1 m from the slit?
(1) 0.68° (2) 1.02°
(3) 0.34° (4) None of these

**27.** A beam of light of wavelength 600 nm from a distant source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. The distance between the first dark fringes on either side of the central bright fringe is
(1) 1.2 mm (2) 1.2 cm
(3) 2.4 cm (4) 2.4 mm

**28.** Direction of the first secondary maximum in the Fraunhofer diffraction pattern at a single slit is given by ($a$ is the width of the slit)
(1) $a\sin\theta = \frac{\lambda}{2}$ (2) $a\cos\theta = \frac{3\lambda}{2}$
(3) $a\sin\theta = \lambda$ (4) $a\sin\theta = \frac{3\lambda}{2}$

**29.** A parallel monochromatic beam of light is incident normally on a narrow slit. A diffraction pattern is formed on a screen placed perpendicular to the direction of incident beam. At the first maximum of the diffraction pattern the phase difference between the rays coming from the edges of the slit is
(1) 0 (2) $\frac{\pi}{2}$
(3) $\pi$ (4) $2\pi$

**30.** A light wave is incident normally over a slit of width $24\times10^{-5}$ cm. The angular position of second dark fringe from the central maxima is 30°. What is the wavelength of light?
(1) 6000 Å (2) 5000 Å
(3) 3000 Å (4) 1500 Å

**31.** A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a single narrow slit of width 0.001 mm. The light is focused by a convex lens on a screen placed on the focal plane. The first minimum will be formed for the angle of diffraction equal to
(1) 0° (2) 15°
(3) 30° (4) 60°

**32.** In the far field diffraction pattern of a single slit under polychromatic illumination, the first minimum with the wavelength $\lambda_1$ is found to be coincident with the third maximum at $\lambda_2$. So
(1) $3\lambda_1 = 0.3\lambda_2$ (2) $3\lambda_1 = \lambda_2$
(3) $\lambda_1 = 3.5\lambda_2$ (4) $0.3\lambda_1 = 3\lambda_2$

**33.** Light of wavelength $\lambda$ = 5000 Å falls normally on a narrow slit. A screen placed at a distance of 1 m from the slit and perpendicular to the direction of light. The first minima of the diffraction pattern is situated at 5 mm from the centre of central maximum. The width of the slit is
(1) 0.1 mm (2) 1.0 mm
(3) 0.5 mm (4) 0.2 mm

**34.** A single slit of width $a$ is illuminated by violet light of wavelength 400 nm and the width of the diffraction pattern is measured as $y$. When half of the slit width is covered and illuminated by yellow light of wavelength 600 nm, the width of the diffraction pattern is
(1) the pattern vanishes and the width is zero
(2) $y/3$
(3) $3y$
(4) none of these

**35.** The angle of polarization for any medium is 60°, what will be critical angle for this?

(1) $\sin^{-1}\sqrt{3}$ (2) $\tan^{-1}\sqrt{3}$

(3) $\cos^{-1}\sqrt{3}$ (4) $\sin^{-1}\frac{1}{\sqrt{3}}$

36. A polaroid is placed at 45° to an incoming light of intensity $I_0$. Now the intensity of light passing through polaroid after polarization would be?

(1) $I_0$ (2) $I_0/2$

(3) $I_0/4$ (4) Zero

37. Two Nicols are oriented with their principal planes making an angle of 60°. The percentage of incident unpolarized light which passes through the system is

(1) 50% (2) 100%

(3) 12.5% (4) 37.5%

38. Unpolarized light falls on two polarizing sheets placed one on top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of the final transmitted light is one-third the maximum intensity of the first transmitted beam?

(1) 75° (2) 55°

(3) 35° (4) 15°

39. When unpolarized light beam is incident from air onto glass ($n = 1.5$) at the polarizing angle

(1) Reflected beam is polarized 100 percent

(2) Reflected and refracted beams are partially polarized

(3) The reason for (1) is that almost all the light is reflected

(4) All of the above

40. When the angle of incidence on a material is 60°, the reflected light is completely polarized. The velocity of the refracted ray inside the material is (in $ms^{-1}$)

(1) $3\times10^8$ (2) $\left(\frac{3}{\sqrt{2}}\right)\times10^8$

(3) $\sqrt{3}\times10^8$ (4) $0.5\times10^8$

41. Two polaroids are placed in the path of unpolarized beam of intensity $I_0$ such that no light is emitted from the second polaroid. If a third polaroid whose polarization axis makes an angle $\theta$ with the polarization axis of first polaroid is placed between these polaroids then the intensity of light emerging from the last polaroid will be

(1) $\left(\frac{I_0}{8}\right)\sin^2 2\theta$ (2) $\left(\frac{I_0}{4}\right)\sin^2 2\theta$

(3) $\left(\frac{I_0}{2}\right)\cos^4\theta$ (4) $I_0\cos^4\theta$

42. Angular width of central maxima in the Fraunhofer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, the angular width decreases by 30%. The wavelength of this light will be

(1) 6000 Å (2) 4200 Å

(3) 3000 Å (4) 1800 Å

43. A beam of natural light falls on a system of 6 polaroids, which are arranged in succession such that each polaroid is turned through 30° with respect to the preceding one. The percentage of incident intensity that passes through the system will be

(1) 100% (2) 50%

(3) 30% (4) 12%

44. If $I_0$ is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled?

(1) $I_0$ (2) $\frac{I_0}{2}$

(3) $2I_0$ (4) $4I_0$

45. A beam of electron is used in an YDSE. The slit width is $d$. When the velocity of electron is increased, then

(1) no interference is observed

(2) fringe width increases

(3) fringe width decreases

(4) fringe width remains same

46. The time period of rotation of the Sun is 25 days and its radius is $7\times10^8$ m. The Doppler shift for the light of wavelength 6000 Å emitted from the surface of the Sun will be

(1) 0.04 Å (2) 0.40 Å

(3) 4.00 Å (4) 40.0 Å

47. In hydrogen spectrum the wavelength of $H_\alpha$ line is 656 nm whereas in the spectrum of a distant galaxy, $H_\alpha$ line wavelength is 706 nm. Estimated speed of the galaxy with respect to earth is

(1) $2\times10^8$ m/s (2) $2\times10^7$ m/s

(3) $2\times10^6$ m/s (4) $2\times10^5$ m/s

48. A rocket is going towards moon with a speed $v$. The astronaut in the rocket sends signals of frequency $v$ towards the moon and receives them back on reflection from the moon. What will be the frequency of the signal received by the astronaut? (Take $v<<c$)

(1) $\frac{c}{c-v}v$ (2) $\frac{c}{c-2v}v$

(3) $\frac{2v}{c}v$ (4) $\frac{2c}{v}v$

49. The periodic time of rotation of a certain star is 22 days and its radius is $7\times10^8$ metre. If the wavelength of light emitted by its surface be 4320 Å, the Doppler shift will be (1 day = 86400 sec)

(1) 0.033 Å (2) 0.33 Å

(3) 3.3 Å (4) 33 Å

50. In a single slit diffraction of light of wavelength $\lambda$ by a slit of width $d$, the size of the central maximum on a screen at a distance $b$ is

(1) $2b\lambda + d$ (2) $\frac{2b\lambda}{d}$

(3) $\frac{2b\lambda}{d} + d$ (4) $\frac{2b\lambda}{d} - d$

51. In a single slit diffraction experiment first minimum for red light (660 nm) coincides with first maximum of some other wavelength $\lambda'$. The value of $\lambda'$ is

(1) 4400 Å (2) 6600 Å

(3) 2000 Å (4) 3500 Å

52. The ratio of intensities of consecutive maxima in the diffraction pattern due to a single slit is

(1) 1:4:9 (2) 1:2:3

(3) $1:\frac{4}{9\pi^2}:\frac{4}{25\pi^2}$ (4) $1:\frac{1}{\pi^2}:\frac{9}{\pi^2}$

**53.** The ratio of the intensity at the centre of a bright fringe to the intensity at a point one-quarter of the distance between two fringe from the centre is

(1) 2 (2) 1/2

(3) 4 (4) 16

**54.** A beam of plane polarized light falls normally on a polarizer of cross-sectional area $3 \times 10^{-4}$ m$^2$. Flux of energy of incident ray in $10^{-3}$ W. The polarizer rotates with an angular frequency of 31.4 rad/s. The energy of light passing through the polarizer per revolution will be

(1) $10^{-4}$ joule (2) $10^{-3}$ joule

(3) $10^{-2}$ joule (4) $10^{-1}$ joule

**55.** Unpolarized light of intensity 32 Wm$^{-2}$ passes through three polarizers such that transmission axes of the first and second polarizer makes an angle 30° with each other and the transmission axis of the last polarizer is crossed with that of the first. The intensity of final emerging light will be

(1) 32 Wm$^{-2}$ (2) 3 Wm$^{-2}$

(3) 8 Wm$^{-2}$ (4) 4 Wm$^{-2}$

# Archives

## JEE Main

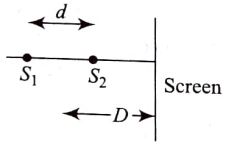
### Single Correct Answer Type

**1.** A beam of unpolarised light of intensity $I_0$ is passed through a polaroid $A$ and then through another polaroid $B$ which is oriented so that its principal plane makes an angle of 45° relative to that of $A$. The intensity of the emergent light is

(1) $I_0/2$ (2) $I_0/4$

(3) $I_0/8$ (4) $I_0$ **(JEE Main 2013)**

**2.** Two beams, $A$ and $B$, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam $A$ has maximum intensity (and beam $B$ has zero intensity), a rotation of polaroid through 300 makes the two beams appear equally bright. If the initial intensities of the two beams are $I_A$ and $I_B$ respectively, then $I_A/I_B$ equals




(1) 1 (2) 1/3

(3) 3 (4) 3/2 **(JEE Main 2014)**

**3.** Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is

(1) 1 μm (2) 30 μm

(3) 100 μm (4) 300 μm

**(JEE Main 2015)**

**4.** The box of a pin hole camera, of length $L$, has a hole of radius $a$. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength $\lambda$ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say $b_{min}$) when

(1) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(2) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

(3) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

(4) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$ **(JEE Main 2016)**

**5.** The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance ? (i.e. distance between the centres of each slit)

(1) 100 μm (2) 25 μm

(3) 50 μm (4) 75 μm **(JEE Main 2018)**

**6.** Unpolarized light of intensity $I$ passes through an ideal polarizer $A$. Another identical polarizer $B$ is placed behind $A$. The intensity of light beyond $B$ is found to be $I/2$. Now another identical polarizer $C$ is placed between $A$ and $B$. The intensity beyond $B$ is now found to be $I/8$. The angle between polarizer $A$ and $C$ is

(1) 60° (2) 0°

(3) 30° (4) 45° **(JEE Main 2018)**

# Answers Key

## EXERCISES

### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| **1.** (2) | **2.** (1) | **3.** (1) | **4.** (1) | **5.** (3) |
| **6.** (1) | **7.** (2) | **8.** (2) | **9.** (4) | **10.** (4) |
| **11.** (2) | **12.** (3) | **13.** (4) | **14.** (4) | **15.** (3) |
| **16.** (1) | **17.** (3) | **18.** (1) | **19.** (1) | **20.** (3) |
| **21.** (4) | **22.** (2) | **23.** (2) | **24.** (1) | **25.** (2) |
| **26.** (1) | **27.** (4) | **28.** (4) | **29.** (4) | **30.** (1) |
| **31.** (3) | **32.** (3) | **33.** (1) | **34.** (3) | **35.** (4) |
| **36.** (2) | **37.** (3) | **38.** (2) | **39.** (1) | **40.** (3) |
| **41.** (1) | **42.** (2) | **43.** (4) | **44.** (4) | **45.** (2) |
| **46.** (1) | **47.** (2) | **48.** (2) | **49.** (1) | **50.** (3) |
| **51.** (1) | **52.** (3) | **53.** (1) | **54.** (1) | **55.** (2) |

## ARCHIVES

### JEE Main

### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| **1.** (2) | **2.** (2) | **3.** (2) | **4.** (3) | **5.** (2) |
| **6.** (4) | | | | |

# 5 Dual Nature of Radiation and Matter

## INTRODUCTION

While studying classical physics we learned that the laws that govern the characteristics of particles and waves are fundamentally different.

Louis de-Broglie propounded that under proper circumstances, particles can behave as waves while waves may behave as particles. Thus dual nature, i.e. particle and wave, was established and a fascinating branch of physics, called **quantum mechanics**, evolved. In quantum mechanics, some of the classically inspired notions of the distinction between particles and waves are discarded. Quantum mechanics (or wave mechanics) involves concepts that are often strange and counter to our intuition. Einstein was one of many who contributed to the development of a radical revision of our understanding of particle and wave character by developing quantum theory.

We shall now show that a particle or wave characteristic is not an inherent property but depends upon the type of experimental observation.

## PHOTONS

The concept of photons and quanta comes from quantum mechanics and quantum theory. Quantum mechanics is a mathematical model that describes the behaviour of particles on an atomic and subatomic scale. It demonstrates that matter and energy are quantized, or come in small discrete bundles.

In 1900, Planck proposed that electromagnetic radiation (or light) is quantized and exists in elementary amounts (or quanta) that we now call photons. According to this proposal, the quantum of light wave of frequency $f$ has the energy: $E = hf$.

Here $h$ is Planck's constant and it has the value,

$$h = 6.63 \times 10^{-34} \text{ J s} = 4.14 \times 10^{-15} \text{ eV s}$$

We can say that the light energy from any source is always an integral multiple of a smaller energy value called quantum of light. Hence, energy:

$Q = NE$, where $N$ (number of photons) = 1, 2, 3, ...

Here energy is quantized. $E = hf$ *is* the quantum of energy, it is a packet of energy called photon.

Also $E = hf = \dfrac{hc}{\lambda}$ (in joules) ...(i)

So, the least energy a light of frequency $f$ can have is $hf$. The light cannot have energy like $1.5hf$ or $\pi hf$.

The energy of a photon generally expressed in electron-volt (eV).

In Eq. (i), $hc = 12400$ (eV-Å) $\Rightarrow$ $E = \dfrac{12400}{\lambda(\text{in Å})}$ eV

## PROPERTIES OF PHOTONS

A photon can be considered as a particle of electromagnetic radiation defined as a discrete bundle (or quantum) of electromagnetic (or light) energy. A photon describes the particle properties of an electromagnetic wave instead of the overall wave itself. In other words, we can picture an electromagnetic wave as being made up of individual particles called photons.

- A source of radiation emits energy in the form of photons and these photons travel in straight line with the speed of light. Photon is not a material particle, it is a packet of energy.
- Energy of a photon depends upon its frequency and it does not change with change in medium.
- With change in medium, the speed and wavelength of the photon change but frequency does not change.
- Photons are electrically neutral. They are not defected by electric or magnetic fields.
- A photon can collide with material particles like electron. During these collisions, the total energy and total momentum remain constant.
- A photon does not exist at rest. Its rest mass is zero.
- Equivalent mass of photon: $E = mc^2 = hf$

  $\Rightarrow m = \dfrac{hf}{c^2} = \dfrac{h}{c\lambda}$
- Photons have momentum: $p = mc = \dfrac{h}{c\lambda}c = \dfrac{h}{\lambda}$
- (Intensity of a light beam) $\propto$ (number of photons present in it).

### ILLUSTRATION 5.1

What is the energy (in eV) of a photon of wavelength 12400 Å?

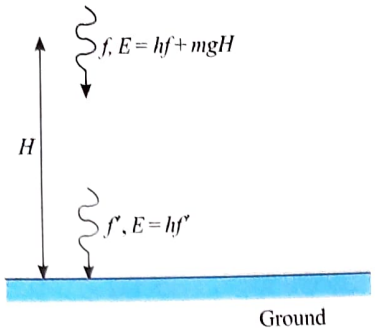
**Sol.** Using Plank's formula, we have $E = hf = \dfrac{hc}{\lambda}$

$$E = \frac{12400}{\lambda(\text{Å})}\text{eV} = \frac{12400}{12400}\text{eV} = 1\text{ eV}$$

### ILLUSTRATION 5.2

If a photon frequency $f$ falls on the surface of earth from a height $H$, then what will be its frequency on the surface of earth.

**Sol.** Change in frequency of the photon takes place.




Let $f$ be the frequency of photon at height $H$.

Then the mass of photon, $m = \dfrac{hf}{c^2}$

Mass of the photon depends on its frequency, but we will consider the mass to be constant as difference in $f'$ and $f$ is very small.

Energy conservation gives Initial energy = final energy

$$\Rightarrow \quad hf + mgH = hf'$$

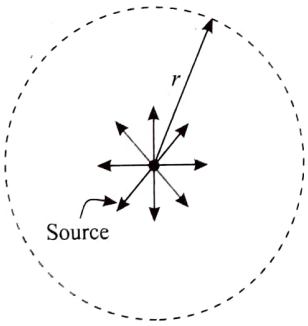
$$\Rightarrow \quad hf + \left(\frac{hf}{c^2}\right)gH = hf'$$

$$\Rightarrow \quad f' = f\left(1 + \frac{gH}{c^2}\right)$$

## PHOTON COUNTS EMITTED BY A SOURCE PER SECOND

Consider a point source of power $P$ watt as the source of radiation energy (see figure). If the wavelength of radiation emitted by the source is $\lambda$, then energy of each photon emitted by the source can be given as

$$E = hf = \frac{hc}{\lambda} \qquad \ldots(i)$$




The power of the source is $P$ watt, we can say that the source is emitting radiation energy $P$ joule per second in the form of photons. If it is 100% efficient, then the number of photons emitted per second by the source can be given as

$$n = \frac{\text{Power of the source}}{\text{Energy of photon}} = \frac{P}{E} = \frac{P\lambda}{hc} \qquad \ldots(ii)$$

These photons are assumed to be emitted uniformly in all directions. Here, we can consider that all the light energy emitted by the source is uniformly distributed in the spherical region with center at the source.

### ILLUSTRATION 5.3

Calculate the number of photons emitted in 10 h by a 60 W sodium lamp ($\lambda = 6200$ Å).

**Sol.** The energy of a photon (in eV) can be given by

$$E = \frac{12400}{\lambda(\text{Å})}\text{eV} = \frac{12400}{6200} = 2\text{eV}$$

Hence the energy of a photon in joules,

$$E = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}\,\text{J}$$

Therefore, the number of photons emitted by sodium lamp in 10 h

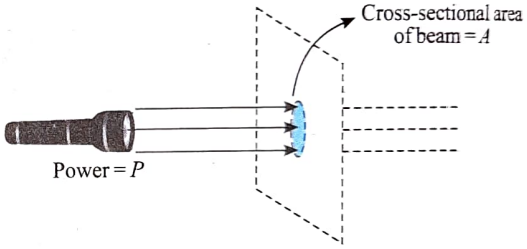
$$N = \frac{\text{Total energy released in 10 h}}{\text{Energy of a photon}}$$

$$\Rightarrow \quad N = \frac{60 \times 10 \times 3600}{3.2 \times 10^{-19}} = 6.75 \times 10^{24}$$

## INTENSITY OF RADIATION

The radiation energy crossing per unit area per unit time normal to the direction of propagation is called the **intensity of a wave (radiation)**.

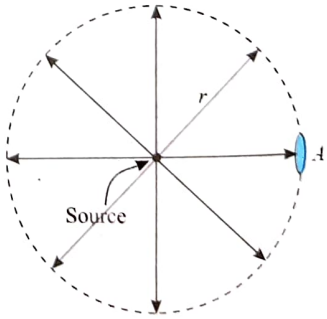
Consider a source of light emits a uniform cylindrical light beam of cross-sectional area $A$. If the source emits a total power $P$ in the beam as shown in figure.




then the intensity of light beam is $I = \dfrac{P}{A}\ \text{W m}^{-2}$

If cross-sectional area of the beam remain constant throughout the beam intensity at every point will remain constant.

Now let us find the intensity of light (radiation) due to a point isotropic source as shown in figure.




If the power of source is $P$ watt emitting radiation in all directions uniformly. If we want to calculate intensity of light at a point, at a distance $r$ from the source, then it can be given as

$$I = \frac{P}{4\pi r^2}\ \text{W m}^{-2}$$

Here, we can assume that the whole power $P$ is incident on the normal area of a hypothetical sphere of radius $r$ passing through points with center at the source as shown.

**Note:** The radiation power ($P'$) crossing through a small area $A$ at a distance $r$, $P' = IA = \left(\frac{P}{\pi r^2}\right)A$

## PHOTON FLUX

It is defined as the number of photons incident on a normal surface per second per unit area. Consider a light beam of intensity $I$ W m$^{-2}$ having wavelength $\lambda$ incident on a surface. Then, number of photons per second per unit area in the beam can be given as

$$\text{Photon flux, } \phi = \frac{\text{Intensity}}{\text{Energy of a photon}} = \frac{I}{hc/\lambda} = \frac{I\lambda}{hc} \quad ...(i)$$

If we consider a point source of power $P$ watt which emits light in all directions, then it produces photons per second at a rate

$$n = \frac{\text{Power of a source}}{\text{Energy of a photon}} = \frac{P}{(hc/\lambda)} = \frac{P\lambda}{hc} \quad ...(ii)$$

All these photons are distributed in the three-dimensional spherical space around the source. If we find the photon flux at a distance $r$ from the point source, it can be given as

$$\phi = \frac{\text{Number of photons per second}}{\text{Surface area of sphere of radius } r} = \frac{n}{4\pi r^2} \quad ...(iii)$$

### ILLUSTRATION 5.4

A bulb lamp emits light of mean wavelength of 4500 Å. The lamp is rated at 150 W and 8% of the energy appears as emitted light. How many photons are emitted by the lamp per second?

**Sol.** The energy of photon $= \frac{hc}{\lambda} = \frac{(6.63\times10^{-34})(3\times10^{8})}{4500\times10^{-10}}$

According to the given problem, power of the lamp is 150 W. As 8% of the light energy is utilized for emission of light, hence the utilized energy for emission per second is given by

$$\text{Useful power} = 150\times\frac{8}{100} = 12 \text{ W} = 12 \text{ J s}^{-1}$$

Therefore, number of photons emitted per second by the lamp

$$\frac{\text{Useful power}}{\text{Energy of photon}} = \frac{12\times4500\times10^{-10}}{(6.63\times10^{-34})(3\times10^{8})} = 27.17\times10^{18}$$

### ILLUSTRATION 5.5

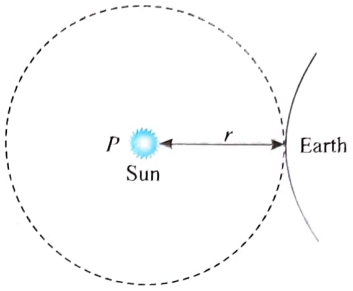
The power of light emitted by the sun is $3.9 \times 10^{26}$ W. Assuming the mean wavelength of sunlight to be 6000 Å, calculate the photon flux arriving at earth perpendicular to light radiations. The average radius of earth's orbit around sun is $1.49 \times 10^{11}$ m.

**Sol.** Power of light emitted by the sun, $P = 3.9\times10^{26}$ W

Earth sun mean distance $r = 1.49\times10^{11}$ m

Mean wavelength of sun light, $\lambda = 6000$ Å $= 6\times10^{-7}$ m

Intensity of light an earth, $I = \frac{W}{4\pi r^2}$




The energy of a photon, $E = \frac{hc}{\lambda}$

Hence photon-flux, $\phi = \frac{I}{hc/\lambda} = \frac{P\lambda}{4\pi r^2 hc}$

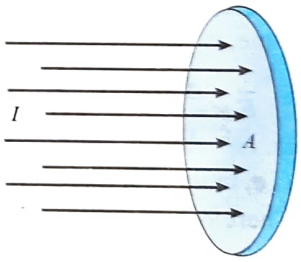
$$\Rightarrow \quad \phi = \frac{(3.9\times10^{26})\times(6\times10^{-7})}{4\pi(1.49)^2\times10^{22}\times6.63\times10^{-34}\times3\times10^{8}}$$

or $\phi = 4.22\times10^{21}$ photon/s m$^2$

## FORCE EXERTED ON A SURFACE DUE TO RADIATION

When photons fall on a surface, they exert a force and pressure on the surface. The force (pressure) experienced by surface exposed to radiation is known as radiation force (pressure). This force can be calculated using photon theory of radiation.

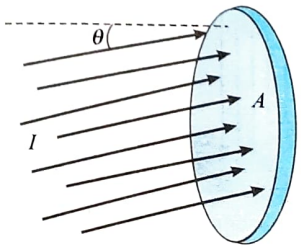
Let us consider a surface of area $A$ exposed to radiation of intensity $I$ as shown in figure. The radiation is falling normally on the surface.




Assume absorption and reflection coefficients of surface are $a$ and $r$, respectively. The absorption and reflection coefficients means the fraction of the photons being absorbed and reflected by the incident surface respectively. Assume no transmission. Energy received by the surface per second, $P = IA$. It is also the power received by the surface. Number of photons received by surface per second,

$$n = \frac{P}{\text{Energy of one photon}} = \frac{P}{hc/\lambda} = \frac{IA\lambda}{hc}$$

If radiation incident on the surface at certain angle, then power incident on the surface, $P = IA\cos\theta$

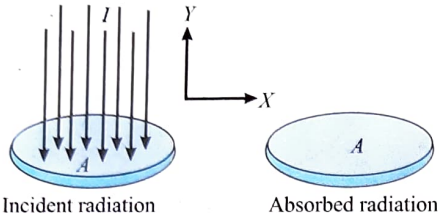


The average force imparted by radiation can be expressed

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

Here $\Delta \vec{p}$ is the change in momentum of the photons incident on the surface.

**Case I. The radiation falling on the surface normally and absorbed by the surface:** For this, absorption coefficient $a = 1$, reflection coefficient $r = 0$.




Incident radiation Absorbed radiation

The momentum of a photon $p = \frac{h}{\lambda}$

Initial momentum of one photon $\vec{p}_{\text{initial}} = \left(\frac{h}{\lambda}\right)(-\hat{j})$

(i.e. in downward direction)

Final momentum of the photon $\vec{p}_{\text{final}} = 0$

Change in momentum of one photon,

$$\Delta\vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}} = 0 - \left(\left(\frac{h}{\lambda}\right)(-\hat{j})\right) = \frac{h}{\lambda}(\hat{j})$$

(i.e. in upward direction)

Energy incident per unit time (or power), $P = IA$

Number of photons incident per unit time, $n = \frac{IA}{hf} = \frac{IA\lambda}{hc}$

Therefore, the total change in momentum of photons per unit time, it is also force acting on photons

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = n\frac{h}{\lambda}(\hat{j}) = \frac{IA\lambda}{hc} \times \frac{h}{\lambda}(\hat{j}) = \frac{IA}{c}(\hat{j})$$

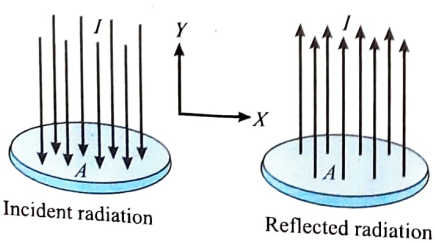
(i.e. in upward direction),

Therefore, force on plate due to photons,

$$F = \frac{IA}{c} \quad \text{(downward)}$$

Therefore, radiation pressure $= \frac{F}{A} = \frac{IA}{cA} = \frac{I}{c}$

**Case II. The radiation falling on the surface normally and reflected by the surface:** For this, reflection coefficient $r = 1$, absorption coefficient $a = 0$.




Incident radiation Reflected radiation

Initial momentum of one photon, $\vec{p}_{\text{initial}} = \left(\frac{h}{\lambda}\right)(-\hat{j})$
(i.e. in downward direction)

Final momentum of the photon, $\vec{p}_{\text{final}} = \left(\frac{h}{\lambda}\right)(\hat{j})$

Change in momentum of one photon,

$$\Delta\vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}} = = \left(\frac{h}{\lambda}\right)(\hat{j}) - \left(\frac{h}{\lambda}\right)(-\hat{j}) = \frac{2h}{\lambda}(\hat{j})$$

(i.e. in upward direction)

Energy incident per unit time (or power) $p = IA$

Number of photons incident per unit time, $n = \frac{IA}{hf} = \frac{IA\lambda}{hc}$

Therefore, the total change in momentum of photons per unit time, it is also force acting on photons

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t} = n\frac{2h}{\lambda}(\hat{j}) = \frac{IA\lambda}{hc} \times \frac{2h}{\lambda}(\hat{j}) = \frac{2IA}{c}(\hat{j})$$

(i.e in upward direction)

Therefore, force on plate due to photons, $F = \frac{2IA}{c}$ (downward)

Hence radiation pressure, $P = \frac{F}{A} = \frac{2IA}{cA} = \frac{2I}{c}$

**Case III. When some part of radiation is absorbed and the remaining part is reflected:** For this, reflection coefficient $r$, absorption coefficient $a$.

Force on plate due to reflected photons, $F_r = r\left(\frac{2IA}{c}\right)$
(downward direction)

Force on plate due to absorbed photons, $F_a = a\left(\frac{IA}{c}\right)$
(downward direction)

Total force on plate, $F = F_r + F_a = r\frac{2IA}{c} + a\frac{IA}{c}$ (downward)

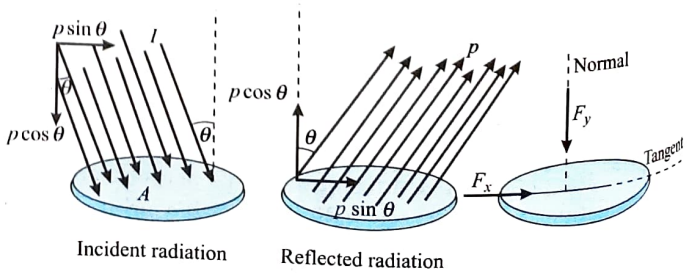
But, $a + r = 1$, so $a = 1 - r$

Hence, $F = r\frac{2IA}{c} + (1-r)\frac{IA}{c} = (1+r)\frac{IA}{c}$ (downward)

Hence radiation pressure, $P = \frac{F}{A} = \frac{I}{c}(1+r)$

**Case IV. The radiation falling on the surface at certain angle and reflected by the surface:** In this case the momentum of the photons will remain unchanged in horizontal direction (in $x$-direction)

Hence, $F_x = 0$




Incident radiation Reflected radiation

Now considering in vertical direction:

The momentum of a photon, $p = \frac{h}{\lambda}$

Initial momentum of one photon, $(\vec{p}_{\text{initial}})_y = \left(\frac{h}{\lambda}\cos\theta\right)(-\hat{j})$ (i.e. in downward direction)

Final momentum of the photon, $(\vec{p}_{\text{final}})_y = \left(\frac{h}{\lambda}\cos\theta\right)(\hat{j})$

Change in momentum of one photon in vertical direction,

$$(\Delta\vec{p})_y = (\vec{p}_{\text{final}})_y - (\vec{p}_{\text{initial}})_y$$

$$(\Delta\vec{p})_y = \left(\frac{h}{\lambda}\cos\theta\right)(\hat{j}) - \left(\left(\frac{h}{\lambda}\cos\theta\right)(-\hat{j})\right) = \frac{2h}{\lambda}\cos\theta(\hat{j})$$

(i.e. in upward direction)

Energy incident per unit time (or power), $E = IA\cos\theta$

Number of photons incident per unit time,

$$n = \frac{IA\cos\theta}{hf} = \frac{IA\cos\theta.\lambda}{hc}$$

Therefore, the total change in momentum of photons per unit time, i.e., force acting on photons,

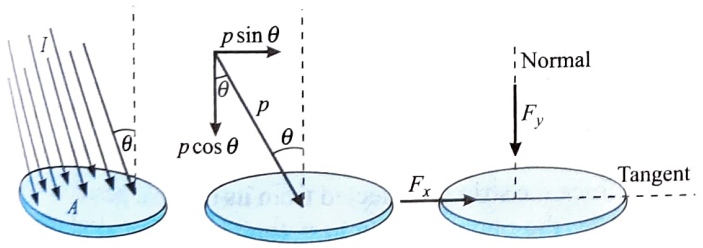
$$\vec{F}_y = \frac{\Delta\vec{p}_y}{\Delta t} = n\frac{2h}{\lambda}\cos\theta(\hat{j})$$

$$\Rightarrow \vec{F}_y = \frac{IA\cos\theta.\lambda}{hc}\times\frac{2h}{\lambda}\cos\theta(\hat{j}) = \frac{2IA}{c}\cos^2\theta(\hat{j})$$

(i.e. in upward direction),

Hence radiation pressure, $P = \frac{F}{A} = \frac{2IA}{cA}\cos^2\theta = \frac{2I}{c}\cos^2\theta$

**Case V: The radiation falling on the surface at certain angle and absorbed by the surface:** In this case the momentum of the photons will change in horizontal direction ($x$-direction) and vertical direction ($y$-direction).




Momentum of a photon, $p = \frac{h}{\lambda}$

Considering in horizontal direction.

Initial momentum of one photon, $(\vec{p}_{\text{initial}})_x = \left(\frac{h}{\lambda}\sin\theta\right)(\hat{i})$ (i.e. in rightward direction)

Final momentum of the photon, $(\vec{p}_{\text{final}})_x = 0$

Change in momentum of one photon in horizontal direction,

$$(\Delta\vec{p})_x = (\vec{p}_{\text{final}})_x - (\vec{p}_{\text{initial}})_x$$

$$(\Delta\vec{p})_x = 0 - \left(\left(\frac{h}{\lambda}\sin\theta\right)(-\hat{i})\right) = -\frac{h}{\lambda}\sin\theta(\hat{i})$$

(i.e. in leftward direction)

Therefore, the total change in momentum of photons per unit time, i.e. force acting on photons,

$$\vec{F}_x = \frac{\Delta\vec{p}_x}{\Delta t} = n\left(\frac{h}{\lambda}\sin\theta\right)(-\hat{i})$$

$$\Rightarrow \vec{F}_x = \frac{IA\cos\theta.\lambda}{hc}\times\left(\frac{h}{\lambda}\sin\theta\right)(-\hat{i}) = \frac{IA}{c}\sin\theta\cos\theta(-\hat{i})$$

(i.e. in leftward direction)

Hence the horizontal component of the radiation force on the surface is $\frac{IA}{c}\sin\theta\cdot\cos\theta$

Considering in vertical direction:

Initial momentum of one photon, $(\vec{p}_{\text{initial}})_y = \left(\frac{h}{\lambda}\cos\theta\right)(-\hat{j})$ (i.e. in downward direction)

Final momentum of the photon, $(\vec{p}_{\text{final}})_y = 0$

Change in momentum of one photon in vertical direction,

$$(\Delta\vec{p})_y = (\vec{p}_{\text{final}})_y - (\vec{p}_{\text{initial}})_y$$

$$(\Delta\vec{p})_y = 0 - \left(\left(\frac{h}{\lambda}\cos\theta\right)(-\hat{j})\right) = \frac{h}{\lambda}\cos\theta(\hat{j})$$

(i.e. in upward direction)

Energy incident per unit time (or power), $E = IA\cos\theta$

Number of photons incident per unit time,

$$n = \frac{IA\cos\theta}{hf} = \frac{IA\cos\theta.\lambda}{hc}$$

Therefore, the total change in momentum of photons per unit time, i.e. force acting on photons,

$$\vec{F}_y = \frac{\Delta\vec{p}_y}{\Delta t} = n\frac{h}{\lambda}\cos\theta(\hat{j})$$

$$\Rightarrow \vec{F}_y = \frac{IA\cos\theta.\lambda}{hc}\times\frac{h}{\lambda}\cos\theta(\hat{j}) = \frac{IA}{c}\cos^2\theta(\hat{j})$$

(i.e. in upward direction)

Hence the vertical component of the radiation force on the surface will be $\frac{2I}{c}\cos^2\theta$ in downward direction.

Hence radiation pressure, $P = \frac{F}{A} = \frac{2IA}{cA}\cos^2\theta = \frac{2I}{c}\cos^2\theta$

## ILLUSTRATION 5.6

**(a)** How many photons of a radiation of wavelength $\lambda = 5 \times 10^{-7}$ m must fall per second on a blackened plate in order to produce a force of $6.62 \times 10^{-5}$ N?

**(b)** At what rate will the temperature of plate rise if its mass is 19.86 kg and specific heat is equal to 2500 J kg$^{-1}$ K$^{-1}$?

**Sol.**

**(a)** If $n$ is the number of photons falling per second on the plate, then the total momentum per second of the incident photons is $p = n \times \frac{h}{\lambda}$.

Since the plate is blackened, all photons are absorbed by it.

$$\therefore \quad \frac{\Delta P}{\Delta t} = n\frac{h}{\lambda}$$

Since, $F = \frac{\Delta P}{\Delta t} = n\frac{h}{\lambda} \Rightarrow n = \frac{F\lambda}{h}$

or $$n = \frac{6.62 \times 10^{-5} \times 5 \times 10^{-7}}{6.62 \times 10^{-34}} = 5 \times 10^{22}$$

**(b)** Energy of each photon $= \frac{hc}{\lambda}$

Since $n$ photons fall on the plate per second, the total energy absorbed by the plates in one second is

$$E = n \times \frac{hc}{\lambda} = 1986 \text{ J s}^{-1}$$

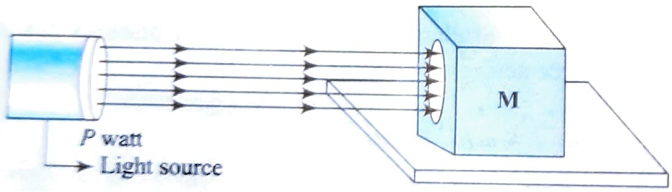
i.e., $$\frac{dQ}{dt} = 1986 \text{ J s}^{-1}$$

$$mc\frac{dT}{dt} = 1986$$

$$\Rightarrow \quad \frac{dT}{dt} = 1986/(19.86 \times 2500) = 4 \times 10^{-2} \text{ °C s}^{-1}$$

## ILLUSTRATION 5.7

A source of light of power $P$ is shown in figure. Find the force on the block placed in the path of the light rays. The surface of body on which the light beam is incident is having a reflection coefficient $a_r = 0.7$ and absorption coefficient $a_a = 0.3$.

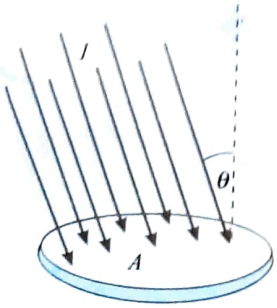



**Sol.** In this case, 70% of the incident photons are reflected back and 30% are absorbed by the body. Thus, the photon which is absorbed will impart a momentum $h/\lambda$ to the body and the photon which is reflected will impart the change in momentum $2h/\lambda$ to the body. Thus, the net force acting on body can be given as

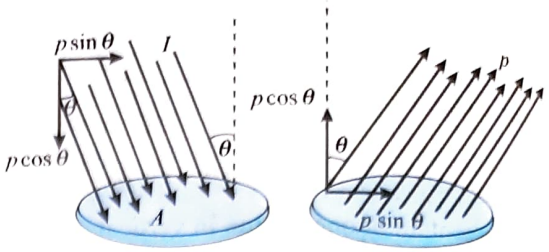
$$F = \frac{0.7P\lambda}{hc} \times \frac{2h}{\lambda} + \frac{0.3P\lambda}{hc} \times \frac{h}{\lambda} = \frac{1.7P}{c}$$

## ILLUSTRATION 5.8

A plane light wave of intensity $I$ falls on a plane mirror surface with reflection coefficient $\rho$. The angle of incidence is $\theta$. In terms of corpuscular theory, find the magnitude of the normal pressure exerted on that surface.




**Sol.** The normal component of the radiation force will be responsible for radiation pressure.




Incident radiation Reflected radiation

The momentum corresponding to incident photons normal to the surface,

$$\left(\frac{dp}{dt}\right)_{\text{incident}} = \frac{I}{c}dA\cos^2\theta$$

Since reflection coefficient is $\rho$, so the momentum of the reflected photons per second normal to surface,

$$\left(\frac{dp}{dt}\right)_{\text{reflected}} = -\frac{I}{c}dA\,\rho\cos^2\theta$$

Hence, rate of change of momentum of the photons,

$$\left(\frac{dp}{dt}\right)_{\text{photons}} = -\frac{I}{c}dA(\rho+1)\cos^2\theta$$

From Newton's third law, $\left(\frac{dP}{dt}\right)_{\text{surface}} = -\left(\frac{dp}{dt}\right)_{\text{photons}}$

Hence, pressure exerted on surface, $P = \frac{dF}{dA} = \frac{I}{c}(\rho+1)\cos^2\theta$

## ILLUSTRATION 5.9

Figure shows a small, plane strip of mass $m$ suspended from a fixed support through a string. A continuous beam of monochromatic light is incident horizontally on the strip and is completely absorbed. The energy falling on the strip per unit time is $W$. Find the deflection of the string from the vertical, if the strip stays in equilibrium.

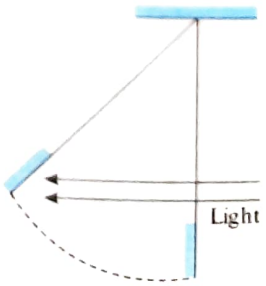
**Sol.** The radiation incident on the strip apply force on it, because of this force the strip is deflected from its original position. Force exerted by the photons of light = (number of incident photons per second) × (change in momentum of each photon)

Let $n$ be the number of incident photons per second.

The momentum of photon incident on the strip per second,

$$P_{\text{incident}} = \frac{W}{c}$$

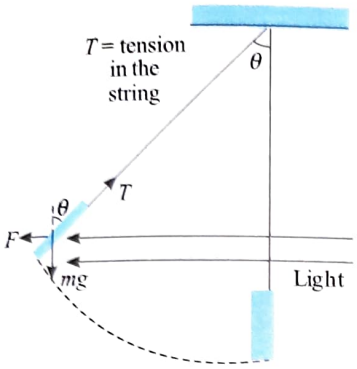
As the photon is completely absorbed, hence its final momentum should be zero.



The change in momentum of each photon per second, $\left|\frac{\Delta p}{\Delta t}\right| = \frac{W}{c}$

Force acting on the strip due to radiation, $F = \frac{W}{c}$

If the string makes an angle $\theta$ with the vertical when the strip is at equilibrium, then for the equilibrium of the strip,




$T\sin\theta = F$, in horizontal direction, and $T\cos\theta = mg$, in vertical direction.

Dividing the above equations, we get

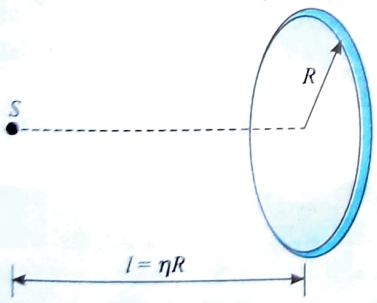
$$\tan\theta = \frac{F}{mg} = \frac{W/c}{mg} = \frac{W}{cmg}$$

$$\therefore \quad \tan\theta = \frac{W}{cmg}$$

$$= \quad \theta = \tan^{-1}\left(\frac{W}{cmg}\right)$$

## ILLUSTRATION 5.10

An isotropic point source having radiation power $P$ is located on the axis of an ideal mirror plate. The distance between the source and the plate exceeds the radius of the plate $\eta$-fold. Find the force that light exerts on the plate.



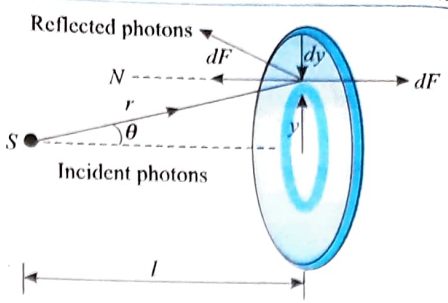

**Sol.** Intensity of radiation at distance of $r$ from source,

$$I = \frac{P}{4\pi r^2}\text{; here } P \text{ is the power of the source} \qquad \ldots(i)$$

If radius of the mirror is $R$ and distance of the point source from the mirror is $l$, then according to the example, $\frac{l}{R} = \eta$.

Momentum per second of photons striking the mirror in normal direction,

$$\left(\frac{dp}{dt}\right)_{n,\text{incident}} = \frac{I}{c}\,dA\cos^2\theta$$




Momentum per second of reflected photons,

$$\left(\frac{dp}{dt}\right)_{n,\text{reflected}} = -\frac{I}{c}\,dA\rho\cos^2\theta$$

where $\rho$ is reflection coefficient.

Hence rate of change of momentum of photons,

$$\left(\frac{dp}{dt}\right)_{n,\text{photons}} = -\frac{I}{c}\,dA(\rho+1)\cos^2\theta$$

From Newton's third law, rate of change of momentum of mirror,

$$\left(\frac{dp}{dt}\right)_{n,\text{mirror}} = \frac{I}{c}\,dA(\rho+1)\cos^2\theta$$

Since mirror plate is perfect reflector, force on its surface is

$$dF = \frac{2I}{c}dA\cos^2\theta = \frac{2I}{c}2\pi y\,dy\cos^2\theta$$

where $dA$ is the area of differential ring element,

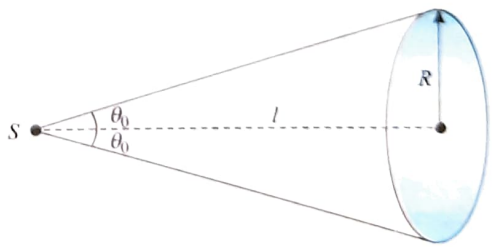
$$y = l\tan\theta \Rightarrow dy = l\sec^2\theta\,d\theta$$

$$dF = \frac{2I}{c}2\pi l\tan\theta\, l\sec^2\theta\,d\theta\cos^2\theta \left[\because \frac{l^2}{r^2} = \cos^2\theta\right]$$

$$= 2\left(\frac{P}{c4\pi r^2}\right)2\pi l^2\tan\theta\,d\theta$$

$$dF = \frac{P}{c}\sin\theta\cos\theta\,d\theta$$

$$F = \int_0^{\theta_0}\frac{P}{c}\sin\theta\cos\theta\,d\theta = \frac{P}{2c}\sin^2\theta_0$$

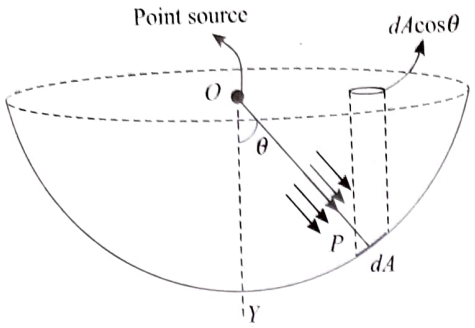



$$\Rightarrow \quad F = \frac{P}{2c}\frac{R^2}{R^2+l^2} = \frac{P}{2c}\left[\frac{1}{1+(l/R)^2}\right] = \frac{P}{2c}\left(\frac{1}{1+\eta^2}\right)$$

## ILLUSTRATION 5.11

A point source of light is placed at the centre of curvature of a hemispherical surface. The radius of curvature is $r$ and the inner surface is completely reflecting. Find the force on the hemisphere due to the light falling on it if the source emits a power $W$.

**Sol.** The intensity of the light at the hemisphere surface will be uniform and should be equal to, $I = \dfrac{W}{(4\pi r^2)}$.




Let us take a small elemental area $dA$ at the point $P$ of the hemisphere. The power incident on this elemental area

$$dW = I \cdot dA = \left(\frac{W}{4\pi r^2}\right) dA$$

The corresponding momentum incident on this elemental area per unit time is $\dfrac{dp}{dt} = \dfrac{dW}{c} = \dfrac{W\,dA}{4\pi r^2 c}$

As the light is reflected back, the change in momentum per unit time, i.e., the force on $dA$ is

$$dF = \frac{2W\,dA}{4\pi r^2 c}$$

Suppose the radius $OP$ through the area $dA$ makes an angle $\theta$ with the symmetry axis $OY$. The force on $dA$ is along this radius. By symmetry, the resultant force on the hemisphere is along $OY$. The component of $dF$ along $OY$ is

$$dF\cos\theta = \frac{2W\,dA}{4\pi r^2 c}\cos\theta$$

If we project the area $dA$ on the plane containing the rim, the projection is $dA\cos\theta$. Thus, the component of $dF$ along $OY$ is

$$dF\cos\theta = \frac{2W}{4\pi r^2 c}\ (\text{projection of } dA)$$

The net force along $OY$ is $F = \dfrac{2W}{4\pi r^2 c}\ (\text{projection of } dA)$

When all the small areas $dA$ are projected, we get the area enclosed by the rim which is $\pi r^2$.

Thus, $F = \dfrac{2W}{4\pi r^2 c} \times \pi r^2 = \dfrac{W}{2c}$

**ILLUSTRATION 5.12**

In the path of a uniform light beam of large cross-sectional area and intensity $I$, a solid sphere of radius $R$ which is perfectly reflecting is placed. Find the force exerted on this sphere due to the light beam.

**Sol.** The front surface of the sphere will be illuminated by the light as shown in figure, as angle of incidence of light is different at different position of the front face of the sphere.

We can consider a small elemental circular strip (ring) of angular width $d\theta$ on its surface at an angle $\theta$ from its horizontal diameter as shown. Let the area of this elemental strip is $dS$, then

$$dS = 2\theta R \sin\theta\, R d\theta \quad \ldots(i)$$

$R\sin\theta$ is the radius of this circular strip (ring).

We take the projection of area of this slant elemental circular strip in vertical plane. Let $dA$ be the projection of the slant strip area $dS$ along the cross-sectional plane of the light beam and it is given as

$$dA = dS\cos\theta \quad \ldots(ii)$$

Hence, the power of light incident on this strip is $dP = IdA$

The momentum of photons per second incident on this strip is

$$dp = \frac{dP}{c} = \frac{IdA}{c} \quad \ldots(iii)$$

These photons are incident at an angle $\theta$ to the normal $N$ of this strip. As the surface of the sphere is perfectly reflecting, the photons will be reflected at the same angle $\theta$ to $N$ as shown in the figure.

Hence, the change in linear momentum of photons is along the normal and thus force exerted on this strip along the normal is

$$dF = 2dp\cos\theta = \frac{2IdA}{c}\cos\theta \quad \ldots(iv)$$

Thus, the net force on sphere will be along the $x$-axis only. Thus, the force on the sphere will be given as

$$F = \int dF\cos\theta = \int \frac{2IdA}{c}\cos^2\theta$$

$$= \int_0^{\pi/2} \frac{2I}{c}(2\pi R\sin\theta\cos\theta\, R d\theta)\cos^2\theta$$

$$= \frac{4I\pi R^2}{c}\int_0^{\pi/2}\cos^3\theta\sin\theta\, d\theta$$

$$= \frac{4I\pi R^2}{c}\left[-\frac{\cos^4\theta}{4}\right]_0^{\pi/2}$$

$$= \frac{I\pi R^2}{c}[1-0] = \frac{I\pi R^2}{c} \quad \ldots(v)$$

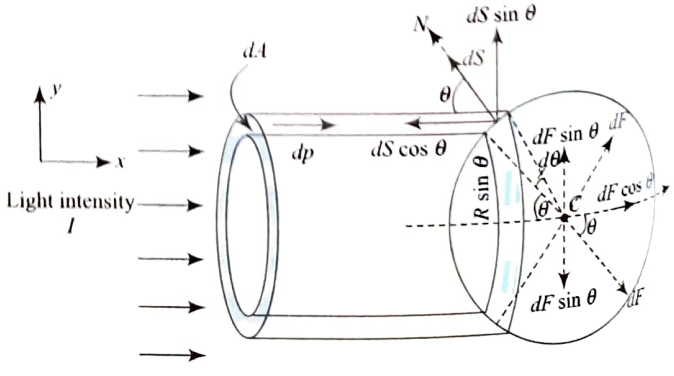
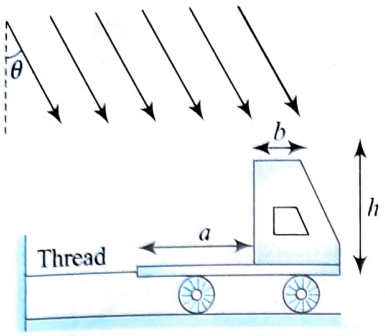


**ILLUSTRATION 5.13**

A toy truck has dimensions as shown in figure and its width, normal to the plane of this paper, is $d$. The sun rays are incident on it as shown in the figure. If intensity of rays is $I$ and all surfaces of truck are perfectly black, calculate tension in the thread used to keep the truck stationary. Neglect friction.




**Sol.** First, we have to calculate power incident on the truck and then momentum of light photons incident per second. Since the truck surfaces are perfectly black, therefore momentum finally reduces to zero. It means that the rate of change of momentum of light is equal to the momentum of photons incident per second.

Area of top of the driving cabin = $bd$.

Light is incident due to the component $I \cos\theta$ of intensity $I$.

Power incident on top of the cabin = $bdI \cos\theta$

Similarly, power incident on rear horizontal part of the truck is $adI \cos\theta$ and power incident on rear vertical wall of the driving cabin is $hdI \sin\theta$.

Total power incident on the truck is

$$P = (b\cos\theta + a\cos\theta + h\sin\theta)Id$$

Momentum of these photons is $p = P/c$ (where $c$ is speed of light in vacuum).

But rate of change in momentum of photons,

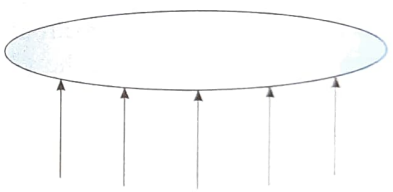
$$p = (Id/c)(b\cos\theta + a\cos\theta + h\sin\theta)$$

This momentum is inclined at angle $\theta$ with the vertical. Its vertical component $p\cos\theta$ is balanced by normal reaction of the floor and horizontal component $p\sin\theta$ is balanced by tension in the thread.

Tension $= p\sin\theta = (Id/c)(b\cos\theta + a\cos\theta + h\sin\theta)\sin\theta$

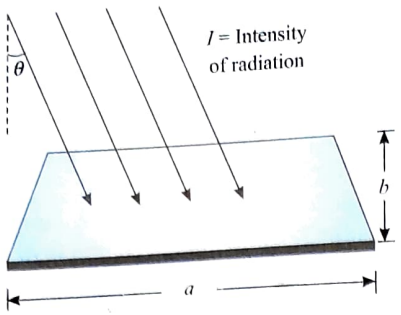
## CONCEPT APPLICATION EXERCISE 5.1

1. The intensity of direct sunlight before it passes through the earth's atmosphere is 1.4 $kWm^{-2}$. If it is completely absorbed, find the corresponding radiation pressure.
2. Monochromatic light of wavelength 632.8 nm is produced by a helium–neon laser. The power emitted is 9.42 mW.
   (a) Find the energy and momentum of each photon in the light beam.
   (b) How many photons per second, on an average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross section, which is less than the target area.)
   (c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon? Given that mass of hydrogen atom is $1.66 \times 10^{-27}$ kg.
3. An electron and a photon each has a wavelength of 1.0 nm. Find **(a)** their momenta, **(b)** the energy of the photon, and **(c)** the kinetic energy of electron. Take $h = 6.63 \times 10^{-34}$ J s.
4. A plate is kept in front of a beam of photons. The plate reflects 40% of the incident photons and absorbs the remainder. If the plate absorbs energy at a rate 1200 $Js^{-1}$, find the net force acting on it.
5. A sensor is exposed for 0.1 s to a 200 W lamp 10 m away. The sensor has an opening that is 20 mm in diameter. How many photons enter the sensor if the wavelength of light is 600 nm? Assume that all the energy of the lamp is given off as light.
6. Photons of wavelength $\lambda = 662$ nm are incident normally on a perfectly reflecting screen. Calculate their number per second falling on the screen such that the exerted force is 1 N.
7. The sun delivers about 1.4 kW $m^{-2}$ of electromagnetic flux to the earth's surface. Calculate:
   **(a)** the total power incident on a roof of dimensions 8 m × 20 m.
   **(b)** the solar energy in joules incident on the roof in 1 h.
   **(c)** the radiation pressure and force assuming the roof to be a perfect absorber.
8. A plate of mass 10 g is in equilibrium in air due to the force exerted by a light beam on the plate. Calculate power of the beam. Assume that the plate is perfectly absorbing.



9. The sun gives light at the rate of 1400 $Wm^{-2}$ of area perpendicular to the direction of light. Assume $\lambda$ (sunlight) = 6000 Å. Calculate
   **(a)** the number of photons per second arriving at 1 $m^2$ area at earth.
   **(b)** the number of photons emitted from the sun per second assuming the average radius of the Earth's orbit is $1.49 \times 10^{11}$ m.
10. A radiation of wavelength 200 nm is propagating in the form of a parallel surface. The intensity of the beam is 5 mW and its cross-sectional area is 1.0 $mm^2$. Find the pressure exerted by radiation on the metallic surface if the radiation is completely reflected.
11. A laser emits a light pulse of duration $T = 0.10$ ms and energy $E = 10$ J. Find the mean pressure exerted by such a pulse when it is focused on a spot of diameter $d = 10$ μm on a surface perpendicular to the beam and with a reflection coefficient $\rho = 0.5$.
12. A plane light wave of intensity $I = 0.50$ W $cm^{-2}$ falls on a plane mirror of reflection coefficient $\rho = 0.8$. The angle of incidence $\theta = 45°$. Find the normal pressure exerted by light on that surface.
13. A plank of mass $m$ is lying on a rough surface having coefficient of friction as $\mu$ in situation as shown in

figure. Find the acceleration of the plank assuming that it slips and the surface of body exposed to radiation is black body.




**ANSWERS**

1. $4.66 \times 10^{-6}\ \text{Nm}^{-2}$
2. (a) 1.963 eV; $1.046 \times 10^{-27}\ \text{kg m s}^{-1}$ (b) $3 \times 10^{16}$ (c) $0.63\ \text{m s}^{-1}$
3. (a) $6.63 \times 10^{-25}\ \text{kg m s}^{-1}$ (b) 1243.1 eV (c) 1.51 eV
4. $1.86 \times 10^{-5}$ N
5. $1.53 \times 10^{13}$
6. $5 \times 10^{26}$
7. (a) 224 kW (b) 806.4 MJ (c) $4.7 \times 10^{-6}\ \text{Nm}^{-2}$; $7.52 \times 10^{-4}$ N
8. $3 \times 10^{-7}$ W
9. (a) $n = 4.22 \times 10^{21}$ (b) $1.178 \times 10^{45}$
10. $3.33 \times 10^{-5}\ \text{Nm}^{-2}$
11. $6.37 \times 10^{6}\ \text{Nm}^{-2}$
12. $1.25 \times 10^{-5}\ \text{Nm}^{-2}$
13. $\dfrac{F\sin\theta - \mu(mg + F\cos\theta)}{m}$

# ELECTRON EMISSION

We know that the electrons in the outermost orbit of an atom are at maximum distance from the nucleus and hence most loosely bound to it. These are called free electrons. The electrons in metals which are free to move within the volume of metal are called free electrons. These electrons do not get ejected out of the surface of metal on their own. This is due to the reason that whenever an electron tries to leave the surface, the surface acquires a positive charge which pulls back the electron. To escape from the surface, an electron has to do a definite amount of work to overcome the force restraining it. To do this work, the electron must possess a certain minimum energy which can be imparted to it by an external source. This minimum energy is called the **work function** of the metal and is denoted by $\phi_0$.

The minimum energy that must be supplied to liberate the most weakly bound surface electrons from a metal without giving them any velocity is called the **work function** of the metal.

Work function is measured in electron volt (eV) where

$$1\text{eV} = 1\text{e} \times 1\text{V} = (1.6 \times 10^{-19}\text{C})(1\text{V}) = 1.6 \times 10^{-19}\ \text{J}$$

### Work functions of some photosensitive metals

| Metal | Work function (eV) | Metal | Work function (eV) |
|---|---|---|---|
| Cesium | 1.9 | Calcium | 3.2 |
| Potassium | 2.2 | Copper | 4.5 |
| Sodium | 2.3 | Silver | 4.7 |
| Lithium | 2.5 | Platinum | 5.6 |

When a free electron gets extra energy ($\geq$ work function) from an external agent, it is able to overcome potential barrier and is ejected out. There are a number of ways in which energy from outside can be supplied. Hence, there are different ways in which electron emission can take place. These ways are listed below:

**Photoelectric emission:** When electromagnetic radiations of suitable wavelength (or frequency) are incident on a metallic surface, then electrons are emitted, this phenomenon is called photoelectric effect.

**Thermionic emission:** In this case, extra energy is given to electrons to overcome potential barrier in the form of heat by passing current through a filament (Joule's heating effect).

**Field emission:** In this case, metal is placed in a strong electric field due to which the electrons are accelerated to such a speed that the corresponding kinetic energy is sufficient to overcome potential barrier.

**Secondary emission:** Electrons from metal can also be ejected by impinging a beam of high-speed electrons so that through collision, electrons acquire sufficient kinetic energy to overcome potential barrier. This type of electron emission is called secondary emission.

## PHOTOELECTRIC EFFECT

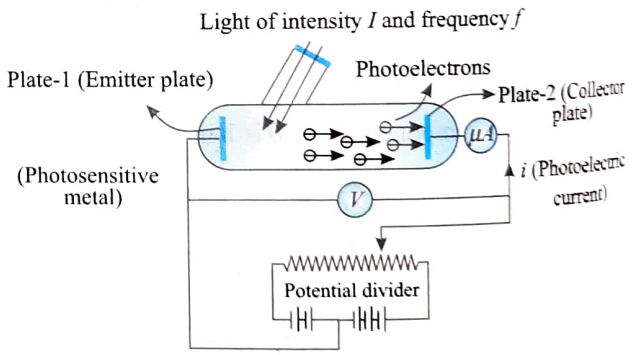
The discovery of photoelectric effect is due to the extensive investigations carried out by W. Smith, Heinrich Hertz, Wilhelm Hallwachs, Elster, Geital, J.J. Thomson and P. Lenard.

When electromagnetic radiations of suitable wavelength (or frequency) are incident on a metallic surface then electrons are emitted, this phenomenon is called **photoelectric effect**. The electrons so emitted are known as **photoelectrons**. And the current so obtained is known as **photoelectric current**.

Alkali metals Li, Na, K, Cs, etc., show photoelectric emission with visible light. Zn, Cd, Mg, etc., show photoelectric emission with ultraviolet light.

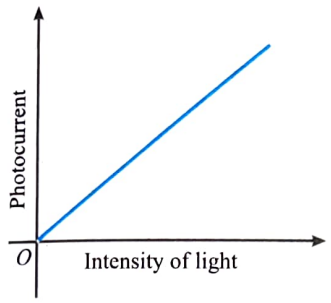
### Study of Photoelectric Effect

Figure shows the experimental setup to study the photoelectric effect.

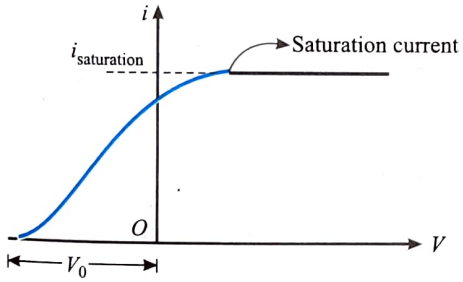



When monochromatic light of sufficiently short wavelength falls on the plate-1 called cathode which is made of some photosensitive material, electrons are emitted from it. The electrons are collected by the plate-2 called anode and a current flow in the circuit. Both these electrodes (cathode and anode) are enclosed in an evacuated glass tube which is provided with a quartz window $W$. Quartz is used because we have to use UV light to which glass is opaque. The anode can be maintained at a suitable positive or negative potential with the help of a potential divider. The potential difference between the electrodes is measured with the help of voltmeter. When light of suitable wavelength falls on cathode, electrons are emitted. When anode is positive w.r.t. cathode, electrons are attracted to it and he resulting photoelectric current ($i$) is measured with a microammeter.

**Experiment I. Effect of intensity of light on photoelectric current:** By keeping the frequency of light constant, we study the variation of photoelectric current ($i$) with the intensity of light ($I$). The anode is kept at a fixed positive potential ($V$) w.r.t. the cathode. If the intensity of radiation ($I$) is varied (or more number of photons are allowed to fall per unit time), a graph between intensity of light and photoelectric current (PEC) is a straight line passing through the origin as shown in figure. The photoelectric current is directly proportional to the intensity of incident radiation.




**Experiment II. Effect of potential on photoelectric current:** By keeping the intensity and frequency of incident light constant, we study the variation of photoelectric current with potential difference between cathode and anode. The shape of graph between intensity ($I$) and frequency ($f$) is as shown in the figure. The portion of the graph towards the right of the current ($i$) axis shows that as the collector plate (anode) is made even slightly positive, almost all the electron ejected by the emitter plate (cathode) are collected by anode and soon the current attains its maximum value. The maximum value of the photoelectric current is called the **saturation current**.




The portion of the graph towards left of the current axis shows the effect of making collector plate negative with respect to emitter plate. Since the current does not immediately drop to zero, it is obvious that the photoelectrons are emitted with velocities ranging from zero to a certain maximum. When we give positive potential given to collector plate, we call it as accelerating potential, and if we give negative potential given to collector plate, we call it retarding potential.

The retarding potential (given to the collector) for which the photoelectric current becomes zero is called the **cut-off** or **stopping potential** ($V_0$) for the corresponding frequency of the incident light.

If $m$ is the mass of the electron and $v_{max}$ is the maximum velocity of the most energetic ejected electron,

$$\frac{1}{2}mv_{max}^2 = eV_0 \qquad ...(i)$$

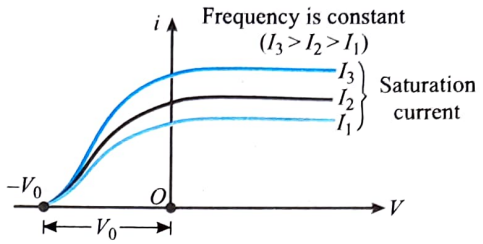
If $K_{max}$ is the corresponding kinetic energy,

$$K_{max} = \frac{1}{2}mv_{max}^2 \qquad ...(ii)$$

Thus, from Eqs. (i) and (ii),

$$K_{max} = eV_0 \qquad ...(iii)$$

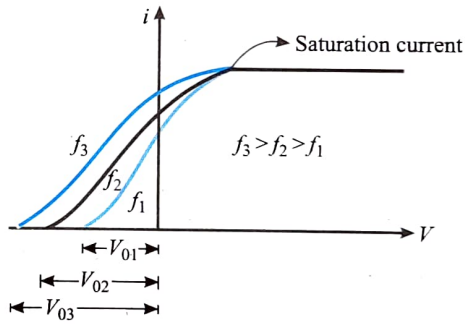
If the experiment is now repeated with incident lights of the same frequency ($f$) but of higher intensities $I_2$ and $I_3$ ($I_3 > I_2 > I_1$), it is found that stopping potential remains the same, i.e., $V_0$, though the saturation current increases as shown in figure.




**Important Point:**

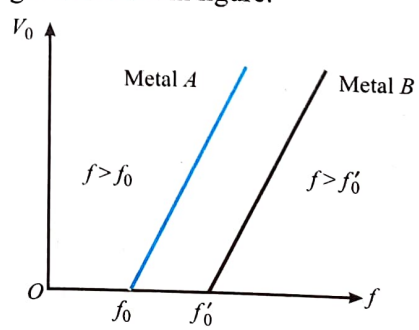
As stopping potential is a measure of $K_{max}$, it is obvious that: Maximum kinetic energy of photoelectron ($K_{max}$) does not depend on the intensity of light of a given frequency.

**Experiment III. Effect of frequency of incident light on stopping potential:** If we keep the intensity of light constant, we study the variation of photoelectric current with potential difference between emitter plate and collector plate. If light of different frequencies (keeping the intensity same) is used, then plots are obtained which are shown in figure.




We find that stopping potential for each frequency is different and $V_{03} > V_{02} > V_{01}$ (in magnitude). The higher the frequency of incident light, greater is the stopping potential. It means, the more is the energy of the photoelectrons ejected, i.e., $K_{max}$ of ejected electrons increases with frequency of incident light.

The graph between frequency of radiation ($f$) and stopping potential ($V_0$) it is found to be **straight** line which is not passing through the origin as shown in figure.



**Important Points:**

- As stopping potential ($V_0$) varies linearly with frequency of radiation ($f$), $K_{max}$ also varies linearly with frequency.
- The intercept of the graph on the frequency axis gives the frequency ($f_0$) that would just liberate electrons out of the emitter. If light of frequency less than $f_0$ is used, no electrons would be ejected.
- The minimum value of the frequency of light below which the photoelectric emission stops completely, howsoever large may be the intensity of light, is called the **threshold frequency**.
- The number of photoelectrons emitted per second is proportional to the intensity of the incident light.
- There is a lower limit of frequency called threshold frequency below which no emission takes place, however high the intensity of the incident radiation may be.
- Above the threshold frequency, the maximum velocity with which electrons emerge is dependent solely on the frequency and not on the intensity of the incident light.
- The emission of photoelectrons is an instantaneous process. It means time difference between incidence of light and emission of photoelectrons is very small, may be even less than $10^{-9}$ s.

**ILLUSTRATION 5.14**

Will photoelectrons be emitted from a copper surface, of work function 4.4 eV, when illuminated by a visible light?

**Sol.** The threshold wavelength $\lambda_0$ corresponding to work function $W$ is given by

$$\lambda_0 = \frac{hc}{W}$$

or $$\lambda_0 = \frac{12400}{4.4} = 2820 \text{ Å}$$

Since $\lambda_0$ does not lie in the visible range (4000 Å to 7500 Å), therefore it cannot eject photoelectrons from copper.

**ILLUSTRATION 5.15**

Find the number of photons emitted per second by a 25 W source of monochromatic light of wavelength 6600 Å. What is the photoelectric current assuming 3% efficiency for photoelectric effect ? Given $h = 6.6\times10^{-34}$ Js

**Sol.** Energy of each photon $= \frac{hc}{\lambda}$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{6600\times10^{-10}} = 3\times10^{-19} \text{ J}$$

Total energy emitted per second by 25 W source = 25 J

Number of photons emitted per second $= \frac{25}{3\times10^{-19}} = 8.33\times10^{19}$

Photoelectric current = 3% of photons emitted per second × charge on electron $= \frac{3}{100}\times\frac{25}{3\times10^{-19}}\times1.6\times10^{-19} = 0.4 \text{ Å}$

## Failure of Wave Theory of Light in Explaining Photoelectric Effect

### *Intensity Problem*

Maxwell's electromagnetic wave theory of light says that the average intensity of light is directly proportional to the square of the electric field of the light wave. If we increase the intensity of light it results in increase in the electric field intensity. It pushes the surface electrons with more force. If more force acts in an electron it does more work, increasing the kinetic energy of the electron to greater extent. As a result, the emitted electrons will possess more kinetic energy if the intensity of incident radiation is more. However, this opposes the experimental observation that the maximum kinetic energy of photoelectrons does not depend on intensity of light.

### *Frequency Problem*

When the radiation falls on the metallic surface as electromagnetic wave continuously and speed (distributed) uniformly, the electrons of the metallic surface will absorb the incident energy continuously sooner or later each electron will accumulate enough energy to overcome the potential barrier of the metal and escape from it. Hence photoelectric effect should occur at any frequency of incident light. Again this prediction of wave theory of light contradicts the experimental observation that the photoelectric effect can take place above a cutoff (threshold) frequency of incident light (not at all frequencies for a given a metal).

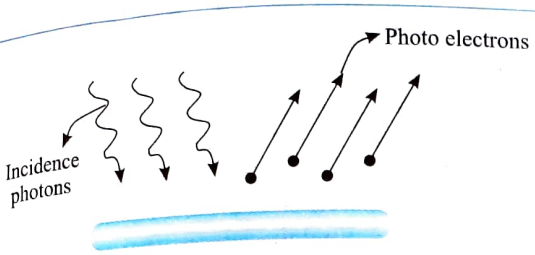
### *Time Delay Problem*

If the radiation falls continuously and uniformly and the atoms will absorb the incident radiation continuously, each atom will receive radiation energy proportional to their cross-sectional area. Since the cross-section of an atom is very small, each atom will receive very little energy in each second. Therefore, it takes a long time for an atom to accumulate sufficient energy to eject an electron from it. However, this prediction also proved to be incorrect because observation says that the photoelectrons start coming out of the metal almost instantaneously after the light falls on it.

## Einstein's Explanation for Photoelectric Effect

The most authentic explanation of photoelectric effect was given by Einstein. Einstein explained the Planck's concept of quantisation of electromagnetic waves more clearly. He assumed that if a radiation of frequency $f$ propagating in vacuum, it can be considered a stream of photons. Each photon travels at speed of light and has energy $hf$. He put his theory for photoelectric effect as following way:

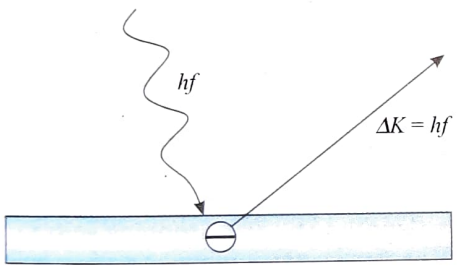
(i) The emission of photoelectron is instantaneous. The term instantaneous (or sudden) about the emission of electrons implies that "certain body" is colliding the electrons promptly. That "certain body" can be imagined as a pulse or packet of radiation. As radiation is an electromagnetic wave, we can imagine the light hitting the metal as electromagnetic wave pulse. Each pulse can be termed as photon (bundles or packet or quartz of radiation).




(ii) When a photon interacts with an electron, it gives its entire energy to the electron and then exists no longer. It is absorbed by a single electron. The chance of two photons hitting the same electron simultaneously is practically zero as there is one-to-one interaction between the photons and the electrons.

(iii) The emission of photoelectrons stops below certain frequency of incident radiation. This signifies that electron cannot absorb a fraction of an incident photon and cannot increase its kinetic energy by continuous absorption of photons one by one. In other words, each photon must be absorbed completely by an electron inside the metal.

(iv) The maximum kinetic energy of (or stopping potential for) photoelectrons increases linearly (proportionality) with the frequency of the incident light. It means that the energy of each photon which is converted to (directly proportional to) the maximum kinetic energy of the photoelectrons must be varying linearly with the frequency of incident radiation. It can be shown that, the energy of a photon is



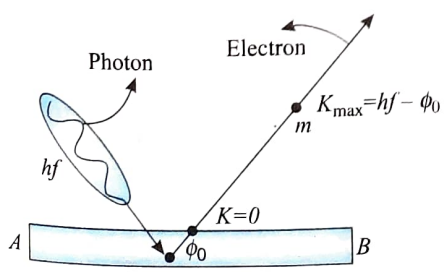

A photon kicks an electron increasing its KE by *hf*.

$$E_{photon} = hf$$

(v) If the intensity of light increases, the photoelectric current increases proportionally. This means that each photon of a monochromatic light must carry equal energy, that is, *hf*. If the intensity of light increases, the number of photons striking the metal per second will increase. Eventually, more electrons will come out of the metal per second. In short, the rate of emission for photoelectrons is directly proportional to the rate of incident photons.

## Einstein's Photoelectric Equation

According to Einstein, photon energy (*hf*) falling on a metal is utilized for two purposes:

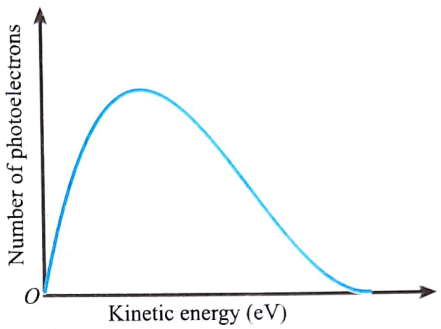



1. Partly for getting the electron free from the atom and away from the metal surface. This energy is known as the photoelectric work function of the metal and is represented by $W_0$ (or $\phi_0$).
2. The balance of the photon energy is used up in giving the electron a kinetic energy, $K_{max}$ being the maximum value of this kinetic energy.

So, we can write:

$$hf = W_0 + K_{max} \Rightarrow \quad K_{max} = hf - W_0 \qquad ...(i)$$

Whenever photoelectric effect takes place, electrons are ejected out with kinetic energies ranging from 0 to $K_{max}$. The energy distribution of photoelectrons is as shown in figure.




Let us consider a photon which has got only that much energy which can just eject an electron from the metal surface. The frequency of such a photon is obviously the threshold frequency ($f_0$). Thus, when $f = f_0, K_{max} = 0$. From Eq. (i),

$$W_0 = hf_0 \qquad ...(ii)$$

Combining Eqs. (i) and (ii)

$$K_{max} = hf - hf_0 = h(f - f_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \qquad ...(iii)$$

Equation (iii) is called Einstein's photoelectric equation.

**Important Point:**

In Eq. (iii), if we write wavelengths, $\lambda$ and $\lambda_0$ in Å, the kinetic energy in electron-volts can be expressed as

$$K_{max} = 12400\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)\text{eV}$$

### *Finding Plank's Constant from Einstein's Photoelectric Equation*

Robert Millikan verified Einstein's photoelectric equation experimentally and provided irrefutable evidence of the photon model of light.

According to Einstein's photoelectric equation,

$$K_{max} = hf - W_0 \qquad ...(i)$$

Also, $K_{max} = eV_0$ ...(ii)

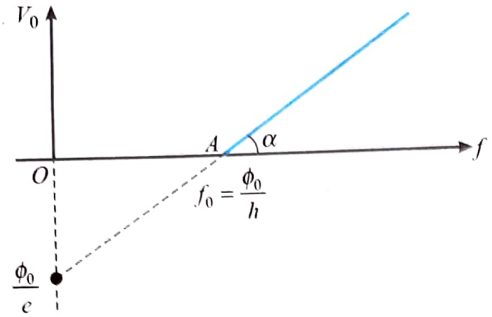
where $V_0$ is the stopping potential.

From Eqs. (i) and (ii),

$$eV_0 = hf - W_0 \qquad ...(iii)$$

or $$V_0 = \left(\frac{h}{e}\right)f - \frac{W_0}{e} \qquad ...(iv)$$

Equation (iv) implies that $V_0$ versus $f$ curve is a straight line with slope $(h/e)$, which is independent of the nature of the material. Further, slope of $V_0$ versus $f$ graph



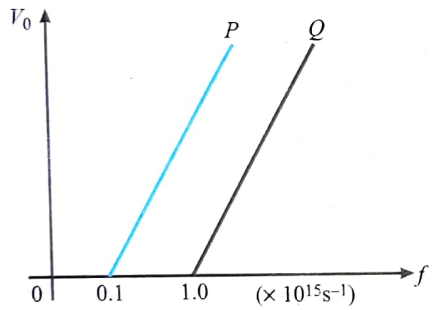

$$\tan\alpha = \frac{h}{e}$$

or $$h = e\tan\alpha \quad \ldots(v)$$

Measuring $\alpha$ and using the known value of $e$, Millikan calculated the value of $h$ and the value so obtained agreed fairly well with that obtained from black body radiation experiments.

## ILLUSTRATION 5.16

Figure shows the variation of stopping potential $V_0$ with the frequency $f$ of the incident radiation for two photosensitive metals $P$ and $Q$.




(a) Explain which metal has smaller threshold wavelength.
(b) Explain, giving reason, which metals emits photoelectrons having smaller kinetic energy, for the same wavelength of incident radiation.
(c) If the distance between the light source and metal $P$ is doubled, how will the stopping potential change?

**Sol.**

**(a)** Threshold wavelength, $\lambda_0 = \frac{c}{f_0}$

As $f_{o(Q)} > f_{0(P)}$

$\therefore \lambda_{0(Q)} < \lambda_{0(P)}$

Thus the metal $Q$ has smaller threshold wavelength.

**(b)** According to Einstein's photoelectric equation,

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + \text{KE of photoelectron}$$

For the same $\lambda$ of incident radiation, L.H.S. is constant. So metal $Q$ with smaller value of $\lambda_0$ will emit photoelectrons of smaller KE

**(c)** Stopping potential $V_0$ will remain same because it is independent of intensity and hence on distance between the light source and the metal surface.

## ILLUSTRATION 5.17

The threshold frequency of a metal is $f_0$. When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of electrons emitted is $f_1$. When the frequency of the incident radiation is increased to $5f_0$, the maximum velocity of electrons emitted is $f_2$. Find the ratio of $f_1$ to $f_2$.

**Sol.** The minimum amount of energy required to eject an electron from a metal surface (without imparting it any kinetic energy) is called work function of the metal. As $f_0$ is the threshold frequency, so $W = hf_0$

From Einstein's photoelectric equation, the maximum kinetic energy of emitted electron is given by

$$\frac{1}{2}mv_1^2 = h\times 2f_0 - W = h\times 2f_0 - hf_0 = hf_0$$

and $$\frac{1}{2}mv_2^2 = h\times 5f_0 - hf_0 = 4hf_0$$

$$\therefore \quad \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{hf_0}{4hf_0}$$

or $$\frac{v_1}{v_2} = \frac{1}{2} = 1:2$$

## ILLUSTRATION 5.18

Light is incident on the cathode of a photocell and the stopping voltages are measured for light of two different wavelengths. From the data given below, determine the work functions of the metal of the cathode in eV.

| Wavelength (Å) | Stopping voltage (volt) |
|---|---|
| 4000 | 1.3 |
| 500 | 0.9 |

**Sol.** As $K_{max} = hf - W_0$

or $$eV_0 = \frac{hc}{\lambda} - W$$

or $$V_0 = \frac{hc}{e\lambda} - \frac{W_0}{e}$$

$$\therefore \quad \Delta V_0 = (V_0)_2 - (V_0)_1$$

$$= \left[\frac{hc}{e\lambda_2} - \frac{W_0}{e}\right] - \left[\frac{hc}{e\lambda_1} - \frac{W_0}{e}\right] = \frac{hc}{e}\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right]$$

$$\Delta V_0 = \frac{hc}{e}\left[\frac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right]$$

$$\therefore \quad \frac{hc}{e} = \frac{\Delta V_0 \cdot \lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

$$= \frac{(1.3-0.9)\times 4000\times 10^{-10}\times 4500\times 10^{-10}}{500\times 10^{-10}}$$

$$= 1.44\times 10^{-6}\ \text{Vm}$$

Also, $$V_0 = \frac{hc}{e\lambda} - \frac{W_0}{e}$$

$$\therefore \quad \frac{W_0}{e} = \frac{hc}{e\lambda} - V_0 = \frac{1.44\times10^{-6}}{4000\times10^{-10}} - 1.3 = 2.3\,\text{V}$$

$$\Rightarrow \quad W_0 = 2.3\,\text{eV}$$

## ILLUSTRATION 5.19

A small piece of cesium metal ($\phi = 1.9$ eV) is kept at a distance of 17.7 cm from a large metal plate having a charge density of $1.0\times10^{-9}\,\text{C/m}^2$ on the surface facing the cesium piece. A monochromatic light of wavelength 400 nm is incident on the cesium piece. Find the minimum and maximum kinetic energy of the photoelectrons reaching the large metal plate. Neglect any change in electric field due to the small piece of cesium present.

**Sol.** Electric field between the plate, $E = \frac{\sigma}{\varepsilon_0}$

The potential difference between the plates $V = E\cdot d = \frac{\sigma}{\varepsilon_0}\cdot d$

$$\Rightarrow \quad V = \frac{1.0\times10^{-9}\times17.7}{8.85\times10^{-12}\times100} = 20\,\text{V}$$

From Einstein equation $K_{max} = \frac{hc}{\lambda} - W_0 = \frac{12400}{4000} - 1.9 = 1.2\,\text{eV}$

But $K_{max} = eV_0$

$$\Rightarrow \quad 1.2\,\text{eV} = eV_0$$

$$\Rightarrow \quad V_0 = 1.2\,\text{V}$$

As $V_0$ is less than the potential difference between the plates (20 V). Hence the minimum energy required to reach the charged plate should be $K_{min} = 20$ eV.

For maximum kinetic energy, the potential difference between the plates should be an accelerating one. Hence maximum kinetic energy of electron should be

$$K_{max} = 20 + 1.2 = 21.2\,\text{eV}$$

## ILLUSTRATION 5.20

Calculate the velocity of a photoelectron if the work function of the target material is 1.24 eV and the wavelength of incident light is $4.36\times10^{-7}$ m. What retarding potential is necessary to stop the emission of the electrons?

**Sol.** We know that

$$\frac{1}{2}mv^2 = hf - \phi \text{ or } \frac{1}{2}mv^2 = \frac{hf}{\lambda} - \phi$$

Subsituting the values, we get

$$v = 7.43\times10^5\ \text{m s}^{-1}$$

Let $V_s$ be the retarding potential required to stop the emission of photoelectrons. Then, we have

$$eV_s = \frac{1}{2}mv^2$$

$$\Rightarrow \quad V_s = \frac{\frac{1}{2}mv^2}{e} = \frac{2.511\times10^{-19}}{1.6\times10^{-19}} = 1.58\,\text{V}$$

## ILLUSTRATION 5.21

The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength 5893 Å is 0.36 V. Calculate the maximum kinetic energy of photoelectrons, the work function of the surface, and the threshold frequency.

**Sol.** We know that $K_{max} = hf - \phi = \left(\frac{hc}{\lambda}\right) - \phi$

or $\quad \phi = \frac{hc}{\lambda} - K_{max}$

Also, $\quad K_{max} = eV_s = 0.36$ eV

$$\Rightarrow \quad \phi = \frac{(6.62\times10^{-34})\times(3\times10^8)}{5893\times10^{-10}} - 0.36\times1.6\times10^{-19}$$

$$= 1.746\,\text{eV}$$

The threshold frequency is given by

$$f_0 = \frac{\phi}{h} = \frac{2.794\times10^{-19}}{6.62\times10^{-34}} = 4.22\times10^{14}\ \text{Hz}$$

## ILLUSTRATION 5.22

Photoelectric threshold of silver is $\lambda$ = 3800 Å. Ultraviolet light of $\lambda$ = 2600 Å is incident on a silver surface. Calculate:

**(a)** the value of work function in joule and in eV.

**(b)** maximum kinetic energy of the emitted photoelectrons.

**(c)** the maximum velocity of the photoelectrons.

(Mass of the electrons = $9.11\times10^{-31}$ kg)

**Sol.**

**(a)** $\lambda_0 = 3800$ Å

$$W_0 = hf_0 = h\frac{c}{\lambda_0} = \frac{6.633\times10^{-34}\times3\times10^8}{3800\times10^{-10}}\ \text{J}$$

$$= 5.23\times10^{-19}\ \text{J} = 3.27\ \text{eV}$$

**(b)** Incident wavelength, $\lambda$ = 2600 Å

Therefore, $f$ = incident frequency = $\frac{3\times10^8}{2600\times10^{-10}}$ Hz

Then, $K_{max} = hf - W_0$

$$hf = \frac{6.63\times10^{-34}\times3\times10^8}{2600\times10^{-10}} = 7.65\times10^{-19}\,\text{J} = 4.78\,\text{eV}$$

$$K_{max} = hf - W_0 = 4.78\ \text{eV} - 3.27\ \text{eV} = 1.51\ \text{eV}$$

**(c)** $K_{max} = \frac{1}{2}mv_{max}^2$

$$\Rightarrow \quad v_{max} = \sqrt{\frac{2K_{max}}{m}} = \sqrt{\frac{2\times2.42\times10^{-19}}{9.11\times10^{-31}}}$$

$$= 0.7289\times10^6\ \text{m s}^{-1}$$

## ILLUSTRATION 5.23

When a surface is irradiated with light of wavelength 4950 Å, a photocurrent appears which vanishes if a retarding potential greater than 0.6 V is applied across the photo tube. When a different source of light is used, it is found that the critical retarding potential is changed to 1.1 V. Find the work function of the emitting surface and the wavelength of the second source.

**Sol.** We know that $hf_1 = \phi + \frac{1}{2}mv_1^2$ and $\frac{1}{2}mv_1^2 = eV_1$

$$\phi = hf_1 - eV_1 = \frac{hc}{\lambda_1} - eV_1$$

$$= \frac{(6.6\times10^{-34})\times(3\times10^{8})}{4950\times10^{-10}} - (1.6\times10^{-19})(0.6)$$

$$= 3.04\times10^{-19}\ \text{V}$$

$$hf_2 = \phi + eV_2$$

$$\frac{hc}{\lambda_2} = 3.04\times10^{-19} + (1.6\times10^{-19})\times1.1 = 4.8\times10^{-19}$$

$$\Rightarrow \quad \lambda_2 = \frac{(6.6\times10^{-34})\times(3\times10^{8})}{4.8\times10^{-19}} = 4125\ \text{Å}$$

## ILLUSTRATION 5.24

A beam of light has three wavelengths 4144 Å, 4972 Å, and 6216 Å with a total intensity of $3.6\times10^{-3}$ $\text{Wm}^{-2}$ equally distributed among the three wavelengths. The beams fall normally on an area 1.0 $\text{cm}^2$ of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in 2 s.

**Sol.** We know that threshold wavelength $(\lambda_0) = \frac{hc}{\phi}$.

$$\lambda_0 = \frac{(6.63\times10^{-34})(3\times10^{8})}{2.3\times(1.6\times10^{-19})} = 5.404\times10^{-7}\ \text{m} = 5404\ \text{Å}$$

Thus, wavelengths 4144 Å and 4972 Å will emit electrons from the metal surface.

Energy incident on the surface per unit time for each wavelength
= Intensity of each wavelength × Area of the surface

$$= \frac{3.6\times10^{-3}}{3}\times(1.0\ \text{cm}^2) = 1.2\times10^{-7}\ \text{W}$$

Energy incident on the surface for each wavelength in 2 s,
$E = (1.2\times10^{-7})\times(2) = 2.4\times10^{-7}$ J

Number of photons $n_1$ due to wavelength 4144 Å
$(= 4144\times10^{-10}\ \text{m})$,

$$n_1 = \frac{(2.4\times10^{-7})(4144\times10^{-10})}{(6.63\times10^{-34})(3\times10^{8})} = 0.5\times10^{12}$$

Number of photons $n_2$ due to wavelength 4972 Å,

$$n_2 = \frac{(2.4\times10^{-7})(4972\times10^{-10})}{(6.63\times10^{-34})(3\times10^{8})} = 0.575\times10^{12}$$

$$N = n_1 + n_2 = 0.5\times10^{12} + 0.575\times10^{12} = 1.075\times10^{12}$$

## ILLUSTRATION 5.25

A photocell is operating in saturation mode with a photocurrent 4.8 μA when a monochromatic radiation of wavelength 3000 Å and power 1 mW is incident. When another monochromatic radiation of wavelength 1650 Å and power 5 mW is incident, it is observed that maximum velocity of photoelectron increases to two times. Assuming efficiency of photoelectron generation per incident to be same for both the cases, calculate:

**(a)** threshold wavelength for the cell
**(b)** efficiency of photoelectron generation
[(Number of photoelectrons emitted per incident photon) × 100]
**(c)** saturation current in the second case.

**Sol.**

**(a)** $K_1 = \frac{12400}{3000} - W = 4.13 - W$ ...(i)

$$K_2 = \frac{12400}{1650} - W = 7.51 - W$$ ...(ii)

Since $V_2 = 2V_1$, so $K_2 = 4K_1$ ...(iii)

Solving above equations, we get $W = 3$ eV

Therefore, threshold wavelength, $\lambda_0 = \frac{12400}{3} = 4133$ Å

**(b)** Energy of a photon in the first case $= \frac{12400}{3000} = 4.13$ eV
or $E_1 = 6.6\times10^{-19}$ J
Rate of incident photons (number of photons per second)

$$\frac{P_1}{E_1} = \frac{10^{-3}}{6.6\times10^{-19}} = 1.52\times10^{15}\ \text{s}^{-1}$$

Number of electrons ejected $= \frac{4.8\times10^{-6}}{1.6\times10^{-19}}\ \text{s}^{-1} = 3.0\times10^{13}\ \text{s}^{-1}$

Therefore, efficiency of photoelectron generation,

$$\gamma = \frac{3.0\times10^{13}}{1.52\times10^{15}}\times100 = 1.97\%$$

**(c)** Energy of photon in the second case,

$$E_2 = \frac{12400}{1650} = 7.51\ \text{eV} = 12\times10^{-19}\ \text{J}$$

Therefore, number of photons incident per second.

$$n_2 = \frac{P_2}{E_2} = \frac{5.0\times10^{-3}}{12\times10^{-19}} = 4.17\times10^{15}\ \text{s}^{-1}$$

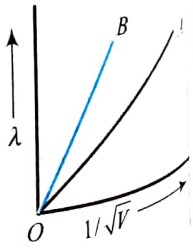
Number of electrons emitted per second is

$$\frac{1.97}{100}\times4.7\times10^{15} = 9.27\times10^{13}\ \text{s}^{-1}$$

Therefore, saturation current in the second case
$i = (9.27\times10^{13})\ (1.6\times10^{-19})\ \text{A} = 14.8\ \mu\text{A}$

## CONCEPT APPLICATION EXERCISE 5.2

1. An electron is accelerated by a potential difference of 25 V. Find the de Broglie wavelength associated with it.
2. Why are de Broglie waves associated with a moving football not apparent to us?
3. The de Broglie wavelength of a particle of kinetic energy $K$ is $\lambda$. What would be the wavelength of the particle, if its kinetic energy were $K/4$?
4. The two lines $A$ and $B$ in figure show the plot of de Broglie wavelength $(\lambda)$ as a function of $1/\sqrt{V}$ ($V$ is the accelerating potential) for two particles having the same charge. Which of the two represents the particle of heavier mass?



5. In a photoelectric experiment, the collector plate is at 2.0 V with respect to the emitter plate made of copper ($W_0$ = 4.5 eV). The emitter is illuminated by a source of monochromatic light of wavelength 200 nm. Find the minimum and maximum kinetic energy of the photoelectrons reaching the collector.
6. Radiations of frequencies $f_1$ and $f_2$ are made to fall in turn, on a photo-sensitive surface. The stopping potentials required for stopping the most energetic emitted photo-electrons in the two cases are $V_1$ and $V_2$ respectively. Obtain a formula for determining Planck's constant and the threshold frequency in terms of these parameters.
7. If the frequency of incident light on a metal surface is doubled, will the kinetic energy of the photoelectrons be doubled?
8. A light of wavelength 1240 Å falls on a metallic sphere of radius 1 m and work function $W_0$ = 3 eV. Find the maximum number of electrons left from the sphere till the photoelectric effect stops.
9. A small metal plate (work function $W_0$) is kept at a distance $d$ from a singly ionized, fixed ion. A monochromatic light beam is incident on the metal plate and photoelectrons are emitted. Find the maximum wavelength of the light beam so that some of the photoelectrons may go round the ion along a circle.

**ANSWERS**

1. 2.5 Å **3.** $2\lambda$ **4.** Line $A$ **5.** 2 eV; 3.7 eV
8. $4.68 \times 10^9$ **9.** $\dfrac{8\pi\varepsilon_0 hcd}{e^2 + 8\pi\varepsilon_0 dW_0}$

# WAVE NATURE OF MATTER: DE-BROGLIE WAVES OR MATTER WAVES

The experiments related to interference, diffraction and polarization confirm the wave nature of light, whereas the photoelectric effect and Compton effect put the corpuscular or particle nature of light. It means light act as wave and particle both. This ideal of dual behaviour of the photon inspired Louis de-Broglie, to put forth a revolutionary hypothesis. He suggested that nature loves symmetry. If a light wave can behave like a particle on some occasions, it is reasonable to suppose that a particle also has a dual nature. In some circumstances, a particle may behave like a wave rather than a particle.

## DE-BROGLIE WAVES EQUATION

The two physical quantities which govern all the forms of the physical universe are mass and energy. These quantities are connected through the Einstein's mass–energy relationship:

$$E = mc^2 \quad \text{...(i)}$$

where $E$ = energy of a photon

$m$ = relativistic mass of a photon

and $c$ = velocity of light in vacuum.

Above equation shows that there is a complete equivalence between matter (mass) and radiation (energy). There must be a mutual symmetry between matter (mass) and radiation.

According to the quantum theory of radiation,

$$E = hf \quad \text{...(ii)}$$

where $h$ = Planck's constant

$f$ = frequency of the photon

From Eq. (i), relativistic mass of a photon of energy ($E$) is given by $m = \dfrac{E}{c^2}$

If a photon moves with the velocity of light, momentum of the photon, i.e.,

$$p = mc = \frac{E}{c^2} \times c = \frac{E}{c} \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), $p = \dfrac{hf}{c}$ ...(iv)

As $c = f\lambda$, $\dfrac{f}{c} = \dfrac{1}{\lambda}$

where $\lambda$ is the wavelength of the photon.

From Eq. (iv), $p = \dfrac{h}{\lambda}$ ...(v)

A quantum of light, i.e., photon should thus be able to act like particle and possess momentum.

Now, if there is an electron (or any other particle) of mass $m$ moving with velocity $v$, its momentum is given by

$$p = mv \quad \text{...(vi)}$$

From Eqs. (v) and (vi), we get $\dfrac{h}{mv} = \lambda$

or $$\lambda = \frac{h}{mv} = \frac{h}{p} \quad \text{...(vii)}$$

This is known as de-Broglie's wave equation.

Also, $K = \dfrac{p^2}{2m} \Rightarrow \lambda = \dfrac{h}{\sqrt{2mK}}$

Thus, from here we conclude that a particle of mass $m$ moving with a velocity $v$, can be considered to be a wave of wavelength $\lambda$. This equation explains the dual nature of matter as it connects the wave characteristic '$\lambda$' with the particle characteristic '$p$'.

From de-Broglie's equation, we find that

1. The wavelength of a moving particle is inversely proportional to its momentum.
   i.e., $\lambda \propto \dfrac{1}{p}$
2. If $v = 0$, then $\lambda = \infty$. This implies that waves are associated with material particles only when they are in motion.
3. To be associated with a de-Broglie wave, a particle need not have a charge. That is why, de-Broglie waves are also known as matter waves.
4. de-Broglie waves cannot be electromagnetic in nature because electromagnetic waves are only associated with accelerated charged particles.

**Important Point:**

The de-Broglie wavelength is independent of the charge and nature of the material particle.

de-Broglie wavelength, $\lambda \propto 1/m$. This wavelength is significantly measurable (of the order of atomic-planes spacing in crystals) only in case of subatomic particles like electrons, protons, etc.; due to the smallness of their masses. But the de-Broglie wavelength of large moving objects is very small, quite beyond measurement, due to their large masses. That is why the **macroscopic objects in our daily life do not show wave-like properties.**

**ILLUSTRATION 5.26**

Calculate the de-Broglie wavelengths for electrons and protons, if their speed is $10^{15}$ m/s.

**Sol.** We are given that $v = 10^5$ m/s.

(i) Mass of an electron, $m = 9.1\times10^{-31}$ kg

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{(9.1\times10^{-31})10^5}$$

or $\lambda = 72.8\times10^{-10}\text{m} = 72.8\,\text{Å}$

(ii) Mass of proton, $m = 1.67 \times 10^{-27}$ kg

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{(1.67\times10^{-27})10^5}$$

or $\lambda = 0.03968\times10^{-10}\text{ m} = 0.03968\,\text{Å}$

**ILLUSTRATION 5.27**

An electron and a photon have the same de-Broglie wavelength. Which one of these has higher kinetic energy?

**Sol.** Let $\lambda$ be the de-Broglie wavelength of the electron and the photon. If $m$ and $v$ are the mass and the velocity of the electron, then de-Broglie wavelength of the electron

$$\lambda = \frac{h}{mv}$$

The photon has zero rest mass. Therefore, energy of the photon is totally kinetic in nature. Since the wavelength of the photon is same as that of the electron, the kinetic energy of the photon having wavelength $\lambda$,

$$E_1 = \frac{hc}{\lambda} = \frac{hc}{h/mv}$$

or $E_1 = mvc$ ...(i)

Now, the kinetic energy of the electron,

$$E_2 = \frac{1}{2}mv^2 = mv \times \frac{v}{2}$$ ...(ii)

Since $c > v/2$ (as $c > v$), from the results (i) and (ii), it follows that

$$E_1 > E_2$$

i.e., kinetic energy of the photon is greater than that of the electron. As it moves with the speed $c$, it is faster than electron.

**ILLUSTRATION 5.28**

A photon and an electron have the same de-Broglie wavelength. Which has greater total energy? Explain.

**Sol.** Let $\lambda$ be the de-Broglie wavelength of the electron and also the wavelength of the photon. If $m$ and $v$ are the mass and the velocity of the electron, then de-Broglie wavelength of the electron $\lambda = \dfrac{h}{mv}$

Energy of the photon having wavelength $\lambda$, $E_1 = \dfrac{hc}{\lambda}$

Since wavelength of the photon is same as that of the electron,

$$E_1 = hc \times \frac{mv}{h}$$

or $E_1 = mvc$ ...(i)

According to Einstein's mass–energy equivalence relation, the energy of the electron (mass $m$)

$$E_2 = mc^2 = mc \times (c)$$

Since $c > v$, from Eqs. (i) and (ii), it follows that ...(ii)

$$E_2 > E_1$$

i.e., the total energy of electron is greater than that of the photon.

## DE-BROGLIE WAVELENGTH OF AN ELECTRON

Consider an electron of mass $m$ and charge $e$. Let $v$ be the final velocity attained by the electron when it is accelerated from rest through a potential difference of $V$ volts. Then kinetic energy gained by the electron equals the work done on the electron by the electric field.

KE gained by the electron, $K = \dfrac{1}{2}mv^2 = \dfrac{p^2}{2m}$

Work done on the electron = eV

$$\therefore \quad K = \frac{p^2}{2m} = eV$$

or $p = \sqrt{2mK} = \sqrt{2meV}$

Hence the de-Broglie wavelength of the electron is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$$

Now $h = 6.63\times10^{-34}$ Js

$m = 9.1\times10^{-31}$ kg

$e = 1.6\times10^{-19}$ C

$$\therefore \quad \lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\,\text{V}}}$$

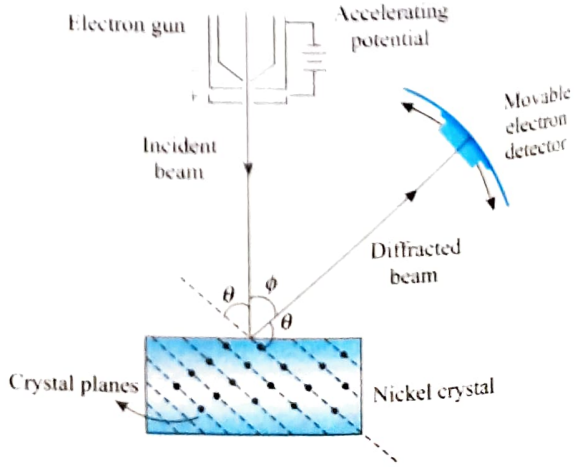
$$\Rightarrow \quad \lambda = \frac{12.27\times10^{-10}}{\sqrt{V}}\text{m} = \frac{12.27}{\sqrt{V}}\,\text{Å}$$

For an accelerating potential of 120 V, we find $\lambda = 0.112$ nm. This wavelength is of the same order as the spacing between the atomic planes in crystals. This suggested that matter waves associated with electrons could be detected by electron diffraction experiments.

## EXPERIMENTAL DEMONSTRATION OF WAVE NATURE OF ELECTRONS

### Davisson and Germer Experiment

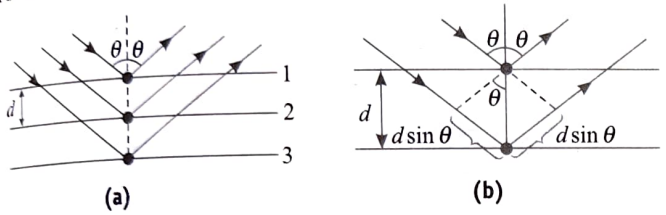
Davisson and Germer confirmed the de-Broglie theory about the wave nature of particles with direct experiment.



**Experimental arrangement:** The apparatus used by Davisson and Germer is shown in figure. Here the electrons emitted by the hot filament of an electron gun are accelerated by applying a suitable potential difference. These electrons are directed at the **nickel crystal**. The crystal is capable of rotation about an axis perpendicular to the plane of paper. The electrons, scattered in different directions by the atoms of Ni crystal, are received by a movable detector which is just an electron collector. Thus, we measure scattered electron intensity as a function of the scattering angle $\phi$, the angle between the incidence and the scattered electron beam. The experiment is repeated for different accelerating potentials $V$.

**Observations and inference:** Above result satisfies the Bragg's law of diffraction of $X$-rays.

According to Bragg's, $2d\sin\theta = n\lambda$




where $\theta$ = diffraction angle
$d$ = lattice constant, distance between crystal planes
$n$ = order of maxima

The value of $d$, as determined by X-ray diffraction by Ni-crystal, is 0.91 Å. As $\theta = 65°$ for first order diffraction maxima,

so $2d\sin\phi = n\lambda$

or $2\times 0.91\ \text{Å}\ \sin 65° = 1\lambda$

$\Rightarrow \lambda = 1.65$ Å

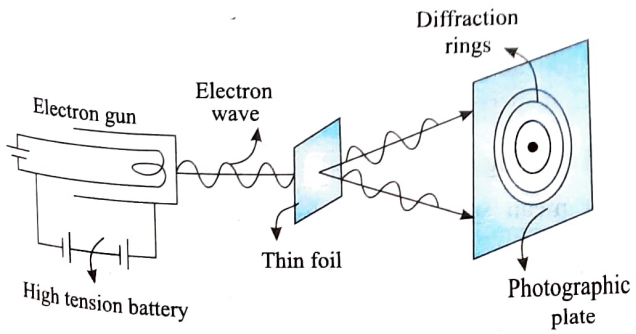
Now according to de-Broglie hypothesis, the wavelength for electrons accelerated through a potential difference of 54 volt is given by

$$\lambda = \frac{12.27}{\sqrt{V}}\text{Å} = \frac{12.27}{\sqrt{54}} = 1.67\ \text{Å}$$

Thus, the two results obtained are in remarkable agreement with each other. We conclude that particles, like waves, can be diffracted. Thus, Davisson–Germer experiment provides a direct verification of the de-Broglie hypothesis regarding the wave nature of particles.

### G.P. Thomson's Experiment

G.P. Thomson was able to obtain a diffraction pattern of an electron beam. The experimental set-up is shown in figure. From an electron gun, a beam of electrons accelerated through a potential difference of 10 to 50 kV is incident on a thin platinum foil.




The emergent beam is received on a photographic plate. The electron beam is diffracted at the spacings between the randomly oriented crystals of the thin foil. On the photographic plate, we get a circular diffraction pattern similar to Laue's X-ray diffraction pattern. This conclusively proves that **the electron beams behave like waves**.

## DE-BROGLIE WAVELENGTH ASSOCIATED WITH DIFFERENT CHARGED PARTICLES

The de Broglie wavelength of a charged particle is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$$

Here, $h = 6.63\times10^{-34}$ Js

| Particle | Mass | Charge | Wavelength |
|---|---|---|---|
| Protons | $1.67\times10^{-27}$ kg | $1.6\times10^{-19}$ C | $\lambda = \frac{0.286}{\sqrt{V}}$ Å |
| Deuterons | $2\times1.67\times10^{-27}$ kg | $1.6\times10^{-19}$ C | $\lambda = \frac{0.202}{\sqrt{V}}$ Å |
| α-particles | $4\times1.67\times10^{-27}$ kg | $2\times1.6\times10^{-19}$ C | $\lambda = \frac{0.101}{\sqrt{V}}$ Å |

## DE-BROGLIE WAVELENGTH ASSOCIATED WITH UNCHARGED PARTICLES

1. For neutrons ($m_n = 1.67\times10^{-27}$ kg):

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62\times10^{-34}}{\sqrt{2\times1.67\times10^{-27}\ E}}$$

2. For thermal neutrons at ordinary temperatures:

$E = kT$

$$\lambda = \frac{h}{\sqrt{2mkT}} = \frac{30.835}{\sqrt{T}}\text{Å}$$

3. For gas molecules: $\lambda = \frac{h}{m\times C_{rms}}$

4. For gas molecules at $T$ K: $E = \frac{3}{2}kT \Rightarrow \lambda = \frac{h}{\sqrt{3mkT}}$

### ILLUSTRATION 5.29

Calculate the de-Broglie wavelength associated with an α-particle accelerated through a potential difference of 200 V. Given $m_p = 1.67\times10^{-27}$ kg.

**Sol.** Mass of α-particle, $m = 4m_p = 4\times1.67\times10^{-27}$ kg

Charge on α-particle = $2e$

If the α-particle acquires velocity $v$, then $qV = \frac{1}{2}mv^2$

or $2eV = \frac{1}{2m}m^2v^2$

$\therefore\quad mv = \sqrt{4meV}$

$$\Rightarrow \quad \lambda = \frac{h}{mv} = \frac{h}{\sqrt{4meV}}$$

$$\Rightarrow \quad \lambda = \frac{6.6\times10^{-34}}{\sqrt{4\times4\times1.67\times10^{-27}\times1.6\times10^{-19}\times200}}\text{ m}$$

$$= 0.07138\times10^{-11}\text{ m} = 0.007138\text{ Å}$$

## ILLUSTRATION 5.30

Find the typical de-Broglie wavelength of an electron in a metal at 27°C and compare it with the mean separation between two electrons in a metal which is given to be about $2\times10^{-10}$ m.

**Sol.** Mass of an electron is $m = 9.11\times10^{-31}$ kg

and $T = 27 + 273 = 300$ K

$\therefore$ de-Broglie wavelength of electron is

$$\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.63\times10^{-34}}{\sqrt{3\times9.11\times10^{-31}\times1.38\times10^{-23}\times300}}\text{ m}$$

$$\Rightarrow \quad \lambda = 6.2\times10^{-9}\text{ m}$$

Mean separation between two electrons in a metal is

$r = 2\times10^{-10}$ m

$$\therefore \quad \frac{\lambda}{r} = \frac{6.2\times10^{-9}}{2\times10^{-10}} = 31$$

Thus the de-Broglie wavelength is much greater than the given interelectron separation.

## ILLUSTRATION 5.31

The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which **(a)** an electron and **(b)** a neutron would have the same de Broglie wavelength. Given that mass of neutron = $1.66\times10^{-27}$ kg.

**Sol.** Here, $\lambda = 589\text{ nm} = 589\times10^{-9}$ m

The de Broglie wavelength of a particle having energy $E$ is given by

$$\lambda = \frac{h}{\sqrt{2mE}} \quad \text{or} \quad E = \frac{h^2}{2m\lambda^2}$$

**(a)** For electron: Here, $\lambda = 589\text{ nm} = 589\times10^{-9}$ m and $m = 9.1\times10^{-27}$ kg

$$\therefore \quad E = \frac{h^2}{2m\lambda^2} = \frac{(6.62\times10^{-34})^2}{2\times9.1\times10^{-31}\times(589\times10^{-9})^2}$$

$$= 6.94\times10^{-25}\text{ J}$$

**(b)** For neutron: Here $\lambda = 589\text{ nm} = 589\times10^{-9}$ m and $m = 1.66\times10^{-27}$ kg

$$\therefore \quad E = \frac{h^2}{2m\lambda^2} = \frac{(6.62\times10^{-34})^2}{2\times1.66\times10^{-27}\times(589\times10^{-9})^2}$$

$$= 3.81\times10^{-28}\text{ J}$$

## ILLUSTRATION 5.32

**(a)** For what kinetic energy of a neutron will the associated de-Broglie wavelength be $1.40\times10^{-10}$ m?

**(b)** Also, find the de-Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of (3/2) $kT$ at 300 K. Given that mass of neutron = $1.66\times10^{-27}$ kg and $k = 1.38\times10^{-23}$ J kg$^{-1}$.

**Sol.**

**(a)** Here $\lambda = 1.40\ \lambda\times10^{-10}$ m; $m = 1.66\times10^{-27}$ kg

The energy of neutron is given by $E = \frac{h^2}{2m\lambda^2}$

$$= \frac{(6.62\times10^{-34})^2}{2\times1.66\times10^{-27}\times(1.40\times10^{-10})^2} = 6.735\times10^{-21}\text{ J}$$

**(b)** Here $k = 1.38\times10^{-23}$ J kg$^{-1}$ and $T = 300$ K

Therefore, energy of neutron

$$E = \frac{3}{2}kT = \frac{3}{2}\times1.38\times10^{-23}\times300 = 6.21\times10^{-21}\text{ J}$$

Now, $\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62\times10^{-34}}{\sqrt{2\times1.66\times10^{-27}\times6.21\times10^{-21}}}$

$$= 1.46\times10^{-10}\text{ m} = 1.46\text{ Å}$$

## ILLUSTRATION 5.33

Determine the de-Broglie wavelength of a proton, whose kinetic energy is equal to the rest of the mass energy of an electron. Given that the mass of an electron is $9.1\times10^{-31}$ kg and the mass of a proton is 1837 times as that of the electron.

**Sol.** Here mass of electron, $m_0 = 9.1\times10^{-31}$ kg

Mass of proton, $m = 1837\times9.1\times10^{-31} = 1.67\times10^{-27}$ kg

Let $v$ be the velocity of the proton. Then, $\frac{1}{2}mv^2 = m_0c^2$

or $\quad m^2v^2 = 2mm_0c^2$

or $\quad mv = \sqrt{2mm_0c}$

$$= \sqrt{2\times1.67\times10^{-27}\times9.1\times10^{-31}}\times3\times10^8$$

$$= 1.654\times10^{-20}\text{ kg m s}^{-1}$$

Therefore, de-Broglie wavelength of the proton,

$$\lambda = \frac{h}{mv} = \frac{6.62\times10^{-34}}{1.654\times10^{-20}} = 4\times10^{-14}\text{ m}$$

## ILLUSTRATION 5.34

Find the ratio of de Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27°C and 127°C, respectively.

**Sol.** de-Broglie wavelength is given by $\lambda = h/mv$.

Root mean square velocity of a gas particle at the given temperature ($T$) is given as, $\frac{1}{2}mv^2 = \frac{3}{2}kT$

$$v = \sqrt{\frac{3kT}{m}}$$

where $k$ = Boltzmann's constant, $m$ = mass of the gas particle, and $T$ = temperature of the gas in K.

$$mv = \sqrt{3mkT}$$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{3mkT}}$$

$$\frac{\lambda_H}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_H T_H}} = \sqrt{\frac{(4)(273+127)}{(2)(273+27)}} = \sqrt{\frac{8}{3}}$$

**ILLUSTRATION 5.35**

The extent of localization of a particle is determined roughly by its de Broglie wave. If an electron is localized within the nucleus (of size about $10^{-14}$ m) of an atom, what is its energy? Compare this energy with the typical binding energies (of the order of a few MeV) in a nucleus and hence argue why electrons cannot reside in a nucleus.

**Sol.** The de Broglie wavelength of electron,

$\lambda$ = size of atom = $10^{-14}$ m

Corresponding to this wavelength, the electron possesses energy,

$$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{10^{-14}} = 1.986\times10^{-11}\text{ J}$$

$$= \frac{1.986\times10^{-11}}{1.6\times10^{-13}} = 124.1\text{ MeV}$$

If the electrons possess energy of the order of 124 MeV, they will not be able to stay inside the nucleus. It is because Coulomb's force cannot provide such high value of binding energy to the electrons. Hence, electrons cannot reside inside the nucleus.

## PROPERTIES OF MATTER WAVES

1. Matter waves are different from electromagnetic waves. This is an obvious fact as matter waves are related to moving particles and independent of the fact whether particle is neutral or charged.
2. Matter waves travel with speed more than that of light.

$$v_{\text{matter waves}} = f\lambda = \frac{E}{h}\times\frac{h}{p} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

As velocity of particle can never exceed speed of light, hence $v_{\text{matter waves}} > c$. As a matter of fact, matter waves travel with the body in a group. Group velocity is same as velocity of body, whereas the individual waves constituting the group may theoretically move with speed greater than that of light.

3. In ordinary situation, de-Broglie wavelength is very small and wave nature of matter can be ignored. To appreciate this point, let us calculate de-Broglie wavelength of 46 g of golf ball moving with a speed of 30 m s$^{-1}$.

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{(0.046)(30)} = 4.8\times10^{-34}\text{ m}$$

The wavelength of golf ball is so small compared with its dimensions that we would not expect to detect any wave aspects in its behavior using ordinary instrument.

4. The wave and particle aspects of moving bodies can never be observed at the same time, i.e., the two nature are mutually exclusive. Which nature dominates in a given situation will be directed by how its de Broglie wavelength compares with its dimensions and dimensions of whatever it interacts with.
5. The square of the amplitude of de Broglie wave at any point is proportional to the probability of finding the particle at that point.

# ELECTRON MICROSCOPE

Electron microscope is an important application of de-Broglie waves designed to study very minute objects like viruses, microbes and the crystal structure of the solids.

**Principle:** Its working is based on the following facts:

1. Like light radiations, electron beams behave as waves but with much smaller wavelengths.
2. By using electric and magnetic fields, electron beams can be focused just as ordinary light beams are focused by glass lenses.

**Theory:** The de-Broglie wavelength associated with an electron accelerated through a potential difference of $V$ volts is given by

$$\lambda = \frac{12.3}{\sqrt{V}}\text{ Å}$$

The magnifying power of a microscope is inversely related with the wavelength of radiation used. With an optical microscope, a magnification of $\approx$ 1500 is possible. This limit is due to the large wavelength of visible radiation. However, by selecting a suitable value of potential difference $V$, one can have an electron beam of as small wavelength as desired. That is why an electron microscope can have a very high magnification of $\approx$100,000. In an electron microscope, the electrons are focused with the help of electric and magnetic lenses.

**ILLUSTRATION 5.36**

The speed of electrons in an electron microscope is $10^8$ m s$^{-1}$. If protons with the same speed are used instead of electrons, what additional advantage such a proton microscope has over an electron microscope?

**Sol.** de-Broglie wavelength, $\lambda = \frac{h}{mv}$

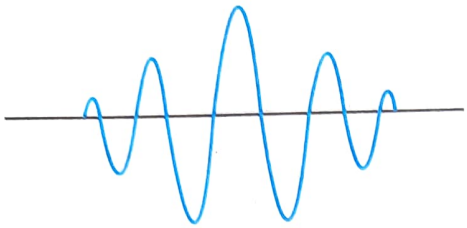
As the mass of a proton is 1836 times that of an electron, so for the same speed, the wavelength of proton beam will be $\frac{1}{1836}$ times that of an electron beam.

But resolving power of a microscope $\propto \frac{1}{\text{wavelength}}$.

Hence the resolving power of a proton microscope will be 1836 times that of an electron microscope.

# MATTER WAVES AND HEISENBERG'S UNCERTAINTY PRINCIPLE

The de-Broglie wave associated with a moving particle should be regarded as a wave packet instead of a continuous waves (see figure). A wave packet is a localized wave consisting of continuous range of frequencies



**The wave packet description of an electron**

The matter wave picture given by de Broglie elegantly incorporates the **Heisenberg's uncertainty** principle. According to this principle, it is not possible to measure both the position and momentum of a subatomic particle at the same time exactly. The product of the uncertainly in position ($\Delta x$) and the uncertainty in momentum ($\Delta p$) is never less than $h/4\pi$,

i.e., $\Delta x \times \Delta p \geq \frac{h}{4\pi}$

It must be noted that the uncertainties in $x$ and $p$ are not due to imperfections in the apparatus. In fact, they are due to the fact that the quantities involved ($x$ and $p$) are not precise.

**Important Points:**

- Heisenberg's uncertainty principle provides complementary descriptions of the same phenomenon. That is, to obtain the complete picture of any event, we need both the wave and the particle properties of matter, but because of the uncertainty principle, it is impossible to design an experiment that will show both of them in detail at the same time. Any one experiment will reveal the details of either the wave or the particle character, according to the purpose for which the experiment is designed.
- When the momentum of the particle is accurately known, $\Delta p = 0$ and $\Delta x \approx \infty$. Such a particle is represented by a wave of constant amplitude extending over all space as shown in figure.

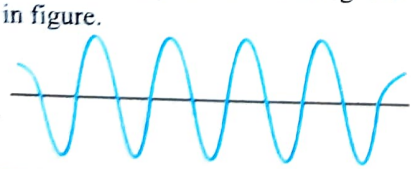


**ILLUSTRATION 5.37**

Applying uncertainty principle, calculate the minimum, kinetic energy of an electron confined in a region of length $l = 0.2$ nm.

**Sol.** The uncertainty in position of the electron is $\Delta x = l$

Then, the uncertainty in momentum of the electron is $p \geq \frac{h}{2\pi\Delta x}$

or $p \geq \frac{h}{2\pi l}$

Hence, the minimum momentum is $p_m = \frac{h}{2\pi l}$

Then, the minimum KE of the electron is $K_{min} = \frac{p_m^2}{2m}$

or $K_{min} = \frac{h^2}{8\pi^2 m l^2} = \left[\frac{(6.3\times10^{-34})^2}{8\times(3.14)^2(9.1\times10^{-27})(0.2\times10^{-9})^2}\right]$ J

$\simeq 8.6\times10^{-5}$ eV $\quad (\because 1\text{eV} = 1.6\times10^{-19}\text{ J})$

## CONCEPT APPLICATION EXERCISE 5.3

1. A proton and an alpha particle are accelerated through the same potential. Which one of the two has **(i)** greater value of de-Broglie wavelength associated with it, and **(ii)** less kinetic energy? Justify your answer.
2. An electron and a proton are accelerated through the same potential. Which one of the two has **(i)** greater value of de-Broglie wavelength associated with it and **(ii)** less momentum? Justify your answer.
3. An electron, $\alpha$-particle and a proton have the same kinetic energy. Which of these particles has the shortest de-Broglie wavelength?
4. An $\alpha$-particle and a proton are accelerated from rest through the same potential difference $V$. Find the ratio of de-Broglie wavelengths associated with them.
5. Calculate the ratio of the accelerating potential required to accelerate a proton and an $\alpha$-particle to have the same de-Broglie wavelength associated with them.
   [Given: Mass of proton = $1.6\times10^{-27}$ kg; Mass of $\alpha$-particle = $6.4\times10^{-27}$ kg]
6. An electron of mass $m$ and charge $e$ initially at rest gets accelerated by a constant electric field $E$. Find the rate of change of de-Broglie wavelength of this electron at time $t$.
7. Find the de Broglie's wavelength of a hydrogen molecule 27°C at its most probable speed.

**ANSWERS**

**4.** $1:2\sqrt{2}$ **5.** 8:1 **6.** $\frac{-h}{eEt^2}$ **7.** $1.2\times10^{-7}$ m

## Solved Examples

**EXAMPLE 5.1**

The wavelength of a photon is 1.4 Å. It collides with an electron. Its wavelength after the collision is 2.0 Å. Calculate the energy of the scattered electron.

**Sol.** We are given that, initial wavelength of the photon,

$\lambda = 1.4\text{ Å} = 1.4\times10^{-10}$ m

Final wavelength of the photon, $\lambda' = 2.0\text{ Å} = 2\times10^{-10}$ m

Initial energy of the photon $= hf = \frac{hc}{\lambda}$

Final energy of the photon $= hf' = \frac{hc}{\lambda'}$

If $E$ is the energy of the scattered photon,

$$E = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda'}\right] = hc\left[\frac{\lambda' - \lambda}{\lambda\lambda'}\right]$$

$$= (6.63\times10^{-34})(3\times10^{8})\left[\frac{2\times10^{-10} - 1.4\times10^{-10}}{(2\times10^{-10})(1.4\times10^{-10})}\right]$$

$$= 4.26\times10^{-16}\ \text{J}$$

## EXAMPLE 5.2

A monochromatic light of wavelength $\lambda$ is incident on an isolated metallic sphere of radius $a$. The threshold wavelength is $\lambda_0$ which is greater than $\lambda$. Find the number of photoelectrons emitted before the emission of photoelectrons will stop.

**Sol.** When electrons are ejected from the isolated sphere, it becomes positively charged. If its potential is raised to $V$ before the emission of photoelectrons stops, $hf = \phi_0 + \text{eV} = hf_0 + eV$, where $\phi_0$ is the work function of the material of the sphere and $eV$ is the attractive pull on the electron due to positively charged sphere.

Thus, $\frac{hc}{\lambda} = \frac{hc}{\lambda_0} + eV$

or $V = \frac{hc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$

As $Q = CV = (4\pi\varepsilon_0 a)V = \frac{4\pi\varepsilon_0 ahc}{e}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$,

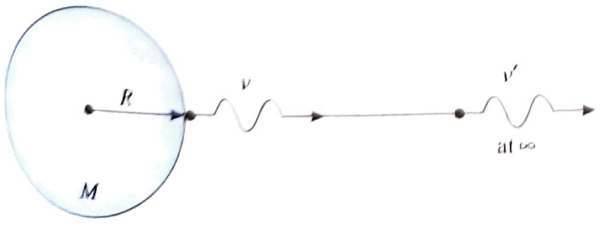
Number of electrons emitted, i.e., $n = \frac{Q}{e} = \frac{4\pi\varepsilon_0 ahc}{e^2}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$

## EXAMPLE 5.3

A planet of mass $M$ and radius $R$ emits a photon of frequency $f$. What will be the frequency of photon when it covers an infinite distance?

**Sol.** Let $f'$ be the frequency photon at an infinite distance.

$G$ = Gravitational constant




Mass of the photon, $m = \frac{hf}{c^2}$

Energy of photon on the surface, $E_i = hf - G\frac{Mm}{R}$

$$= hf - \frac{GM}{R}\left(\frac{hf}{c^2}\right)$$

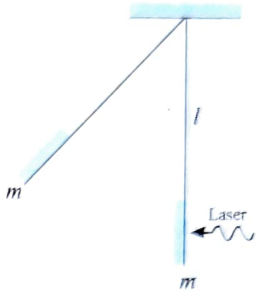
Energy of photon at infinity $E_f = hf'$

Energy must be conserved, therefore

$$hf - \frac{GM}{R}\left(\frac{hf}{c^2}\right) = hf' \Rightarrow f' = f\left[1 - \frac{GM}{Rc^2}\right]$$

## EXAMPLE 5.4

A small size perfectly reflecting mirror of mass is suspended by a weightless thread of length $l$ as shown in the figure. Find the angle through which the thread will be deflected when a short laser with energy $E$ is shot in horizontal direction at right angles to the mirror.




**Sol.** The laser beam incident on the mirror apply force on it, because of this force the mirror is deflected from its original position. The mirror is perfectly reflecting, hence the impulse applied by laser on the mirror $J = \frac{2E}{c}$.

Initial momentum of the mirror = 0

Let, final momentum of the mirror $= mv$

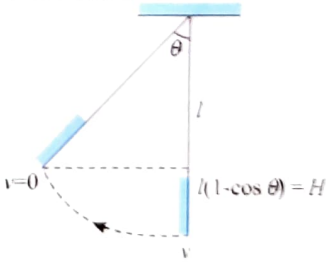
Change in momentum of the mirror due to impact,

$\Delta p = 0 - (-mv)$,

where $v$ is the speed of mirror just after impact.

But, impulse momentum theorem gives,

Impulse = change in momentum




$$J = \Delta p \Rightarrow \frac{2E}{c} = mv$$

or $v = \frac{2E}{mc}$ ...(i)

Total mechanical energy of the mirror will be conserved after impact.

Thus, Loss in kinetic energy = Gain in potential energy

$$\frac{1}{2}mv^2 = mgH \Rightarrow v^2 = 2gH$$

$$\frac{4E^2}{m^2c^2} = 2gH \qquad [\text{as } v = \frac{2E}{mc} \text{ from Eq. (i)}]$$

$$\frac{2E^2}{m^2c^2} = gH \Rightarrow \frac{2E^2}{m^2c^2} = gl(1-\cos\theta),$$

[As $H = l(1-\cos\theta)$]

or $1-\cos\theta=\dfrac{2E^2}{m^2c^2gl}$

$$2\sin^2\left(\frac{\theta}{2}\right)=\frac{2E^2}{m^2c^2gl} \qquad \left[\text{As } (1-\cos\theta)=2\sin^2\left(\frac{\theta}{2}\right)\right]$$

$$\sin\frac{\theta}{2}=\frac{E^2}{mc\sqrt{gl}} \Rightarrow \theta=2\sin^{-1}\left(\frac{E^2}{mc\sqrt{gl}}\right)$$

## EXAMPLE 5.5

Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature (27°C) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.

**Sol.** Mass of the atom is given by

$$m=\frac{\text{Atomic wt. of He}}{\text{Avogadro's number}}=\frac{4\times10^{-3}}{6\times10^{23}}=\frac{2}{3}\times10^{-26}\text{ kg}$$

$$T=27+273=300\text{ K}$$

Average KE of a He atom at absolute temperature $T$ is

$$\frac{1}{2}mv^2=\frac{3}{2}kT \qquad \therefore\ m^2v^2=3mkT$$

or $p^2=3mkT$ or $p=\sqrt{3mkT}$

$$\therefore\quad \lambda=\frac{h}{p}=\frac{h}{\sqrt{3mkT}}$$

$$\Rightarrow\quad \lambda=\frac{6.63\times10^{-34}}{\sqrt{3\times\frac{2}{3}\times10^{-26}\times1.38\times10^{-23}\times300}}$$

$$\Rightarrow\quad \lambda=0.73\times10^{-10}\text{ m}$$

For one mole of a gas, $PV=RT$

or $PV=kNT \qquad \left[\therefore k=\dfrac{R}{N}\right]$

or $\dfrac{V}{N}=\dfrac{kT}{P}$

$$\therefore\quad \text{Mean separation, } r=\left[\frac{\text{Molar volume}}{\text{Avogadro's number}}\right]^{1/3}=\left[\frac{kT}{P}\right]^{1/3}$$

Given $T=300\text{ K}$, $P=1\text{ atm}=1.01\times10^5\text{ Pa}$

$$k=1.38\times10^{-23}\text{ J mol}^{-1}\text{ K}^{-1}$$

$$\text{Hence } r=\left[\frac{1.38\times10^{-23}\times300}{1.01\times10^5}\right]^{1/3}\text{m}=\left[\frac{138\times30}{101}\right]^{1/3}\times10^{-19}\text{ m}$$

$$\Rightarrow\quad r=3.4\times10^{-19}\text{ m}$$

We find that $r>\lambda$, i.e., the wave packets associated with He atoms do not overlap and hence He atoms can be distinctly seen.

## EXAMPLE 5.6

X-rays of wavelength $\lambda$ fall on photo-sensitive surface, emitting electrons. Assuming that the work function of the surface can be neglected. Find the de-Broglie wavelength of electrons emitted.

**Sol.** As the work function of the metal can be neglected, so KE of emitted electron = Energy of X-ray photon

or $\dfrac{1}{2}mv^2=hf$ or $\dfrac{p^2}{2m}=\dfrac{hc}{\lambda}$

or $p=\sqrt{\dfrac{2mhc}{\lambda}}$

de-Broglie wavelength of emitted electrons, $\lambda_1=\dfrac{h}{p}=\dfrac{h}{\sqrt{\frac{2mhc}{\lambda}}}$

or $\lambda_1=\sqrt{\dfrac{h\lambda}{2mc}}$

## EXAMPLE 5.7

In a photocell the plates $P$ and $Q$ have a separation of 10 cm, which are connected through a galvanometer without any cell. Bichromatic light of wavelengths 4000 Å and 6000 Å are incident on plate $Q$ whose work function is 2.39 eV. If a uniform magnetic field $B$ exists parallel to the plates, find the minimum value of $B$ for which the galvanometer shows zero deflection.

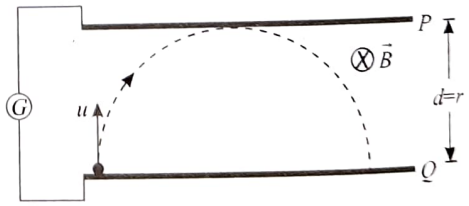
**Sol.** Energy of photons corresponding to light of wavelength $\lambda_1=4000$ Å is

$$E_1=\frac{12400}{4000}=3.1\text{eV}$$

and that corresponding to $\lambda_2=6000$ Å is $E_2=\dfrac{12375}{6000}=2.06\text{eV}$

As $E_2<W_0$ and $E_1>W_0$ ($W_0$ = Work function)

Photoelectric emission is possible with $\lambda_1$ only.




Photoelectrons experience magnetic force and move along a circular path. The galvanometer will indicate zero deflection if the photoelectrons just complete semicircular path before reaching the plate $P$.

Thus, $d=r=10$ cm,

$\therefore\quad r=10\text{ cm}=0.10\text{ m}$

Further, $r=\dfrac{mv}{Bq}=\dfrac{\sqrt{2Km}}{Bq}$

$$\therefore\quad B_{\min}=\frac{\sqrt{2Km}}{rq}$$

Here $K = E_1 - W = (3.1 - 2.39) = 0.71$ eV

Substituting the values, we have

$$B_{min} = \frac{\sqrt{2 \times 0.71 \times 1.6 \times 10^{-19} \times 9.10 \times 10^{-31}}}{(0.10)(1.6 \times 10^{-19})}$$

$$\Rightarrow \quad B_{min} = 2.85 \times 10^{-5} \text{ tesla}$$

## EXAMPLE 5.8

When a beam of 10.6 eV photons of intensity 2.0 $Wm^{-2}$ falls on a platinum surface of area $1.0 \times 10^{-4}$ $m^2$ and work function 5.6 eV, 0.53% of the incident photons ejects photoelectrons. Find the number of photoelectrons emitted per second, and their minimum and maximum energies (in eV). Take 1 eV $= 1.6 \times 10^{-19}$ J.

**Sol.** Energy incident on the surface per second

$$P = IA = 2.0 \times 1.0 \times 10^{-4} = 2 \times 10^{-4} \text{ J s}^{-1}$$

Energy of each photon = 10.6 eV = $10.6 \times 1.6 \times 10^{-19}$ J

$$\therefore \text{Number of photons incident on the surface} = \frac{2 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}}$$

$$\text{Number of photoelectrons emitted} = \frac{0.53}{100} \times \frac{2 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}}$$

$$= 6.25 \times 10^{11}$$

According to Einstein's photoelectric equation, maximum KE of photoelectrons,

$$K_{max} = E - W_0 = 10.6 \text{ eV} - 5.6 \text{ eV} = 5 \text{ eV}$$

Minimum kinetic energy of photoelectrons = zero

## EXAMPLE 5.9

Find the frequency of light which ejects electrons from a metal surface. Fully stopped by a retarding potential of 3 V, the photoelectric effect begins in this metal at a frequency of $6 \times 10^{14}$ Hz. Find the work function for this metal. (Given: $h = 6.63 \times 10^{-34}$ J s).

**Sol.** According to Einstein's photoelectric equation,

$$K_{max} = hf - W_0$$

If $V_s$ is the retarding or stopping potential and $\upsilon_0$ the threshold frequency, then above equation becomes

$$eV_s = hf - hf_0$$

or $$hf = eV_s + hf_0$$

or $$f = \frac{eV_s}{h} + f_0$$

Hence, $e = 1.6 \times 10^{-10}$ C, $V_s = 3$ V, and $f_0 = 6 \times 10^{14}$ Hz

$$\text{Required frequency, } f = \frac{1.6 \times 10^{-10} \times 3}{6.63 \times 10^{-34}} + 6 \times 10^{14}$$

$$= 7.24 \times 10^{14} + 6 \times 10^{14}$$

$$= 1.324 \times 10^{15} \text{ Hz}$$

## EXAMPLE 5.10

Light of wavelength of 2000 Å falls on an aluminium surface. In aluminium, 4.2 eV are required to remove an electron from its surface. What is the kinetic energy, in electron volt, of **(a)** the fastest, and **(b)** the slowest emitted photo-electrons. **(c)** What is the stopping potential? **(d)** What is the cut-off wavelength for aluminium? (Planck's constant $h = 6.6 \times 10^{-34}$ J s, and speed of light $c = 3 \times 10^8$ m $s^{-1}$)

**Sol.** Energy corresponding to incident photon

$$hf = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2000 \times 10^{-10}}$$

$$= 9.9 \times 10^{-19} \text{ J}$$

$$= \frac{9.9 \times 10^{-19}}{1.6 \times 10^{-19}} \text{eV} = 6.2 \text{ eV}$$

**(a)** The kinetic energy of fastest electrons,

$$K_{max} = hf - W_0$$

or $$K_{max} = 6.2 \text{ eV} - 4.2 \text{ eV} = 2 \text{ eV}$$

**(b)** The kinetic energy of the slowest electrons is zero, since the emitted electrons have all possible energies from 0 to certain maximum value $K_{max}$.

**(c)** If $V_s$ is the stopping potential, then $K_{max} = eV_s$

or $$V_S = \frac{K_{max}}{e} = \frac{2\text{eV}}{e} = 2\text{V}$$

**(d)** If $\lambda_0$ is the cut-off wavelength for aluminium, then

$$W_0 = (hc/\lambda_0)$$

or $$\lambda_0 = (hc/W_0) = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.2 \times 1.6 \times 10^{-19}}$$

$$= 3000 \times 10^{-10} \text{ m} = 3000 \text{ Å}$$

## EXAMPLE 5.11

**(a)** A stopping potential of 0.82 V is required to stop the emission of photoelectrons from the surface of a metal by light of wavelength 4000 Å. For light of wavelength 3000 Å, the stopping potential is 1.85 V. Find the value of Planck's constant. [1 electron volt (eV) $= 1.6 \times 10^{-19}$ J]

**(b)** At stopping potential, if the wavelength of the incident light is kept fixed at 4000 Å, but the intensity of light increases two times, will photoelectric current be obtained? Give reasons for your answer.

**Sol.**

**(a)** In case I: $$\frac{hc}{\lambda_1} = W_0 + eV_1 \quad \text{...(i)}$$

In case II: $$\frac{hc}{\lambda_2} = W_0 + eV_2 \quad \text{... (ii)}$$

Subtracting, we get

$$\frac{hc}{\lambda_2} - \frac{hc}{\lambda_1} = e(V_2 - V_1)$$

Here, $c = 3 \times 10^8$ m s$^{-1}$,

$\lambda_1 = 4000$ Å $= 4 \times 10^{-7}$ m, $V_1 = 0.82$ V

$\lambda_2 = 3000$ Å $= 3 \times 10^{-7}$ m, $V_2 = 1.85$ V

$$\therefore \quad h \times 3 \times 10^8 \left( \frac{1}{3 \times 10^{-7}} - \frac{1}{4 \times 10^{-7}} \right)$$

$$= 1.6 \times 10^{-19} (1.85 - 0.82)$$

or $$h \times 3 \times 10^8 \times \frac{1}{12 \times 10^{-7}} = 1.6 \times 10^{-19} \times 1.03$$

Therefore, Planck's constant,

$$h = \frac{1.6 \times 10^{-19} \times 1.03 \times 12 \times 10^{-7}}{3 \times 10^8} = 6.592 \times 10^{-34} \text{ Js}$$

**(b)** No, because stopping potential does not depend on intensity of incident light.

## EXAMPLE 5.12

A 40 W ultraviolet light source of wavelength 2480 Å illuminates a magnesium (Mg) surface placed 2 m away. Determine the number of photons emitted from the surface per second and the number incident on unit area of Mg surface per second. The photoelectric work function for Mg is 3.68 eV. Calculate the kinetic energy of the fastest electrons ejected from the surface. Determine the maximum wavelength for which the photoelectric effect can be observed with Mg surface.

**Sol.** Energy of each photon, $E = \frac{hc}{\lambda}$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2480 \times 10^{-10}} \text{ J} = 8.0 \times 10^{-19} \text{ J}$$

Number of photons emitted per second, $N = \frac{P}{E}$

$$= \frac{40 \text{ J s}^{-1}}{8.0 \times 10^{-19} \text{ J}} = 5.0 \times 10^{19} \text{ s}^{-1}$$

These photons spread in all directions over surface area $4\pi r^2$, therefore the number of photons incident per unit area per second,

$$N_i = \frac{N}{4\pi r^2} = \frac{5.0 \times 10^{19}}{4 \times 3.14 \times (2)^2} = 9.95 \times 10^{13} \text{ s}^{-1} \text{ (As } r = 2 \text{ m)}$$

From Einstein's photoelectric equation, $K_{max} = hf - W_0$

$$\Rightarrow \quad E = hf = \frac{hc}{\lambda} = 8.0 \times 10^{-19} \text{ J} = \frac{8.0 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 5.0 \text{ eV}$$

$$\therefore \quad K_{max} = 5.0 \text{ eV} - 3.68 \text{ eV} = 1.32 \text{ eV}$$

Threshold (or maximum) wavelength for photoelectrons emission,

$$\lambda_0 = \frac{hc}{W_0} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3.68 \times 1.6 \times 10^{-19}} \text{ m} = 3.373 \times 10^{-7} \text{ m} = 3373 \text{ Å}$$

## EXAMPLE 5.13

Ultraviolet light of wavelengths 800 Å and 700 Å when allowed to fall on hydrogen atoms in their ground states is found to liberate electrons with kinetic energies 1.8 eV and 4.0 eV, respectively. Find the value of Planck's constant.

**Sol.** The energy of incident photon $= (hc/\lambda)$

If $W_i$ is the ionization energy and $E_k$ the kinetic energy of emitted electron, then we have

$$\frac{hc}{\lambda_1} = W_i + E_{K_1}$$

Therefore, for incident photon of wavelength

$\lambda_1 = 800$ Å $= 8 \times 10^{-8}$ m

$$\frac{hc}{\lambda} = W_i + E_{K_1} \quad \text{...(i)}$$

And for incident photon of wavelength

$\lambda_2 = 700$ Å $= 7 \times 10^{-8}$ m,

$$\frac{hc}{\lambda} = W_i + E_{K_2} \quad \text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$hc\left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) = E_{K_2} - E_{K_1}$$

or $$h = \frac{(E_{K_2} - E_{K_1}) \lambda_1 \lambda_2}{c(\lambda_1 - \lambda_2)}$$

Here, $E_{K_1} = 1.8 \text{ eV} = 1.8 \times 1.6 \times 10^{-19}$ J

and $E_{K_2} = 4.0 \text{ eV} = 4.0 \times 1.6 \times 10^{-19}$ J

Substituting given values, we get

$$h = \frac{(4.8 - 1.8) \times 1.6 \times 10^{-19} \times 8 \times 10^{-8} \times 7 \times 10^{-8}}{3 \times 10^{-8} (8 \times 10^{-8} - 7 \times 10^{-8})}$$

$$= 6.57 \times 10^{-34} \text{ J s}$$

## EXAMPLE 5.14

A beam of light has three wavelengths 4144 Å, 4972 Å and 6216 Å with a total intensity of $3.6 \times 10^{-3}$ Wm$^{-2}$ equally distributed amongst the three wavelengths. The beam falls normally on an area 1.0 cm$^2$ if a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each energetically capable photon ejects one electron. Calculate the number of photoelectrons liberated in two seconds.

**Sol.** Energy of photon having wavelength 4144 Å,

$$E_1 = \frac{12375}{4144} \text{ eV} = 2.99 \text{ eV}$$

Similarly, $E_2 = \frac{12375}{4972} \text{ eV} = 2.49 \text{ eV}$ and

$$E_3 = \frac{12375}{6216} \text{ eV} = 1.99 \text{ eV}$$

Since only $E_1$ and $E_2$ are greater than the work function $W = 2.3$ eV, only first two wavelengths are capable for ejecting photoelectrons. Given intensity is equally distributed in all wavelengths. Therefore, intensity corresponding to each wavelength is $\frac{3.6 \times 10^{-3}}{3} = 1.2 \times 10^{-3}$ W/m$^2$

or energy incident per second in the given area ($A = 1.0\ cm^2 = 10^{-4}\ m^2$) is

$$P = 1.2\times10^{-3}\times10^{4} = 1.2\times10^{-7}\ J/s$$

Let $n_1$ be the number of photons incident per unit time in the given area corresponding to first wavelength. Then,

$$n_1 = \frac{P}{E_1} = \frac{1.2\times10^{-7}}{2.99\times1.6\times10^{-19}} = 2.5\times10^{11}$$

Similarly, $n_2 = \frac{P}{E_2} = \frac{1.2\times10^{-7}}{2.49\times1.6\times10^{-19}} = 3.0\times10^{11}$

Since, each energetically capable photon ejects electron, total number of photoelectrons liberated in 2 s

$$= 2(n_1 + n_2) = 2(2.5\times3.0)\times10^{11} = 1.1\times10^{12}$$

## EXAMPLE 5.15

In a photoelectric effect set-up a point source of light of power $3.2\times10^{-3}$ W emits monoenergetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV and of radius $8.0\times10^{-3}$ m. The efficiency of photoelectron emission is one for every $10^6$ incident photons. Assume that the sphere is isolated and initially neutral and that photoelectrons are instantly swept away after emission.

(a) Calculate the number of photoelectrons emitted per second.

(b) Find the ratio of the wavelength of incident light to the de-Broglie wavelength of the fastest photoelectrons emitted.

(c) It is observed that the photoelectron emission stops at a certain time $t$ after the light source is switched on, why?

(d) Evaluate the time $t$.

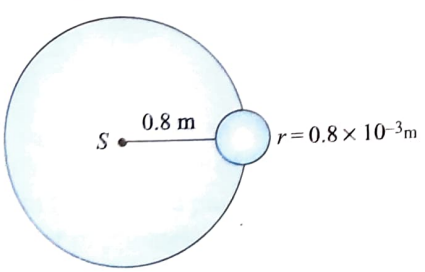
**Sol.**

(a) Energy of emitted photons,

$$E_1 = 5.0\ eV = 5.0\times1.6\times10^{-19}\ J = 8.0\times10^{-19}\ J$$

Power of the point source is $3.2\times10^{-3}$ watt or $3.2\times10^{-3}$ J/s

Therefore, energy emitted per second, $E_2 = 3.2\times10^{-3}\ J$




Hence number of photons emitted per second, $n_1 = \frac{E_2}{E_1}$

or $n_1 = \frac{3.2\times10^{-3}}{8.0\times10^{-19}}$

$$n_1 = 4.0\times10^{15}\ \text{photons/s}$$

Number of photons incident on unit area at a distance of 8.0 m from the source $S$ will be

$$n_2 = \frac{n_1}{4\pi(0.8)^2} = \frac{4.0\times10^{15}}{4\pi(0.64)} \approx 5.0\times10^{14}\ \text{photon/s m}^2$$

The area of metallic sphere over which photons will fall is

$$A = \pi r^2 = \pi(8\times10^{-3})^2\ m^2 = 2.01\times10^{-4}\ m^2$$

Therefore, number of photons incident on the sphere per second are

$$n_3 = n_2 A = (5.0\times10^{14}\times2.01\times10^{-4}) \approx 10^{11}\ /\ s$$

But since one photoelectron is emitted for every $10^6$ photons hence number of photoelectrons emitted per second,

$$n = \frac{n_3}{10^6} = \frac{10^{11}}{10^6} = 10^5\ \text{per second}$$

or $n = 10^5$ per second

**(b)** Maximum kinetic energy of photoelectrons,

$K_{max}$ = Energy of incident photons – work function

$$K_{max} = (5.0 - 3.0)\ eV = 2.0\ eV = 2.0\times1.6\times10^{-19}\ J$$

$$K_{max} = 3.2\times10^{-19}\ J$$

The de-Broglie wavelength of these photoelectrons will be

$$\lambda_1 = \frac{h}{p} = \frac{h}{\sqrt{2\,K_{max}m}}$$

(Here $h$ = Planck's constant and $m$ = mass of electron)

$$\therefore\quad \lambda_1 = \frac{6.63\times10^{-34}}{\sqrt{2\times3.2\times10^{-19}\times9.1\times10^{-31}}}$$

$$= 8.68\times10^{-10}\ m = 8.68\ Å$$

Wavelength of incident light, $\lambda_2\,(\text{in Å}) = \frac{12375}{E_1\,(\text{in eV})}$

or $\lambda_2 = \frac{12375}{5}\ Å = 2476\ Å$

Therefore, the desired ratio is $\frac{\lambda_2}{\lambda_1} = \frac{2475}{8.68} = 285.1$

**(c)** As soon as electrons are emitted from the metal sphere, it gets positively charged and acquires positive potential. The positive potential gradually increases as more and more photoelectrons are emitted from its surface. Emission of photoelectrons is stopped when its potential is equal to the stopping potential required for fastest moving electrons.

**(d)** As discussed in part (c), emission of photoelectrons is stopped when potential on the metal sphere is equal to the stopping potential of fastest moving electrons.

Since $K_{max} = 2.0$ eV

Therefore, stopping potential $V_0 = 2$ volt. Let $q$ be the charge required for the potential on the sphere to be equal to stopping potential or 2 volt. Then

$$2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} = (9.0\times10^{9})\frac{9}{8.0\times10^{-3}}$$

$$\therefore\quad q = 1.78\times10^{-12}\ C$$

Photoelectrons emitted per second = $10^5$ [part (a)]

or charge emitted per second $= (1.6\times10^{-19})\times10^5\ C$

$$= (1.6\times10^{-14})\ C$$

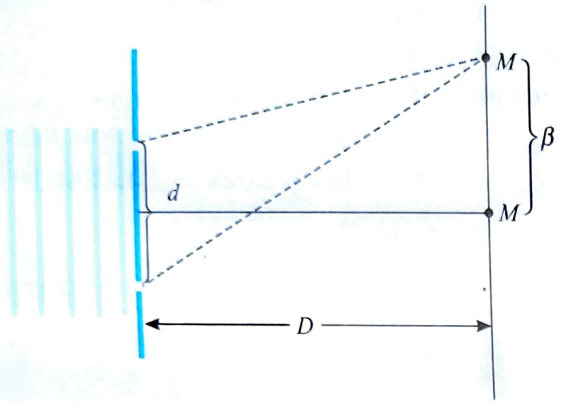
Therefore, time required to acquire to charge $q$ will be

$$t = \frac{q}{1.6\times10^{-14}}\ s = \frac{1.78\times10^{-12}}{1.6\times10^{-14}}\ s$$

or $t \approx 111\ s$

## EXAMPLE 5.16

A beam of electrons hit the slits of separation $d$ after passing through a potential difference $V$. They will form interference pattern on a screen situated at a distance $D$ from the slits, **(a)** Find the distance $\beta$ between two adjacent maxima, **(b)** Putting $d = 50 \times 10^{-6}$ m, $D = 1$ m and $V = 25$ volts, find $\beta$.




**Sol.**

**(a)** The distance between two adjacent maxima is

$$\beta = \frac{\lambda D}{d} \quad \text{...(i)}$$

The de-Broglie's wavelength of the electrons is

$$\lambda = \frac{h}{p} \quad \text{...(ii)}$$

The momentum of the electron just before hitting the slits is

$$p = \sqrt{2mK} \quad \text{...(iii)}$$

The KE of the electron just before hitting the slits is

$$K = eV \quad \text{...(iv)}$$

Using all equations we get $\beta = \dfrac{Dh}{d\sqrt{2meV}}$

**(b)** $\beta = \dfrac{(1\text{m})(6.3\times10^{-34}\text{ Js})}{(50\times10^{-6})\text{m}\sqrt{2\times9.1\times10^{-31}\text{kg}\times1.6\times10^{-19}\text{C}\times25\text{ volt}}}$

$= 4.7\times10^{-6}\text{m}$

# Exercises

## Single Correct Answer Type

1. An electron beam accelerated from rest through a potential difference of 5000 V in vacuum is allowed to impinge on a surface normally. The incident current is 50 μA and if the electrons come to rest on striking the surface the force on it is
(1) $1.1924 \times 10^{-8}$ N (2) $2.1 \times 10^{-8}$ N
(3) $1.6 \times 10^{-8}$ N (4) $1.6 \times 10^{-6}$ N

2. $10^{-3}$ W of 5000 Å light is directed on a photoelectric cell. If the current in the cell is 0.16 μA, the percentage of incident photons which produce photoelectrons, is
(1) 40% (2) 0.04%
(3) 20% (4) 10%

3. When a certain metallic surface is illuminated with monochromatic light of wavelength $\lambda$, the stopping potential for photoelectric current is $3V_0$ and when the same surface is illuminated with light of wavelength $2\lambda$, the stopping potential is $V_0$. The threshold wavelength of this surface for photoelectric effect is
(1) $6\lambda$ (2) $4\lambda/3$
(3) $4\lambda$ (4) $8\lambda$

4. Threshold frequency for a certain metal is $\nu_0$. When light of frequency $2\nu_0$ is incident on it, the maximum velocity of photoelectrons is $4 \times 10^8$ cm s$^{-1}$. If frequency of incident radiation is increased to $5\nu_0$, then the maximum velocity of photoelectrons, in cm s$^{-1}$, will be
(1) $(4/5) \times 10^8$ (2) $2 \times 10^8$
(3) $8 \times 10^8$ (4) $20 \times 10^8$

5. Light of wavelength 0.6 μm from a sodium lamp falls on a photocell and causes the emission of photoelectrons for which the stopping potential is 0.5 V. With light of wavelength 0.4 μm from a mercury vapor lamp, the stopping potential is 1.5 V. Then, the work function [in electron volts] of the photocell surface is
(1) 0.75 eV (2) 1.5 eV
(3) 3 eV (4) 2.5 eV

6. Ultraviolet light of wavelength 300 nm and intensity 1.0 W m$^{-2}$ falls on the surface of a photosensitive material. If one per cent of the incident photons produce photoelectrons, then the number of photoelectrons emitted per second from an area of 1.0 cm$^2$ of the surface is nearly
(1) $9.61 \times 10^{14}$ s$^{-1}$ (2) $4.12 \times 10^{13}$ s$^{-1}$
(3) $1.51 \times 10^{12}$ s$^{-1}$ (4) $2.13 \times 10^{11}$ s$^{-1}$

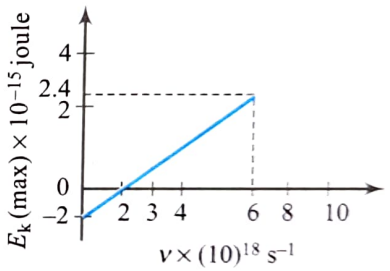
7. In a photocell, with excitation wavelength $\lambda$, the faster electron has speed $v$. If the excitation wavelength is changed to $3\lambda/4$, the speed of the fastest electron will be
(1) $v(3/4)^{1/2}$ (2) $v(4/3)^{1/2}$
(3) less than $v(4/3)^{1/2}$ (4) greater than $v(4/3)^{1/2}$

8. Two identical photocathodes receive light of frequencies $f_1$ and $f_2$. If the velocities of the photoelectrons (of mass $m$) coming out are $v_1$ and $v_2$, respectively, then
(1) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$ (2) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$
(3) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$ (4) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 - f_2)$

9. If the intensity of radiation incident on a photocell be increased four times, then the number of photoelectrons and the energy of photoelectrons emitted respectively become
(1) four times, doubled
(2) doubled, remains unchanged
(3) remains unchanged, doubled
(4) four times, remains unchanged

10. If a surface has work function of 3.00 eV, the longest wavelength of light which will cause the emission of electrons is
(1) $4.8 \times 10^{-7}$ m (2) $5.99 \times 10^{-7}$ m
(3) $4.13 \times 10^{-7}$ m (4) $6.84 \times 10^{-7}$ m

11. In the experiment on photoelectric effect, the graph between $E_K$(max) and $\nu$ is found to be a straight line as shown in figure.




The threshold frequency and Planck's constant according to this graph are
(1) $3.33 \times 10^{18}$ s$^{-1}$, $6 \times 10^{-34}$ Js
(2) $6 \times 10^{18}$ s$^{-1}$, $6 \times 10^{-34}$ Js
(3) $2.66 \times 10^{18}$ s$^{-1}$, $4 \times 10^{-34}$ Js
(4) $4 \times 10^{18}$ s$^{-1}$, $3 \times 10^{-34}$ Js

12. Monochromatic light incident on a metal surface emits electrons with kinetic energies from zero to 2.6 eV. What is the least energy of the incident photon if the tightly bound electron needs 4.2 eV to remove?
(1) 1.6 eV (2) From 1.6 eV to 6.8 eV
(3) 6.8 eV (4) More than 6.8 eV

13. The work function for sodium surface is 2.0 eV and that for aluminium surface is 4.2 eV. The two metals are illuminated with appropriate radiations so as to cause photoemission. Then
(1) the threshold frequency for sodium will be less than that for aluminium
(2) the threshold frequency of sodium will be more than that of aluminium
(3) both sodium and aluminium will have the same threshold frequency
(4) none of the above

**14.** Representing the stopping potential $V$ along $y$-axis and $(1/\lambda)$ along $x$-axis for a given photocathode, the curve is a straight line, the slope of which is equal to

(1) $e/hc$ (2) $hc/e$

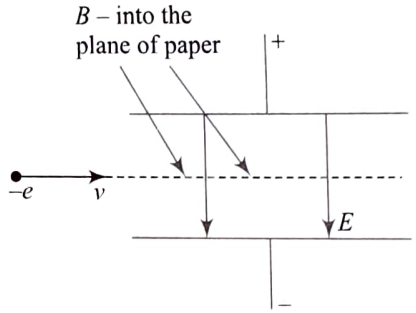
(3) $ec/h$ (4) $he/c$

**15.** In Q. 14, the intercept on the $y$-axis is equal to

(1) $+W/e$ (2) $-W/e$

(3) $-We$ (4) $e/W$

**16.** Electrons traveling at a velocity of $2.4 \times 10^6$ m s$^{-1}$ enter a region of crossed electric and magnetic fields as shown in figure. If the electric field is $3.0 \times 10^6$ Vm and the flux density of the magnetic field is 1.5 T, the electrons upon entering the region of the crossed fields will




(1) continue to travel undeflected in their original direction

(2) be deflected upward in the plane of the diagram

(3) be deflected downward on the plane of the diagram

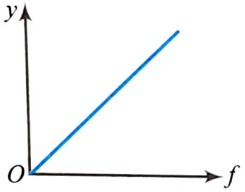
(4) none of the above

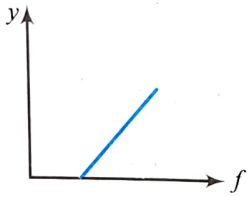
**17.** A surface irradiated with light $\lambda = 480$ nm gives out electrons with maximum velocity $v$ m s$^{-1}$, the cut off wavelength being 600 nm. The same surface would release electrons with maximum velocity $2v$ m s$^{-1}$ if it is irradiated by light of wavelength.

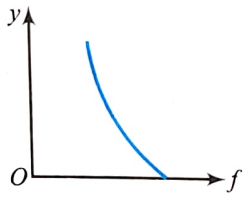
(1) 325 nm (2) 360 nm

(3) 384 nm (4) 300 nm

**18.** In an experiment on the photoelectric effect, an evacuated photocell with a pure metal cathode is used. Which graph best represents the variation of $V$, the minimum potential difference needed to prevent current from flowing, when $x$, the frequency of the incident light, is varied?

(1)



(2)



(3)
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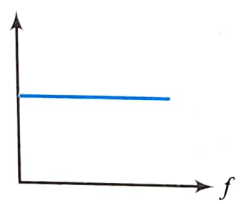
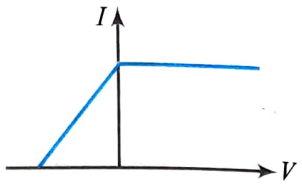
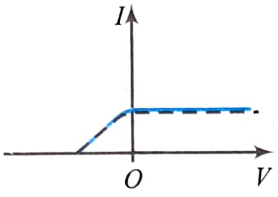



**19.** A metal surface in an evacuated tube is illuminated with monochromatic light causing the emission of photoelectrons which are collected at an adjacent electrode. For a given intensity of light, the way in which the photocurrent $I$ depends in the potential difference $V$ between the electrodes is shown by approximate graph in figure.




If the experiment were repeated with light of twice the intensity but the same wavelength, which of the graphs below would best represent the new relation between $I$ and $V$? (In these graphs, the result of the original experiment is indicated by a broken line.)

(1)
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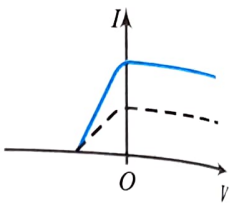


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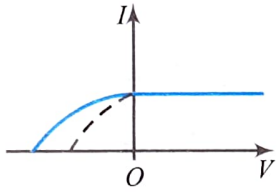


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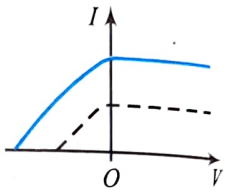



**20.** If stopping potentials corresponding to wavelengths 4000 Å and 4500 Å are 1.3 V and 0.9 V, respectively, then the work function of the metal is

(1) 0.3 eV (2) 1.3 eV

(3) 2.3 eV (4) 5 eV

**21.** The photoelectric threshold of a certain metal is 3000 Å. If the radiation of 2000 Å is incident on the metal

(1) electrons will be emitted

(2) positrons will be emitted

(3) protons will be emitted

(4) electrons will not be emitted

**22.** The frequency and the intensity of a beam of light falling on the surface of a photoelectric material are increased by a factor of two. This will

(1) increase the maximum kinetic energy, the photoelectrons, as well as photoelectric current by a factor of 2

(2) increase the maximum kinetic energy of the photoelectron and would increase the photoelectric current by a factor of 2

(3) increase the maximum kinetic energy of the photoelectrons by a factor of 2 and will have no effect on the magnitude of the photoelectric current produced

(4) not produce any effect on the kinetic energy of the emitted electrons but will increase the photoelectric current by a factor of 2

**23.** The frequency of incident light falling on a photosensitive metal plate is doubled, the KE of the emitted photoelectrons is

(1) double the earlier value (2) unchanged

(3) more than doubled (4) less than doubled

**24.** Lights of two different frequencies whose photons have energies 1 and 2.5 eV, respectively, successively illuminate a metal whose work function is 0.5 eV. The ratio of the maximum speeds of the emitted electrons will be

(1) 1:5 (2) 1:4
(3) 1:2 (4) 1:1

25. A proton when accelerated through a potential difference of $V$ volt has a wavelength $\lambda$ associated with it. An $\alpha$-particle in order to have the same $\lambda$ must be accelerated through a potential difference of
(1) $V$ volt (2) $4V$ volt
(3) $2V$ volt (4) $(V/8)$ volt

26. A cesium photocell, with a steady potential difference of 60 V across it, is illuminated by a small bright light placed 1 m away. When the same light is placed 2 m away, the electrons crossing the photocell
(1) each carry one-quarter of their previous momentum
(2) each carry one-quarter of their pervious energy
(3) are one-quarter as numerous
(4) are half as numerous

27. If 5% of the energy supplied to a bulb is irradiated as visible light, how many quanta are emitted per second by a 100 W lamp? Assume wavelength of visible light as $5.6 \times 10^{-5}$ cm.
(1) $1.4 \times 10^{19}$ (2) $3 \times 10^{3}$
(3) $1.4 \times 10^{-19}$ (4) $3 \times 10^{4}$

28. Five volt of stopping potential is needed for the photoelectrons emitted out of a surface of work function 2.2 eV by the radiation of wavelength
(1) 1719 Å (2) 3444 Å
(3) 861 Å (4) 3000 Å

29. The work functions for tungsten and sodium are 4.5 eV and 2.3 eV, respectively. If the threshold wavelength $\lambda$ for sodium is 5460 Å, the value of $\lambda$ for tungsten is
(1) 5893 Å (2) 10683 Å
(3) 2791 Å (4) 528 Å

30. The photoelectric threshold for some material is 200 nm. The material is irradiated with radiations of wavelength 40 nm. The maximum kinetic energy of the emitted photoelectrons is
(1) 2 eV (2) 1 eV
(3) 0.5 eV (4) none of these

31. Photoelectric work function of a metal is 1 eV. Light of wavelength $\lambda = 3000$ Å falls on it. The photoelectrons come out with velocity
(1) 10 m $s^{-1}$ (2) $10^3$ m $s^{-1}$
(3) $10^4$ m $s^{-1}$ (4) $10^6$ m $s^{-1}$

32. If a surface has a work function 4.0 eV, what is the maximum velocity of electrons liberated from the surface when it is irradiated with ultraviolet radiation of wavelength 0.2 μm?
(1) $4.4 \times 10^5$ m $s^{-1}$ (2) $8.8 \times 10^7$ m $s^{-1}$
(3) $8.8 \times 10^5$ m $s^{-1}$ (4) $4.4 \times 10^7$ m $s^{-1}$

33. A modern 200 W sodium street lamp emits yellow light of wavelength 0.6 μm. Assuming it to be 25% efficient in converting electrical energy to light, the number of photons of yellow light it emits per second is
(1) $62 \times 10^{20}$ (2) $3 \times 10^{19}$
(3) $1.5 \times 10^{20}$ (4) $6 \times 10^{18}$

34. An electron of mass $m_e$ and a proton of mass $m_p$ are accelerated through the same potential difference. The ratio of the de Broglie wavelength associated with an electron to that associated with proton is
(1) 1 (2) $m_p/m_e$
(3) $m_e/m_p$ (4) $\sqrt{m_p/m_e}$

35. A material particle with a rest mass $m_o$ is moving with a velocity of light $c$. Then, the wavelength of the de Broglie wave associated with it is
(1) $(h/m_o c)$ (2) zero
(3) $\infty$ (4) $(m_o c/h)$

36. If a photocell is illuminated with a radiation of 1240 Å, then stopping potential is found to be 8 V. The work function of the emitter and the threshold wavelength are
(1) 1 eV, 5200 Å (2) 2 eV, 6200 Å
(3) 3 eV, 7200 Å (4) 4 eV, 4200 Å

37. Silver has a work function of 4.7 eV. When ultraviolet light of wavelength 100 mm is incident upon it, a potential of 7.7 V is required to stop the photoelectrons from reaching the collector plate. How much potential will be required to stop the photoelectrons when light of wavelength 200 mm is incident upon silver?
(1) 1.5 V (2) 3.85 V
(3) 2.35 V (4) 15.4 V

38. Out of a photon and an electron, the equation $E = pc$, is valid for
(1) both (2) neither
(3) photon only (4) electron only

39. When a centimeter thick surface is illuminated with light of wavelength $\lambda$, the stopping potential is $V$. When the same surface is illuminated by light of wavelength $2\lambda$, the stopping potential is $V/3$. Threshold wavelength for the metallic surface is
(1) $4\lambda/3$ (2) $4\lambda$
(3) $6\lambda$ (4) $8\lambda/3$

40. Light of wavelength $\lambda$ strikes a photoelectric surface and electrons are ejected with kinetic energy $K$. If $K$ is to be increased to exactly twice its original value, the wavelength must be changed to $\lambda'$ such that
(1) $\lambda' < \lambda/2$ (2) $\lambda' > \lambda/2$
(3) $\lambda > \lambda' > \lambda/2$ (4) $\lambda' = \lambda/2$

41. The ratio of momenta of an electron and an $\alpha$-particle which are accelerated from rest by a potential difference of 100 V is
(1) 1 (2) $\sqrt{2m_e/m_\alpha}$
(3) $\sqrt{m_e/m_\alpha}$ (4) $\sqrt{m_e/2m_\alpha}$

42. The kinetic energy of an electron is $E$ when the incident wavelength is $\lambda$. To increase the KE of the electron to $2E$, the incident wavelength must be
(1) $2\lambda$
(2) $\lambda/2$
(3) $(hc\lambda)/(E\lambda + hc)$
(4) $(hc)/(E\lambda + hc)$

**43.** The KE of the photoelectrons is $E$ when the incident wavelength is $\lambda/2$. The KE becomes $2E$ when the incident wavelength is $\lambda/3$. The work function of the metal is

(1) $hc/\lambda$ (2) $2hc/\lambda$

(3) $3hc/\lambda$ (4) $hc/3\lambda$

**44.** If $\lambda_0$ stands for mid-wavelength in the visible region, the de Broglie wavelength for 100 V electrons is nearest to

(1) $\lambda_0/5$ (2) $\lambda_0/50$

(3) $\lambda_0/500$ (4) $\lambda_0/5000$

**45.** When a metallic surface is illuminated by a light of frequency $8 \times 10^{14}$ Hz, photoelectron of maximum energy 0.5 eV is emitted. When the same surface is illuminated by light of frequency $12 \times 10^{14}$ Hz, photoelectron of maximum energy 2 eV is emitted. The work function is

(1) 0.5 eV (2) 2.85 eV

(3) 2.5 eV (4) 3.5 eV

**46.** The de Broglie wavelength of neutrons in thermal equilibrium is (given $m_n = 1.6 \times 10^{-27}$ kg)

(1) $30.8/\sqrt{T}$ Å (2) $3.08/\sqrt{T}$ Å

(3) $0.308/\sqrt{T}$ Å (4) $0.0308/\sqrt{T}$ Å

**47.** A particle of mass $M$ at rest decays into two masses $m_1$ and $m_2$ with non-zero velocities. The ratio $\lambda_1/\lambda_2$ of de Broglie wavelengths of the particles is

(1) $m_2/m_1$ (2) $m_1/m_2$

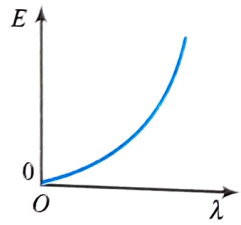
(3) $\sqrt{m_1}/\sqrt{m_2}$ (4) 1 : 1

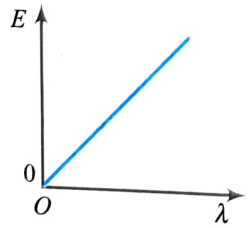
**48.** If $\lambda_1$ and $\lambda_2$ denote the wavelengths of de Broglie waves for electrons in the first and second Bohr orbits in a hydrogen atom, then $\lambda_1/\lambda_2$ is equal to

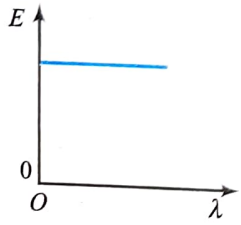
(1) 2/1 (2) 1/2

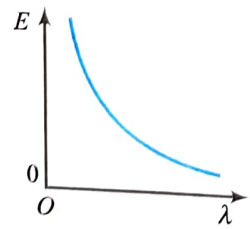
(3) 1/4 (4) 4/1

**49.** Which curve shows the relationship between the energy $E$ and the wavelength $\lambda$ of a photon of electromagnetic radiation?

(1) 


(2) 


(3) 


(4) 


**50.** Work function of nickel is 5.01 eV. When ultraviolet radiation of wavelength 200 Å is incident on it, electons are emitted. What will be the maximum velocity of emitted electrons?

(1) $3 \times 10^8$ m s$^{-1}$ (2) $6.46 \times 10^5$ m s$^{-1}$

(3) $10.36 \times 10^5$ m s$^{-1}$ (4) $8.54 \times 10^6$ m s$^{-1}$

**51.** An electron is accelerated through a potential difference of $V$ volt. It has a wavelength $\lambda$ associated with it. Through what potential difference an electron must be accelerated so that its de Broglie wavelength is the same as that of a proton? Take mass of proton to be 1837 times larger than the mass of electron.

(1) $V$ volt (2) $1837V$ volt

(3) $V/1837$ volt (4) $\sqrt{1837}\,V$ volt

**52.** The kinetic energy of most energetic electrons emitted from a metallic surface is doubled when the wavelength $\lambda$ of the incident radiation is changed from 400 nm to 310 nm. The work function of the metal is

(1) 0.9 eV (2) 1.7 eV

(3) 2.2 eV (4) 3.1 eV

**53.** Two identical metal plates show photoelectric effect. Light of wavelength $\lambda_A$ falls on plate $A$ and $\lambda_B$ falls on plate $B$, $\lambda_A = 2\lambda_B$. The maximum KE of the photoelectrons are $K_A$ and $K_B$, respectively. Which one of the following is true?

(1) $2K_A = K_B$ (2) $K_A = 2K_B$

(3) $K_A < K_B/2$ (4) $K_A > 2K_B$

**54.** The potential difference applied to an X-ray tube is $V$. The ratio of the de Broglie wavelength of electron to the minimum wavelength of X-ray is directly proportional to

(1) $V$ (2) $\sqrt{V}$

(3) $V^{3/2}$ (4) $V^{7/2}$

**55.** The maximum velocity of electrons emitted from a metal surface is $v$. What would be the maximum velocity if the frequency of incident light is increased by a factor of 4?

(1) $2v$ (2) $> 2v$

(3) $< 2v$ (4) between $2v$ and $4v$

**56.** The eye can detect $5 \times 10^4$ photons $(\text{m}^2\,\text{s})^{-1}$ of green light ($\lambda = 5000$ Å), while ear can detect $10^{-13}$ W m$^{-2}$. As a power detector, which is more sensitive and by what factor?

(1) Eye is more sensitive by a factor of 5.00.

(2) Ear is more sensitive by a factor of 5.00.

(3) Both are equally sensitive.

(4) Eye is more sensitive by a factor of $10^{-1}$.

**57.** A photon of wavelength 0.1 Å is emitted by a helium atom as a consequence of the emission of photon. The KE gained by helium atom is

(1) 0.05 eV (2) 1.05 eV

(3) 2.05 eV (4) 3.05 eV

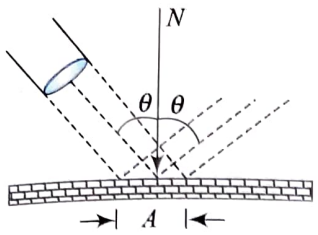
**58.** How many photons of a radiation of wavelength $\lambda = 5 \times 10^{-7}$ m must fall per second on a blackened plate in order to produce a force of $6.62 \times 10^{-5}$ N?

(1) $3 \times 10^{19}$ (2) $5 \times 10^{22}$

(3) $2 \times 10^{22}$ (4) $1.67 \times 10^{18}$

**59.** A nozzle throws a stream of gas against a wall with a velocity $v$ much larger than the thermal agitation of the molecules. The wall deflects the molecules without changing the magnitude of their velocity. Also, assume that the force exerted on the

wall by the molecules is perpendicular to the wall. (This is not strictly true for a rough wall). Find the force exerted on the wall.




(1) $Anmv^2 \cos^2\theta$ (2) $2Anmv^2 \cos^2\theta$

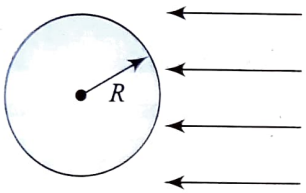
(3) $2Anmv^2 \sin^2\theta$ (4) $Anmv^2 \cos\theta$

60. A plane light wave of intensity $I = 0.20$ W cm$^{-2}$ falls on a plane mirror surface with reflection coefficient $\rho = 0.8$. The angle of incidence is 45°. In terms of corpuscular theory, find the magnitude of the normal pressure exerted on that surface.

(1) 1.2 N cm$^{-2}$ (2) 0.2 N cm$^{-2}$

(3) 2.6 N cm$^{-2}$ (4) 0.5 N cm$^{-2}$

61. A plane wave of intensity $I = 0.70$ W cm$^{-2}$ illuminates a sphere with ideal mirror surface. The radius of sphere is $R = 5.0$ cm. From the standpoint of photon theory, find the force that light exerts on the sphere.




(1) 0.8 μN (2) 0.2 μN

(3) 0.5 μN (4) 1.2 μN

62. Light of wavelength $\lambda$ from a small 0.5 mW He–Ne laser source, used in the school laboratory, shines from a spacecraft of mass 1000 kg. Estimate the time needed for the spacecraft to reach a velocity of 1.0 km s$^{-1}$ from rest. The momentum $p$ of a photon of wavelength $\lambda$ is given by $p = h/\lambda$, where $h$ is Planck's constant.

(1) $6 \times 10^{18}$ (2) $3 \times 10^{17}$

(3) $6 \times 10^{17}$ (4) $2 \times 10^{15}$

63. What is the de Broglie wavelength of the wave associated with an electron that has been accelerated through a potential difference of 50.0 V?

(1) $2.7 \times 10^{-10}$ (2) $1.74 \times 10^{-10}$

(3) $3.6 \times 10^{-9}$ (4) $4.9 \times 10^{-11}$

64. An $\alpha$-particle and a proton are fired through the same magnetic field which is perpendicular to their velocity vectors. The $\alpha$-particle and the proton move such that radius of curvature of their paths is same. Find the ratio of their de Broglie wavelengths.

(1) 2:3 (2) 3:4

(3) 5:7 (4) 1:2

65. Find the ratio of de Broglie wavelength of a proton and an $\alpha$-particle which have been accelerated through same potential difference.

(1) $2\sqrt{2}:1$ (2) 3:2

(3) $3\sqrt{2}:1$ (4) 2:1

66. All electrons ejected from a surface by incident light of wavelength 200 nm can be stopped before traveling 1 m in the direction of a uniform electric field of 4 NC$^{-1}$. The work function of the surface is

(1) 4 eV (2) 6.2 eV

(3) 2 eV (4) 2.2 eV

67. If the short wavelength limit of the continous spectrum coming out of a Coolidge tube is 10 Å, then the de Broglie wavelength of the electrons reaching the target metal in the Coolidge tube is approximately

(1) 0.3 Å (2) 3 Å

(3) 30 Å (4) 10 Å

68. In a photoelectric effect, electrons are emitted

(1) with a maximum velocity proportional to the frequency of the incident radiation

(2) at a rate that is independent of the intensity of the incident radiation

(3) only if the frequency of the incident radiation is above a certain threshold value

(4) only if the temperature of the emitter is high

69. A 60 W bulb is placed at a distance of 4 m from you. The bulb is emitting light of wavelength 600 nm uniformly in all directions. In 0.1 s, how many photons enter your eye if the pupil of the eye is having a diameter of 2 mm? [Take $hc = 1240$ eV-nm]

(1) $2.84 \times 10^{12}$ (2) $2.84 \times 10^{11}$

(3) $9.37 \times 10^{11}$ (4) $6.48 \times 10^{11}$

70. Two identical non-relativistic particles $A$ and $B$ move at right angles to each other, processing de Broglie wavelengths $\lambda_1$ and $\lambda_2$, respectively. The de Broglie wavelength of each particle in their centre of mass frame of reference is

(1) $\lambda_1 + \lambda_2$ (2) $2\lambda_1\lambda_2 \Big/ \left(\sqrt{\lambda_1^2 + \lambda_2^2}\right)$

(3) $\lambda_1\lambda_2 \Big/ \left(\sqrt{|\lambda_1^2 + \lambda_2^2|}\right)$ (4) $(\lambda_1 + \lambda_2)/2$

71. A sodium metal piece is illuminated with light of wavelength 0.3 $\mu$m. The work function of sodium is 2.46 eV. For this situation, mark out the correct statement(s).

(1) The maximum kinetic energy of the ejected photoelectrons is 1.68 eV

(2) The cut-off wavelength for sodium is 505 nm

(3) The minimum photon energy of incident light for photoelectric effect to take place is 2.46 eV

(4) All of the above

72. The kinetic energy of a particle is equal to the energy of a photon. The particle moves at 5% of the speed of light. The ratio of the photon wavelength to the de Broglie wavelength of the particle is

[No need to use relativistic formula for the particle.]

(1) 40 (2) 4

(3) 2 (4) 80

**73.** The resolving power of an electron microscope operated at 16 kV is $R$. The resolving power of the electron microscope when operated at 4 kV is

(1) $R/4$ (2) $R/2$

(3) $4R$ (4) $2R$

**74.** Two electrons are moving with same speed $v$. One electron enters a region of uniform electric field while the other enters a region of uniform magnetic field, then after some time de Broglie wavelengths of two are $\lambda_1$ and $\lambda_2$, respectively. Now,

(1) $\lambda_1 = \lambda_2$ (2) $\lambda_1 > \lambda_2$

(3) $\lambda_1 < \lambda_2$

(4) $\lambda_1$ can be greater than or less than $\lambda_2$

**75.** The energy of a photon is equal to the kinetic energy of a proton. The energy of photon is $E$. Let $\lambda_1$ be the de Broglie wavelength of the proton and $\lambda_2$ be the wavelength of the photon. Then, $\lambda_1/\lambda_2$ is proportional to

(1) $E^0$ (2) $E^{1/2}$

(3) $E^{-1}$ (4) $E^{-2}$

**76.** Up to what potential $V$ can a zinc ball (work function 3.74 eV) removed from other bodies be charged by irradiating it with light of $\lambda = 200$ nm?

(1) 2.5 V (2) 1.8 V

(3) 2.2 V (4) 3 V

**77.** A sensor is exposed for time $t$ to a lamp of power $P$ placed at a distance $l$. The sensor has an opening that is $4d$ in diameter. Assuming all energy of the lamp is given off as light, the number of photons entering the sensor if the wavelength of light is $\lambda$ is

(1) $N = P\lambda d^2 t/hcl^2$ (2) $N = 4P\lambda d^2 t/hcl^2$

(3) $N = P\lambda d^2 t/4hcl^2$ (4) $N = P\lambda d^2 t/16hcl^2$

**78.** A photon has same wavelength as the de Broglie wavelength of electrons. Given $c$ = speed of light, $v$ = speed of electron. Which of the following relation is correct? [Here $E_e$ = kinetic energy of electron, $E_{ph}$ = energy of photon, $p_e$ = momentum of electron and $p_{ph}$ = momentum of photon]

(1) $E_e/E_{ph} = 2c/v$ (2) $E_e/E_{ph} = v/2c$

(3) $p_e/p_{ph} = 2c/v$ (4) $p_e/p_{ph} = c/v$

**79.** A particle of mass $3m$ at rest decays into two particles of masses $m$ and $2m$ having non-zero velocities. The ratio of the de Broglie wavelengths of the particles $(\lambda_1/\lambda_2)$ is

(1) 1/2 (2) 1/4

(3) 2 (4) none of these

**80.** The radius of the second orbit of an electron in hydrogen atom is 2.116 Å. The de Broglie wavelength associated with this electron in this orbit would be

(1) 6.64 Å (2) 1.058 Å

(3) 2.116 Å (3) 13.28 Å

**81.** Radiation of wavelength 546 nm falls on a photo cathode and electrons with maximum kinetic energy of 0.18 eV are emitted. When radiation of wavelength 185 nm falls on the same surface, a (negative) stopping potential of 4.6 V has to be applied to the collector cathode to reduce the photoelectric current to zero. Then, the ratio $h/e$ is

(1) $6.6 \times 10^{-15}$ J s C$^{-1}$ (2) $4.12 \times 10^{-15}$ J s C$^{-1}$

(3) $6.6 \times 10^{-34}$ J s C$^{-1}$ (4) $4.12 \times 10^{-34}$ J s C$^{-1}$

**82.** The human eye is most sensitive to green light of wavelength 505 nm. Experiments have found that when people are kept in a dark room until their eyes adapt to the darkness, a single photon of green light will trigger receptor cells in the rods of the retina. The velocity of typical bacterium of mass $9.5 \times 10^{-12}$ g, if it had absorbed all energy of photon, is nearly

(1) $10^{-6}$ m s$^{-1}$ (2) $10^{-8}$ m s$^{-1}$

(3) $10^{-10}$ m s$^{-1}$ (4) $10^{-13}$ m s$^{-1}$

**83.** The light sensitive compound on most photographic films is silver bromide AgBr. A film is exposed when the light energy absorbed dissociates this molecule into its atoms. The energy of dissociation of AgBr is $10^5$ J mol$^{-1}$. For a photon that is just able to dissociate a molecule of AgBr, the photon energy is

(1) 1.04 eV (2) 2.08 eV

(3) 3.12 eV (4) 4.16 eV

## Multiple Correct Answers Type

**1.** Photoelectric effect supports the quantum nature of light because

(1) there is a minimum frequency of light below which no photoelectrons are emitted

(2) the maximum KE of photoelectrons depends only on the frequency of light and not on its intensity

(3) even when the metal surface is faintly illuminated by light of wavelength less than the threshold wavelength, the photoelectrons leave the surface immediately

(4) electric charge of photoelectrons is quantized

**2.** In a photoelectric experiment, the wavelength of the incident light is decreased from 6000 Å to 4000 Å. While the intensity of radiations remains the same,

(1) the cut-off potential will decrease

(2) the cut-off potential will increase

(3) the photoelectric current will increase

(4) the kinetic energy of the emitted electrons will increase

**3.** When photons of energy $hc/\lambda$ fall on a metal surface, photoelectrons are ejected from it. If the work function of the surface is $h\upsilon_0$, then

(1) maximum kinetic energy of the electron is $[(hc/\lambda) - h\nu_0]$

(2) maximum kinetic energy of the photoelectron is equal to $(hc/\lambda)$

(3) minimum KE of the photoelectron is zero

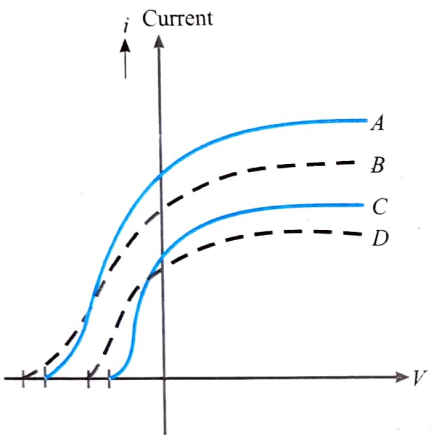
(4) minimum kinetic energy of the photoelectron is equal to $hc/\lambda$

**4.** Threshold wavelength of certain metal is $\lambda_0$. A radiation of wavelength $\lambda < \lambda_0$ is incident on the plate. Then, choose the correct statement from the following.

(1) Initially electrons will come out from the plate
(2) The ejected electrons experience retarding force due to development of positive charges on the plate
(3) After some time, ejection of electrons stops
(4) None of the above

5. Photons of wavelength 248 nm falls on a metal surface whose work function is 2.2 eV. Assume thru each photoelectron inside the metal lattice may come out of the surface or collide with the lattice before coming out. In each collision with the lattice, it loses 20% of its existing energy. Which of the following can be a kinetic energy of an ejected photoelectron?
(1) 2.8 eV (2) 2.24 eV
(3) 1.8 eV (4) 1 eV

6. The figure shows the results of an experiment involving photoelectric effect. The graphs $A$, $B$, $C$ and $D$ relate to a light beam having different wavelengths. Select the correct alternative



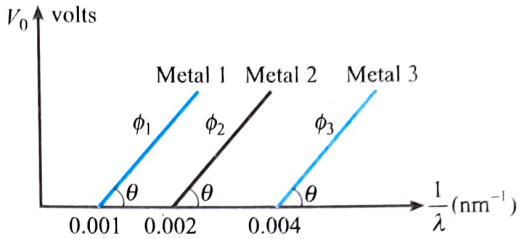

(1) Beam $B$ has highest frequency
(2) Beam $C$ has longest wavelength
(3) Beam $A$ has highest rate of photoelectric emission
(4) Photoelectrons emitted by $B$ have highest momentum

7. The threshold wavelength for photoelectric emission from a material is 5200 Å. Photoelectrons will be emitted when this material is illuminated with monochromatic radiation from a
(1) 50 W infrared lamp
(2) 1 W infrared lamp
(3) 50 W ultraviolet lamp
(4) 1 W ultraviolet lamp

8. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are, respectively, 0.6 V and 18.0 mA. If the same source is placed 0.6 m away from the photoelectric cell, then
(1) the stopping potential will be 0.2 V
(2) the stopping potential will be 0.6 V
(3) the saturation current will be 6.0 mA
(4) the saturation current will be 2.0 mA

9. When photons of energy 4.25 eV strike the surface of metal $A$, the ejected photoelectrons have maximum kinetic energy $T_A$ and de Broglie wavelength $\lambda_A$. The maximum kinetic energy of photoelectrons liberated from another metal $B$ by photons of energy 4.70 eV is $T_B = (T_A \times 1.50)$ eV. If the de Broglie wavelength of these photoelectrons is $\lambda_B = 2\lambda_A$, then
(1) the work function of $A$ is 2.25 eV
(2) the work function of $B$ is 4.20 eV
(3) $T_A = 2.00$ eV
(4) $T_B = 2.75$ eV

10. The graph between the stopping potential ($V_0$) and ($1/\lambda$) is shown in the figure. $\phi_1$, $\phi_2$ and $\phi_3$ are work functions. Then, which of the following is/are correct?




(1) $\phi_1 : \phi_2 : \phi_3 = 1 : 2 : 4$
(2) $\phi_1 : \phi_2 : \phi_3 = 4 : 2 : 1$
(3) $\tan\theta \propto (hc)/e$
(4) Ultraviolet light can be used to emit photoelectrons from metal 2 and metal 3 only

11. A surface has light of wavelength $\lambda_1 = 550$ nm incident on it, causing the ejection of photoelectrons for which the stopping potential is $V_{s_1} = 0.19$ V. If the radiation of wavelength $\lambda_2 = 190$ nm is now incident on the surface,
(1) the stopping potential is $V_{s_2} = 4.47$ V
(2) the stopping potential is $V_{s_2} = 6.54$ V
(3) the work function of the surface is 2.07 eV
(4) the threshold frequency for the surface is $5 \times 10^{14}$ Hz

12. A uniform monochromatic beam of light of wavelength $365 \times 10^{-9}$ m and intensity $10^{-8}$ W m$^{-2}$ falls on a surface having absorption coefficient 0.8 and work function 1.6 eV.
(1) the number of electrons emitted per square meter per second are $1.47 \times 10^{10}$ m$^{-2}$ s$^{-1}$
(2) the number of electrons emitted per square meter per second are $3.75 \times 10^{10}$ m$^{-2}$ s$^{-1}$
(3) power absorbed per m$^2$ is $8 \times 10^{-9}$ Wm$^{-2}$
(4) the maximum kinetic energy of emitted photo electrons is 1.80 eV

13. A parallel beam of monochromatic light of wavelength 500 nm is incident normally on a perfectly absorbing surface. The power through any cross-section of the beam is 10 W.
(1) the number of photons absorbed per second by the surface are $2.52 \times 10^{19}$
(2) the number of photons absorbed per second by the surface are $4.76 \times 10^{19}$
(3) the force exerted by the light beam on the surface is $3.33 \times 10^{-8}$ N
(4) the force exerted by the light beam on the surface is $1.5 \times 10^{-8}$ N

## Linked Comprehension Type

**For Problems 1–3**

Photoelectric threshold of silver is $\lambda = 3800$ Å. Ultraviolet light of $\lambda = 2600$ Å is incident on silver surface. (Mass of the electron $9.11 \times 10^{-31}$ kg)

**1.** Calculate the value of work function in eV.

(1) 1.77 (2) 3.27

(3) 5.69 (4) 2.32

**2.** Calculate the maximum kinetic energy (in eV) of the emitted photoelectrons.

(1) 1.51 (2) 2.36

(3) 3.85 (4) 4.27

**3.** Calculate the maximum velocity (in $ms^{-1}$) of the photoelectrons.

(1) $72.89 \times 10^8$ (2) $57.89 \times 10^8$

(3) $42.93 \times 10^8$ (4) $68.26 \times 10^8$

**For Problems 4–6**

A 100 W point source emits monochromatic light of wavelength 6000 Å.

**4.** Calculate the total number of photons emitted by the source per second.

(1) $5 \times 10^{20}$ (2) $8 \times 10^{20}$

(3) $6 \times 10^{21}$ (4) $3 \times 10^{20}$

**5.** Calculate the photon flux (in SI unit) at a distance of 5 m from the source. Given $h = 6.6 \times 10^{34}$ J s and $c = 3 \times 10^8$ m $s^{-1}$.

(1) $10^{15}$ (2) $10^{18}$

(3) $10^{20}$ (4) $10^{22}$

**6.** 1.5 mW of 4000 Å light is directed at a photoelectric cell. If 0.10 per cent of the incident photons produce photoelectrons, find current in the cell. [Given $h = 6.6 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m $s^{-1}$ and $e = 1.6 \times 10^{-19}$ C]

(1) 0.59 μA (2) 1.16 μA

(3) 0.48 μA (4) 0.79 μA

**For Problems 7–8**

A metallic surface is illuminated alternatively with lights of wavelengths 3000 Å and 6000 Å. It is observed that the maximum speeds of the photoelectrons under these illuminations are in the ratio 3:1.

**7.** The work function of the metal is

(1) 1.45 eV (2) 2.26 eV

(3) 1.23 eV (4) 3.4 eV

**8.** What will be the maximum speed of the photoelectrons so emitted?

(1) $9 \times 10^5$ m $s^{-1}$ (2) $9 \times 10^7$ m $s^{-1}$

(3) $3.6 \times 10^5$ m $s^{-1}$ (4) $4.5 \times 10^7$ m $s^{-1}$

**For Problems 9–10**

A helium–neon laser has a power output of 1 mW of light of wavelength 632.8 nm.

**9.** Calculate the energy of each photon in electron volt.

(1) 2.5 (2) 1.96

(3) 0.53 (4) 3.3

**10.** Determine the number of photons emitted by the laser each second.

(1) $3.18 \times 10^{15}$ (2) $4.5 \times 10^{16}$

(3) $1.2 \times 10^{15}$ (4) $2.9 \times 10^{17}$

**For Problems 11–13**

Photoelectrons are ejected from a surface when light of wavelength $\lambda_1 = 550$ nm is incident on it. The stopping potential for such electrons is $V_{s1} = 0.19$ V. Suppose that radiation of wavelength $\lambda_2 = 190$ nm is incident on the surface.

**11.** Calculate the stopping potential $V_{s2}$.

(1) 4.47 (2) 3.16

(3) 2.76 (4) 5.28

**12.** Calculate the work function of the surface.

(1) 3.75 (2) 2.07

(3) 4.20 (4) 3.60

**13.** Calculate the threshold frequency for the surface.

(1) $500 \times 10^{12}$ Hz (2) $480 \times 10^{13}$ Hz

(3) $520 \times 10^{11}$ Hz (4) $460 \times 10^{13}$ Hz

**For Problems 14–15**

In a photoelectric effect experiment, a metallic surface of work function 2.2 eV is illuminated with a light of wavelength 400 nm. Assume that an electron makes two collisions before being emitted and in each collision 10% additional energy is lost.

**14.** Find the kinetic energy of this electron as it comes out of the metal.

(1) 0.46 eV (2) 0.31 eV

(3) 0.23 eV (4) none of these

**15.** After how many collisions will the electron be unable to come out of the metal?

(1) 2 (2) 6

(3) 4 (4) 8

**For Problems 16–18**

In a photoelectric setup, a point source of light of power $3.2 \times 10^{-3}$ W emits monoenergetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the center of a stationary metallic sphere of work function 3.0 eV and of radius $8.0 \times 10^{-3}$ m. The efficiency of photoelectron emission is 1 for every $10^6$ incident photons. Assume that the sphere is isolated and initially neutral and that photoelectrons are instantaneously swept away after emission.

**16.** Calculate the number of photoelectrons emitted per second.

(1) $10^3$ (2) $10^4$

(3) $5 \times 10^4$ (4) $10^5$

**17.** It is observed that photoelectron emission stops at a certain time $t$ after the light source is switched on. It is due to the retarding potential developed in the metallic sphere due to left over positive charges. The stopping potential ($V$) can be represented as

(1) $2(K_{max}/e)$ (2) $(K_{max}/e)$
(3) $(K_{max}/3e)$ (4) $(K_{max}/2e)$

18. Evaluate time $t$ (in minutes).
(1) 1.85 (2) 2.36
(3) 2.75 (4) 0.78

**For Problems 19–20**

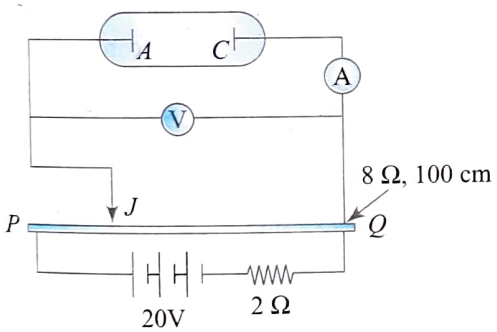
The incident intensity on a horizontal surface at sea level from the sun is about 1 kW $m^{-2}$.

19. Assuming that 50 per cent of this intensity is reflected and 50 per cent is absorbed, determine the radiation pressure on this horizontal surface (in pascals).
(1) $8.2 \times 10^{-2}$ (2) $5 \times 10^{-6}$
(3) $3 \times 10^{-5}$ (4) $6 \times 10^{-5}$

20. Find the ratio of this pressure to atmospheric pressure $p_0$ (about $1 \times 10^5$ Pa) at sea level.
(1) $5 \times 10^{-11}$ (2) $4 \times 10^{-8}$
(3) $6 \times 10^{-12}$ (4) $8 \times 10^{-11}$

**For Problems 21–25**

An experimental setup of verification of photoelectric effect is shown in figure. The voltage across the electrodes is measured with the help of an ideal voltmeter, and which can be varied by moving jockey $J$ on the potentiometer wire. The battery used in potentiometer circuit is of 20 V and its internal resistance is 2 Ω. The resistance of 100 cm long potentiometer wire is 8 Ω.




The photocurrent is measured with the help of an ideal ammeter. Two plates of potassium oxide of area 50 $cm^2$ at separation 0.5 mm are used in the vacuum tube. Photocurrent in the circuit is very small, so we can treat the potentiometer circuit as an independent circuit.

The wavelength of various colors is as follows:

| Light | Violet | Blue | Green | Yellow | Orange | Red |
|---|---|---|---|---|---|---|
| λ in Å | 4000 to 5000 | 4500 to 5000 | 5000 to 5500 | 5500 to 6000 | 6000 to 6500 | 6500 to 7000 |

21. Calculate the number of electrons that appear on the surface of the cathode plate, when the jockey is connected at the end $P$ of the potentiometer wire. Assume that no radiation is falling on the plates.
(1) $8.85 \times 10^6$ (2) $11.0625 \times 10^9$
(3) $8.85 \times 10^9$ (4) 0

22. When radiation falls on the cathode plate, a current of 2 μA is recorded in the ammeter. Assuming that the vacuum tube setup follows Ohm's law, the equivalent resistance of vacuum tube operating in this case when jockey is at end $P$ is
(1) $8 \times 10^8$ Ω (2) $16 \times 10^6$ Ω
(3) $8 \times 10^6$ Ω (4) $10 \times 10^6$ Ω

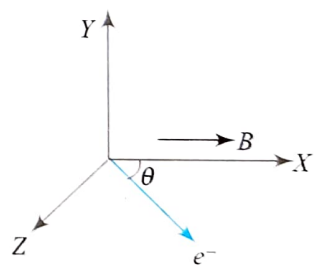
23. It is found that ammeter current remains unchanged (2 μA) even when the jockey is moved from the end $P$ to the middle point of the potentiometer wire. Assuming that all the incident photons eject electrons and the power of the light incident is $4 \times 10^{-6}$ W. Then, the color of the incident light is
(1) green (2) violet
(3) red (4) orange

24. Which of the following colors may not give photoelectric effect for this cathode?
(1) green (2) violet
(3) red (4) orange

25. When other light falls on the anode plate, the ammeter reading remains zero till jockey is moved from the end $P$ to the middle point of the wire $PQ$. Thereafter, the deflection is recorded in the ammeter. The maximum kinetic energy of the emitted electron is
(1) 16 eV (2) 8 eV
(3) 4 eV (4) 10 eV

**For Problems 26–28**

Light having photon energy $h\nu$ is incident on a metallic plate having work function $\phi$ to eject the electrons. The most energetic electrons are then allowed to enter in a region of uniform magnetic field $B$ as shown in figure. The electrons are projected in $X$-$Z$ plane making an angle $\theta$ with $X$-axis and magnetic field is $\vec{B} = B_0\hat{i}$ along $X$-axis.




Maximum pitch of the helix described by an electron is found to be $p$. Take mass of electron as $m$ and charge as $q$.

Based on the above information, answer the following questions:

26. The correct relation between $p$ and $B_0$ is
(1) $qpB_0 = 2\pi \cos\theta\sqrt{2(h\nu - \phi)m}$
(2) $qpB_0 = 2\pi \cos\theta\sqrt{\dfrac{2(h\nu - \phi)}{m}}$
(3) $pqB_0 = 2\pi\sqrt{2(h\nu - \phi)m}$
(4) $p = \dfrac{2\pi m}{qB_0} \times \sqrt{h\nu - \phi}$

27. Considering the instant of crossing origin at $t = 0$, the $Z$-coordinate of the location of electron as a function of time is

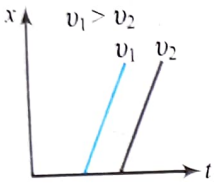
(1) $-\frac{\sqrt{2m(h\nu-\phi)}}{qB_0}\times\sin\theta\left[1-\cos\left(\frac{qB_0t}{m}\right)\right]$

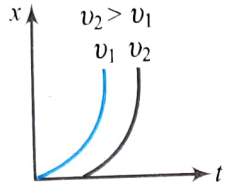
(2) $\frac{\sqrt{2m(h\nu-\phi)}}{qB_0}\times\sin\theta\times\sin\left[\frac{qB_0t}{m}\right]$

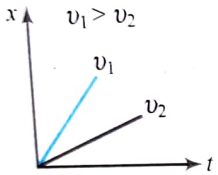
(3) $\frac{-\sqrt{2m(h\nu-\phi)}}{qB_0}\times\sin\theta\times\sin\left[\frac{qB_0t}{m}\right]$

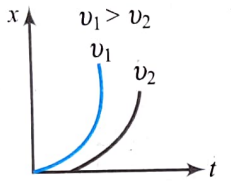
(4) $\frac{\sqrt{2m(h\nu-\phi)}}{qB_0}\times\sin\left(\frac{qB_0t}{m}\right)$

**28.** The plot between $X$-coordinate of the location of electron as a function of time for different frequencies $\upsilon$ of the incident light is

(1) 


(2) 


(3) 


(4) 


**For Problems 29–31**

When a high frequency electromagnetic radiation is incident on a metallic surface, electrons are emitted from the surface. Energy of emitted photoelectrons depends only on the frequency of incident electromagnetic radiation and the number of emitted electrons depends only on the intensity of incident light.

Einstein's photoelectric equation [$K_{max} = h\nu - \phi$] correctly explains the PE, where $\upsilon$ = frequency of incident light and $\phi$ = work function.

**29.** Light of wavelength 3300 is incident on two metals $A$ and $B$, whose work functions are 4 eV and 2 eV, respectively. Then

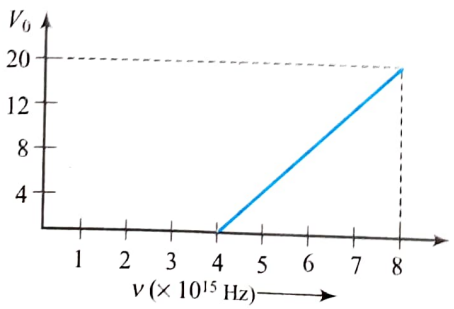
(1) $A$ will emit photoelectrons but $B$ will not

(2) $B$ will emit photoelectrons, but $A$ will not

(3) both $A$ and $B$ will not emit photoelectrons

(4) neither $A$ nor $B$ will emit photoelectrons

**30.** For photoelectric effect in a metal, the graph of the stopping potential $V_0$ (in volt) versus frequency $\upsilon$ (in hertz) of the incident radiation is shown in figure. The work function of the metal (in eV) is




(1) 12.5 (2) 14.5

(3) 16.5 (4) 18.5

**31.** The slope of the graph shown in figure [here $h$ is the Planck's constant and $e$ is the amount of charge on an electron] is

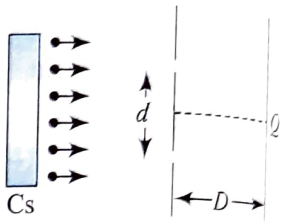
(1) $\frac{h}{e}$ (2) $eh$

(3) $h$ (4) $\frac{e}{h}$

**For Problems 32–34**

A Cs plate is irradiated with a light of wavelength $\lambda = hc/\phi$, $\phi$ being the work function of the plate, $h$ Planck's constant, and $c$ the velocity of light in vacuum.

Assume all the photoelectrons are moving perpendicular to the plate toward a YDSE setup when accelerated through a potential difference $V$. Take charge on a proton = $e$ and mass of an electron = $m$.




**32.** The fringe width due to the electron beam is

(1) $\lambda D/d$ (2) $\lambda D/2d$

(3) $hD/(d\sqrt{2emV})$ (4) none of these

**33.** If the wavelength of light used in photoemission is less than $\lambda$, then the fringe width will

(1) increase (2) decrease

(3) remain same (4) cannot be decided

**34.** Instead of moving perpendicular to the plate, if the electrons were moving randomly, then the central maximum would shift

(1) upward

(2) downward

(3) no shift

(4) no fringes will be formed

**For Problems 35–37**

A pushed dye laser emits light of wavelength 585 nm. Because this wavelength is strongly absorbed by the haemoglobin in the blood, the method is especially effective for removing various types of blemishes due to blood. To get a reasonable estimate of the power required for such laser surgery, we can model the blood as having the same specific heat and heat of vaporization as water. [$S = 4.2 \times 10^3$ J (kg K)$^{-1}$, $L = 2.25 \times 10^6$ J kg$^{-1}$]

**35.** Suppose that each pulse must remove 2 μg of blood by evaporating it starting at 30°C. The energy that each pulse must deliver to the blemish is nearly

(1) 5.1 J (2) 5.1 mJ

(3) 5.1 μJ (4) 5.1 kJ

36. The power output of the laser must be
(1) 5.5 W (2) 11 W
(3) 16.5 W (4) 22 W

37. The number of photons that each pulse delivers to the blemish is
(1) $1.5 \times 10^{16}$ (2) $1.5 \times 10^{8}$
(3) $3 \times 10^{16}$ (4) $3 \times 10^{8}$

**For Problems 38–39**

Some phenomenon of light can be explained by wave theory and some other phenomenon show the particle nature of light. de-Broglie proposed a hypothesis based on the dual nature of light. According to de-Broglie each moving particle has a wave associated with it. This wave is known as de-Broglie wave or matter wave and its wavelength is given by $\lambda = \frac{h}{p}$. Actually this expression correlate wave and particle nature. Wavelength ($\lambda$) represents the wave nature and momentum ($p$) represents the particle nature.

38. Proton, deuteron and $\alpha$ particles are accelerated through the same potential difference, then the ratio of their wavelength is
(1) $1:\sqrt{2}:1$ (2) 1:1:1
(3) $1:2:2\sqrt{2}$ (4) $2\sqrt{2}:2:1$

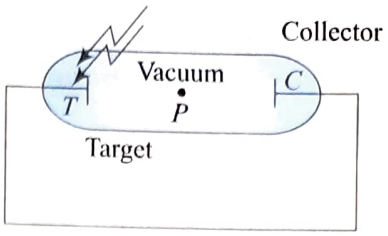
39. When electron accelerated through the 150 volt potential difference. The wavelength associated with it
(1) 1 Å (2) 2 Å
(3) 1.2 Å (4) 4 Å

## Matrix Match Type

1. With respect to photoelectric effect experiment, match the entries of Column I with the entries of Column II.

| Column I | Column II |
|---|---|
| i. If $f$ (frequency) is increased keeping $I$ (intensity) and $\phi$ (work function) constant | a. stopping potential increases |
| ii. If $I$ is increased keeping $f$ and $\phi$ constant | b. saturation photocurrent increases |
| iii. If the distance between anode and cathode increases | c. maximum KE of the photoelectrons increases |
| iv. If $\phi$ is decreased keeping $f$ and $I$ constant. | d. stopping potential remains the same |

2. In a photoelectric experimental arrangement, light of frequency $f$ is incident on a metal target whose work function is $\phi = hf/3$ as shown. In Column I, KE of photoelectron is mentioned at various locations/instants and in Column II, the corresponding values. Match the entries of Column I with the entries of Column II.

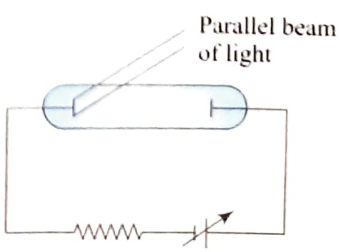



| Column I | Column II |
|---|---|
| i. Maximum KE of photoelectrons just after emission from target | a. zero |
| ii. KE of photoelectrons just after emission from target | b. $hf/3$ |
| iii. KE of photoelectron when they are halfway between the target and collector | c. $hf/2$ |
| iv. KE of photoelectrons as they reach the collector | d. $2hf/3$ |

3. Related to photoelectric effect, in Column I, some physical quantities change while in Column II effects of these changes are given. Match the entries of Column I with the entries of Column II.

| Column I | Column II |
|---|---|
| i. Intensity of incident light changes | a. $K_{max}$ of emitted photo-electrons changes |
| ii. Frequency of incident light changes | b. Stopping potential changes |
| iii. Target material changes | c. Saturation current changes |
| iv. Potential difference between the emitter and collector changes | d. Time delay in emission of photoelectrons changes |

4. In the shown experimental setup to study photoelectric effect, two conducting electrodes are enclosed in an evacuated glass-tube as shown. A parallel beam of monochromatic light falls on photosensitive electrode. The emf of battery shown is high enough such that all photoelectrons ejected from left electrode will reach the right electrode. Under initial conditions, photoelectrons are emitted. As changes are made in each situation of Column I. Match the statement in Column I with results in Column II.



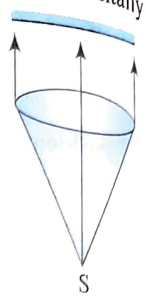
| Column I | Column II |
|---|---|
| i. If frequency of incident light is increased keeping its intensity constant | a. magnitude of stopping potential will increase |
| ii. If frequency of incident light is increased and its intensity is decreased | b. current through the circuit may stop |
| iii. If work function of photon sensitive electrode is increased | c. maximum kinetic energy of ejected photoelectrons will increase |
| iv. If intensity of incident light is increased keeping its frequency constant | d. saturation current will increase |

5. In Column I, the nature of light is given and in Column II the information about the photons are mentioned. Match the entries of Column I with the entries of Column II.

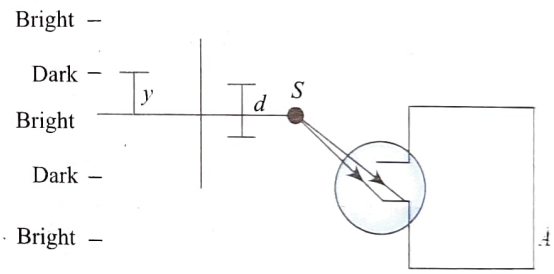
| Column I | Column II |
|---|---|
| i. A bichromatic light source | a. Few photons have same energy and momenta |
| ii. A point source of white light emitting light uniformly in all directions | b. Few photons have different energy and different momenta |
| iii. A point source of monochromatic light emitting light uniformly in all directions | c. Few photons have same energy and different momenta |
| iv. Laser light source | d. Few photons have different energy and same momenta |

## Numerical Value Type

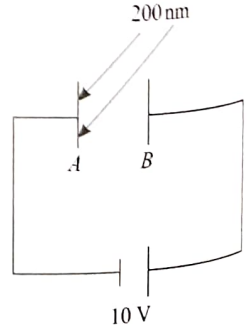
1. The radius of an $\alpha$-particle moving in a circle in a constant magnetic field is half of the radius of an electron moving in circular path in the same field. The de-Broglie wavelength of $\alpha$-particle is $n$ times that of the electron. Find $n$ (an integer).
2. The de-Broglie wavelength of an electron moving with a velocity of $1.5 \times 10^6$ m s$^{-1}$ is equal to that of a photon. Find the ratio of the kinetic energy of the photon to that of the electron.
3. An element of atomic number 9 emits $K_\alpha$ X-ray of wavelength $\lambda$. Find the atomic number of the element which emits $K_\alpha$ X-ray of wavelength $4\lambda$.
4. A monochromatic source of light operating at 200 W emits $4 \times 10^{20}$ photons per second. Find the wavelength of the light (in $\times 10^{-7}$m).
5. A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is $1.0 \times 10^{19}$. Calculate the force exerted by light beam on the mirror (in $10^{-8}$N).
6. A totally reflecting, small plane mirror placed horizontally faces a parallel beam of light as shown in the figure. The mass of the mirror is 20 g. Assume that there is no absorption in the lens and that 30% of the light emitted by the source goes through the lens. Find the power (in $\times 10^8$ W) of the source needed to support the weight of the mirror. Take $g = 10$ m s$^{-2}$.



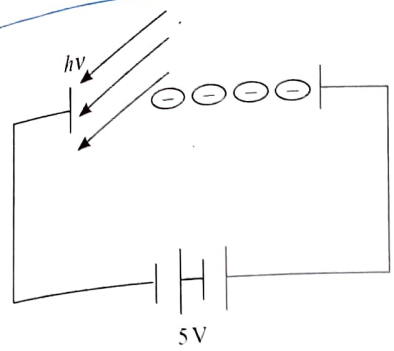

7. A silver ball of radius 4.8 cm is suspended by a thread in a vacuum chamber. Ultraviolet light of wavelength 200 nm is incident on the ball for some time during which a total light energy of $1.0 \times 10^{-7}$ J falls on the surface. Assuming that on the average, one photon out of ten thousand photons is able to eject a photoelectron, find the electric potential (in $\times 10^{-1}$V) at the surface of the ball assuming zero potential at infinity.
8. In the arrangement shown in figure, $y = 1.0$ mm, $d = 0.24$ mm and $D = 1.2$ m. The work function of the material of the emitter is 2.2 eV. Find the stopping potential $V$ needed to stop the photocurrent (in $\times 10^{-1}$V).




9. An electron of mass $m$ when accelerated through a potential difference $V$, has de Broglie wavelength $\lambda$. The de-Broglie wavelength associated with a particle of mass $m/4$ and charge $e$ accelerated through the same potential difference is $n\lambda$. Then find the value of $n$.
10. In figure, electromagnetic radiations of wavelength 200 nm are incident on a metallic plate $A$. The photoelectrons are accelerated by a potential difference of 10 V. These electrons strike another metal plate $B$ from which electromagnetic radiations are emitted. The minimum wavelength of emitted photons is 100 nm. If the work function of metal $A$ is found to be $(x \times 10^{-1})$ eV, then find the value of $x$. (Given $hc = 1240$ eV nm)




11. Photons of energy 5 eV are incident on the cathode. Electrons reaching the anode have kinetic energies varying from 6 eV to 8 eV Find the work function of the metal (in eV).




12. A plane light wave of intensity $I = 0.20$ W cm$^{-2}$ falls on a plane mirror surface with reflection coefficient $\rho = 0.8$. The angle of incidence is 45°. In terms of corpuscular theory, find the magnitude of the normal pressure exerted on that surface (in N cm$^{-2}$).
13. Light of wavelength 180 nm ejects photoelectrons from a plate of a metal whose work function is 2 eV. If a uniform magnetic field of $5 \times 10^{-5}$ T is applied parallel to the plate, what would be the radius of the path (in m) followed by electrons ejected normally from the plate with maximum energy.
14. Calculate de-Broglie wavelength (in Å) for an average helium atom in a furnace at 400 K. Given mass of helium = 4.002 amu.
15. What amount of energy (in eV) should be added to an electron to reduce its de-Broglie wavelength from 100 to 50 pm?
16. A cylindrical rod of some laser material $5 \times 10^{-2}$ m long and $10^{-2}$ m in diameter contains $2 \times 10^{25}$ ions per m$^3$. If on excitation all the ions are in the upper energy level and de-excite simultaneously emitting photons in the same direction, calculate the maximum energy contained in a pulse of radiation of wavelength $6.6 \times 10^{-7}$ m. If the pulse lasts for $10^{-7}$ s, calculate the average power (in W) of the laser during the pulse.

# Archives

## JEE MAIN

### Single Correct Answer Type

1. The surface of a metal is illuminated with a light of wavelength 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is ($hc = 1240$ eV nm)
(1) 3.09 eV (2) 1.41 eV
(3) 1.51 eV (4) 1.68 eV
**(AIEEE 2009)**

2. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements
**Statement 1:** When ultraviolet light is incident on a photocell, its stopping potential is $V_0$ and the maximum kinetic energy of the photoelectrons is $K_{max}$. When the ultraviolet light is replaced by X-rays, both $V_0$ and $K_{max}$ increase.
**Statement 2:** Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.
(1) Statement 1 is true, statement 2 is true; statement 2 is the correct explanation for statement 1.
(2) Statement 1 is true, statement 2 is true; statement 2 is not the correct explanation for statement 1.
(3) Statement 1 is false but statement 2 is true.
(4) Statement 1 is true but statement 2 is false.
**(AIEEE 2010)**

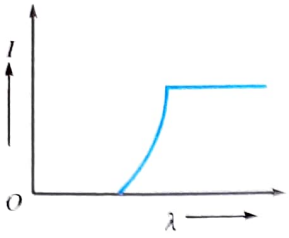
3. If a source of power 4 kW produces $10^{20}$ photons per second, the radiation belongs to a part of the spectrum called
(1) X-rays (2) ultraviolet rays
(3) microwaves (4) $\gamma$-rays **(AIEEE 2010)**

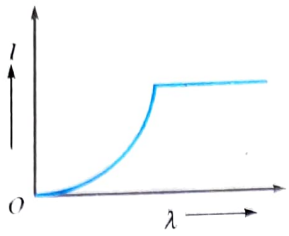
4. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = (a/x^{12}) - (b/x^6)$, where $a$ and $b$ are constants and $x$ is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, $D$ is
(1) $\frac{b^2}{2a}$ (2) $\frac{b^2}{12a}$
(3) $\frac{b^2}{4a}$ (4) $\frac{b^2}{6a}$ **(AIEEE 2010)**

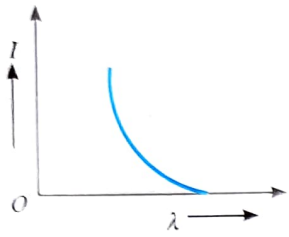
5. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements
**Statement 1:** A metallic surface is irradiated by a monochromatic light of frequency $f > f_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are $K_{max}$ and $V_0$ respectively. If the frequency incident on the surface is doubled, both the $K_{max}$ and $V_0$ are also doubled.
**Statement 2:** The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.
(1) Statement 1 is true but statement 2 is false.
(2) Statement 1 is true, statement 2 is true, statement 2 is the correct explanation of statement 1.
(3) Statement 1 is true, statement 2 is true, statement 2 is not the correct explanation of statement 1.
(4) Statement 1 is false but statement 2 is true.
**(AIEEE 2011)**

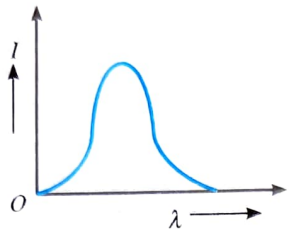
6. **Statement 1:** Davisson–Germer experiment established the wave nature of electrons.
**Statement 2:** If electrons have wave nature, they can interfere and show diffraction.
(1) Statement 1 is false but statement 2 is true.
(2) Statement 1 is true but statement 2 is false
(3) Statement 1 is true, statement 2 is true, statement 2 is the correct explanation for statement 1
(4) Statement 1 is true, statement 2 is true, statement 2 is not the correct explanation of statement 1
**(AIEEE 2012)**

7. The anode voltage of a photocell is kept fixed. The wavelength $\lambda$ of the light falling on the cathode is gradually changed. The plate current $I$ of the photocell varies as follows:

(1) 


(2) 


(3) 


(4) 


**(JEE Main 2013)**

8. Match List I (Fundamental Experiment) with List II (its conclusion) and select the correct option from the choices given below the list:

| List I | | List II | |
|---|---|---|---|
| (p) | Franck–Hertz experiment | (i) | Particle nature of light |
| (q) | Photo electric experiment | (ii) | Discrete energy levels of atom |
| (r) | Davison–Germer experiment | (iii) | Wave nature of electron |
| | | (iv) | Structure of atom |

(1) (p) - (i), (q) - (iv), (r) - (iii)
(2) (p) - (ii), (q) - (iv), (r) - (iii)
(3) (p) - (ii), (q) - (i), (r) - (iii)
(4) (p) - (iv), (q) - (iii), (r) - (ii) **(JEE Main 2015)**

9. Radiation of wavelength $\lambda$, is incident on a photocell. The fastest emitted electron has speed $v$. If the wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be

(1) $> v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (2) $< v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

(3) $= v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (4) $= v\left(\frac{3}{4}\right)^{\frac{1}{2}}$

**(JEE Main 2016)**

10. A particle $A$ of mass $m$ and initial velocity $v$ collides with a particle $B$ of mass $m/2$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths $\lambda_A$ to $\lambda_B$ after the collision is

(1) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (2) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

(3) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (4) $\frac{\lambda_A}{\lambda_B} = 2$

**(JEE Main 2017)**

## JEE ADVANCED

### Single Correct Answer Type

1. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is $3 \times 10^8$ m/s. The final momentum of the object is

(1) $0.3 \times 10^{-17}$ kg ms$^{-1}$ (2) $1.0 \times 10^{-17}$ kg ms$^{-1}$

(3) $0.2 \times 10^{-17}$ kg ms$^{-1}$ (4) $9.0 \times 10^{-17}$ kg ms$^{-1}$

**(JEE Advanced 2013)**

2. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are $u_1$ and $u_2$, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and $hc = 1240$ eV nm, the work function of the metal is nearly

(1) 3.7 eV (2) 3.2 eV

(3) 2.8 eV (4) 2.5 eV

**(JEE Advanced 2014)**

3. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength ($\lambda$) of incident light and the corresponding stopping potential ($V_0$) are given below:

| $\lambda$ (μm) | $V_0$ (Volt) |
|---|---|
| 0.3 | 2.0 |
| 0.4 | 1.0 |
| 0.5 | 0.4 |

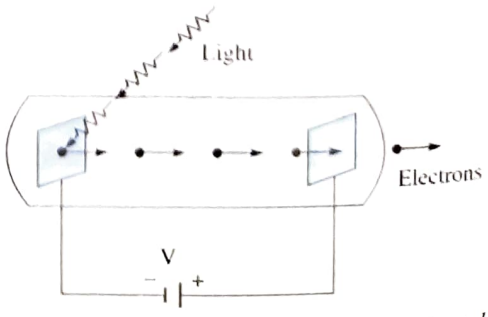
Given that $c = 3 \times 10^8$ ms$^{-1}$ and $e = 1.6 \times 10^{-19}$ C. Planck's constant (in units of J s) found from such an experiment is

(1) $6.0 \times 10^{-34}$ (2) $6.4 \times 10^{-34}$

(3) $6.6 \times 10^{-34}$ (4) $6.8 \times 10^{-34}$

**(JEE Advanced 2016)**

4. Light of wavelength $\lambda_{ph}$ falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is $\phi$ and the anode is a wire mesh of conducting material kept at a distance $d$ from the cathode. A potential difference $V$ is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is $\lambda_e$, which of the following statement(s) is (are) true?




(1) $\lambda_e$ increases at the same rate as $\lambda_{ph}$ for $\lambda_{ph} < hc/\phi$.
(2) For large potential difference ($V >> \phi/e$), $\lambda_e$ is approximately halved if $V$ is made four times

(3) $\lambda_e$ is approximately halved, if $d$ is doubled

(4) $\lambda_e$ decreases with increase in $\phi$ and $\lambda_{ph}$

**(JEE Advanced 2016)**

5. A photoelectric material having work–function $\phi_0$ is illuminated with light of wavelength $\lambda\left(\lambda < \frac{hc}{\phi_0}\right)$.

The fastest photoelectron has a de-Broglie wavelength $\lambda_d$. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in $\lambda_d$. Then the ratio $\frac{\Delta\lambda_d}{\Delta\lambda}$ is proportional to

(1) $\frac{\lambda_d^3}{\lambda^2}$ (2) $\frac{\lambda_d^3}{\lambda}$

(3) $\frac{\lambda_d^2}{\lambda^2}$ (4) $\frac{\lambda_d}{\lambda}$

**(JEE Advanced 2017)**

## Linked Comprehension Type

**For Problems 1–3**

When a particle is restricted to move along $x$-axis between $x = 0$ and $x = a$, where $a$ is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum $p$ of the particle according to the de Broglie relation. The energy of the particle of mass $m$ is related to its linear momentum as $E = p^2/2m$. Thus, the energy of the particle can be denoted by a quantum number '$n$' taking values 1, 2, 3, … ($n = 1$, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving along the line from $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ J s and $e = 1.6 \times 10^{-19}$ C.

**(IIT-JEE 2009)**

1. The allowed energy for the particle for a particular value of $n$ is proportional to

(1) $a^{-2}$ (2) $a^{-3/2}$

(3) $a^{-1}$ (4) $a^2$

2. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to

(1) 0.8 meV (2) 8 meV

(3) 80 meV (4) 800 meV

3. The speed of the particle that can take discrete values is proportional to

(1) $n^{-3/2}$ (2) $n^{-1}$

(3) $n^{1/2}$ (4) $n$

## Numerical Value Type

1. An $\alpha$-particle and a proton are accelerated from rest by a potential difference of 100 V. After this, their de Broglie wavelengths are $\lambda_\alpha$ and $\lambda_p$, respectively. The ratio $\lambda_p/\lambda_\alpha$, to the nearest integer, is

**(IIT-JEE 2010)**

2. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (where $1 < A < 10$). The value of "$Z$" is **(IIT-JEE 2011)**

3. A proton is fired from very far away towards a nucleus with charge $Q = 120e$, where $e$ is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de-Broglie wavelength (in units of fm) of the proton at its start is (take the proton mass, mp = (5/3) × $10^{-27}$ kg; $h/e = 4.2 \times 10^{-15}$ Js/C; $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ m/F; 1fm = $10^{-15}$ m) **(IIT-JEE 2012)**

4. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is **(JEE Advanced 2013)**

5. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy.

Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of $n$ is.....

Mass of the electron $m_e = 9 \times 10^{-31}$ kg and

1.0 eV = $1.6 \times 10^{-19}$ J.

**(JEE Advanced 2018)**

# Answers Key

## EXERCISES

### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| 1. (1) | 2. (2) | 3. (3) | 4. (3) | 5. (2) |
| 6. (3) | 7. (4) | 8. (2) | 9. (4) | 10. (3) |
| 11. (1) | 12. (3) | 13. (1) | 14. (2) | 15. (2) |
| 16. (3) | 17. (4) | 18. (2) | 19. (2) | 20. (3) |
| 21. (1) | 22. (2) | 23. (3) | 24. (3) | 25. (4) |
| 26. (3) | 27. (1) | 28. (1) | 29. (3) | 30. (4) |
| 31. (4) | 32. (3) | 33. (3) | 34. (4) | 35. (2) |
| 36. (2) | 37. (1) | 38. (3) | 39. (2) | 40. (3) |
| 41. (4) | 42. (3) | 43. (1) | 44. (4) | 45. (3) |
| 46. (1) | 47. (4) | 48. (2) | 49. (4) | 50. (2) |
| 51. (3) | 52. (3) | 53. (3) | 54. (2) | 55. (2) |
| 56. (1) | 57. (3) | 58. (2) | 59. (1) | 60. (4) |
| 61. (1) | 62. (3) | 63. (2) | 64. (4) | 65. (1) |

66. (4) 67. (1) 68. (3) 69. (2) 70. (2)
71. (4) 72. (1) 73. (2) 74. (4) 75. (2)
76. (1) 77. (1) 78. (2) 79. (4) 80. (1)
81. (2) 82. (4) 83. (1)

### Multiple Correct Answers Type

1. (1),(2),(3) 2. (2),(4) 3. (1),(3)
4. (1),(2) 5. (1),(3),(4) 6. (1),(2),(3),(4)
7. (3),(4) 8. (2),(4) 9. (1),(2),(3)
10. (1),(3) 11. (1),(3),(4) 12. (1),(3),(4)
13. (1),(3)

### Linked Comprehension Type

1. (2) 2. (1) 3. (1) 4. (4) 5. (2)
6. (3) 7. (3) 8. (1) 9. (2) 10. (1)
11. (1) 12. (2) 13. (1) 14. (4) 15. (3)
16. (4) 17. (2) 18. (1) 19. (2) 20. (1)
21. (3) 22. (3) 23. (4) 24. (3) 25. (2)
26. (1) 27. (2) 28. (3) 29. (2) 30. (3)
31. (1) 32. (3) 33. (2) 34. (3) 35. (2)
36. (2) 37. (1) 38. (4) 39. (1)

### Matrix Match Type

1. i. → a., c.; ii. → b., d.; iii. → d.; iv. → a., c.
2. i. → d.; ii. → a., b., c., d.; iii. → b., c., d.; iv. → a., c., d.
3. i. → c.; ii. → a., b.; iii. → a., b.; iv. → d.
4. i. → a., c.; ii. → a., c.; iii. → b.; iv. → d.
5. i. → a., b., c., d.; ii. → a., b., c., d.; iii. → a., c.; iv. → a.

### Numerical Value Type

1. (1) 2. (4) 3. (5) 4. (4) 5. (1)
6. (1) 7. (3) 8. (9) 9. (2) 10. (38)
11. (2.00) 12. (0.50) 13. (0.148) 14. (0.63) 15. (450)
16. (23.55)

## ARCHIVES

### JEE Main

#### Single Correct Answer Type

1. (2) 2. (4) 3. (1) 4. (3) 5. (4)
6. (3) 7. (3) 8. (3) 9. (1) 10. (4)

### JEE Advanced

#### Single Correct Answer Type

1. (2) 2. (1) 3. (2) 4. (2) 5. (1)

#### Linked Comprehension Type

1. (1) 2. (2) 3. (4)

#### Numerical Value Type

1. (3) 2. (7) 3. (7) 4. (1) 5. (24)

# 6 Atomic Physics

## INTRODUCTION

The idea that matter is made up of discrete units is a very old idea, appearing in many ancient cultures such as Greece and India. The term "atom" comes from the Greek word for indivisible, because it was once thought that atoms were the smallest form of matter in the universe and could not be divided. Over the past two centuries, physicists and chemists developed the concept of an atom. In the nineteenth century, though still unseen, atoms were used to explain a large variety of natural phenomena. Eventually, atoms were found to have structure and to be divisible, though not by means of ordinary physical and chemical processes. Today, we can produce images of individual atoms by using a scanning tunneling electron microscope. Thus, atoms which were once only a philosopher's dream, have become a physical reality.

In this chapter, we will learn the atomic models of the past two centuries and the Bohr model of the atom in early 1900s. In fact, the atom as we understand it today is based on quantum mechanics which is radically different from Newton's classical mechanics.

## ATOMIC STRUCTURE

### DALTON'S ATOMIC THEORY

John Dalton, in 1808, put forward his theory, according to which

(i) Every material is composed of minute particles known as atom. Atom is indivisible i.e. it cannot be subdivided. It can neither be created nor be destroyed.

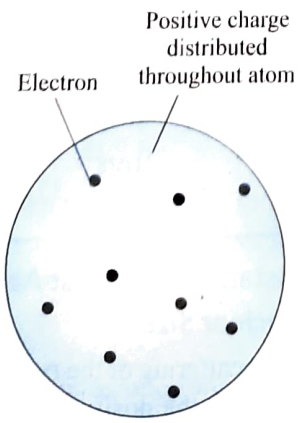
(ii) All atoms of same element are identical physically as well as chemically, whereas atoms of different elements are different in properties.

(iii) The atom is stable and electrically neutral.

Nobody seriously questioned Dalton's theory until about the beginning of the twentieth century.

### THOMSON'S ATOMIC MODEL

In 1897, the English physicist Joseph J. Thomson suggested a model that describes the atom as a region in which positive charge is spread out in space with electrons embedded throughout the region, much like the seeds in a watermelon or raisins in thick pudding. The atom as a whole would then be electrically neutral. In Thomson's view

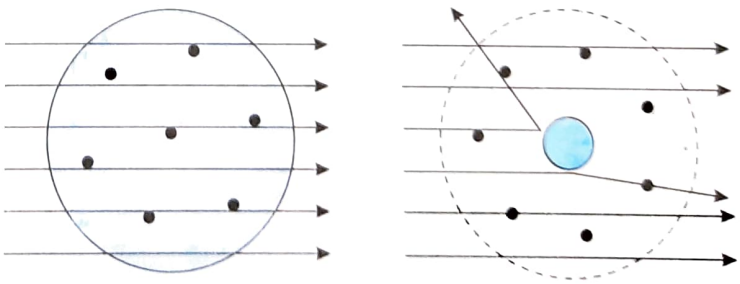



1. Atom is a positively charged sphere of radius of the order of $10^{-10}$ m.
2. There was no nucleus at the centre of an atom. Instead, the positive charge (and also mass) was assumed to be spread throughout the atom, forming a kind of "paste" or "pudding." The negative electrons were assumed to be suspended like "plums."
3. The number of electrons is such that their total negative charge is equal to the total positive charge. Hence, atom is electrically neutral.

The "plum-pudding" model was discredited in 1911 when the New Zealand physicist Ernest Rutherford (1871–1937) published the experimental results that this model could not explain the atom.

### RUTHERFORD'S ATOMIC MODEL

Rutherford and his co-workers directed a beam of alpha particles at a thin metal foil made of gold. Alpha particles are positively charged particles (the nuclei of helium atoms, although this was not recognized at the time) emitted by some radioactive materials. If the "plum-pudding" model were correct, the $\alpha$-particles would be expected to pass nearly straight through the foil. After all, there is nothing in this model to deflect the relatively massive $\alpha$-particles, since the electrons have a comparatively small mass and the positive charge is spread out in a diluted "pudding." Using a zinc sulphide screen, which flashed briefly when struck by an $\alpha$-particle, Rutherford and co-workers were able to determine that not all the $\alpha$-particles passed straight through the foil. Instead, some particles were deflected at large angles, even backward.

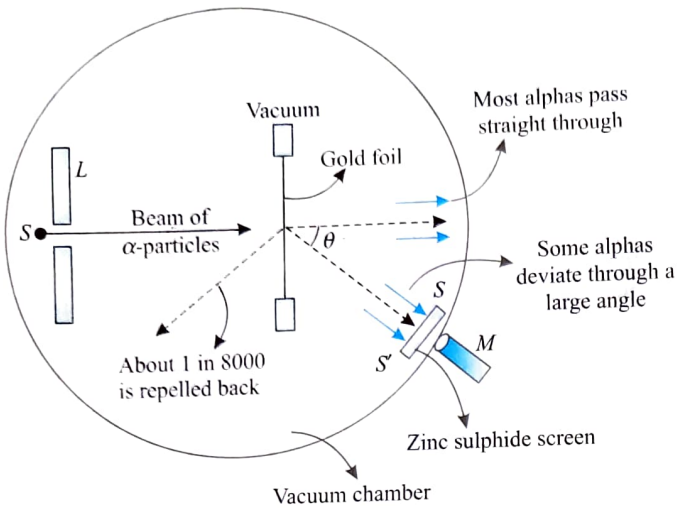


Thomson's model | Rutherford's model

#### Experimental Arrangement

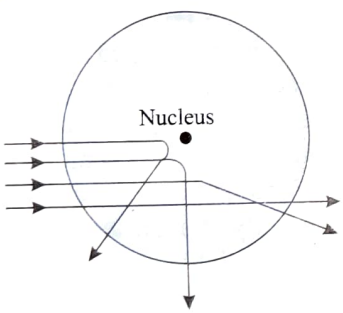
The experimental arrangement is shown in figure. $\alpha$-particles are emitted by some radioactive material (polonium), kept inside a thick lead box. A very fine beam of $\alpha$-particles passes through a small hole in the lead screen. This well collimated beam is then allowed to fall on a thin gold foil. While passing through the gold foil, $\alpha$-particles are scattered through different angles.

A zinc sulphide screen was placed out the other side of the gold foil. This screen was movable, so as to receive the $\alpha$-particles, scattered from the gold foil at angles varying from 0 to 180°. When an $\alpha$-particle strikes the screen, it produces a flash of light and it is observed by the microscope.




### *Observations*

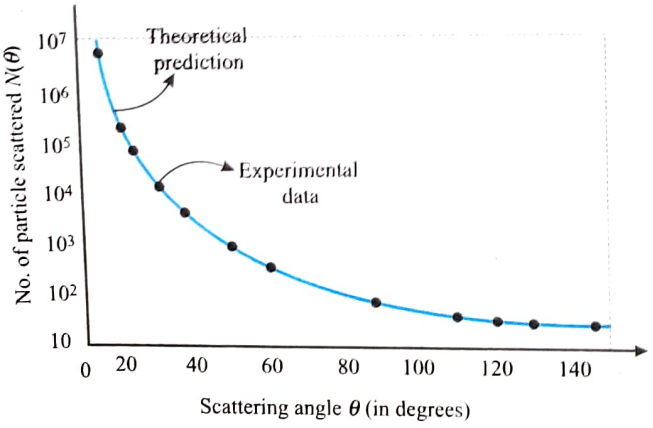
The beam of $\alpha$-particles is scattered in various directions. While most of the $\alpha$-particles passed straight through the foil, some of them bounced off at small angles, whereas a few underwent deflections through very large angles




According to **Rutherford scattering formula**,

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

where $N(\theta)$ is the number of $\alpha$-particles per unit area that reach the screen at a scattering angle of $\theta$. A graph between $N(\theta)$ and $(\theta)$ is shown in the figure. The experimental findings follow this curve as only 0.14 percent of the incident $\alpha$-particles are scattered by more than 1°.




The above graph reveals the following facts:

1. Most of the $\alpha$-particles pass straight through the gold foil or suffer only small deflections.
2. A few $\alpha$-particles, about 1 in 8,000, get deflected through 90° or more.
3. Occasionally, an $\alpha$-particle gets rebounded from the gold foil, suffering a deflection of nearly 180°.

**Conclusions:** Since $\alpha$-particles are relatively heavy and have high initial speeds, strong electrical forces are required to deflect them through large angles. This led Rutherford to postulate that:

1. The whole positive charge of the atom and almost its entire mass is concentrated in a small region of the atom. Rutherford named this region as the **nucleus**.
2. The nucleus is surrounded by electrons. As the atom on the whole is neutral, the total negative charge of the electrons is equal to the positive charge on the nucleus.
3. Using high speed $\alpha$-particles, the nuclear diameter has been found to be of the order of $10^{-14}$ m.
4. For the stability of the atom, Rutherford assumed that the electrons are revolving at high speeds around the nucleus in closed circular orbits so that the force of attraction between the nucleus and the electrons is balanced by the centrifugal force acting on the electrons.

   If the electrons in an atom were stationary, these would fall into the nucleus due to the electrostatic force of attraction between the electrons and the nucleus.
5. The existence of sufficient empty space within the atom explains why most of the $\alpha$-particles go undeflected. Small angle of scattering is accounted for by the fact that the nucleus occupies only a fraction of the total volume of the atom.

   Thus, the shortcomings of Thomson's model were removed with the discovery of nucleus.

**ILLUSTRATION 6.1**

The number of particles scattered at 60° is 100 per minute in an $\alpha$-particle scattering experiment, using gold foil. Calculate the number of particles per minute scattered at 90° angle.

**Sol.** Number of $\alpha$-particles scattered at an angle $\theta$.

$$N \propto \frac{1}{\sin^4(\theta/2)} \quad \text{i.e.,} \quad N = \frac{K}{\sin^4(\theta/2)},$$

where $K$ is a proportionality constant

$$\therefore \quad \frac{N_{90^\circ}}{N_{60^\circ}} = \frac{\sin^4(60^\circ/2)}{\sin^4(90^\circ/2)}$$

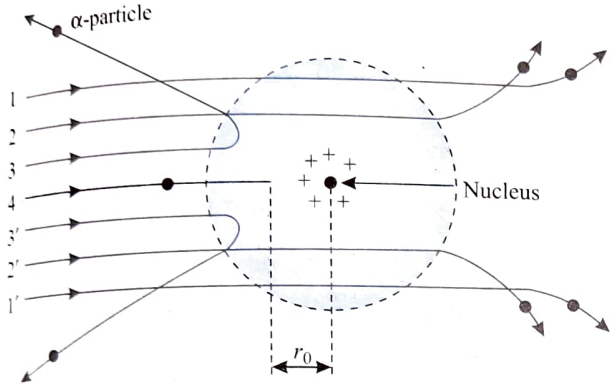
$$\text{or} \quad N_{90^\circ} = \frac{\sin^4 30^\circ}{\sin^4 45^\circ} \times N_{60^\circ} = \left[\frac{1/2}{1/\sqrt{2}}\right]^4 \times 100$$

$$= \frac{100}{4} = 25 \,\text{particle}\,\text{min}^{-1}$$

## Distance of Closest Approach: Estimation of Nuclear Size

The scattering of the $\alpha$-particles is due to the Coulombic repulsion between the positively charged $\alpha$-particles and the nuclei. In the

figure $\alpha$-particles like 1 or 1′, passing through the atom at large distance from the nucleus, experiences small repulsion and passes almost undeflected. But the $\alpha$-particles like (2, 2′), (3, 3′) which pass closer to the nucleus, experience large repulsive forces and hence scatter through large angles. Very rarely, an $\alpha$-particle like 4 travels head-on towards the nucleus. At a certain distance from the nucleus, the strong repulsive force slows down the $\alpha$-particle, which is finally stopped and then repelled back along its original path, i.e., it is scattered through an angle of 180°. The distance $r_0$ is called the **distance of closest approach**. At this distance $r_0$, the entire initial kinetic energy of the α-particle gets converted into electrostatic potential energy.




Let an $\alpha$-particle (of charge $+2e$). is moving with velocity $v$ is directed towards a nucleus (of charge $+Ze$), here $Z$ is the atomic number of foil atoms.

At the distance ($r_0$) of nearest approach, the kinetic energy of the alpha particle is completely converted into the potential energy of the system.

Electrostatic PE of $\alpha$-particle and nucleus at distance $r_0$,

$$U = k\frac{q_1 q_2}{r_0} = k\frac{2e\cdot Ze}{r_0} = k\frac{2Ze^2}{r_0}$$

By conservation of energy, $K_\alpha = U$

$$K_\alpha = k\frac{2Ze^2}{r_0}$$

or $$r_0 = k\frac{2Ze^2}{K_\alpha} \qquad \ldots(i)$$

where $k = \dfrac{1}{4\pi\epsilon_0} = 9\times10^9\ \text{Nm}^2\text{C}^{-2}$

It is observed that in case of gold nucleus ($Z$ = 79), head-on collision takes place when the kinetic energy of the $\alpha$-particle is 5.5 MeV, i.e., when

$$K = 5.5\,\text{MeV} = 5.5(1.6\times10^{-13}\ \text{J}) = 8.8\times10^{-13}\ \text{J}$$

Clearly, from Eq. (i), $r_0 = (9\times10^9)\times\dfrac{2(79)(1.6\times10^{-19})^2}{8.8\times10^{-13}}$

$$= 4.13\times10^{-14}\ \text{m} = 41.3\ \text{fm}$$

This distance is considerably larger than the sum of the radii of the gold nucleus and the $\alpha$-particle. The radius of $\alpha$ nucleus is of the order of a fermi, where 1 fermi (fm) = $10^{-15}$ m.

## Impact Parameter

The trajectory traced by an α-particle depends upon the impact parameter of collision.

**Impact parameter** is defined as the perpendicular distance of the velocity vector of $\alpha$-particle from the centre of the nucleus (when $\alpha$-particle is far away from the atom).

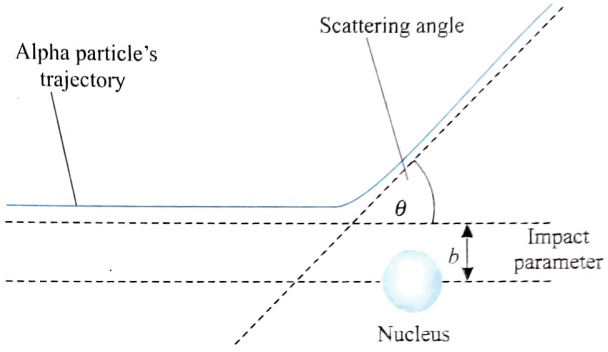
It is denoted by $b$ as shown in figure.

**Angle of scattering ($\theta$)** is defined as the angle between the direction of approach and the direction of recede of the $\alpha$-particle. It is denoted by $\theta$. Rutherford calculated that

$$\cot(\theta/2) = \frac{2b}{r_0} \qquad \ldots(i)$$

where $$r_0 = k_e\frac{2Ze^2}{K}$$

Thus, $$\cot(\theta/2) = \frac{2b}{k_e\dfrac{2Ze^2}{K}} = \frac{4\pi\varepsilon_0 K}{Ze^2}b$$




### ILLUSTRATION 6.2

In a Geiger-Marsden experiment, what is the distance of closest approach to the nucleus of a 7.7 MeV $\alpha$-particle before it comes momentarily to rest and reverses its direction?

**Sol.** Here $K = 7.7\,\text{MeV} = 7.7\times1.6\times10^{-13}\,\text{J}, Z(\text{Au}) = 79$

At the distance of closed approach of an $\alpha$-particle,

$$K = \frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze\cdot 2e}{r_0} \qquad [q_1 = Ze,\ q_2 = 2e]$$

$$\therefore\quad r_0 = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K}$$

$$= \frac{9\times10^9\times2\times79\times(1.6\times10^{-19})^2}{7.7\times1.6\times10^{-13}}\ \text{m}$$

$$= 29.5\times10^{-15}\ \text{m} \approx 30\ \text{fm} \qquad [1\ \text{fm} = 10^{-15}\ \text{m}]$$

The radius of gold nucleus is less than 30 fm. In fact, the actual radius of gold nucleus 6 fm.

### ILLUSTRATION 6.3

An α-particle after passing through a potential difference of $2\times10^6$ V falls on a silver foil. The atomic number of silver is 47. Calculate **(i)** the kinetic energy of the α-particle at the time of falling on the foil, **(ii)** the kinetic energy of the $\alpha$-particle at a distance of $5\times10^{-14}$ m from the nucleus and **(iii)** the shortest distance from the nucleus of silver to which the α-particle reaches.

**Sol.**

**(i)** Charge on $\alpha$-particle, $q = 2e$,

$V = 2 \times 10^6$ V

KE of $\alpha$-particle,

$$K = qV = 2\times1.6\times10^{-19}\times2\times10^{6} = 6.4\times10^{-13}\text{ J}$$

**(ii)** Charge on silver nucleus $= Ze = 47e$

PE of the $\alpha$-particle at a distance of $5 \times 10^{-1}$ m from the silver nucleus

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{47e\times2e}{5\times10^{-14}}$$

$$= \frac{9\times10^{9}\times94\times(1.6\times10^{-19})^2}{5\times10^{-14}} = 4.3\times10^{-13}\text{ J}$$

So, $4.3 \times 10^{-3}$ J of KE gets converted into PE

$\therefore$ KE of the $\alpha$-particle at a distance of $5 \times 10^{-14}$ m from the silver nucleus $= 6.4\times10^{-13} - 4.3\times10^{-13} = 2.1\times10^{-13}$ J

**(iii)** Distance of closest approach,

$$r_0 = \frac{2kZe^2}{E} = \frac{2\times9\times10^{9}\times47\times(1.6\times10^{-10})^2}{6.4\times10^{-13}}$$

$$= 3.4\times10^{-14}\text{ m}$$

## ILLUSTRATION 6.4

For scattering by an 'inverse-square' field (such as that produced by a charged nucleus in Rutherford's model) the relation between impact parameter $b$ and the scattering angle $\theta$ is given by

$$b = \frac{Ze^2\cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$$

**(a)** What is the scattering angle for $b = 0$?

**(b)** For a given impact parameter $b$, does the angle of deflection increase or decrease with increase in energy?

**(c)** What is the impact parameter at which the scattering angle is 90° for $Z = 79$ and initial energy equal to 10 MeV?

**(d)** Why is it that the mass of the nucleus does not enter the formula above but its charge does?

**(e)** For a given energy of the projectile,, does the scattering angle increase or decrease with decrease in impact parameter?

**Sol.** In Rutherford's model, the impact parameter is given by

$$b = \frac{Ze^2\cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$$

**(a)** Given $b = 0$

$$\therefore\quad \frac{Ze^2\cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)} = 0$$

or $\cot\frac{\theta}{2} = 0$ [$\because$ All other quantities are finite]

$$\frac{\theta}{2} = 90°$$

or $\theta = 180°$

which is the value expected physically for a head-on collision.

**(b)** For a given value of $b$, $\dfrac{Ze^2\cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)} = \text{constant}$

$\therefore$ As the energy $\left(\frac{1}{2}mv^2\right)$ increases, the value of $\cot\frac{\theta}{2}$ increases and hence the value of scattering angle $\theta$ decreases, as expected.

**(c)** Given $\theta = 90°$, $Z = 79$, $e = 1.6\times10^{-19}$ C

$$E = \frac{1}{2}mv^2 = 10\text{ MeV}$$

$$= 10\times10^{6}\times1.6\times10^{-19}\text{ J}$$

$$= 1.6\times10^{-12}\text{ J}$$

$$\therefore\quad b = \frac{Ze^2\cot\theta/2}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$$

$$= \frac{9\times10^{9}\times79\times(1.6\times10^{-19})^2\cot45°}{1.6\times10^{-12}}\text{ m}$$

$$= 9\times79\times1.6\times10^{-11}\text{ m}$$

$$= 11376\times10^{-11}\text{ m} \simeq 1.1\times10^{-14}\text{ m}$$

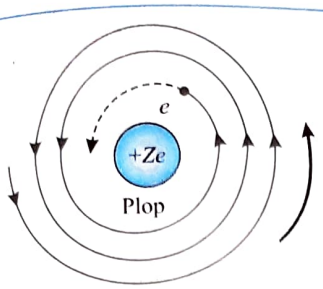
**(d)** It is the charge on the nucleus which provides the electrostatic field and due to which scattering of $\alpha$-particles occurs. If $Z = 0$, then from given formula we have, $\theta = 0°$. This means that scattering does not occur when nucleus carries no charge. Mass of nucleus does not appear in the expression for $b$, because recoil of the nucleus is being ignored, i.e., the nucleus is assumed to be at rest during its interaction with the $\alpha$-particle.

**(e)** For a given energy $\left(\frac{1}{2}mv^2\right)$ of the projectile, the decrease in impact parameter $b$ implies a decrease in the value of cot $\theta/2$ and hence an increase in the scattering angle $\theta$.

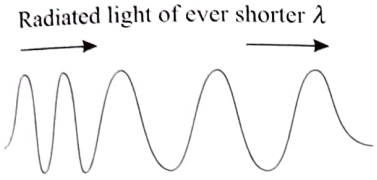
### Drawbacks of Rutherford's Atomic Model

The Rutherford's atomic model in spite of strong experimental support, has the following drawbacks.

(i) According to electromagnetic theory, an accelerated charged particle always radiates energy. The negatively charged electron revolving around the nucleus possesses centripetal acceleration and would lose its energy continuously. The radius of its orbit, therefore, would go on decreasing and it would finally spiral into the nucleus, resulting in an ultimate catastrophic collapse of the atom (called "plop") as shown in figure. But in practice, atoms do not collapse.



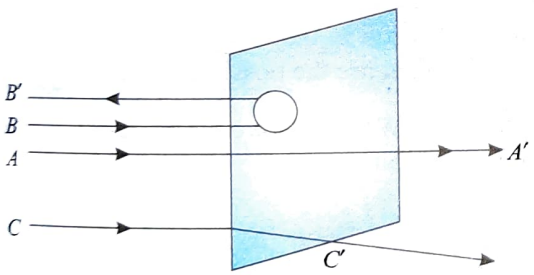

(ii) As an electron can revolve in any orbit, according to em theory, it must emit continuous radiations of all wavelengths, i.e., (frequencies) as shown in the figure. Actually, the elements are found to emit spectral lines of definite frequencies and not all of them.




To account for the drawbacks of Rutherford atom model, Bohr suggested another model which has come to be known as Bohr's atomic model.

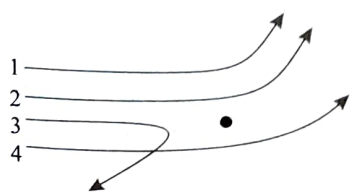
## CONCEPT APPLICATION EXERCISE 6.1

1. A beam of fast moving alpha particles were directed towards a thin film of gold. The parts $A'$, $B'$ and $C'$ of the transmitted and reflected beams corresponding to the incident parts $A$, $B$ and $C$ of the beam, are shown in the adjoining diagram. The number of alpha particles in




(a) $B'$ will be minimum and in $C'$ maximum
(b) $A'$ will be maximum and in $B'$ minimum
(c) $A'$ will be minimum and in $B'$ maximum
(d) $C'$ will be minimum and in $B'$ maximum

2. The diagram shows the path of four $\alpha$-particles of the same energy being scattered by the nucleus of an atom simultaneously. Which of these are/is not physically possible?




(a) 3 and 4 (b) 2 and 3
(c) 1 and 4 (d) 4 only

3. If scattering particles are 56 for 90° angle then this will be at 60° angle
(a) 224 (b) 256
(c) 98 (d) 108

4. An $\alpha$-particle of 5 MeV energy strikes with a nucleus of uranium at stationary at an scattering angle of 180°. The nearest distance up to which $\alpha$-particle reaches the nucleus will be of the order of
(a) 1 Å (b) $10^{-10}$ cm
(c) $10^{-12}$ cm (d) $10^{-15}$ cm

5. What is the distance of closest approach when a 5.0 MeV proton approaches a gold nucleus?

6. In a head-on collision between an $\alpha$-particle and a gold nucleus, the distance of closest approach is 41.3 fermi. Calculate the energy of the particle. (1 fermi = $10^{-15}$ m).

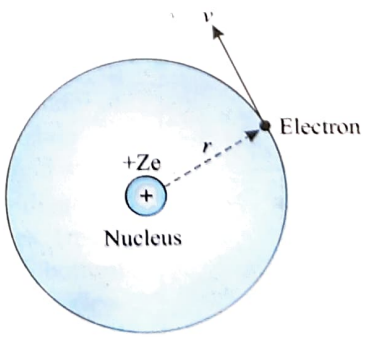
### ANSWERS

**1.** (b) **2.** (d) **3.** (a) **4.** (c)
**5.** $2.3 \times 10^{-14}$ m **6.** 5.51 MeV

## BOHR MODEL OF THE HYDROGEN ATOM

In 1913, Niels Bohr explained the hydrogen atom spectrum by applying the quantum theory of radiation to Rutherford's atomic model. Bohr's theory begins with Rutherford's picture of an atom as a nucleus surrounded by electrons moving in circular orbits, the necessary centripetal force to electrons is provided by the electrostatic force of attraction between nucleus and electrons. In analyzing this picture, Bohr made a number of assumptions in order to combine the new quantum ideas of Planck and Einstein with the traditional description of a particle in uniform circular motion. Thus, with the introduction of quantum concept, the entire picture of atom became complete. Bohr's theory is based on the following three postulates.

**Postulate 1:** There is a positively charged nucleus at the centre of the atom around which the electron revolves in circular orbits. The necessary centripetal force is provided by the Coulomb's force of attraction exerted by the positively charged nucleus on the negatively charged electron.




i.e. $$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)e}{r^2} = \frac{mv^2}{r} \quad \ldots(i)$$

$\varepsilon_0$ = Absolute permittivity of free space
$= 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$
$m$ = Mass of electron

$v$ = Velocity (linear) of electron
$r$ = Radius of the orbit in which electron is revolving.
$Z$ = Atomic number of hydrogen like atom. For hydrogen $Z=1$.

**Postulate 2:** The electron moves in certain discrete orbits, called the stationary orbits, for which the total angular momentum of the moving electron is an integral multiple of $h/2\pi$, $h$ being the Planck's constant.

The magnitude of the angular momentum is $L = mvr$ for a particle with mass $m$ moving with speed $v$ in a circle of radius $r$. So, according to Bohr's postulate,

$$mvr = \frac{nh}{2\pi} \quad (n = 1, 2, 3\ldots) \qquad \ldots(ii)$$

where $n = 1, 2, 3,\ldots$ and is called the **principal quantum number**.

Equation (ii) is known as the **quantum condition** and it limits the number of permissible orbits.

Adopting Planck's idea of quantized energy levels, Bohr hypothesized that in a hydrogen atom there can be only certain values of the total energy (electron kinetic energy plus potential energy) for the electrons. These allowed energy levels correspond to different orbits for the electron as it moves around the nucleus, the larger orbits being associated with larger total energies. In addition, Bohr assumed that an electron in one of these orbits does not radiate electromagnetic wave. For this reason, the orbits are called stationary orbits or stationary states. Bohr recognized that radiation less orbits violated the laws of physics. But the assumption of such orbits was necessary because the traditional laws indicated that an electron radiates electromagnetic waves as it accelerates around a circular path, and the loss of the energy carried by the waves would lead to the collapse of the orbit.

**Postulate 3:** Electrons in outer orbits have greater energy than those in inner orbits. The orbiting electron emits energy when it jumps from an outer orbit (higher energy states, $E_i$) to an inner orbit (lower energy states $E_f$), it radiates energy as a single photon of frequency $f$.

$$E_i - E_f = hf \qquad \ldots(iii)$$

Here a question arises, how do electrons get into the higher energy orbits? They get there by picking up energy when atoms collide, which happens more often when a gas is heated, or by acquiring energy when a high voltage is applied to a gas.

## Radius of Orbit

We have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_o}\frac{(Ze)(e)}{r^2} \qquad \ldots(i)$$

and $$mvr = \frac{nh}{2\pi} \qquad \ldots(ii)$$

From Eq. (ii), $v = \dfrac{nh}{2\pi mr}$

Substituting the value of $v$ in Eq. (i), we get

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2Z}$$

or $$r_n = (0.53)\frac{n^2}{Z}\ \text{Å}$$

So, for H-like atoms, $r_n \propto \dfrac{n^2}{Z}$.

Stationary orbits are not equally spaced.

## Velocity of Electron in nth Orbit

Since $v = \dfrac{nh}{2\pi mr}$ and $r = \dfrac{n^2h^2\varepsilon_0}{\pi me^2Z}$

Put the value of $r$ to get

$$v = \frac{Ze^2}{2\varepsilon_0 nh} \quad \text{or} \quad v = \left(\frac{e^2}{2\varepsilon_0 ch}\right)\left(\frac{cZ}{n}\right)$$

or $$v = \alpha\left(\frac{cZ}{n}\right)$$

where $\alpha = e^2/2h\varepsilon_0 c$ is Summerfield's fine structure constant (a pure number) whose value is 1/137. Therefore,

$$v = \frac{1}{137}\left(\frac{cZ}{n}\right)$$

i.e., velocity of electron in Bohr's first orbit of hydrogen ($Z = 1$) is $c/137$, in second orbit is $c/274$, and so on.

## Orbital Frequency of Electron

$$f = \frac{v}{2\pi r}$$

Put the value of $v$ and $r$ to get: $f = \dfrac{mZ^2e^4}{4\varepsilon_0^2 n^3h^3} \Rightarrow f \propto \dfrac{Z^2}{n^3}$

### ILLUSTRATION 6.5

The electron in a hydrogen atom makes a transition $n_1 \to n_2$ where $n_1$ and $n_2$ are the principal quantum numbers of the two energy states. Assume Bohr's model to be valid. The time period of the electron in the initial state is eight times that in the final state. What are the possible values of $n_1$ and $n_2$?

**Sol.** Here, $Z = 1$. We can derive the expression for time period of electron in $n$th orbit as

$$T_n = \frac{1}{f_n} \propto n^3$$

Hence $\dfrac{T_1}{T_2} = \dfrac{n_1^3}{n_2^3}$. As $T_1 = 8T_2$, therefore we get

$$8 = \left(\frac{n_1}{n_2}\right)^3 \quad \Rightarrow \quad n_1 = 2n_2$$

Thus, the possible values of $n_1$ and $n_2$ are $n_1 = 2$, $n_2 = 1$; $n_1 = 4$, $n_2 = 2$; $n_1 = 6$, $n_2 = 3$; and so on.

### ILLUSTRATION 6.6

How many times does the electron go round the first Bohr orbit of hydrogen atoms in 1 s?

**Sol.** Here, we have to find the frequency which is given as

$$f = \frac{mZ^2e^4}{4\varepsilon_0^2 n^3h^3}$$

$$= \frac{9.1\times10^{-31}\times1^2\times(1.6\times10^{-19})^4}{4\times(8.85\times10^{-12})^2\times1^3\times(6.6\times10^{-34})^3} = 6.62\times10^{15}\ \text{Hz}$$

**ILLUSTRATION 6.7**

What is the angular momentum of an electron in Bohr's hydrogen atom whose energy is –3.4 eV?

**Sol.** First, we need to identify the quantum number of the energy level.

As $E = -\frac{13.6}{n^2}$, we have $-3.4 = -\frac{13.6}{n^2}$ or $n = 2$

The angular momentum quantization gives, $L = mvr = \frac{nh}{2\pi}$

Substituting $n = 2$, we get $L = \frac{2h}{2\pi} = \frac{h}{\pi}$

**ILLUSTRATION 6.8**

If the average life time of an excited state of hydrogen is of the order of $10^{-8}$ s, estimate how many orbits an electron makes when it is in the state $n = 2$ and before it suffers a transition to state $n = 1$ (Bohr radius $a_0 = 5.3 \times 10^{-11}$m)?

**Sol.** The angular momentum of an electron in $n$th orbit is $nh/2\pi$.

By Bohr's hypothesis: $mvr = nh/2\pi$ where $r$ is the radius of the orbit.

or $v = \frac{nh}{2\pi mr}$ ...(i)

The time period of completing an orbit

$$T = \frac{2\pi r}{v} = \frac{2\pi r(2\pi mr)}{nh}$$

or $T = \frac{4\pi^2 mr^2}{nh}$ ...(ii)

Since the radius of the orbit is proportional to $n^2$, hence $r = a_0 n^2$

$$\therefore \quad T = \frac{4\pi^2 m a_0^2 n^4}{nh} = \frac{4\pi^2 m a_0^2 n^3}{h}$$

Number of orbits completed in $10^{-8}$ s

$$= \frac{10^{-8}}{T} = \frac{10^{-8} \times h}{4\pi^2 m a_0^2 n^3}$$

$$= \frac{10^{-8} \times (6.6 \times 10^{-34})}{4(3.14)^2 (9.1 \times 10^{-31})(5.3 \times 10^{-11})^2 (2)^3} = 8 \times 10^6$$

## Energy of Electron in $n$th Orbit

**Kinetic energy:** Since we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r^2} \quad \text{or} \quad \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

or $K = \frac{Ze^2}{8\pi\varepsilon_0 r}$

**Potential energy:** $U = -\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(e)}{r}$ or $U = -\frac{Ze^2}{4\pi\varepsilon_0 r}$

**Total energy:** $E = K + U = \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r}$

or $E = -\frac{Ze^2}{8\pi\varepsilon_0 r}$

So we conclude that total energy $= -K = \frac{U}{2}$

Using $r = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2 Z}$, we get

$$E = -\left(\frac{me^4}{8h^2\varepsilon_0^2}\right)\frac{Z^2}{n^2} = -(13.6)\frac{Z^2}{n^2}\text{ eV}$$

Also $E = -\left(\frac{me^4}{8\varepsilon_0^2 ch^3}\right)ch\frac{Z^2}{n^2} = -(Rch)\frac{Z^2}{n^2}$

where $R$ = Rydberg's constant $= \frac{me^4}{8\varepsilon_0^2 ch^3} = 1.097 \times 10^7\ \text{m}^{-1}$

Substituting values of $m$, $e$, $\varepsilon_0$, and $h$ with $n = 1$, we get the least energy of the atom in first orbit, which is –13.6 eV.

$Rch$ = Rydberg's energy $\approx 2.17 \times 10^{-18}$ J $\approx$ 13.6 eV is the electron energy in first orbit of $H$ atom.

Hence, $E_1 = -13.6$ eV ...(i)

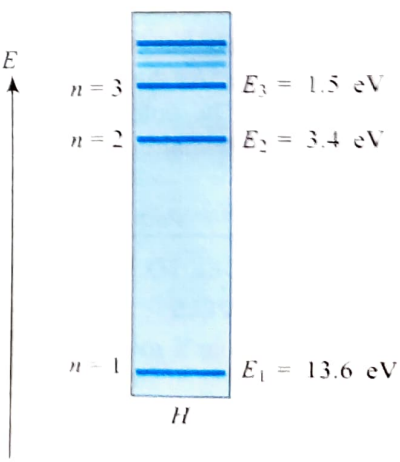
and $E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2}\text{eV}$ ...(ii)

Substituting $n$ = 2, 3, 4, ..., etc., we get energies of atom in different orbits.

The significance of negative sign in Eq. (ii) is that the electron is bound to the nucleus by attractive forces and to separate the electron from the nucleus energy must be supplied to it. Giving different values to $n$, we can calculate the orbital energy or binding energy of the electron in different orbitals.

$E_1 = -13.6$ eV where $n = 1$ ($K$-shell)
$E_2 = -3.4$ eV $n = 2$ ($L$-shell)
$E_3 = -1.5$ eV $n = 3$ ($M$-shell)
⋮
$E_\infty = 0$ eV $n = \infty$ (Limiting case)

Now, we shall consider the energy level diagram. This diagram can be drawn in terms of various electron orbits to the scale of their radii but it is customary to draw horizontal lines to the energy scale as shown in figure.




The diagram is known as energy level diagram. The lowest energy level ($n$ = 1) corresponds to normal unexcited state of hydrogen. This state is also called as ground state. In energy level diagram, the lower energies (more negative) are at the bottom while higher energies (less negative) are at the top. By such a consideration, the various electron jumps between allowed orbits will be vertical arrows between different energy levels. The energy of radiated photon is greater when the length of arrow is greater.

## Frequency of Emitted Radiation

If electron makes a transition (jumps) from final state $n_f$ to the initial state $n_i$, then frequency of emitted radiation $f$ is given by

$$hf = E_f - E_i \quad \text{or} \quad f = \frac{E_f - E_i}{h} = -Z^2 Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

If $\lambda$ is the wavelength of emitted radiation, then

$$f = \frac{c}{\lambda} = -Z^2 Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

So reciprocal of wavelength or wave number ( $\bar{\nu}$ ) is given by

$$\bar{\nu} = \frac{1}{\lambda} = -Z^2 R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

This relation holds for radiations emitted by hydrogen-like atoms, i.e.,

$$H(Z = 1),\ He^{+}(Z = 2),\ Li^{++}(Z = 3),\ \text{and}\ Be^{+++}(Z = 4)$$

If the electron makes a transition from $n = 1$ to the higher states, then absorption of radiation takes place.

**ILLUSTRATION 6.9**

Consider energy level diagram of a hydrogen atom. How will the kinetic energy and potential energy of electron vary if the electron moves from a lower level to a higher level?

**Sol.** Kinetic energy of an electron in a hydrogen atom is given by

$$K_n = \frac{e^2}{8\pi\varepsilon_0 r_n} = \frac{me^4}{8\varepsilon_0^2 n^2 h^2} = \frac{K_0}{n^2}, \quad \text{where} \quad K_0 = \frac{me^4}{8\varepsilon_0^2 h^2}$$

Potential energy is given by

$$U_n = -\frac{e^2}{4\pi\varepsilon_0 r_n} = -\frac{me^4}{4\varepsilon_0^2 n^2 h^2} = -\frac{2K_0}{n^2}$$

The kinetic energy is always positive and it is inversely proportional to square of $n$. Hence, as the electron is excited to higher states, its kinetic energy decreases.

The potential energy is always negative for a particle in bound state. The magnitude of potential energy is twice that of kinetic energy. As the electron is raised to higher energy level, $n$ increases. The potential energy becomes less negative that means the potential energy actually increases. Potential energy is maximum for $n \to \infty$, which corresponds to infinite separation between nucleus and electron.

## Bohr Model to Define Hypothetical Atomic Energy Levels

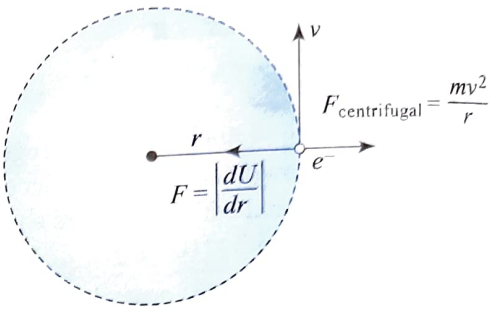
Originally, Bohr's model was to define only different energy levels for hydrogen atom.

According to Bohr's first postulate: $\frac{kZe^2}{r_n^2} = \frac{mv_n^2}{r_n}$ ...(i)

$$\left[\text{where}\ \ k = \frac{1}{4\pi\varepsilon_0}\right]$$

and from Bohr's second postulate: $mv_n r_n = \frac{nh}{2\pi}$ ...(ii)

Equation (ii) gives explanation for balancing the inward Coulombian force on electron by the outward centrifugal force on it in the rotating frame of reference. There may be some situation in which it is given that electron of an atom is revolving under the influence of a hypothetical potential energy field function given as $U = f(r)$ [non-Coulombian field] and using Bohr model, we can develop the properties of the electron in this new atom. For this, first, we will develop the first postulate equation for this new atom. As the electron is orbiting in a new potential energy field given as $U = f(r)$, it will experience an inward force toward the center of orbit as shown in figure.




As we know, $F = -\frac{\partial U}{\partial r}$

The magnitude of force can be written as

$$F = \left|\frac{dU}{dr}\right| = \left|\frac{d}{dr}[f(r)]\right| \quad \text{...(iii)}$$

Thus, for a stable $n^{th}$ orbit if electron is moving with speed $v_n$ in the orbit of radius $r_n$, we have

$$\left|\frac{d}{dr}[f(r_n)]\right| = \frac{mv_n^2}{r_n} \quad \text{...(iv)}$$

Equation (iv) will be the new equation for the first postulate and from Bohr's model the second postulate is based on quantization of angular momentum of electron, its equation will remain same as

$$mv_n r_n = \frac{nh}{2\pi} \quad \text{...(v)}$$

Now, using Eqs. (iv) and (v), we can derive all the properties for electron motion such as radius of $n^{th}$ orbit, velocity of electron in $n^{th}$ orbit, angular velocity, frequency, time period, current, magnetic induction, magnetic moment, and the total energy of energy levels for this hypothetical atom in the same way we have derived these for properties for a general hydrogenic atom. Let us discuss a few examples to understand these concepts in a better way.

**ILLUSTRATION 6.10**

Suppose potential energy between electron and proton at separation $r$ is given by $U = k \log r$, where $k$ is a constant. For such a hypothetical hydrogen atom, calculate the radius of $n^{th}$ Bohr's orbit and its energy levels.

**Sol.** For a conservative force field, $F = -\frac{dU}{dr} = -\frac{k}{r}$

This force $F = -k/r$ provides the centripetal force for the circular motion of electron.

...(i)

So, $\frac{mv^2}{r} = \frac{k}{r} \Rightarrow m^2v^2 = mk \Rightarrow mv = \sqrt{mk}$

Applying Bohr's quantization rule, $mvr = \frac{nh}{2\pi}$ ...(ii)

From Eqs. (i) and (ii), we get $r = \frac{nh}{2\pi\sqrt{mk}}$

From Eq. (i), KE of electron $= \frac{1}{2}mv^2 = \frac{1}{2}k$

Total energy of electron

$$= \text{KE} + \text{PE} = \frac{1}{2}k + k \log r = \frac{k}{2}\left[1 + \log \frac{n^2h^2}{4\pi^2 mk}\right]$$

## HYDROGEN-LIKE ATOMS

We can extend the Bohr model to other one-electron atoms, such as singly ionized helium ($He^+$), doubly ionized lithium ($Li^{2+}$), and so on. Such atoms are called **hydrogen-like atoms**.

The Bohr model of hydrogen can be extended to hydrogen-like atoms, i.e., one-electron atoms, the nuclear charge is $+Ze$, where $Z$ is the atomic number, equal to the number of protons in the nucleus.

The effect in the previous analysis is to replace $e^2$ everywhere by $Ze^2$. Thus, the equations for $r_n$, $v_n$ and $E_n$ are altered as under:

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m Z e^2} = a_0 \frac{n^2}{Z} \quad \text{or} \quad r_n \propto \frac{n^2}{Z} \qquad \text{...(i)}$$

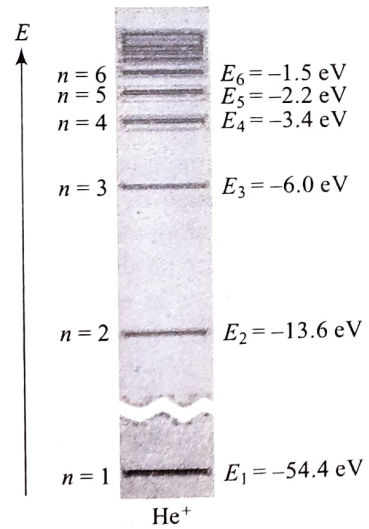
where $a_0 = 0.529$ Å (radius of first orbit of H)

$$v_n = \frac{Ze^2}{2\varepsilon_0 nh} = \frac{Z}{n}v_1 \quad \text{or} \quad v_n \propto \frac{Z}{n} \qquad \text{...(ii)}$$

where $v_1 = 2.19 \times 10^6$ m s$^{-1}$ (speed of electron in first orbit of H)

$$E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2 n^2 h^2} = \frac{Z^2}{n^2}E_1 \quad \text{or} \quad E_n \propto \frac{Z^2}{n^2} \qquad \text{...(iii)}$$

where $E_1 = -13.60$ eV (energy of atom in first orbit of H)




**Energy level diagram of He**

**Important Point:**

**For single-electron system**

**Ground state:** Lowest energy state of any atom or ion is called ground state of the atom.

Ground state energy of H atom = –13.6 eV
Ground state energy of $He^+$ ion = –54.4 eV
Ground state energy of $Li^{++}$ ion = –122.4 eV

### ILLUSTRATION 6.11

Determine the minimum wavelength of a photon that can cause ionization of $He^+$ ion.

**Sol.** The ground state energy of $He^+$ is given by the equation

$$E_n = -(13.6)\left(\frac{Z^2}{n^2}\right)$$

With $n = 1$ and $Z = 2$, we get $E_1 = -(13.6)(4) = -54.4$ eV

Thus, to ionize the $He^+$ ion, we should require 54.4 eV of energy. The minimum wavelength of photon that can cause ionization will have energy $E = hf = 54.4$ eV and wavelength

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{(54.4)(1.6 \times 10^{-19})} = 22.8 \text{ nm}$$

If the atom absorbed a photon of greater energy (wavelength less than 22.8 nm), the atom could still be ionized and the freed electron would have some kinetic energy when it is away from the influence of $He^+$ ion.

### ILLUSTRATION 6.12

Find the quantum number $n$ corresponding to the excited state of $He^+$ ion if on transition to the ground state that ion emits two photons in succession with wavelengths 1026.7 and 304 Å. ($R = 1.096 \times 10^7$ m$^{-1}$)

**Sol.** Given $\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = RZ^2\left(1 - \frac{1}{n^2}\right)$

$$\frac{1}{n^2} = 1 - \left[\frac{\lambda_1 + \lambda_2}{\lambda_1\lambda_2} \times \frac{1}{RZ^2}\right]$$

$$= 1 - \frac{1330.7 \times 10^{-10}}{1026.7 \times 304 \times 10^{-20} \times 4 \times 1.096 \times 10^7}$$

Thus, $\frac{1}{n^2} = 0.0275 \Rightarrow n = 6.03$

Hence, the quantum number = 6

### ILLUSTRATION 6.13

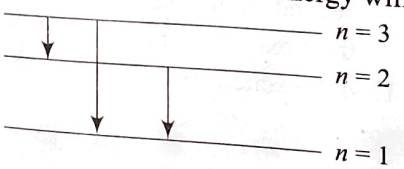
A gas of hydrogen-like atoms can absorb radiations of 68 eV. Consequently, the atoms emit radiations of only three different wavelengths. All the wavelengths are equal to or smaller than that of the absorbed photon.

**(a)** Determine the initial state of the gas atoms.
**(b)** Identify the gas atoms.
**(c)** Find the minimum wavelength of the emitted radiation.
**(d)** Find the ionization energy and the respective wavelength for the gas atoms.

**Sol.**

**(a)** Since three radiations are emitted, therefore, the final excited state of the gas is $n = 3$.

The initial state of the gas atoms is $N = 2$ as all the wavelengths are smaller and the energy will be higher.



**(b)** $13.6Z^2\left[\frac{1}{2^2}-\frac{1}{3^2}\right]=68$

or $13.6Z^2\left[\frac{5}{36}\right]=68 \Rightarrow Z=6$

**(c)** The minimum wavelength corresponds to the transition $n=3$ to $n=1$

$$\frac{1}{\lambda_{min}}=RZ^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right]$$

or $\lambda_{min}=\frac{9}{8RZ^2}=\frac{9}{8(1.097\times10^7)(6)^2}=28.5\ \text{Å}$

**(d)** The ionization energy of the gas atoms is

$$E=13.6Z^2=(13.6)(6)^2=489.6\ \text{eV}$$

$$\lambda=\frac{1}{RZ^2}=\frac{1}{(1.097\times10^7)(6)^2}=25.32\ \text{Å}$$

## Ionization Energy (IE) and Ionization Potential (IP)

**Ionization energy (IE):** Total energy zero of a hydrogen atom corresponds to infinite separation between electron and nucleus. Total positive energy implies that the atom is ionized and electron is in unbound (isolated) state moving with certain kinetic energy. Minimum energy required to move an electron from ground state to $n=\infty$ is called ionization energy of the atom or ion.

$$E_{ionization}=E_\infty-E_n=-E_n=\frac{13.6Z^2}{n^2}\ \text{eV}$$

Ionization energy of H atom = 13.6 eV
Ionization energy of $He^+$ ion = 54.4 eV
Ionization energy of $Li^{++}$ ion = 122.4 eV

**Ionization potential (IP):** Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionization energy of the atom is called ionization potential of the atom.

$$V_{ionization}=\frac{E_n}{e}=\frac{13.6Z^2}{n^2}\ \text{V}$$

IP of H atom = 13.6 V
IP of $He^+$ ion = 54.4 V
IP of $Li^{++}$ ion = 122.4 V

## Excitation Energy and Excitation Potential

**Excitation:** The process of absorption of energy by an electron so as to raise it from a lower energy level to some higher energy level is called excitation.

**Excited state:** The states of an atom other than the ground state are called its excited states.

| | |
|---|---|
| $n=2$, | first excited state |
| $n=3$, | second excited state |
| $n=4$, | third excited state |
| $n=n_0+1$, | $n_0$th excited state |

**Excitation energy:** Energy required to move an electron from ground state of the atom to any other excited state of the atom is called excitation energy of that state.

$$E_{excitation}=E_{higher}-E_{lower}$$

Energy in ground state of H atom = –13.6 eV
Energy in first excited state of H atom = –3.4 eV
The first excitation energy = – 3.4 – (– 13.6) = 10.2 eV.

**Excitation potential:** Potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to excitation energy of any state is called excitation potential of that state.

$$V_{excitation}=\frac{E_{excitation}}{e}$$

First excitation energy = 10.2 eV, so First excitation potential = 10.2 V.

| | | |
|---|---|---|
| $n=1$, | $E_1=-13.6$ eV | This is the ground state energy. |
| $n=2$, | $E_2=-3.4$ eV | This is the first excited level. |
| $n=3$, | $E_3=-1.51$ eV | This is the second excited level. |
| ⋮ | ⋮ | ⋮ |
| $n=\infty$, | $E_\infty=0$ | The atom is said to be ionized. |

## Binding Energy or Separation Energy

1. Energy liberated when constituents of a system are brought from infinity to assemble the system. The binding energy is negative of ionization energy.
$$E_{binding}=E_n$$
2. Energy required to move an electron from any state to $n=\infty$ is called binding energy of that state or energy released during formation of an H-like atom/ion from $n=\infty$ to some particular state $n$ is called binding energy of that state. Binding energy of ground state of H atom is 13.6 eV.

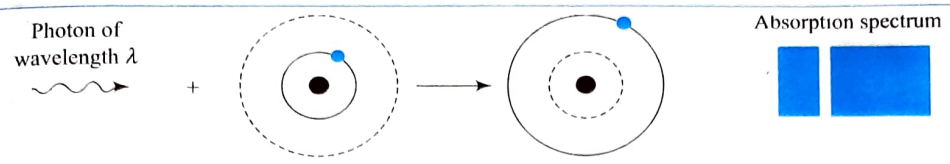
# ATOMIC EXCITATION

An atom can be excited to an energy level above its ground state by the following two ways: (1) by photon absorption and (2) by collision.

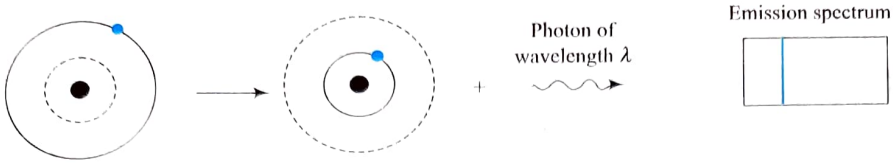
When an atom absorbs photon, it becomes excited and returns to its ground state in an average of $10^{-6}$ s by emitting one or more photons (see figure). An atom will not absorb photon of any arbitrary energy. For a photon to be absorbed by an atom, it must have energy equal to difference of energy of ground state and any excited state. For example, when H atom absorbs a photon of wavelength 121.7 nm (10.2 eV), it will bring up H atom from $n=1$ state to $n=2$ state. This explains the origin of absorption spectra. On subsequent de-excitation, a photon of wavelength 121.7 nm is emitted when H atom drops from $n=2$ to $n=1$ state.

The second type of mechanism for excitation of atoms is by collision. For atomic excitation, collision has to be inelastic and kinetic energy lost in the process is utilized for atomic excitation. Numerical problems based on atomic excitation by collision are worked out using the following principles:

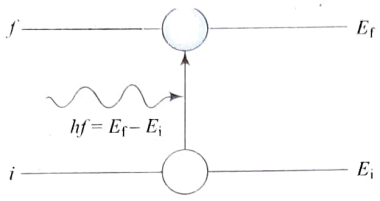
1. Conservation of linear momentum (COLM)
2. Conservation of energy (COE)




**Excitation of electron by photon absorption**




**De-excitation of electron**




**Energy digram**

## ILLUSTRATION 6.14

Find the ratio of ionization energy of Bohr's hydrogen atom and doubly ionized lithium ion ($Li^{2+}$).

**Sol.** The energy of ground state of Bohr's hydrogen-like atom is $E_n = -(13.6)Z^2$.

The ionization energy is equal in magnitude to energy of ground state; so $E_{\text{ionization}} = (13.6)Z^2$

$$\frac{(E_{\text{ionization}})_H}{(E_{\text{ionization}})_{Li^{2+}}} = \frac{(Z_H)^2}{(Z_{Li^{2+}})^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

## ILLUSTRATION 6.15

The Bohr model does not apply when more than one electron orbit the nucleus because it does not account for the electrostatic force that the electrons exert on one another. For instance, an electrically neutral lithium atom (Li) contains three electrons in orbit around a nucleus that includes three protons ($Z = 3$), and Bohr's analysis is not applicable. However, the Bohr model can be used for the doubly charged positive ion of lithium ($Li^{2+}$) that results when two electrons are removed from the neutral atom, leaving only one electron to orbit the nucleus. Obtain the ionization energy that is needed to remove the remaining electron from $Li^{2+}$.

**Sol.** The lithium ion $Li^{2+}$ contains three times the positive nuclear charge as that of the hydrogen atom. Therefore, the orbiting electron is attracted more strongly to the nucleus in $Li^{2+}$ than in the hydrogen atom. As a result, we expect that more energy is required to ionize $Li^{2+}$ than the 13.6 eV required for atomic hydrogen.

The Bohr energy levels for $Li^{2+}$ are given by $E_n = -(13.6\text{ eV})\dfrac{Z^2}{n^2}$

With $Z = 3$ and $n = 1$, $E_1 = -(13.6\text{ eV})\dfrac{3^2}{1^2} = -122.4\text{ eV}$

To remove the electron from $Li^{2+}$, 122.4 eV of energy must be supplied. So the ionization energy for $Li^{2+}$ is 122.4 eV.

## ILLUSTRATION 6.16

The energy levels of a hypothetical one electron atom are given by

$$E_n = -\frac{18.0}{n^2}\text{ eV}, \quad \text{where} \quad n = 1, 2, 3,\ldots$$

**(a)** Compute the four lowest energy levels and construct the energy level diagram.

**(b)** What is the excitation potential of the state $n = 2$?

**(c)** What wavelengths (Å) can be emitted when these atoms in the ground state are bombarded by electrons that have been accelerated through a potential difference of 16.2 V?

**(d)** If these atoms are in the ground state, can they absorb radiation having a wavelength of 2000 Å?

**(e)** What is the photoelectric threshold wavelength of this atom?

**Sol.**

**(a)** $E_1 = -\dfrac{18.0}{1^2} = -18.0\text{ eV}$, $E_2 = -\dfrac{18.0}{2^2} = -4.5\text{ eV}$,

$E_3 = -\dfrac{18.0}{3^2} = -2.0\text{ eV}$, $E_4 = -\dfrac{18.0}{4^2} = -1.125\text{ eV}$

The energy level diagram is shown in the figure.

$E_4 = -1.125$ eV

$E_3 = -2.0$ eV

$E_2 = -4.5$ eV

$E_1 = -18.0$ eV

**(b)** The excitation potential of state $n = 2$ is:

$18.0 - 4.5 = 13.5$ V

**(c)** Energy of the electron accelerated by a potential difference of 16.2 V is 16.2 eV. With this energy, the electron can excite the atom from $n = 1$ to $n = 3$ as

$E_4 - E_1 = 1.125 - (-18.0) = 16.875\text{ eV} > 16.2\text{ eV}$

and $E_3 - E_1 = 2.0 - (-18.0) = 16.0\text{ eV} < 16.2\text{ eV}$

$$\lambda_{32} = \frac{12375}{E_3 - E_2} = \frac{12375}{-2.0 - (-4.5)} = 4950\text{ Å},$$

$$\lambda_{31} = \frac{12375}{E_3 - E_2} = \frac{12375}{16} = 773 \text{ Å}$$

and $$\lambda_{21} = \frac{12375}{E_2 - E_1} = \frac{12375}{-4.5 - (-18.0)} = 917 \text{ Å}$$

**(d)** No. The energy corresponding to $\lambda = 2000$ Å;

$$E = \frac{12375}{2000} = 6.1875 \text{ eV}$$

The minimum excitation energy is 13.5 eV ($n = 1$ to $n = 2$).

**(e)** Threshold wavelength for photoemission to take place from such an atom is, $\lambda_{max} = \dfrac{12375}{18} = 687.5$ Å

### ILLUSTRATION 6.17

A doubly ionized lithium atom is hydrogen-like with atomic number 3. Find the wavelength of the radiation required to excite the electron in $Li^{++}$ from the first to third Bohr orbit. The ionization energy of the hydrogen atom is 13.6 eV.

**Sol.** $E_n = -\dfrac{13.6Z^2}{n^2}$ eV, $E_1 = -\dfrac{13.6 \times 3^2}{1^2} = -122.4$ eV,

$$E_3 = -\frac{13.6 \times 3^2}{3^2} = -13.6 \text{ eV}$$

The corresponding wavelength is $\lambda = \dfrac{12375}{E_3 - E_1} = 113.74$ Å

### ILLUSTRATION 6.18

A single electron orbits a stationary nucleus of charge $+Ze$, where $Z$ is a constant and $e$ is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second orbit to the third Bohr orbit. Find:

**(a)** the value of $Z$.

**(b)** the energy required to excite the electron from the third to the fourth Bohr orbit.

**(c)** the wavelength of electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.

**(d)** find the KE, PE, and angular momentum of electron in the first Bohr orbit.

**(e)** the radius of the first Bohr orbit.

[The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = $5.3 \times 10^{-11}$ m, velocity of light = $3 \times 10^8$ m s$^{-1}$, Planck's constant = $6.6 \times 10^{-34}$ J-s]

**Sol.** The energy required to excite the electron from $n_1$ to $n_2$ orbit revolving round the nucleus with charge $+Ze$ is given by

$$E_{n_2} - E_{n_1} = Z^2 \times 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \text{ electron volt.}$$

**(a)** Since 47.2 eV energy is required to excite the electron from $n_1 = 2$ to $n_2 = 3$ orbit, therefore,

$$47.2 = Z^2 \times 13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\Rightarrow \quad Z^2 = \frac{47.2 \times 36}{13.6 \times 5} = 25 \quad \text{or} \quad Z = 5$$

**(b)** The energy required to excite the electron from $n_1 = 3$ to $n_2 = 4$ orbit is given by

$$E_4 - E_3 = 25 \times 13.6 \times \left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{25 \times 13.6 \times 7}{144} = 16.53 \text{ eV}$$

**(c)** The energy required to remove the electron from the first Bohr orbit to infinity ($\infty$) is given by

$$E_\infty - E_1 = 13.6 \times Z^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 13.6 \times 25 \text{ eV}$$

In order to calculate the wavelength of radiation, we use Bohr's frequency relation

$$hf = \frac{hc}{\lambda} = 13.6 \times 25 \times (1.6 \times 10^{-19}) \text{ J}$$

or $$\lambda = \frac{(6.6 \times 10^{34}) \times (3 \times 10^8)}{13.6 \times 25 \times (1.6 \times 10^{-19})} = 36.5 \text{ Å}$$

**(d)** KE = – Total energy = $-(-13.6 \times 5^2) \times 1.6 \times 10^{-19}$ J
$= 5.44 \times 10^{-17}$ J

PE = $-2 \times$ KE = $-2 \times 5.44 \times 10^{-17} = 10.88 \times 10^{-17}$ J

Angular momentum = $mv_1r_1 = h/2\pi = 1.05 \times 10^{-13}$ Js

**(e)** The radius $r_1$ of the first Bohr's orbit is given by

$$r_1 = \frac{\varepsilon_0 h^2}{\pi m e^2}\frac{1}{Z} = \frac{0.53 \times 10^{-10}}{5} = 0.106 \times 10^{-10} \text{ m}$$

$= 0.106$ Å

$$\left(\because \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53 \times 10^{-10}\right)$$

### ILLUSTRATION 6.19

In a transition from state $n$ to a state of excitation energy 10.19 eV, a hydrogen atom emits a 4890 Å photon. Determine the binding energy of the initial state.

**Sol.** The energy of the emitted photon is $hf = \dfrac{12400}{4890} = 2.54$ eV

The excitation energy is the energy to excite the atom to a level above the ground state. Therefore, the energy of the level to which the transition occurs is:

$E_x = -13.6 + 10.19 = -3.41$ eV

The photon arises from the transition between energy states (say $n$ to $x$), then

$E_n - E_x = hf$, hence $E_n - (-3.41) = hf$

$\Rightarrow \quad E_n - (-3.41) = 2.54$ or $E_n = -0.87$ eV

Therefore, the binding energy of an electron in the state is 0.87 eV.

Note that the transition is from

$$n = \sqrt{\frac{E_1}{E_n}} = \sqrt{\frac{13.6}{0.87}} = 4 \quad \text{to} \quad x = \sqrt{\frac{E_1}{E_x}} = \sqrt{\frac{13.6}{3.41}} = 2$$

### ILLUSTRATION 6.20

The first excitation potential of a hypothetical hydrogen-like atom is 15 V. Find the third excitation potential of the atom.

**Sol.** Let energy of ground state = $-E_0$, then $E_n = -\dfrac{E_0}{n^2}$

Given: $E_2 - E_1 = 15$ eV $\Rightarrow -\dfrac{E_0}{4} + E_0 = 15 \Rightarrow \dfrac{3E_0}{4} = 15$

$\Rightarrow E_0 = 20$ eV

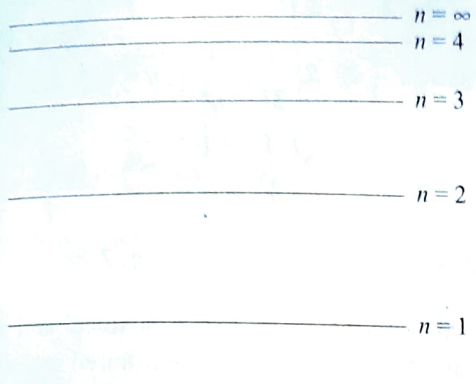
For the third excitation potential: $n = 4$, then

$$E_4 - E_1 = -\frac{E_0}{16} + E_0 = \frac{75}{4}\text{ eV}$$

Therefore, the third excitation potential is (75/4) V.

**ILLUSTRATION 6.21**

The energy level diagram for a hydrogen-like atom is shown in figure.




(a) Find the value of $Z$.

(b) If initially the atom is in the ground state, then
  (i) determine its first excitation potential, and
  (ii) determine its ionization potential.

(c) Can it absorb a photon of 42 eV?

(d) Can it absorb a photon of 56 eV?

(e) Calculate the radius of its first Bohr orbit.

(f) Calculate the kinetic energy and potential energy of an electron in the first orbit.

**Sol.**

(a) We know that $E_n = -13.6\frac{Z^2}{n^2}\text{eV}$

Here, for $n = 1$, $E_1 = -54.4$ eV, therefore,

$$-54.4 = -13.6\frac{Z^2}{1^2}$$

$$\therefore \quad Z = \sqrt{\frac{54.4}{13.6}} = 2$$

(b) (i) The first excitation energy is required to excite the electron from $n = 1$ to $n = 2$. Thus,

$$\Delta E_{12} = E_2 - E_1 = -13.6 - (54.4) = 40.8\text{ eV}$$

Therefore, the first excitation potential is 40.8 V.

(ii) The ionization energy is required to eject the electron from $n = 1$ to $n = \infty$. Thus,

$$\Delta E_{1\infty} = E_\infty - E_1 = 0 - (-54.4) = 54.4\text{ eV}$$

Therefore, the ionization potential is 54.4 V.

(c) No. By absorbing a photon of 42 eV, the energy of electron will become

$$E = -54.4 + 42 = -12.4\text{ eV}$$

This energy level exists between $n = 1$ and $n = 2$ shells which is not possible. Hence, an electron in the first orbit cannot absorb it.

(d) Yes. By absorbing a photon of 56 eV, the electron comes out of the atom where the energy is not quantized. Hence, a photon of 56 eV can be absorbed.

(e) We know that $r_n = 0.53\frac{n^2}{Z}$ (Å)

Here, $n = 1$; $Z = 2$, therefore, $r = 0.53\frac{(1)^2}{2} = 0.265$ (Å)

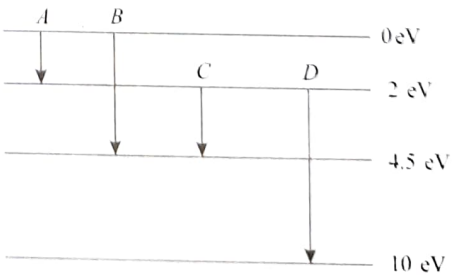
(f) Since $K = -E$ and $U = 2E$, therefore,

$K = 54.4\text{ eV} = 8.7 \times 10^{18}$ J and

$U = -108.8\text{ eV} = 1.74 \times 10^{17}$ J

**CONCEPT APPLICATION EXERCISE 6.2**

1. Which state of the triply ionized beryllium ($Be^{+++}$) has the same orbital radius as that of the ground state of hydrogen? Compare the energies of the two states.
2. Which level of the doubly ionized lithium ($Li^{++}$) has the same energy as the ground state energy of the hydrogen atom? Compare the orbital radii of the two levels.
3. Calculate the frequency of the photon, which can excite the electron to – 3.4 eV from –13.6 eV.
4. The ground state energy of hydrogen atom is –13.6 eV.
   (i) What are the potential energy and kinetic energy of an electron in the 3rd excited state ?
   (ii) If the electron jumps to the ground state from the third excited state, calculate the frequency of photon emitted.
5. A photon of energy 12.09 eV is absorbed by an electron in ground state of a hydrogen atoms. What will be the energy level of electron ? The energy of electron in the ground state of hydrogen atom is –13.6 eV.
6. The first excitation potential of sodium atom is 2.1 V. Calculate the longest wavelength emitted by this atom.
7. Find the frequency of revolution of the electron in the first stationary orbit of H-atom.
8. The electron in a hydrogen atom, initially in a state of quantum number $n_1$, makes a transition to a state whose excitation energy with respect to the ground state is 10.2 eV. If the wavelength associated with the photon emitted in this transition is 487.5 nm, find the (i) energy in eV and (ii) value of the quantum number $n_1$ of the electron in its initial state.
9. (a) The energy levels of an atom are as shown below. Which of them will result in the transition of a photon of wavelength 275 nm ?




   (b) Which transition corresponds to emission of radiation of (i) maximum wavelength and (ii) minimum wavelength?
10. Using the known values for hydrogen atom, calculate:
   (a) radius of the third orbit for $Li^{2+}$.
   (b) speed of electron in the fourth orbit for $He^+$.

**11.** Find the kinetic energy, potential energy, and total energy in the first and second orbits of hydrogen atom if potential energy in the first orbit is taken to be zero.

**12.** A small particle of mass $m$ moves in such a way that the potential energy $U = ar^2$, where $a$ is a constant and $r$ is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of $n^{th}$ allowed orbit.

**ANSWERS**

**1.** $n = 2, \frac{E_{Be^{+++}}}{E_H} 4$ **2.** $n = 3, \frac{r_{Lithium}}{r_{Hydrogen}} = 3$

**3.** $2.47 \times 10^{15}$ Hz **4.** (i) – 1.7 eV, 0.85 eV (ii) $3 \times 10^{15}$ Hz

**5.** $n = 3$ **6.** 5893 Å **7.** $6.6 \times 10^{15}$ Hz

**8.** (i) –0.85 eV (ii) 4

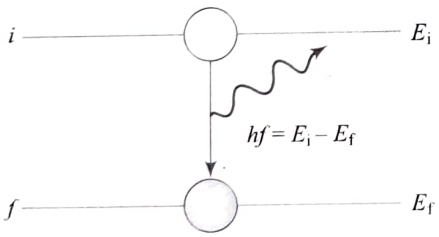
**9.** (a) Transition $B$ (b) (i) Transition $A$ (ii) Transition $D$

**10.** (a) 1.587 Å (b) $1.095 \times 10^6$ m s$^{-1}$ **12.** $\left(\frac{n^2h^2}{8um\pi^2}\right)^{1/4}$

## WAVELENGTH OF PHOTON EMITTED IN DE-EXCITATION

According to Bohr, when an atom makes a transition from a higher energy level to a lower energy level, it emits a photon with energy equal to the energy difference between the initial and final levels. If $E_i$ is the initial energy of the atom before such a transition, $E_f$ is its final energy after the transition, and the photon's energy is $hn = hc/\lambda$, then conservation of energy gives,

$$h\nu = \frac{hc}{\lambda} = E_i - E_f \quad \text{(energy of emitted photon)} \quad ...(i)$$




By 1913, the spectrum of hydrogen had been studied intensively. The visible line with longest wavelength, or lowest frequency is in red and is called $H_\alpha$, the next line, in the blue-green is called $H_\beta$, and so on.

In 1885, Johann Balmer, a Swiss teacher found a formula that gives the wavelengths of these lines. This is now called the Balmer series. Balmer's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad ...(ii)$$

Here, $n = 3, 4, 5, ...$, etc., $R$ = Rydberg constant = $1.097 \times 10^7$ m$^{-1}$; and $\lambda$ is the wavelength of light/photon emitted during transition. For $n = 3$, we obtain the wavelength of $H_\alpha$ line.

Similarly, for $n = 4$, we obtain the wavelength of $H_\beta$ line. For $n = \infty$, the smallest wavelength (= 3646 Å) of this series is obtained. Using the relation $E = hc/\lambda$, we can find the photon energies corresponding to the wavelengths of the Balmar series.

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{Rhc}{2^2} - \frac{Rhc}{n^2}$$

This formula suggests that, $E_n = \frac{Rhc}{n^2}$, $n = 1, 2, 3, ...$ ...(iii)

The wavelengths corresponding to other spectral series (Lyman, Paschen, etc.) can be represented by formula similar to Balmer's formula.

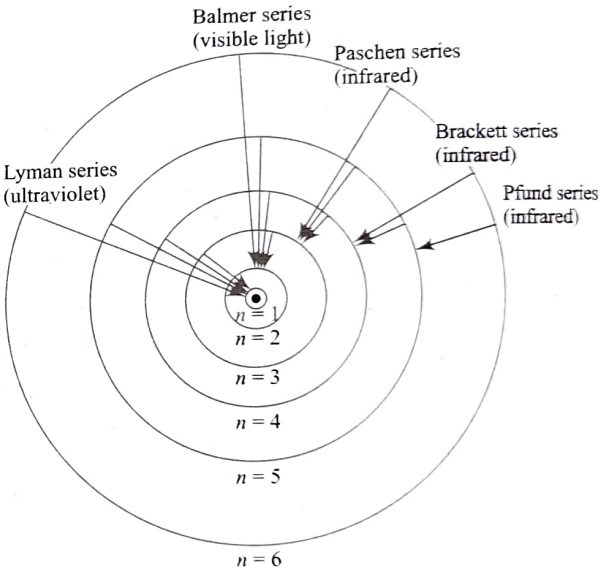
Lyman series: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, $n = 2, 3, 4, ...$

Paschen series: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, $n = 4, 5, 6, ...$

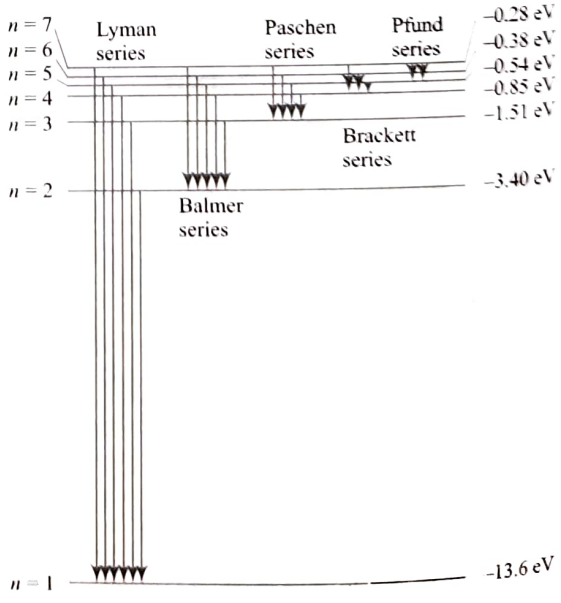
Brackett series: $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, $n = 5, 6, 7, ...$

Pfund series: $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$, $n = 6, 7, 8$

The Lyman series is in the ultraviolet, and the Paschen, Brackett, and Pfund series are in the infrared region.




(a)




(b)

**Hydrogen spectrum**

| | Initial state | Final state | Wavelength formula | First member-second member | Series limit $n_i \to \infty$ to $n_f$ | Maximum wavelength $(n_f+1)$ to $n_f$ | Lines found in |
|---|---|---|---|---|---|---|---|
| Lyman | $n_i = 2, 3, 4, 5, 6, \ldots$ | $n_f = 1$ | $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$ | $n_i = 2$ to $n_f = 1$<br>$n_i = 3$ to $n_f = 1$ | From $\infty$ to 1<br>$\lambda = \frac{4}{R}$<br>$\lambda = 911$ Å | From 2 to 1<br>$\lambda = \frac{4}{3R}$<br>$\lambda = 121$ Å | UV region |
| Balmar | $n_i = 3, 4, 5, 6, 7, \ldots$ | $n_f = 2$ | $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$ | $n_i = 3$ to $n_f = 2$<br>$n_i = 4$ to $n_f = 2$ | From $\infty$ to 2<br>$\lambda = \frac{4}{R}$<br>$\lambda = 3646$ Å | From 3 to 2<br>$\lambda = \frac{36}{5R}$<br>$\lambda = 6563$Å | Visible region |
| Paschen | $n_i = 4, 5, 6, 7, 8, \ldots$ | $n_f = 3$ | $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$ | $n_f = 4$ to $n_f = 3$<br>$n_f = 5$ to $n_f = 3$ | From $\infty$ to 3<br>$\lambda = \frac{9}{R}$<br>$\lambda = 8204$ Å | From 4 to 3<br>$\lambda = \frac{144}{7R}$<br>$\lambda = 18753$ Å | IR region |
| Brackett | $n_i = 5, 6, 7, 8, 9, \ldots$ | $n_f = 4$ | $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right)$ | $n_i = 5$ to $n_f = 4$<br>$n_i = 6$ to $n_f = 4$ | From $\infty$ to 4<br>$\lambda = \frac{16}{R}$<br>$\lambda = 1485$ Å | From 5 to 4<br>$\lambda = \frac{400}{9R}$<br>$\lambda = 40515$ Å | IR region |
| Pfund | $n_i = 6, 7, 8, 9, 10, \ldots$ | $n_f = 5$ | $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_i^2}\right)$ | $n_i = 6$ to $n_f = 5$<br>$n_i = 7$ to $n_f = 5$ | From $\infty$ to 5<br>$\lambda = \frac{25}{R}$<br>$\lambda = 22790$ Å | From 6 to 5<br>$\lambda = \frac{900}{11R}$<br>$\lambda = 74583$ Å | Far IR region |

## ILLUSTRATION 6.22

Brackett series of lines are produced when electrons excited to high energy levels make transitions to the $n = 4$ level.

(a) Determine the longest wavelength in this series.

(b) Determine the wavelength that corresponds to the transition from $n_f = 6$ to $n_i = 4$.

**Sol.**

(a) The longest wavelength corresponds to transition from $n_f = 5$ to $n_i = 4$; the smallest energy change.

So, $$\frac{1}{\lambda_{max}} = (1.097 \times 10^7)\left(\frac{1}{4^2} - \frac{1}{5^2}\right)$$

$$= 2.468 \times 10^5 \text{m}^{-1}$$

$$\lambda_{max} = \frac{1}{2.468 \times 10^5} = 4051 \text{ nm}$$

(b) The wavelength for transition $n_f = 6$ to $n_i = 4$ is

$$\frac{1}{\lambda} = (1.0947 \times 10^{-7})\left(\frac{1}{4^2} - \frac{1}{6^2}\right) = 3.801 \times 10^5$$

$$\Rightarrow \quad \lambda = \frac{1}{3.801 \times 10^5} = 2630 \text{ nm}$$

## ILLUSTRATION 6.23

Electrons of energy 12.09 eV can excite hydrogen atoms. To which orbit is the electron in the hydrogen atom raised and what are the wavelengths of the radiations emitted as it drops back to the ground state?

**Sol.** The energies of the electron in different states are:

$E_1 = -13.6$eV for $n = 1$,

$E_2 = -3.4$ eV for $n = 2$,

and $E_3 = -1.51$ eV for $n = 3$

Evidently, the energy needed by an electron to go to the $E_3$ level ($n = 3$ or M-level) is $13.6 - 1.51 = 12.09$ eV. Thus, the electron is raised to the third orbit of principal quantum number $n = 3$.

Now, an electron in the $n = 3$ level can return to the ground state making the following possible jumps:

(a) $n = 3$ to $n = 2$ and then from $n = 2$ to $n = 1$.

(b) $n = 3$ to $n = 1$.

Thus, the corresponding wavelengths emitted are:

(a) For $n = 3$ to $n = 2$:

$$\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

or $$\lambda_1 = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7} = 6563 \text{ Å}$$

This wavelength belongs to the Balmer series and lies in the visible region.

(b) For $n = 2$ to $n = 1$:

$$\frac{1}{\lambda_2} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3R}{4}$$

or $$\lambda_2 = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} = 1215 \text{ Å}$$

$\lambda_2$ belongs to the Lyman series and lies in the ultraviolet region.

(c) For the direct jump $n = 3$ to $n = 1$:

$$\frac{1}{\lambda_3} = R\left[\frac{1}{1^2} - \frac{1}{3^2}\right] = \frac{8R}{9}$$

or $$\lambda_3 = \frac{9}{8R} = \frac{9}{8 \times 1.097 \times 10^7} = 1026 \text{ Å}$$

which also belongs to the Lyman series and lies in the ultraviolet region.

## ILLUSTRATION 6.24

A hydrogen atom is in the third excited state. It makes a transition to a different state and a photon is either absorbed or emitted. Determine the quantum number $n_f$ of the final state and the energy of the photon if it is

**(a)** emitted with the shortest possible wavelength,

**(b)** emitted with the longest possible wavelength, and

**(c)** absorbed with the longest possible wavelength.

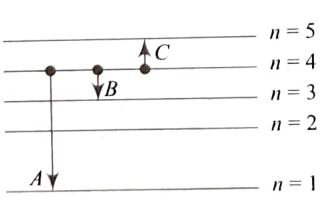
**Sol.**

**(a)** When a photon is emitted, it carries energy with it, so the final energy of the atom is less than its initial energy. So, the final quantum number is less than the initial when a photon is emitted.

An atom gains energy when a photon is absorbed. So, the final quantum number is greater than the initial when a photon is absorbed.

The largest possible energy arises when the electron jumps from the initial state ($n = 4$) to the ground state ($n_1 = 1$) as shown by transition $A$ in figure. Therefore, the quantum number of final state is $n = 1$. The energy of the $n^{th}$ state is given by

$$E_n = \left(\frac{Z^2}{n^2}\right)$$




The energy of the photon emitted corresponding to transition from excited state quantum number $n_u$ to lower energy state $n_l$ can be calculated from

$$\frac{hc}{\lambda} = E_u - E_l = (13.6)(1)^2\left[\frac{1}{n_l^2} - \frac{1}{n_u^2}\right]$$

So, the energy of the photon with minimum wavelength is

$$\Delta E = E_4 - E_1$$
$$= (13.6)(1)^2\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 12.75 \text{ eV}$$

**(b)** The longest possible wavelength photon corresponds to minimum energy. This happens when the electron jumps from the initial state ($n_u = 4$) to the next lower state ($n_l = 3$) as shown by transition $B$ in the figure. Therefore, the quantum number of the final state is $n = 3$. The energy of the photon is

$$\Delta E = E_4 - E_3 = (13.6)(1)^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 0.661 \text{ eV}$$

**(c)** The absorbed photon has the longest possible wavelength when its energy is the smallest, i.e., the electron jumps from the initial state ($n_1 = 4$) to the next higher state ($n_u = 5$). This is shown by transition $C$ in the figure. Therefore, the quantum number of the final state is $n = 5$. The energy of the photon is

$$\Delta E = E_5 - E_4 = (13.6)(1)^2\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = 0.306 \text{ eV}$$

## ILLUSTRATION 6.25

The Balmer series for the hydrogen atom corresponds to electron transitions that terminate in the state of quantum number $n = 2$.

**(a)** Find the longest wavelength photon emitted and determine its energy.

**(b)** Find the shortest wavelength photon emitted in the Balmer series.

**Sol.**

**(a)** The longest wavelength photon in the Balmer series results corresponding to transition from $n = 3$ to $n = 2$. Using the equation

$$\frac{1}{\lambda} = R\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) \Rightarrow \frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36}R$$

$$\Rightarrow \quad \lambda_{max} = \frac{36}{5R} = \frac{36}{5(1.097 \times 10^7)} = 656.3 \text{ nm}$$

This wavelength lies in the red region of the visible spectrum.

The energy of emitted photon is

$$E = \frac{hc}{\lambda_{max}} = \frac{(6.626 \times 10^{-34})(3 \times 10^8)}{(656.3 \times 10^{-9})}$$
$$= 3.03 \times 10^{-19} \text{ J} = 1.89 \text{ eV}$$

**(b)** The shortest wavelength photon in the Balmer series is emitted when the electron makes a transition from $n = \infty$ to $n = 2$.

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{R}{4}$$

$$\Rightarrow \quad \lambda_{min} = \frac{4}{R} = \frac{4}{1.097 \times 10^7} = 364.6 \text{ nm}$$

This wavelength lies in the ultra-violet region.

## ILLUSTRATION 6.26

How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to a state with principal quantum number $n$?

**Sol.** From the $n$th state, the atom may go to $(n-1)^{th}$ state, ..., second state or first state. So, there are $(n-1)$ possible transitions starting from the $n^{th}$ state. The atoms reaching $(n-1)^{th}$ state may make $(n-2)$ different transitions and so on. In general, the total number of possible transitions is

$$(n-1)+(n-2)+(n-3)+\cdots+2+1=\frac{n(n-1)}{2}$$

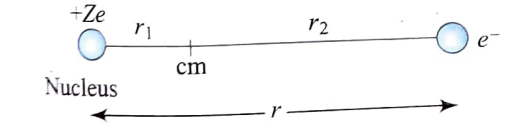
# EFFECT OF NUCLEAR MOTION ON THE ENERGY OF ATOM

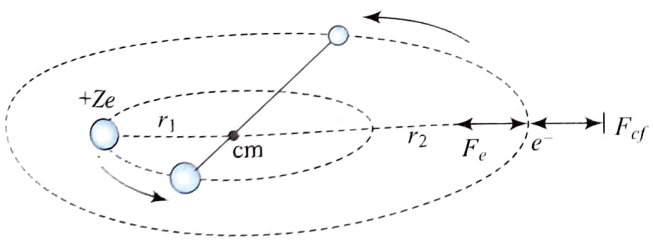
## EFFECT OF MASS OF NUCLEUS ON BOHR MODEL

Up to this level, we have discussed about the structure of a hydrogenic atom by considering nucleus at rest and electron revolving around the nucleus. In fact, we know that as no external force is acting on a nucleus–electron system, hence the centre of mass of the nucleus–electron system must remain at rest. Theoretically, mass of electron is negligible or small compared to that of nucleus and due to this we assume that centre of mass of the atom is almost situated at nucleus. That is why in Bohr atomic model it was assumed that in an atom, nucleus remains at rest and electron revolves around it. But practically the situation is a bit different. Actually, centre of mass of nucleus–electron system is close to nucleus as it is heavy and to keep the centre of mass at rest, both electron and nucleus revolve around their centre of mass like a double star system as shown in figure. If $r$ is the distance of electron from nucleus, the distances of nucleus and electron from the centre of mass, $r_1$ and $r_2$, can be given as

$$r_1=\frac{m_e r}{m_N+m_e}$$

and $$r_2=\frac{m_N r}{m_N+m_e}$$

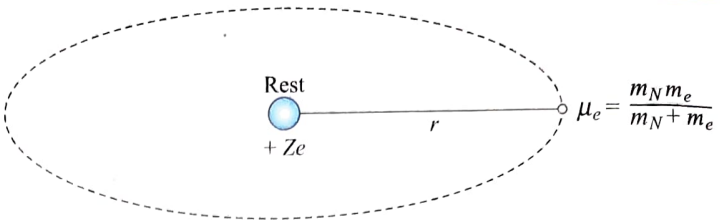



We can see that in the atom, nucleus and electron revolve around their centre of mass in concentric circles of radii $r_1$ and $r_2$ to keep the centre of mass at rest. In above system, we can analyze the motion of electron with respect to nucleus by assuming nucleus to be at rest and the mass of electron replaced by its reduced mass $\mu_e$, given as

$$\mu_e=\frac{m_N m_e}{m_N+m_e}$$

Now, the relative picture of atom will be same what we have considered earlier as shown in figure but electron mass is replaced by its reduced mass.




We can use all those relations which we have derived for the Bohr model just by replacing $m_e$ by $\mu_e$, such as the radius of electron in $n$th orbit of Bohr's model is given as

$$r_n=\frac{n^2h^2}{4\pi^2kZe^2m_e} \qquad ...(i)$$

But if we take the motion of nucleus into account, the radius of $n$th orbit will be given as

$$r_n'=\frac{n^2h^2}{4\pi^2kZe^2\mu_e}\Rightarrow r_n'=\frac{n^2h^2(m_N+m_e)}{4\pi^2kZe^2m_em_N} \qquad ...(ii)$$

or $$r_n'=r_n\times\frac{m_e}{\mu_e}\Rightarrow r=(0.529\text{ Å})\frac{mn^2}{\mu Z} \qquad ...(iii)$$

Similarly, if we find the speed of electron in $n$th Bohr orbit, it can be given by

$$v_n=\frac{2\pi kZe^2}{nh} \qquad ...(iv)$$

Here, we can see that in the above expression of speed the term $m_e$ (mass of electron) is not present. Thus, speed of electron as $n$th orbit does not depend on electron mass. Here, no change will be there in the electron's speed of revolution if we take the motion of nucleus into account.

Similarly, we know the expression of energy of electron in $n$th orbit of the Bohr model is given as

$$E_n=-\frac{2\pi^2k^2Z^2e^4m_e}{n^2h^2} \qquad ...(v)$$

or $$E_n'=-\frac{2\pi^2k^2Z^2e^4m_Nm_e}{n^2h^2(m_N+m_e)} \qquad ...(vi)$$

or $$E_n'=E_n\times\frac{\mu_e}{m_e}\Rightarrow E_n=-(13.6\text{ eV})\frac{Z^2}{n^2}\left(\frac{\mu}{m}\right) \qquad ...(vii)$$

Thus, we can say that the energy of electron will be slightly less compared to what we have derived earlier. But for numerical calculations this small change can be neglected unless in a given problem it is asked to consider the effect of motion of nucleus.

**ILLUSTRATION 6.27**

A positronium "atom" is a system that consists of a positron and an electron that orbit each other. Compare the wavelengths of the spectral lines of positronium with those of ordinary hydrogen.

**Sol.** Here, the two particles have the same mass $m$, so the reduced mass is $\mu=\frac{mm}{m+m}=\frac{m^2}{2m}=\frac{m}{2}$

where $m$ is the electron mass.

We know that $E_n\propto m \Rightarrow \frac{E_n'}{E_n}=\frac{\mu}{m}=\frac{1}{2}$

Hence, energy levels of positronium atom are half of corresponding energy level in a hydrogen atom.

As a result, the wavelengths in the positronium atom spectral lines are all twice those of the corresponding lines in the hydrogen spectrum.

### ILLUSTRATION 6.28

A muon is an unstable elementary particle whose mass is $207m_e$ and whose charge is either $+e$ or $-e$. A negative muon ($\mu^-$) can be captured by a hydrogen nucleus (or proton) to form a muonic atom.

**(a)** Find the radius of the first Bohr orbit of this atom.

**(b)** Find the ionization energy of the atom.

**Sol.**

**(a)** Here $m = 207m_e$ and $M = 1836m_e$; so the reduced mass is

$$\mu = \frac{mM}{m+M} = \frac{(207m_e)(1836m_e)}{207m_e + 1836m_e} = 186\, m_e$$

According to equation $\lambda = h/p$, the orbit radius corresponding to $n = 1$ is $r_1 = \dfrac{h^2\varepsilon_0}{\pi m_e e^2} = 5.29 \times 10^{-11}$ m

Hence, the radius $r'$ that corresponds to the reduced mass $\mu$ is

$$r_1' = \left(\frac{m}{\mu}\right) r_1 = \left(\frac{m_e}{1836\, m_e}\right) r_1 = 2.85 \times 10^{-13} \text{ m}$$

The muon is 186 times closer to the proton than an electron would be in the same orbit.

**(b)** We have $E_1 = -13.6$ eV for $n = 1$

$$E_1' = \left(\frac{\mu}{m}\right) E_1 = 186 \times (-13.6) \text{ eV}$$

$$\Rightarrow \quad E_1 = -2.53 \times 10^3 \text{ eV} = -2.53 \text{ keV}$$

The ionization energy is, therefore, 2.53 keV, 186 times that for an ordinary hydrogen atom.

### ILLUSTRATION 6.29

A particle of charge equal to that of an electron, $-e$, and mass 208 times the mass of electron (called a $\mu$-meson) moves in a circular orbit around a nucleus of charge $+3e$. (Take the mass of the nucleus to be infinite.) Assuming that Bohr's model of the atom is applicable to this system:

**(a)** Derive and expression for the radius of the $n^{th}$ Bohr orbit.

**(b)** Find the value of $n$ for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.

**(c)** Find the wavelength of the radiation emitted when the $\mu$-meson jumps from the third orbit to the first orbit. (Rydberg's constant = 10967800 m)

**Sol.**

**(a)** We have radius of $n^{th}$ orbit of a hydrogen atom as

$$r_n = \frac{n^2h^2}{4\pi^2 kZe^2 m}$$

If electron is replaced by a heavy particle of mass 208 times that of the electron, then the radius is given as

$$r_n = \frac{n^2h^2}{4\pi^2 k(3)e^2(208m)} = \frac{n^2h^2}{2496\pi^2 kZe^2 m} \quad \ldots(i)$$

Here, we have not used reduced mass because it is given that mass of nucleus is assumed to be infinite and here we take $Z = 3$.

**(b)** The radius of first Bohr orbit is given as $r_1 = \dfrac{h^2}{4\pi^2 ke^2 m}$

From Eq. (i), we have $\dfrac{n^2h^2}{2496\pi^2 ke^2 m} = \dfrac{h^2}{4\pi^2 ke^2 m}$

or $\quad n^2 = 624$

or $\quad n \simeq 25$

**(c)** Rydberg constant for hydrogen-like atom is $R = \dfrac{2\pi^2 k^2 e^4 m}{ch^3}$

Now, when electron is replaced by $\mu$-meson, Rydberg constant will change as

$$R' = \frac{2\pi^2 k^2 e^4 (208m)}{ch^3}$$

or $\quad = 208R = 208 \times 1967800 \text{ m}^{-1}$

Now, the wavelength $\lambda$ of the radiation emitted when $\mu$-meson makes a transition from $n_2 = 3$ to $n = 1$ is

$$\frac{1}{\lambda} = R'Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

or $\quad \dfrac{1}{\lambda} = R' \times 9\left[\dfrac{1}{1^2} - \dfrac{1}{3^2}\right]$ or $\dfrac{1}{\lambda} = 8R'$

or $\quad \lambda = \dfrac{1}{8R'}$

$$\lambda = \frac{1}{8 \times 208 \times 10967800} \text{m}^{-1}$$

or $\quad \lambda = 0.548$ Å

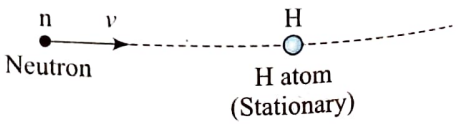
## ATOMIC COLLISION

There are two ways to excite an electron in an atom:

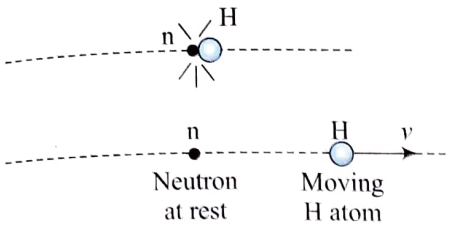
1. By supplying energy to an electron through electromagnetic photons, which we have already discussed in photoelectric effect. An electron absorbs a photon only when the photon energy is equal to the difference in energies of the two energy levels of atom otherwise the photon will not be absorbed.
2. The energy can be supplied to an electron by atomic collisions. In such collisions, assume that the loss in kinetic energy of the system is possible only if it becomes excited or gets ionized.

### COLLISION OF A NEUTRON WITH AN ATOM

To understand the energy supplied by collisions, we consider an example of a head on collision of a moving neutron with a stationary hydrogen atom as shown in figure. Here, for mathematical analysis, let us assume the masses of neutron and H atom to be same.



**Perfectly elastic collision:** Here, when perfectly elastic collision takes place, then (for equal masses) the neutron will come to rest while the hydrogen atom will move with the same speed and kinetic energy with which the neutron was moving initially as shown in figure.




**Perfectly inelastic collision:** Now, if we consider the collision to be perfectly inelastic, then after collision both the neutron and H atom will move together with speed $v_1$ which can be given by law of conservation of momentum as

$$mv_0 = 2mv_1$$

or $$v_1 = \frac{v}{2} \quad ...(i)$$

In this case, the loss of energy can be given as the difference in initial and final kinetic energies of the neutron and H atom, given as

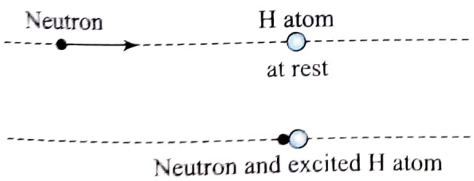
$$\Delta E = E_i - E_f$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2$$

$$= \frac{1}{2}mv^2 - \frac{1}{4}mv^2$$

$$= \frac{1}{4}mv^2 = \frac{1}{2}E_i \quad ...(ii)$$

Thus, half of the initial kinetic energy will be lost in the collision. One important point which we should understand carefully is that in collisions of elementary particles and atoms, no energy can be lost as heat because here we cannot consider the deformation of any lattice in the colliding body. The energy lost can only be absorbed by the atom involved in the collision and may get excited or ionized by this energy loss which takes place in case of inelastic collision.




In our case of collision of a neutron with an H atom, the energy loss is $(1/4)mv^2$ (half of initial energy of the neutron). This loss in energy can be absorbed by H atom only. We know that the minimum energy required to excite an H atom is 10.2 eV for $n = 1$ to $n = 2$. Thus, hydrogen atom can absorb only when this energy loss is equal to 10.2 eV. If this energy loss in perfectly inelastic, collision is more than 10.2 eV, then the H atom may absorb 10.2 eV energy for its excitation and rest of the energy will remain in the colliding particles (neutron and H atom) as their kinetic energy and the collision will not be perfectly inelastic in this case.

**ILLUSTRATION 6.30**

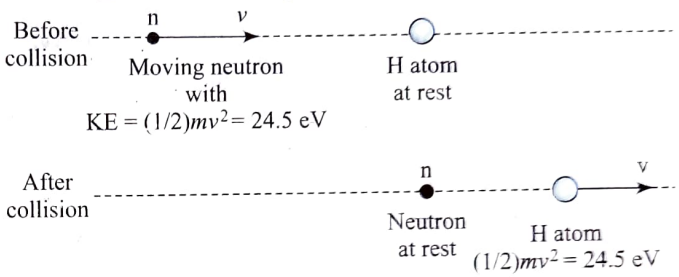
A neutron which is going to collide head on to a stationary H atom has initial kinetic energy of 24.5 eV. Discuss different possibilities of collision.

**Sol.** If perfectly inelastic collision takes place, then the maximum energy loss can be given as

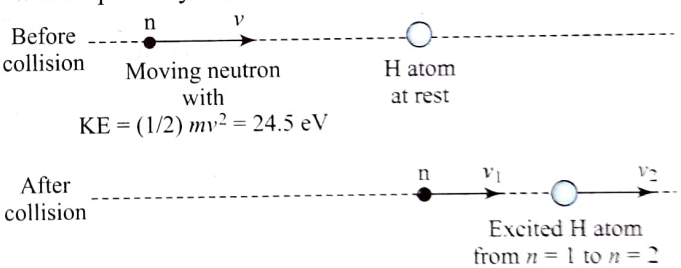
$$\Delta E_{max} = \frac{1}{2}E_i = \frac{1}{2}(24.5)\text{ eV} = 12.25\text{ eV}$$

From this energy, H atom can absorb either 10.2 eV or 12.09 eV for its excitation from $n = 1$ to $n = 2$ or from $n = 1$ to $n = 3$ energy level. Thus, in this case there may be three possibilities for collision. These are:

(a) It may be possible that the collision is perfectly elastic and no energy is absorbed by H atom as shown in figure.




(b) It may be possible that the H atom will absorb 10.2 eV energy during collision and both the neutron and H atom will be moving with kinetic energy 24.5 – 10.2 = 14.3 eV after collision as shown in figure. In this case, the collision will be partially elastic.




Final kinetic energy, $E_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 14.3\text{ eV}$

In this case, the final speeds of neutron and H atom can be obtained by using equations of law of conservation of momentum and energy. Here, by momentum conservation law, we have

$$mv = mv_1 + mv_2$$

or $$v = v_1 + v_2 \quad ...(i)$$

Using energy conservation, we have

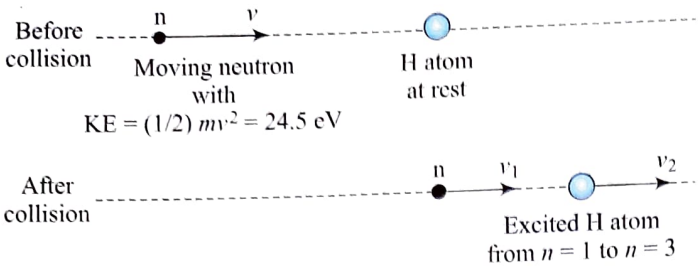
$$\frac{1}{2}mv^2 - 10.2\text{ eV} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$24.5\text{ eV} - 10.2\text{ eV} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 14.3\text{ eV} \quad ...(ii)$$

Now, solving Eqs. (i) and (ii), we will get the values of $v_1$ and $v_2$ which are the possible speeds of neutron and H atom after collision.

(c) It may be possible that H atom will absorb 12.09 eV energy during collision and both the neutron and H atom will be moving with kinetic energy 24.5 – 12.09 = 12.41 eV after collision as shown in figure. In this case, the collision will be partially elastic.




Final kinetic energy, $E_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 12.41$ eV

In this case, the final speeds of neutron and H atom can be obtained by using equations of law of conservation of momentum and energy. Here, by momentum conservation law, we have

$$mv = mv_1 + mv_2$$

or $$v = v_1 + v_2 \quad \text{...(iii)}$$

Using energy conservation, we have

$$\frac{1}{2}mv^2 - 12.09 \text{ eV} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

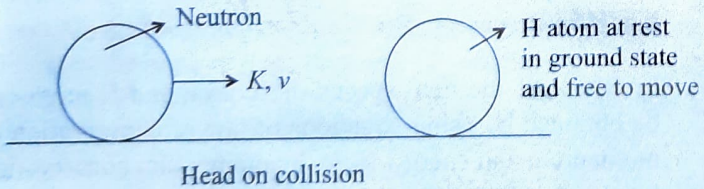
or $$24.5 \text{ eV} - 12.09 \text{ eV} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

or $$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = 12.41 \text{ eV} \quad \text{...(iv)}$$

Now, solving Eqs. (iii) and (iv), we will get values of $v_1$ and $v_2$ which are the possible speeds of neutron and H atom after collision.

**ILLUSTRATION 6.31**

In the figure, what type of collision can be possible, if $K$ = 14 eV, 20.4 eV, 22 eV, 24.18 eV (elastic/inelastic/perfectly inelastic).

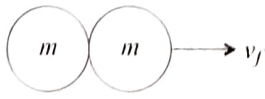



**Sol.** Loss in energy ($\Delta E$) during the collision will be used to excite the atom or electron from one level to another. According to Bohr's theory, for hydrogen atom,

$$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \ldots, 13.6 \text{ eV}\}$$

According to Newtonion mechanics, minimum loss = 0 for elastic collision.

For maximum loss, collision will be perfectly inelastic.




If neutron collides perfectly inelastically, then applying momentum conservation:

$$mv_0 = 2mv_f \Rightarrow v_f = \frac{v_0}{2}$$

$$\text{Final KE} = \frac{1}{2} \times 2m \times \frac{v_0^2}{4} = \frac{\frac{1}{2}mv_0^2}{2} = \frac{K}{2}$$

So, maximum loss $= \frac{K}{2}$

According to classical mechanics, energy loss lies in the range

$$\Delta E = \left[0, \frac{K}{2}\right]$$

(a) $K$ = 14 eV: According to quantum mechanics,
$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}\}$
According to classical mechanics, $\Delta E = [0, 7 \text{ eV}]$
So, possible loss = 0, hence it is an elastic collision.

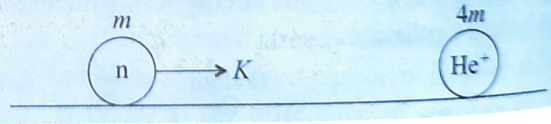
(b) $K$ = 20.4 eV: According to quantum mechanics,
$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \ldots\}$
According to classical mechanics, $\Delta E = [0, 10.2 \text{ eV}]$
If possible loss = 0 $\Rightarrow$ elastic collision,
If loss = 10.2 eV $\Rightarrow$ perfectly inelastic collision

(c) $K$ = 22 eV: According to quantum mechanics,
$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \ldots\}$
According to classical mechanics, $\Delta E = [0, 11 \text{ eV}]$
If possible loss = 0 $\Rightarrow$ elastic collision,
If loss = 10.2 eV $\Rightarrow$ inelastic collision (not perfectly)

(d) $K$ = 24.18 eV: According to quantum mechanics,
$\Delta E = \{0, 10.2 \text{ eV}, 12.09 \text{ eV}, \ldots\}$
According to classical mechanics, $\Delta E = [0, 12.09 \text{ eV}]$
Possible loss = 0 $\Rightarrow$ elastic collision,
If loss = 10.2 eV $\Rightarrow$ inelastic collision (not perfectly),
and If loss = 12.09 eV $\Rightarrow$ perfectly inelastic collision

**ILLUSTRATION 6.32**

A $He^+$ ion is at rest and is in ground state. A neutron with initial kinetic energy $K$ collides head on with the $He^+$ ion. Find minimum value of $K$ so that there can be an inelastic collision between these two particles.




**Sol.** Here, the loss during the collision can only be used to excite the atoms or electrons.

So, according to quantum mechanics, possible losses can be

$$\{0, 40.8 \text{ eV}, 48.36 \text{ eV}, \ldots, 54.4 \text{ eV}\}$$

from $$E_n = -13.6 \text{ eV}\frac{Z^2}{n^2}$$

Now, according to Newtonion mechanics: Minimum loss = 0
Maximum loss will be for perfectly inelastic collision.
Let $v_0$ be the initial speed of neutron and $v_f$ be the final common speed.

So, by momentum conservation: $mv_0 = mv_f + 4mv_f \Rightarrow v_f = \frac{v_0}{5}$

where $m$ = mass of neutron and mass of $He^+$ ion = $4m$

Final kinetic energy of system: $\text{KE} = \frac{1}{2}mv_f^2 + \frac{1}{2}4mv_f^2$

$$= \frac{1}{2}(5m)\frac{v_0^2}{25} = \frac{1}{5}\left(\frac{1}{2}mv_0^2\right) = \frac{K}{5}$$

Maximum loss $= K - \frac{K}{5} = \frac{4K}{5}$

So, loss will be $\left[0, \frac{4K}{5}\right]$ ...(ii)

For inelastic collision, there should be at least one common value other than zero in Eqs. (i) and (ii)

$\therefore \quad \frac{4K}{5} > 40.8 \text{ eV} \quad \Rightarrow \quad K > 51 \text{ eV}$

Hence, minimum value of $K$ for an inelastic collision: $K_{min} = 51$ eV.

### ILLUSTRATION 6.33

In the previous question, find minimum value of $K$ so that all types of collisions are possible.

**Sol.** For all three types of collisions, in set (i) and (ii),

$$\frac{4K}{5} = 48.36 \text{ eV} \quad \Rightarrow \quad K = 60.45 \text{ eV}$$

### ILLUSTRATION 6.34

An H atom in ground state is moving with initial kinetic energy $K$. It collides head on with $He^+$ ion in ground state kept at rest but free to move. Find minimum value of $K$ so that both the particles can excite to their first excited state.

**Sol.** Here, energy loss during the collision is used to excite the atom or ion. Now, according to quantum mechanics, loss in energy ($\Delta E$) for H atom

{0, 10.2 eV, 12.09 eV, ..., 13.06 eV}

For $He^+$ ion: {0, 40.8eV, 48.36 eV, ..., 54.4 eV}

To excite the hydrogen atom and $He^+$ ion in the first excited state, minimum energy = 40.8 + 10.2 = 51 eV

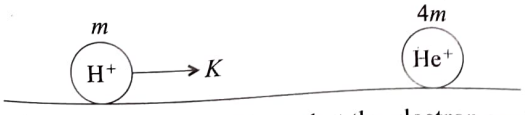
Now, according to Newtonion mechanics, minimum loss is 0 (for elastic collision). Maximum loss will be when there is perfectly inelastic collision.

Now, let mass of H atom be $m$, then mass of $He^+$ ion = $4m$.

Let $v_0$ be the initial speed of H atom and $v_f$ the final common speed. According to momentum conservation:

$$mv_0 = 4mv_f + mv_f \quad \Rightarrow \quad v_f = \frac{v_0}{5}$$

$$\text{Kinetic energy} = \frac{1}{2}(5m)\frac{v_0^2}{25} = \frac{1}{5}\left(\frac{1}{2}mv_0^2\right) = \frac{K}{5}$$




Now, for minimum value of $K$ so that the electron excite to the first excited state of H atom and $He^+$ ion.

$$\frac{4K}{5} = 51 \text{ eV}$$

or $\quad K = \frac{51 \times 5}{4} \text{ eV} \quad$ or $\quad K = 63.75 \text{ eV}$

### ILLUSTRATION 6.35

A moving hydrogen atom makes a head-on collision with a stationary hydrogen atom. Before collision, both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state.

**Sol.** Let $K$ be the kinetic energy of the moving hydrogen atom and $K'$ the kinetic energy of combined mass after collision.

From conservation of linear momentum,

$$p = p' \quad \Rightarrow \quad \sqrt{2Km} = \sqrt{2K'(2m)}$$

or $\quad K = 2K'$ ...(i)

From conservation of energy,

$$K = K' + \Delta E \quad \text{...(ii)}$$

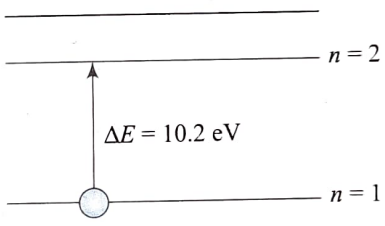
Solving Eqs. (i) and (ii), we get $\Delta E = \frac{K}{2}$

Now, minimum value of $\Delta E$ for hydrogen atom is 10.2 eV.

or $\quad \Delta E \geq 10.2 \text{ eV}$

$\therefore \quad \frac{K}{2} \geq 10.2 \text{ eV}$

$\therefore \quad K \geq 20.4 \text{ eV}$

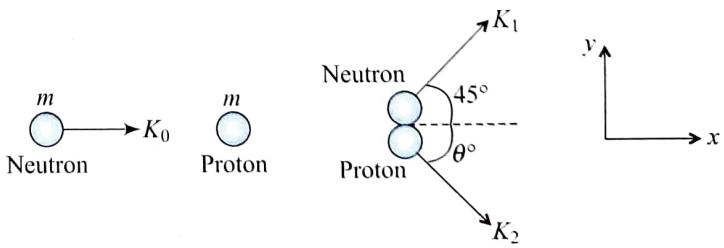



Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV.

### ILLUSTRATION 6.36

A neutron with an energy of 4.6 MeV collides elastically with protons and is retarded. Assuming that upon each collision the neutron is deflected by 45°, find the number of collisions which will reduce its energy to 0.23 eV.

**Sol.** Mass of neutron–mass of proton = $m$




From conservation of momentum in $y$-direction,

$$\sqrt{2mK_1}\ \sin 45^\circ = \sqrt{2mK_2}\ \sin\theta \quad \text{...(i)}$$

In $x$-direction,

$$\sqrt{2mK_0} = \sqrt{2mK_1}\ \cos 45^\circ + \sqrt{2mK_2}\ \cos\theta \quad \text{...(ii)}$$

Squaring and adding Eqs. (i) and (ii), we have

$$K_2 = K_1 + K_0 - \sqrt{2K_0K_1} \quad \text{...(iii)}$$

From conservation of energy,

$$K_2 = K_0 - K_1 \text{ [as collision is elastic]} \quad \text{...(iv)}$$

Solving Eqs. (iii) and (iv), we get $K_1 = \frac{K_0}{2}$

i.e., after each collision energy remains half. Therefore, after $n$ collisions,

$$K_n = K_0\left(\frac{1}{2}\right)^n$$

$$\therefore \quad 0.23 = (4.6 \times 10^6)\left(\frac{1}{2}\right)^n \Rightarrow 2^n = \frac{4.6 \times 10^6}{0.23}$$

Taking log and solving, we get $n \approx 24$

### ILLUSTRATION 6.37

Explain why nearly all H atoms are in the ground state at room temperature and hence emit no light.

**Sol.** According to kinetic theory, the average kinetic energy of atoms or molecules in a gas is given by $K = (3/2)kT$ where $k = 1.38 \times 10^{-23}$ J K$^{-1}$ is Boltzmann's constant and $T$ is Kelvin (absolute) temperature. Room temperature is about $T = 300$ K. Hence,

$$K = \frac{3}{2}(1.38 \times 10^{-23})(300) = 6.2 \times 10^{-21} \text{ J}$$

$$= \frac{6.2 \times 10^{-21}}{1.6 \times 10^{-19}} \text{ eV} = 0.04 \text{ eV}$$

The energy required to raise an electron from ground state to the next higher state is 13.6 – 3.4 = 10.2 eV. The average kinetic energy (0.04 eV) of atom is too small to excite an electron from ground state. Any atom in excited state emits photons and eventually falls to the ground state. Once in ground state, collision with other atoms can transfer energy of nearly 0.04 eV. Very high temperature is required to excite electron to upper status.

### ILLUSTRATION 6.38

Consider a hydrogen-like atom whose energy in $n$th excited state is given by

$$E_n = \frac{13.6Z^2}{n^2}$$

When this excited atom makes a transition from excited state to ground state, most energetic photons have energy $E_{max} = 52.224$ eV and least energetic photons have energy $E_{min} = 1.224$ eV.

Find the atomic number of atom and the initial state of excitation.

**Sol.** Maximum energy is liberated for transition $E_n \to 1$ and minimum energy for $E_n \to E_{n-1}$. Hence,

$$\frac{E_1}{n^2} - E_1 = 52.224 \text{ eV}$$

$$\frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224 \text{ eV}$$

Solving the above equations simultaneously, we get

$$E_1 = -54.4 \text{ eV} \quad \text{and} \quad n = 5$$

Now, $E_1 = -\frac{13.6Z^2}{1^2} = -54.4$ eV

Hence, $Z = 2$, i.e., the gas is helium, originally excited to $n = 5$ energy state.

### ILLUSTRATION 6.39

A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level $A$ and some atoms in a particular upper (excited) energy level $B$ and there are no atoms in any other energy level. The atoms of the gas make transition to higher energy level by absorbing monochromatic light of photon energy 2.7 eV.

Subsequently, the atoms emit radiations of only six different photon energies. Some of the emitted photons have energy 2.7 eV, some have energy more, and some have less than 2.7 eV.

**(a)** Find the principal quantum number of the initially excited level $B$.

**(b)** Find the ionization energy for the gas atoms.

**(c)** Find the maximum and the minimum energies of the emitted photons.

**Sol.**

**(a)** Since only six different transitions take place, the final state is $n = 4$. The energy levels of hydrogen atom are given by

$$E_n = -\frac{13.6}{n^2}\text{eV}$$

If $n_B$ is the principal quantum number of the initially excited state $B$, then

$$E_4 - E_{n_B} = -\frac{13.6}{4^2} - \left(-\frac{13.6}{n_B^2}\right) = 13.6\left[\frac{1}{n_B^2} - \frac{1}{16}\right]$$

$$E_4 - E_{n_B} = 2.7 \text{ eV}$$

$$2.7 = 13.6\left[\frac{1}{n_B^2} - \frac{1}{16}\right]$$

which gives $n_B \approx 2$ (Rounding off to nearest integer)

**(b)** The transition energy is numerically equal to the ground state energy $E_1$ of level $A$.

$$E_4 = \frac{E_1}{16}, \quad E_2 = \frac{E_1}{4}$$

$$E_4 - E_2 = \frac{E_1}{16} - \frac{E_1}{4}$$

$$2.7 \text{ eV} = -\frac{3}{16}E_1$$

$$E_1 = -14.4 \text{ eV}$$

Thus, the ionization energy of the given atom is 14.4 eV

**(c)** Maximum energy of the emitted photon is for the electron transition $n = 4$ to $n = 1$, i.e.,

$$E_4 - E_1 = \frac{E_1}{16} - E_1 = -\frac{15}{16}E_1$$

$$= \frac{15}{16} \times (14.4) = 13.5 \text{ eV}$$

Thus, the maximum energy of the emitted photon is 13.5 eV.

Minimum energy of the emitted photon corresponds to the transition $n = 4$ to $n = 3$, i.e.,

$$E_4 - E_3 = \frac{E_1}{16} - \frac{E_1}{9} = -\frac{7}{144}E_1$$

$$= -\frac{7}{144} \times (-14.4) = 0.7 \text{ eV}$$

### ILLUSTRATION 6.40

A hydrogen-like atom (atomic number $Z$) is in a higher excited state of quantum number $n$. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV, respectively.

Alternatively, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.954 eV, respectively. Determine the values of $n$ and $Z$ (ionization energy of hydrogen atom = 13.6 eV).

**Sol.** For the transition from a higher state $n$ to the first excited state $n_1 = 2$, the total energy released is

$$10.2 + 17.0 \text{ eV} \quad \text{or} \quad 27.2 \text{ eV}.$$

Thus, for $\Delta E = 27.2$ eV, $n_1 = 2$, and $n_2 = n$, we have

$$27.2 = 13.6\, Z^2\left[\frac{1}{4} - \frac{1}{n^2}\right] \qquad ...(i)$$

For the eventual transition to the second excited state $n_1 = 3$, the total energy released is (4.25 + 5.95) eV or 10.2 eV. Thus,

$$10.2 = 13.6\, Z^2\left[\frac{1}{9} - \frac{1}{n^2}\right] \qquad ...(ii)$$

Dividing Eqs. (i) and (ii), we get $\frac{27.2}{10.2} = \frac{9n^2 - 36}{4n^2 - 36}$

Solving, we get $n^2 = 36$ or $n = 6$

Substituting $n = 6$ in any of the above equations, we obtain

$Z^2 = 9$ or $Z = 3$

Thus, $n = 6$ and $Z = 3$

### ILLUSTRATION 6.41

Electrons in a hydrogen-like atom ($Z = 3$) make transitions from the fourth excited state to the third excited state and from the third excited state to the second excited state. The resulting radiations are incident on a metal plate and eject photoelectrons. The stopping potential for photoelectrons ejected by shorter wavelength is 3.95 eV.

Calculate the work function of the metal and stopping potential for the photoelectrons ejected by the longer wavelength.

**Sol.** Energy of a photon corresponding to transition from $n = 5$ (fourth excited state) to $n = 4$ (third excited state) is

$$h\nu = 13.6(3)^2\left[\frac{1}{4^2} - \frac{1}{5^2}\right] = 2.75 \text{ eV} \qquad ...(i)$$

Similarly, energy of a photon corresponding to transition from $n = 4$ to $n = 3$ is

$$h\nu = 13.6(3)^2\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = 5.95 \text{ eV} \qquad ...(ii)$$

From Einstein's photoelectric effect equation,

$$h\nu = \phi + K_{max}$$

$$K_{max} = eV_s = h\nu - \phi$$

The shorter wavelength corresponds to greater energy difference between energy levels involved in the transition. So, shorter wavelength photons are emitted for transition $n = 4$ to $n = 3$. Thus, we have

$$3.95 = 5.95 - \phi \quad \Rightarrow \quad \phi = 2 \text{ eV}$$

and for longer wavelength photon, $eV_s = 2.75 - 2 = 0.75$ eV

So, stopping potential $= \left(\frac{0.75 \text{ eV}}{e}\right) = 0.75$ V

### ILLUSTRATION 6.42

The radiation emitted when an electron jumps from $n = 3$ to $n = 2$ orbit in a hydrogen atom falls on a metal to produce photoelectrons. The electrons from the metal surface with maximum kinetic energy are made to move perpendicular to a magnetic field of (1/320) T in a radius of $10^{-3}$ m. Find **(a)** the kinetic energy of the electrons, **(b)** work function of the metal, and **(c)** wavelength of radiation.

**Sol.**

**(a)** Radius of the circle traced by a charged particle in a magnetic field is given by

$$R = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2mK}}{Bq}$$

Hence, $K = \frac{(BqR)^2}{2m} = \frac{[10^{-3} \times 1.6 \times 10^{-19} \times (1/320)]^2}{2 \times 9.1 \times 10^{-31}}$

$$= \frac{10^{-17}}{72.8} \text{J} = \frac{10^{-17}}{72.8} \times \frac{1}{1.6 \times 10^{-19}} \text{eV} = 0.86 \text{ eV}$$

**(b)** Energy of the photon emitted when the electron makes a transition from $n = 3$ to $n = 2$ energy level is

$$h\nu = E_3 - E_2 = 13.6\left[\frac{1}{2^2} - \frac{1}{3^2}\right]\text{eV} = 1.9 \text{ eV}$$

From Einstein's photoelectric effect equation,

$$h\nu = \phi + K_{max}$$

So, $\phi = (1.9 - 0.86)$eV

$= 1.04$ eV

**(c)** Wavelength of radiation is

$$\lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{1.9 \times 1.6 \times 10^{-19}}\text{m} = 6542 \text{ Å}$$

## DRAWBACKS AND LIMITATIONS OF BOHR'S THEORY

The results of the Bohr's theory agree very well with the experimentally observed facts as:

(i) The value of Rydberg constant obtained earlier agrees with the one obtained on the basis of this theory.

(ii) The spectral lines predicted by this theory were discovered experimentally by Lyman, Brackett and Pfund.

(iii) Ionisation and resonance potentials for hydrogen predicted on the basis of this theory have been verified experimentally.

(iv) A number of phenomena like X-ray production, fluorescence and phosphorescence can easily be explained on the basis of this theory.

But this is not all. This theory has its own **drawbacks and limitations** as listed below.

(a) It is not clear as to why we take only the circular orbits and not elliptical ones. In fact, an electron has also wave properties. It is, therefore, incorrect to talk of sharply defined orbits of an electron.

(b) This theory is successful only in explaining the spectra of hydrogenic atoms. It fails when applied to more complex atoms.

(c) This theory does not predict the relative intensities of various spectral lines.

(d) Even in case of hydrogen, some of the spectral lines have a fine structure, i.e., these lines are a combination of closely packed fine lines. The theory fails to explain this fine structure.

(e) Bohr's theory does not indicate whether an electron from a higher energy level (say 3) falls directly to the lower energy level 1 or successively through intermediate energy levels, i.e., from energy level 3 to 2 and then from 2 to 1.

(f) This theory cannot explain the appearance of a large number of spectral lines when the source is placed under the effect of an electric and magnetic fields **(Stark and Zeeman effects** respectively).

(g) The charge distribution of the electrons comes out to be different from that assumed in this theory.

In short, the weakest aspect of Bohr's theory is its internal logical contradictions. It is neither a consistent classical theory nor a consistent quantum one. This theory is only a transition step towards the creation of a consistent theory of atomic phenomena.

However, some elements of Bohr's theory are a lasting legacy:

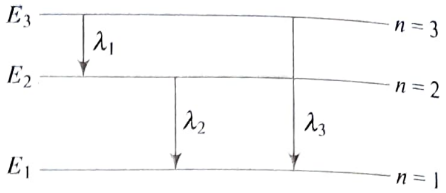
(i) angular momentum is quantised

(ii) only certain discrete energies are allowed for atomic transitions

(iii) electrons undergoing transitions between energy levels emit photons, which carry off the excess energy.

### CONCEPT APPLICATION EXERCISE 6.3

**1.** Three energy levels of an atom are shown in figure. The wavelength corresponding to three possible transitions are $\lambda_1$, $\lambda_2$, and $\lambda_3$. Find the value of $\lambda_3$ in terms of $\lambda_1$ and $\lambda_2$.




**2.** A hydrogen atom in a state of binding energy 0.85 eV makes a transition to a state of excitation energy of 10.2 eV.
**(a)** What is the initial state of the hydrogen atom?
**(b)** What is the final state of the hydrogen atom?
**(c)** What is the wavelength of the photon emitted?

**3.** Calculate **(a)** the wavelength and **(b)** the frequency of the $H_\beta$ line of the Balmer series for hydrogen.

**4.** Find the largest and shortest wavelengths in the Lyman series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

**5.** An imaginary particle has a charge equal to that of an electron and mass 100 times the mass of the electron. It moves in a circular orbit around a nucleus of charge $+4e$. Take the mass of the nucleus to be infinite. Assuming that Bohr's model is applicable to this system:
**(a)** Derive an expression for the radius of $n^{th}$ Bohr orbit.
**(b)** Find the wavelength of the radiation emitted when the particle jumps from fourth orbit to second orbit.

**6.** Hydrogen gas in the atomic state is excited to an energy level such that the electrostatic potential energy of H atom becomes $-1.7$ eV. Now, the photoelectric plate having $W = 2.3$ eV is exposed to the emission spectra of this gas. Assuming all the transitions to be possible, find the minimum de Broglie wavelength of ejected photoelectrons.

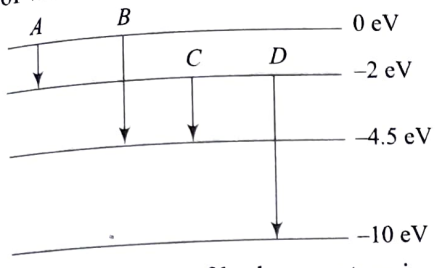
**7.** Taking into account the motion of the nucleus of a hydrogen atom, find the expressions for the electron's binding energy in the ground state and for the Rydberg constant. How much (in percent) do the binding energy and the Rydberg constant, obtained without taking into account the motion of the nucleus, differ from the more accurate corresponding values of these quantities?

**8.** An electron having energy 20 eV collides with a hydrogen atom in the ground state. As a result of the collision, the atom is excited to a higher energy state and the electron is scattered with reduced velocity. The atom subsequently returns to its ground state with emission of radiation of wavelength $1.216 \times 10^{-7}$ m. Find the velocity of the scattered electron.

**9.** According to classical physics, an electron in periodic motion will emit electromagnetic radiation with the same frequency as that of its revolution. Compute this value

for hydrogen atom in $n$th quantum state. Under what conditions does Bohr's quantum theory permit emission of such photons due to transitions between adjoining orbits? Discuss the result obtained.

10. The energy levels of an atom are as shown in figure. Which one of these transitions will result in the emission of a photon of wavelength 275 nm?




11. The ground state energy of hydrogen atom is –13.6 eV.
   (a) What is the kinetic energy of an electron in the second excited state?
   (b) If the electron jumps to the ground state from the second excited state, calculate the wavelength of the spectral line emitted.

12. Suppose that means were available for stripping 28 electrons from $_{29}Cu$ in vapour of this metal.
   (a) Compute the first three energy levels for the remaining electron.
   (b) Find the wavelengths of the spectral lines of the series for which $n_1 = 1$, $n_2 = 2, 3, 4$. What is the ionization potential for the last electron?

**ANSWERS**

1. $\frac{\lambda_1\lambda_2}{\lambda_2+\lambda_2}$ **2.** (a) 4 (b) 2 (c) 4860 Å

3. (a) $4.9 \times 10^{-7}$ m (b) $6.12 \times 10^{14}$ Hz

4. 215 Å, 911 Å; UV region

5. (a) $r_n = \frac{n^2h^2\varepsilon_0}{400\pi me^2}$ (b) 3.03 Å

6. 3.8 Å **7.** 0.055% **8.** $1.86 \times 10^6$ m $s^{-1}$

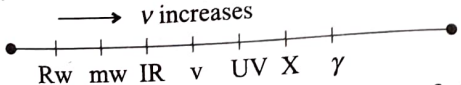
10. Transition $B$ **11.** (a) 1.51 eV (b) 1.027 Å

12. –11.44 keV, –2.86 keV, –1.27 keV; 11.44 kV

## X-RAYS

X-rays are electromagnetic radiations of very short wavelength (0.1 Å to 100 Å) and high energy which are emitted when fast moving electrons or cathode rays strike a target of high atomic mass. These rays are invisible to eye. They are electromagnetic waves and have speed $c = 3 \times 10^8$ m $s^{-1}$ in vacuum.

Its photons have energy around 1000 times more than the visible light.



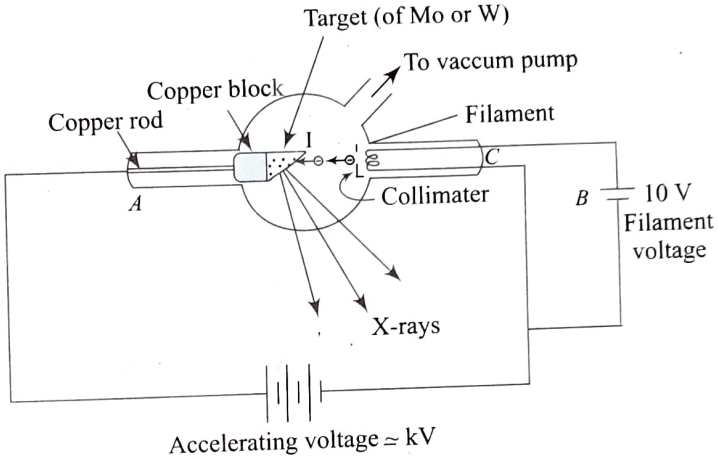

When fast-moving electrons having energy of the order of several keV strike the metallic target, then X-rays are produced.

### DISCOVERY OF X-RAYS

X-rays were discovered by Roentgen (1895) who found that a discharge tube, operating at low pressure and high voltage, emitted a radiation that caused a fluorescent screen in the neighborhood to glow brightly. Crystals of barium platinocyanide also showed fluorescence. Results were same if the discharge tube was wrapped in black paper to prevent visible light. This indicated that some unknown radiation (X-rays) were responsible for fluorescence. Roentgen then confirmed that X-rays are emitted when cathode rays (electrons) strike the wall of discharge tube.

### PRODUCTION OF X-RAYS (COOLIDGE'S TUBE)

X-rays are produced when energetic (fast moving) electrons strike a target such as a metal piece. When electrons collide with the atoms of solid, they loose their kinetic energy which is converted into radiant energy in the form of X-rays. Figure shows the essential features of a modern X-ray tube developed by Coolidge.




Coolidge's X-ray tube consists of a glass bulb exhausted to nearly perfect vacuum. The cathode $C$ is the source of electrons by using a heated filament getting supply from battery $B$. The anode is made of solid copper bar $A$. A high melting metal like platinum or tungsten is embedded at the end of copper rod and serves as target $T$. A high dc voltage $V$(50 kV) is maintained between cathode and anode.

The melting point, specific heat capacity, and atomic number of the target should be high. When voltage is applied across the filament, then the filament on being heated emits electrons from it. For giving the beam shape to electrons, collimator is used. Now when the electrons strike the target, then X-rays are produced.

When electrons strike with the target, some part of energy is lost and converted into heat. Since the target should not melt or it should absorb the heat, so the melting point and specific heat of the target should be high.

Here a copper rod is attached so that heat produced can go behind and it can absorb heat and the target does not get heated very high.

For more energetic electrons, accelerating voltage is increased.

For more number of photons, voltage across the filament is increased.

The energetic electrons strike the target and the X-ray are produced. Only about 1–10% of the energy of the electrons is converted to X-rays and the rest is converted into heat. The target $T$ as a result becomes very hot and therefore should have high melting point. The heat generated is dissipated through the copper rod and the anode is cooled by water flowing through the anode.

The nature of emitted X-rays depends on:

1. the current in the filament $F$, and
2. the voltage between the filament and the anode.

(a) An increase in the filament current increases the number of electrons it emits. Larger number of electrons means an intense beam of X-rays is produced. This way we can control the quantity of X-rays, i.e., intensity of X-rays.

(b) An increase in the voltage of the tube increases the kinetic energy of electrons [$eV = (1/2)\ mv^2$]. When such highly energetic beam of electrons is suddenly stopped by the target, an energetic beam of X-rays is produced. This way we can control the quality of X-rays, i.e., penetration power of X-rays.

(c) Based on penetrating power, X-rays are classified into two types: Hard X-rays and Soft X-rays. The ones having high energy and hence high penetration power are Hard X-rays and the ones with low energy and hence low penetration power are Soft X-rays.

## PROPERTIES OF X-RAYS

1. These are highly penetrating rays and can pass through several materials which are opaque to ordinary light.
2. They ionize the gas through which they pass. While passing through a gas, they knock out electrons from several of the neutral atoms, leaving these atoms with +ve charge.
3. They cause fluorescence in several materials. A plate coated with barium platinocyanide, ZnS (zinc sulphide), etc. becomes luminous when exposed to X-rays.
4. They affect photographic plates especially designed for the purpose.
5. They are not deflected by electric and magnetic fields, showing that they are not charged particles.
6. They show all the properties of the waves except refraction. They show diffraction patterns when passed through a crystal which behaves like a grating.

## APPLICATIONS OF X-RAYS

X-rays have important and useful applications in surgery, medicine, engineering, and studies of crystal structures.

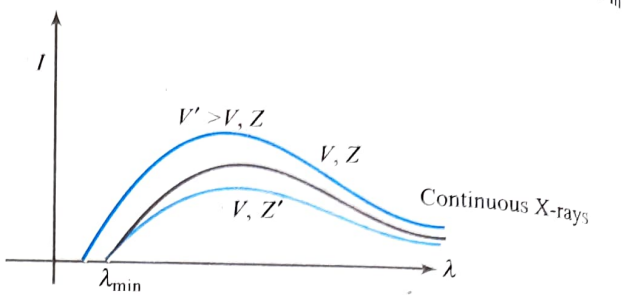
**Scientific applications:** The diffraction of X-rays at crystals opened new dimension to X-rays crystallography. Various diffraction patterns are used in determining internal structure of crystals. The spacing and positions of atoms of a crystal can be precisely determined using Bragg's law.

**Industrial applications:** Since X-rays can penetrate through various materials, they are used in industry to detect defects in metallic structures of big machines, railway tracks and bridges. X-rays are used to analyze the composition of alloys and pearls.

**In radio therapy:** X-rays can cause damage to the tissues of body (cells are ionized and molecules are broken). So, X-rays damage the malignant growths such as cancer and tumors which are dangerous to life, when it is used in proper and controlled intensities.

**In medicine and surgery:** X-rays are absorbed more in heavy elements than the lighter ones. Since bones (containing calcium and phosphorus) absorb more X-rays than the surrounding tissues (containing light elements such as H, C, O), their shadow is casted on the photographic plate. So, the cracks or fracture in bones can be easily located. Similarly, intestine and digestive system abnormalities are also detected by X-rays.

Variation of intensity of X-rays with $\lambda$ is plotted as shown in figure.




The minimum wavelength corresponds to the maximum energy of the X-rays which in turn is equal to the maximum kinetic energy (eV) of the striking electrons. Thus,

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV} = \frac{12400}{V_{(\text{in volts})}}\ \text{Å}$$

We see that cut-off wavelength $\lambda_{min}$ depends only on accelerating voltage applied between the target and filament. It does not depend upon material of the target, it is same for two different metals ($Z$ and $Z'$).

### ILLUSTRATION 6.43

An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.

**Sol.** Kinetic energy acquired by the electron is $K = 20 \times 10^3$ eV. The energy of the photon is $0.05 \times 20 \times 10^3$ eV $= 10^3$ eV

Thus, $\frac{hc}{\lambda} = 10^3$ eV

$$\Rightarrow \quad \lambda = \frac{(4.14 \times 10^{-15}\ \text{eV-s}) \times (3 \times 10^8\ \text{m s}^{-1})}{10^3\ \text{eV}}$$

$$= \frac{1242\ \text{eV-nm}}{10^3\ \text{eV}} = 1.24\ \text{nm}$$

### ILLUSTRATION 6.44

An X-ray tube operates at 20 kV. Find the maximum speed of the electrons striking the anode, given the charge of electron is $1.6 \times 10^{-19}$ coulomb and mass of electron is $9 \times 10^{-31}$ kg.

**Sol.** When an electron of charge $e$ is accelerated through a potential difference $V$, it acquires energy $eV$. If $m$ be the mass of the electron and $v_{max}$ the maximum speed of electron, then

$$\frac{1}{2}mv^2_{max} = eV \text{ or } v_{max} = \sqrt{\left(\frac{2eV}{m}\right)}$$

Substituting the given values, we get

$$v_{max} = \sqrt{\left(\frac{2 \times (1.6 \times 10^{-19}) \times 20{,}000}{9 \times 10^{-31}}\right)} = 8.4 \times 10^7\ \text{m s}^{-1}$$

**ILLUSTRATION 6.45**

The voltage applied to an X-ray tube being increased $\eta = 1.5$ times, the short wave limit of an X-ray continuous spectrum shifts by $\Delta\lambda = 26$ pm. Find the initial voltage applied to the tube.

**Sol.** $\lambda_{min} = \dfrac{ch}{eV}$

Here, $\lambda_1 = \dfrac{ch}{eV_1}$ and $\lambda_2 = \dfrac{ch}{eV_2}$

or $$\frac{ch}{e}\left[\frac{1}{V_2} - \frac{1}{V_1}\right] = \lambda_2 - \lambda_1 = \Delta\lambda = 26 \times 10^{-12}$$

or $$\frac{ch}{e}\left[\frac{2}{3V_1} - \frac{1}{V_1}\right] = 26 \times 10^{-12} \qquad (\because \quad V_2 = 1.5\, V_1)$$

Solving for $V_1$, we get $V_1 = 16000$ V or 16 kV

## X-RAY ABSORPTION

The intensity of X-rays at any point may be defined as the energy falling per second per unit area held perpendicular to the direction of energy flow. The intensity of an X-ray beam decreases during its passage through the sheet of any material. The decrease in the intensity of X-rays is due to the absorption of X-rays by the material.

Let $I_0$ be the intensity of incident beam and $I$ be the intensity of beam after penetrating a thickness $x$ of a material, then $I = I_0 e^{-mx}$; where $m$ is the coefficient of absorption or absorption coefficient of the material. The absorption coefficient depends upon wavelength of X-rays, density of material, and atomic number of material. The elements of high atomic mass and high density absorb X-rays to a higher degree.

## X-RAY SPECTRA AND ORIGIN OF X-RAYS

Experimental observation and studies of spectra of X-rays reveal that X-rays are of two types and so are their respective spectra: characteristic X-rays and continuous X-rays.

### Characteristic X-Rays

These X-rays are called characteristic X-rays because they are characteristic of the element used as target anode. Characteristic X-rays have a line spectral distribution unlike the continuous X-rays. The wavelength spectrum of the X-frequencies corresponding to these lines are the characteristic of the material or the target, i.e., anode material.

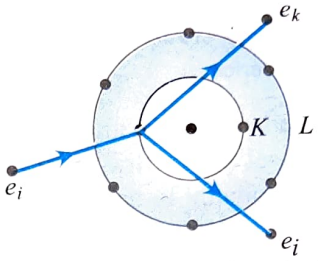
The spectrum of this group consists of several radiations with specific sharp wavelengths and frequency similar to the spectrum (line) of atoms like hydrogen. The wavelengths of this group show characteristic discrete radiations emitted by the atoms of the target material. The characteristic X-rays spectra help us to identify the element of target material.

#### *Origin of Characteristic X-Rays*

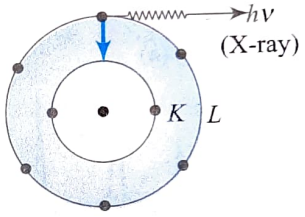
1. When the atoms of the target material are bombarded with high energy electrons (or hard X-rays), which possess enough energy to penetrate into the atom, they knockout the electron of inner shell (say $K$ shell, $n = 1$). When an electron is missing in the $K$ shell, an electron from next upper shell makes a quantum jump to fill the vacancy in the $K$ shell. In the transition process, the electron radiates energy whose frequency lies in the X-ray region. The frequency of emitted radiation (i.e., of photon) is given by

$$\nu = RZ_e^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where $R$ is constant and $Z_e$ is effective atomic number. Generally, $Z_e$ is taken to be equal to $Z - s$, where $Z$ is proton number or atomic number of the element and $s$ is called the screening constant. Due to the presence of other electrons, the charge of the nucleus as seen by the electron will be different in different shells.

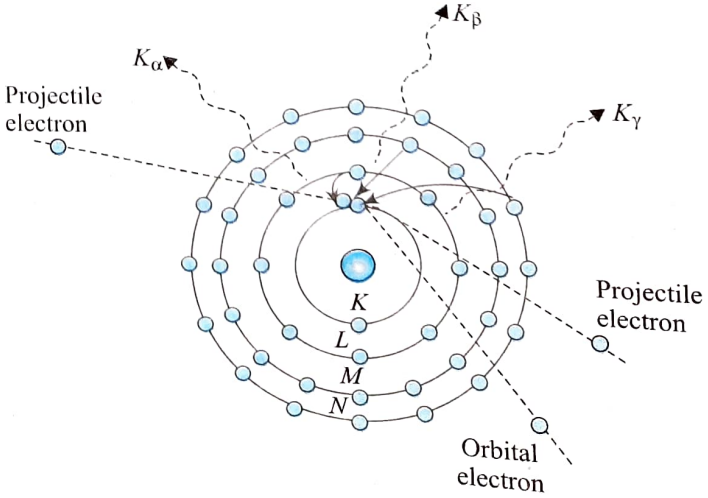



**(a) Knocking out $e'$ of $K$ shell by incident electron $e_i$**

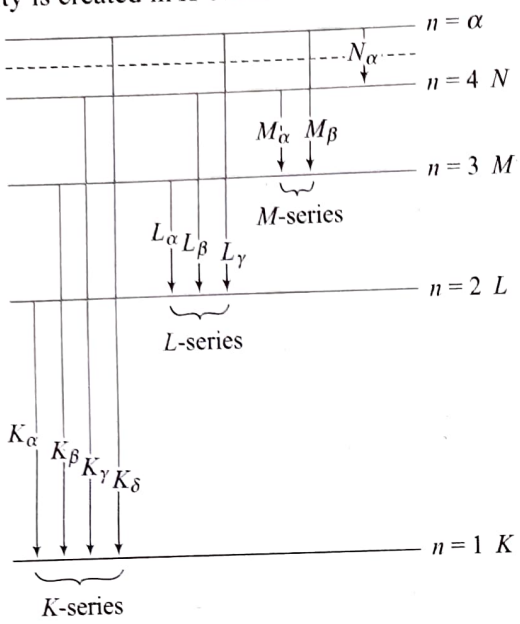



**(b) Emission of X-ray photon ($K_\alpha$ series)**

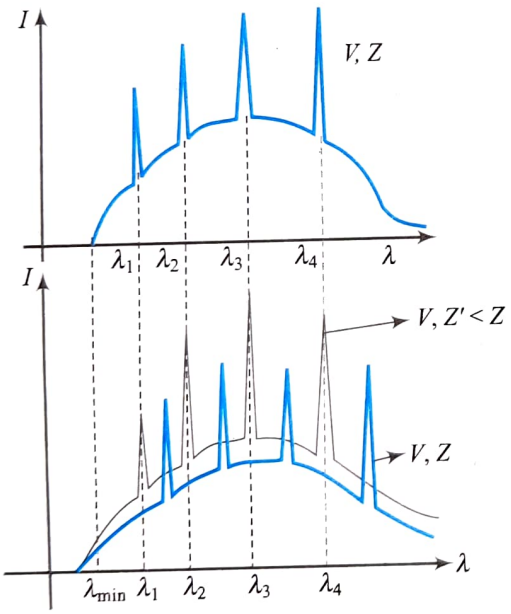
2. Another vacancy is now created in the $L$ shell which is again filled up by another electron jump from one of the upper shell ($M$) which results in the emission of another photon, but of different X-ray frequency. This transition continues till outer shells are reached, thus, resulting in the emission of series of spectral lines.
3. The transitions of electrons from various outer shells to the inner most $K$ shell produces a group of X-ray lines called as $K$-series. These radiations are most energetic and most penetrating. $K$-series is further divided into $K_\alpha$, $K_\beta$, $K_\gamma$,...depending upon the outer shell from which the transition is made (see figure).



4. If cavity is created in $L$ shell, then according to the transition of electrons from high orbits there will be $L_\alpha$, $L_\beta$, ... lines, and this is called $L$-series. There may also be $M$-series as shown in Fig. (a). Figure (b) shows the wavelength spectrum for the characteristic X-rays when cavity is created in $K$-shell.




(a)

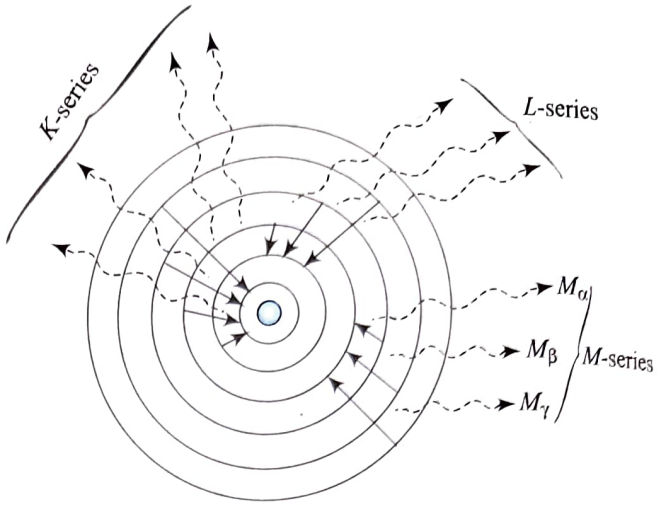



(b)

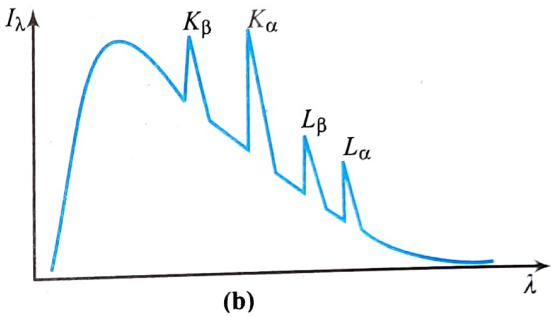
The sharp peaks obtained in graph are known as characteristic X-rays because they are characteristic of the target material.

The characteristic wavelengths of the material having atomic number $Z$ are called **characteristic X-rays** and the spectrum obtained is called **characteristic spectrum**. If a target material of atomic number $Z'$ is used, then peaks are shifted.

Thus, the characteristic lines of the characteristic X-rays depend only on the target material and not on the accelerating voltage. One more thing we can see here that the characteristic X-rays are emitted only when the projectile electron makes a collision with another bound electron of the atom of the target material. When cathode rays pass through the target, then the probability of a projectile electron to collide with the bound electron is very less. Majority of the projectile electrons will pass through the anode without making the collision and they produce continuous X-rays. So, always both types of X-rays, continuous and characteristic, will be emitted. X-rays of only a single type cannot be emitted.



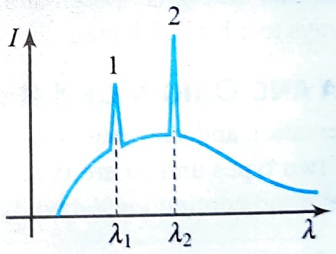

(a)




(b)

**ILLUSTRATION 6.46**

In figure, find which is $K_\alpha$ and $K_\beta$.




**Sol.** $\Delta E = \dfrac{hc}{\lambda}, \lambda = \dfrac{hc}{\Delta E}$

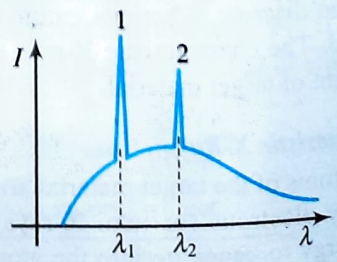
Since energy difference of $K_\alpha$ is less than $K_\beta$,

i.e., $\Delta E_{K_\alpha} < \Delta E_{K_\beta}$, therefore, $\lambda_{K_\alpha} > \lambda_{K_\beta}$

So, peak 1 is $K_\beta$ and peak 2 is $K_\alpha$

**ILLUSTRATION 6.47**

In figure, find which is $K_\alpha$ and $L_\alpha$.




**Sol.** $\Delta E_{K_\alpha} > \Delta E_{L_\alpha} \Rightarrow \lambda_{K_\alpha} < \lambda_{L_\alpha}$, so peak 1 is $K_\alpha$ and peak 2 is $L_\alpha$

**ILLUSTRATION 6.48**

The $K_\alpha$ X-ray emission line of tungsten occurs at $\lambda = 0.021$ nm. What is the energy difference between $K$ and $L$ levels in this atom?

**Sol.** The origin of $K_\alpha$ X-ray can be understood on the basis of the Bohr model of atom. If there is only one electron in the $K$ shell, and fill up this vacancy. An electron from higher shell will get de-excited. In this process of de-excitation of electron, a photon is emitted during this transition which is the X-ray.

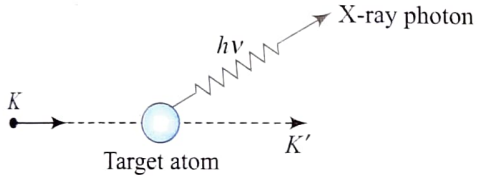
$$\Delta E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{0.021 \times 10^{-9}}$$

$$= 9.46 \times 10^{-15}\ \text{J}$$

$$= 5.9 \times 10^{4}\ \text{eV} = 59\ \text{keV}$$

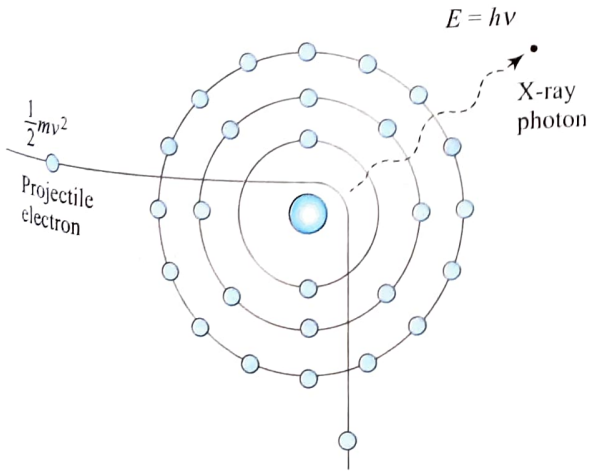
## Continuous X-Rays

In addition to characteristic X-rays, tubes emit a continuous spectrum also. The characteristic line spectra is superimposed on a continuous X-rays spectra of varying intensities. The wavelengths of the continuous X-rays spectra are independent of the material. One important feature of continuous X-rays is that they end abruptly at a certain lower wavelength for a given voltage.




## Origin of Continuous X-Rays

The mechanism for the production of continuous X-rays can be explained on the basis of figure. It shows an atom of the anode material of high atomic weight with its electron configuration. In the Coolidge tube, an electron is projected toward the anode with an accelerating voltage $V$. So, the kinetic energy of the projectile electron will be $eV$. As shown in the figure, when the projectile electron enters into the extremely high electric field of the nucleus of the atom of the anode material, it experiences strong electric force toward the nucleus of the atom and due to this strong attraction the velocity of this electron, when it emerge from the atom, will be highly reduced and negligible compared with the initial velocity of the projectile electron. This electron, in the influence of the highly positive nucleus, experiences a very high acceleration and according to the classical theory every accelerated charge particle emits the electromagnetic radiations, so this electron will also emit electromagnetic radiations. These electromagnetic radiations are called X-rays. According to the law of conservation of energy, the energy of these electromagnetic radiations will be equal to the decrease in the kinetic energy of the projectile electron. This amount can be calculated.




If the initial velocity of the projectile electron is $v$, then as the velocity is gained due to the accelerating voltage $V$, we have

$$eV = \frac{1}{2}mv^2 \qquad \ldots(\text{i})$$

or $$v = \sqrt{\frac{2eV}{m}} \qquad \ldots(\text{ii})$$

When this electron comes out from the atom of anode material, the speed of this electron will be very less as compared to its initial speed. Thus, the difference of kinetic energy of this electron is emitted in the form of an X-ray photon from the anode atom.

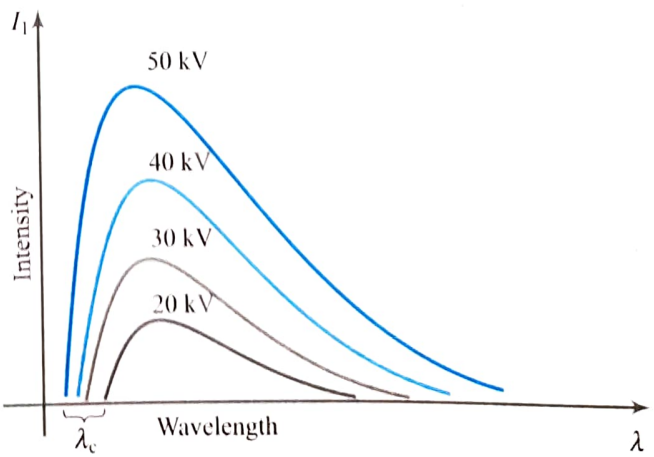
The electrons which pass through the atom very close to the nucleus, will be more accelerated and the energy corresponding to the electrons will be more as compared to those electrons which pass through the atom at relatively large distance from the nucleus. The maximum energy of X-ray photon will be corresponding to that electron which loses almost all of its energy during passing through the atom. The photon corresponding to this electron will have the shortest wavelength among all the photons radiated by other electrons. If this shortest wavelength is $\lambda_c$, then we have

$$\Delta E = \frac{1}{2}mv^2 = eV = \frac{hc}{\lambda_c}$$

or $$\lambda_c = \frac{hc}{eV} = \frac{12431}{V}\ \text{Å} \qquad \ldots(\text{iii})$$

This is the minimum wavelength of X-rays emitted from an X-ray tube. Thus, from Eq. (iii), we can see that the maximum energy or minimum wavelength of X-rays emitted depends only on the potential difference applied across the discharge tube.

Thus, we can obtain X-rays in any range $\lambda_c$ to $\infty$ by applying an appropriate voltage across the discharge tube, which will fix $\lambda_c$ and other photons emitted from the tube will have wavelengths more than $\lambda_c$ and ranging up to $\infty$. That is why these X-rays are called continuous X-rays as shown in figure.



As we can see in the graph, the intensity of emitted X-rays will be maximum (maximum number of photons) for a particular value of wavelength at a particular accelerating voltage across the discharge tube. At a particular voltage, the intensity of X-rays can be varied by changing the current in the circuit because the intensity of X-rays (number of photons) is proportional to the number of electrons attacking the anode. The broad continuous spectrum beyond the peak intensity is referred as "Bremsstrahlung."

### ILLUSTRATION 6.49

When 0.50 Å X-rays strike a material, the photoelectrons from the $K$ shell are observed to move in a circle of radius 23 mm in a magnetic field of $2 \times 10^{-2}$ tesla acting perpendicular to direction of emission of photoelectrons. What is the binding energy of $K$-shell electrons?

**Sol.** The velocity of the photoelectrons is found by the relation:

$$evB = m\frac{v^2}{R} \quad \text{or} \quad v = \frac{e}{m}BR$$

The kinetic energy of the photoelectrons is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{e^2B^2R^2}{m}$$

$$= \frac{1}{2}\frac{(1.6\times10^{-19})^2(2\times10^{-2})^2(23\times10^{-3})^2}{(9.1\times10^{-31})}$$

$$= 2.97 \times 10^{-15}\ \text{J}$$

$$= (2.97\times10^{-15})\frac{1}{1.6\times10^{-19}} = 18.36\ \text{KeV}$$

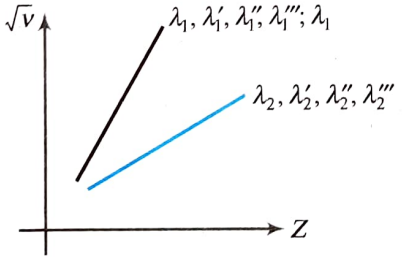
The energy of the incident photon is $E_v = \frac{hc}{\lambda} = \frac{12.4}{0.50} = 24.8$ KeV

The binding energy is the difference between these two values:

$$\text{BE} = E_v - K = 24.8 - 18.6 = 6.2\ \text{keV}$$

## MOSELEY'S LAW

Moseley measured the frequencies of characteristic X-rays for a large number of elements and plotted the square root of frequency against position number in periodic table. He discovered that the plot is very close to a straight line not passing through origin. The relation of straight line is expressed as $\sqrt{\nu} = a(Z-b)$, where $a$ and $b$ are constants. This relation is called **Moseley's law**. It helps to determine the atomic number $Z$ of an atom. Here, $b$ is the screening constant.




Moseley's observations can be mathematically expressed as $\sqrt{\nu} = a(Z-b)$, where $a$ and b are positive constants for one type of X-rays and for all elements (independent of $Z$).

Moseley's law can be derived on the basis of Bohr's theory of atom.

Frequency of X-rays is given by $\sqrt{\nu} = \sqrt{CR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}(Z-b)$

(For multielectron system)

$b$ is known as the screening constant or shielding constant, and $(Z-b)$ is the effective nuclear charge.

For $K_\alpha$ line: $n_1 = 1, n_2 = 2$, therefore $\sqrt{\nu} = \sqrt{\frac{3RC}{4}}(Z-b) = a(Z-b)$

Here $a = \sqrt{\frac{3RC}{4}}$, $\quad [b = 1$ for $K_\alpha$ lines$]$

### ILLUSTRATION 6.50

If the $K_\alpha$ radiation of Mo ($Z = 42$) has a wavelength of 0.71 Å, calculate wavelength of the corresponding radiation of Cu, i.e., $K_\alpha$ for Cu ($Z = 29$) assuming $b = 1$.

**Sol.** According to Moseley's law: $\sqrt{\nu} = a(Z-1)$

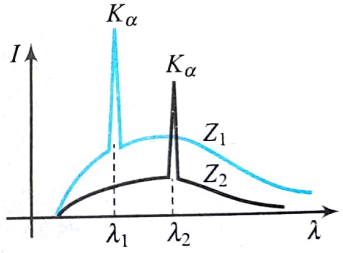
$\Rightarrow \quad (Z-1)^2 \propto \nu$ or $(Z-1)^2 \propto 1/\lambda$

$$\Rightarrow \quad \frac{(Z_{\text{Mo}}-1)^2}{(Z_{\text{Cu}}-1)^2} = \frac{\lambda_{\text{Cu}}}{\lambda_{\text{Mo}}}$$

or $\quad \lambda_{\text{Cu}} = \lambda_{\text{Mo}}\frac{(Z_{\text{Mo}}-1)^2}{(Z_{\text{Cu}}-1)^2} = 0.71 \times \left(\frac{41}{28}\right)^2 = 1.52$ Å

### ILLUSTRATION 6.51

Compare $Z_1$ and $Z_2$.




**Sol.** $\sqrt{\nu} \equiv \sqrt{cR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \cdot (Z-b)$

If $Z$ is greater, then $\nu$ will be greater and hence $\lambda$ will be less. Since $\lambda_1 < \lambda_2$, $Z_1 > Z_2$.

### ILLUSTRATION 6.52

A cobalt target ($Z = 27$) is bombarded with electrons and the wavelengths of its characteristic spectrum are measured. A second, fainter, characteristic spectrum is also found because of an impurity in the target. The wavelength of the $K_\alpha$ lines are 178.9 pm (cobalt) and 143.5 pm (impurity). What is the impurity?

**Sol.** Using Moseley's law and putting $c/\lambda$ for $\nu$ (and assuming $b = 1$), we obtain

$$\sqrt{\frac{c}{\lambda_{\text{Co}}}} = aZ_{\text{Co}} - a \quad \text{and} \quad \sqrt{\frac{c}{\lambda_x}} = aZ_x - a$$

Dividing yields $\sqrt{\frac{\lambda_{\text{Co}}}{\lambda_x}} = \frac{Z_x - 1}{Z_{\text{Co}} - 1}$

Substituting gives us $\sqrt{\frac{178.9\text{ pm}}{143.5\text{ pm}}} = \frac{Z_x - 1}{27 - 1}$

Solving for the unknown, we find $Z_x = 30.0$; the impurity is zinc.

### ILLUSTRATION 6.53

Find the constants $a$ and $b$ in Moseley's equation $\sqrt{\nu} = a(Z - b)$ from the following data.

| Element | Z | Wavelength of $K_\alpha$ X-ray |
|---|---|---|
| Mo | 42 | 71 pm |
| Co | 27 | 178.5 pm |

**Sol.** Moseley's equation is $\sqrt{\nu} = a(Z - b)$

Thus, $\sqrt{\frac{c}{\lambda_1}} = a(Z_1 - b)$ ...(i)

and $\sqrt{\frac{c}{\lambda_2}} = a(Z_2 - b)$ ... (ii)

From Eqs. (i) and (ii), $\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right) = a(Z_1 - Z_2)$

or $a = \frac{\sqrt{c}}{(Z_1 - Z_2)}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right)$

$= \frac{(3\times10^8)^{1/2}}{42 - 27}\left[\frac{1}{(71\times10^{-12})^{1/2}} - \frac{1}{178.5\times10^{-12})^{1/2}}\right]$

$= 5.0 \times 10^7\ (\text{Hz})^{1/2}$

Dividing Eq. (i) by Eq. (ii), $\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b}$ or $\sqrt{\frac{178.5}{71}} = \frac{42 - b}{27 - b}$

or $b = 1.37$

### ILLUSTRATION 6.54

An X-ray tube with a copper target is found to be emitting lines other than those due to copper. The $K_\alpha$ line of copper is known to have a wavelength 1.5405 Å and the other two $K_\alpha$ lines observed have wavelengths 0.7090 Å and 1.6578 Å. Identify the impurities.

**Sol.** According to Moseley's equation for $K_\alpha$ radiation,

$\frac{1}{\lambda} = R(Z - 1)^2\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$ ($\lambda$ = wavelength corresponding to Cu)

Let $\lambda_1$ and $\lambda_2$ be the two other unknown wavelengths, then

$$\frac{\lambda_1}{\lambda} = \frac{(Z-1)^2}{(Z_1-1)^2} = \frac{0.7092}{1.5405}$$

For copper, $Z = 29$, therefore, $(Z_1 - 1) = 28\sqrt{\frac{1.5405}{0.7092}} = 41$

or $Z_1 = 42$ (Molybdenum)

Similarly, $\frac{\lambda_2}{\lambda} = \frac{(28)^2}{(Z_2 - 1)^2} = \frac{1.6578}{1.5405}$

$$(Z_2 - 1) = 28\sqrt{\left(\frac{1.5405}{1.6578}\right)}$$

or $(Z_2 - 1) = 27$ or $Z_2 = 28$ (Nickel)

So, the impurities are molybdenum and nickel.

### ILLUSTRATION 6.55

The $K$-absorption edge of an unknown element is 0.171 Å.

**(a)** Identify the element.

**(b)** Find the average wavelengths of the $K_\alpha$, $K_\beta$, and $K_\gamma$ lines.

**(c)** If a 100 eV electron strikes the target of this element, what is the minimum wavelength of the X-ray emitted?

**Sol.** From Moseley's law, the wavelength of $K$ series of X-rays is given by taking modified Rydberg's formula given as

$\frac{1}{\lambda} = R(Z-1)^2\left(1 - \frac{1}{n^2}\right)$ for $K$ lines, where $n = 2, 3, 4, \ldots$

**(a)** For $K$-absorption edge, we put $n = \infty$ in above expression to get

$(Z - 1) = \sqrt{\frac{1}{\lambda R}}$

or $Z = \sqrt{\frac{1}{(0.171\times10^{-10})(1.097\times10^7)}} + 1 = 74$

The element is tungsten.

**(b)** For $K_\alpha$ line: $\frac{1}{\lambda_{K_\alpha}} = R(74-1)^2\left[1 - \frac{1}{2^2}\right]$

$\Rightarrow \lambda_{K_\alpha} = 0.228$ Å

For $K_\beta$ line: $\frac{1}{\lambda_{K_\beta}} = R(74-1)^2\left[1 - \frac{1}{3^2}\right]$

$\Rightarrow \lambda_{K_\beta} = 0.192$ Å

For $K_\gamma$ line: $\frac{1}{\lambda_{K_\gamma}} = R(74-1)^2\left[1 - \frac{1}{4^2}\right]$

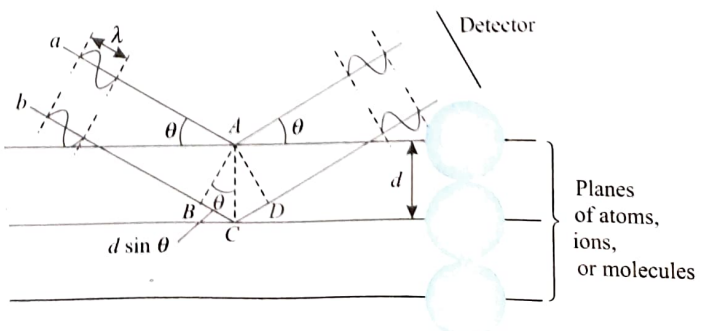
$\Rightarrow \lambda_{K_\gamma} = 0.182$ Å

**(c)** The shortest wavelength corresponding to an electron with kinetic energy 100 eV is given by

$\lambda_c = \frac{hc}{E} = \frac{12431}{100}$ Å $= 124.31$Å

## BRAGG'S LAW

X-rays are electromagnetic waves of high energy with short wavelengths and may be diffracted by suitable diffracting obstacle. However, the diffraction effects are appreciable only when the diffracting apertures are of the order of the wavelength, i.e., of the order of 1 Å. This is almost the size of an atom and it is difficult to construct slits with such small gaps so that X-rays can be appreciably diffracted.



In case of solid crystals, atoms are arranged in fairly regular pattern with interatomic gaps of the order of 1 Å. Almost all the metals at ordinary temperature are crystalline. These metals may act as natural three-dimensional gratings for the diffraction of X-rays.

The structure of a solid can be considered as a series of parallel planes of atoms separated by a inter planer distance $d$. Suppose, an X-ray beam is incident on a solid, making an angle $\theta$ with the planes of the atoms. These X-rays are diffracted by different atoms and the diffracted rays interfere. In certain directions, the interference is constructive and we obtain strong reflected X-rays. The analysis shows that there will be a strong reflected X-ray beam only if

$$2d \sin\theta = n\lambda \quad \ldots(i)$$

where $n$ is an integer. If the X-rays are incident at one of these angles, they are reflected; otherwise they are absorbed. When they are reflected, the laws of reflection are obeyed, i.e., (a) the angle of incidence is equal to the angle of reflection and (b) the incident ray, the reflected ray and the normal to the reflecting plane are coplanar. Equation (i) is known as Bragg's law.

By using a monochromatic X-rays beam and noting the angles of strong reflection, the interplanar spacing $d$ and several informations about the structure of the solid can be obtained.

## CONCEPT APPLICATION EXERCISE 6.4

1. State the following statements as True or False.
   (a) X-rays are electromagnetic waves because these are produced by the deceleration of fast moving electrons.
   (b) X-rays are electromagnetic waves because these produce line spectrum.
   (c) The cut-off wavelength depends on the target material.
   (d) The intensity of $K_\alpha$ radiation is more than that of $K_\beta$.
   (e) The frequency of $K_\alpha$ radiation is more than that of $K_\beta$.
2. Calculate the cut-off wavelength of the X-ray emitted by a Coolidge tube operating at 40 kV.
3. Calculate the wavelength of $K_\alpha$ line for the target made of tungsten ($Z = 74$).
4. Obtain a relation between the frequencies of $K_\alpha$, $K_\beta$, and $L_\alpha$ lines for a target material.
5. An X-rays tube operates at 20 kV. Find the maximum speed of the electrons striking the anticathode, given the charge of electron = $1.6 \times 10^{-19}$ coulomb and mass of electron = $9 \times 10^{-31}$ kg.
6. (a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 Å. What is the maximum energy of a photon in the radiation?
   (b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?
7. The wavelength of characteristic X-ray $K_\alpha$ line emitted from Zinc ($Z = 30$) is 1.415Å. Find the wavelength of the $K_\alpha$ line emitted from molybdenum ($Z = 42$).
8. If the short series limit of the Balmer series for hydrogen is 3644 Å, find the atomic number of the element which gives X-ray wavelengths down to 1 Å. Identify the element.
9. A material whose K-absorption edge is 0.2 Å is irradiated by X-rays of wavelength 0.15 Å. Find the maximum energy of the photoelectrons that are emitted from the $K$ shell.
10. Calculate the wavelength of the emitted characteristic X-ray from a tungsten ($Z = 74$) target when an electron drops from an $M$ shell to a vacancy in the shell.
11. A potential difference of 20 kV is applied to an X-ray tube. Find the minimum wavelength of X-rays generated.
12. The wavelength of X-rays produced by an X-ray tube is 0.76 Å. What is the atomic number of the anode material of the tube?

### ANSWERS

1. (a) True (b) True (c) False (d) True (e) False
2. 0.31 Å
3. 0.228 Å
5. $8.4 \times 10^7$ m s$^{-1}$
6. (a) 27.624 keV (b) 30 kV
7. 0.708 Å
8. Gallium
9. 20.718 keV
11. 0.6215 Å
12. 41

## Solved Examples

### EXAMPLE 6.1

Ultraviolet light of wavelengths 800 Å and 700 Å when allowed to fall on hydrogen atoms in their ground states is found to liberate electrons with kinetic energies 1.8 eV and 4.0 eV, respectively. Find the value of Planck's constant.

**Sol.** The energy of incident photon = $(hc/\lambda)$

If $W_i$ is the ionization energy and $E_K$ the kinetic energy of the emitted electrons, then we have

$$\frac{hc}{\lambda_1} = W_i + E_{K_1}$$

For incident photon of wavelength $\lambda_1 = 800$ Å $= 8 \times 10^{-8}$ m.

$$\frac{hc}{\lambda_1} = W_i + E_{K_1} \quad \ldots(i)$$

And for incident photon of wavelength $\lambda_2 = 700$ Å $= 7 \times 10^{-8}$ m.

$$\frac{hc}{\lambda_2} = W_i + E_{K_2} \quad \ldots(ii)$$

Subtracting Eq. (i) from Eq. (ii), we get $hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right) = E_{K_2} - E_{K_1}$

or $$h = \frac{(E_{K_2} - E_{K_1})\lambda_1\lambda_2}{c(\lambda_1 - \lambda_2)}$$

Here, $E_{K_1} = 1.8\text{eV} = 1.8 \times 1.6 \times 10^{-19}$ J

and $E_{K_2} = 4.0$ eV $= 4.0 \times 1.6 \times 10^{-19}$ J

Substituting the given values, we get

$$h = \frac{(4.0 - 1.8) \times 1.6 \times 10^{-19} \times 8 \times 10^{-8} \times 7 \times 10^{-8}}{3 \times 10^8 (8 \times 10^{-8} - 7 \times 10^{-8})}$$

$$= 6.57 \times 10^{-34} \text{ J-s}$$

### EXAMPLE 6.2

A single electron orbits around a stationary nucleus of charge $+Ze$, where $Z$ is a constant and $e$ is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. Find:

(a) The values of $Z$.
(b) The energy required to excite the electron from the third to the fourth Bohr orbit.
(c) The wavelength of electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.
(d) The kinetic energy, potential energy, and the angular momentum of the electron in the first Bohr orbit.
(e) The radius of the first Bohr orbit.

The ionization energy of the hydrogen atom = 13.6 eV, Bohr radius = $5.3 \times 10^{-11}$ m, speed of light = $3 \times 10^8$ m s$^{-1}$, Planck's constant = $6.63 \times 10^{-34}$ J-s.

**Sol.** The energy of the electron in $n^{th}$ orbit of hydrogen-like atom is given by

$$E_n = -\frac{Z^2\, Rhc}{n^2} \qquad ...(i)$$

(a) The energy required to excite the electron from second ($n = 2$) Bohr orbit to the third ($n = 3$) Bohr orbit is given by

$$\Delta E = E_3 - E_2 = -\frac{Z^2\, Rhc}{3^2} - \left(-\frac{Z^2 Rhc}{3^2}\right)$$

$$= Z^2\, Rhc\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5}{36} Z^2 Rhc$$

Given, ionization energy of hydrogen atom = $Rhc$ = 13.6 eV and $\Delta E = E_3 - E_2 = 47.2$ eV

Thus, we have $47.2 \text{ eV} = \frac{5}{36} Z^2 \times 13.6 \text{ eV}$

or $Z^2 = \frac{36}{5} \times \frac{47.2}{13.6} = 25 \Rightarrow Z = 5$

(b) The energy required to excite the atom from the third to fourth orbit is given by

$$E_4 - E_3 = -\frac{Z^2 Rhc}{4^2} - \left(-\frac{Z^2 Rhc}{3^2}\right)$$

$$= Z^2\, Rhc\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= 5^2 \times (13.6 \text{ eV}) \times \frac{7}{144} = 16.53 \text{ eV}$$

(c) The ionization energy of atom or the energy required to remove the electron from the first Bohr orbit to $\infty$ is

$$E_\infty - E_1 = -\frac{Z^2 Rhc}{(\infty)^2} - \left(-\frac{Z^2 Rhc}{1^2}\right) = Z^2 Rhc$$

If $\lambda$ is the wavelength of corresponding electromagnetic radiation, then

$$\frac{hc}{\lambda} = Z^2 Rhc$$

i.e., $\frac{hc}{\lambda} = 5^2 \times 13.6 \text{ eV} = 340 \text{ eV}$

$$\therefore \quad \lambda = \frac{hc}{340 \text{ eV}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{340 \times 1.6 \times 10^{-19}}$$

$$= 36.056 \times 10^{-10} \text{ m} = 36.056 \text{ Å}$$

(d) Kinetic energy of electron in the first Bohr's orbit,

$$E_{K_1} = -E_1 = 340 \text{ eV}$$

Potential energy of electron in the first Bohr's orbit,

$$U_1 = 2E_1 = -2 \times 340 \text{ V} = -680 \text{ eV}$$

Angular momentum of electron in the first Bohr's orbit

$$= n\frac{h}{2\pi} = 1\frac{h}{2\pi} = \frac{6.63 \times 10^{-34}}{2\pi}$$

$$= 1.0546 \times 10^{-34} \text{ J s}$$

(e) Radius of the first Bohr orbit for given atom

$$= \left(\frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}\right)_{n=1} = \frac{(\varepsilon_0 h^2 / \pi m e^2)}{Z}$$

$$= \frac{\text{Radius of first Bohr orbit of hydrogen}}{Z}$$

$$= \frac{5.3 \times 10^{-11}}{5} \text{m} = 1.06 \times 10^{-11} \text{ m}$$

## EXAMPLE 6.3

The ionization energy of a hydrogen-like Bohr atom is 4 rydbergs.

**(a)** What is the wavelength of radiation emitted when the electron jumps from the first excited state to the ground state?

**(b)** What is the radius of the first orbit for this atom?

Given that Bohr radius of hydrogen atom = $5 \times 10^{-11}$ m and 1 rydberg = $2.2 \times 10^{-18}$ J.

**Sol.** The energy of electron in hydrogen-like atoms in $n^{th}$ orbit is

$$E_n = \frac{Z^2 Rhc}{n^2}$$

We have $Rhc$ = 1 rydberg.

The ionization energy $E_\infty - E_1 = Z^2 Rhc = 4$ rydberg

$$\therefore \quad Z^2 = \frac{4 \text{ rydberg}}{Rhc} = \frac{4 \text{ rydberg}}{1 \text{ rydberg}} = 4$$

$$\therefore \quad Z = 2$$

**(a)** The energy required to excite the electron from $n = 1$ to $n = 2$ is given by

$$E_2 - E_1 = -\frac{Z^2 Rhc}{2^2} - \left(-\frac{Z^2 Rhc}{1^2}\right)$$

$$= Z^2 Rhc\left(1 - \frac{1}{4}\right)$$

$$= \frac{3}{4} Z^2 Rhc = \frac{U}{2} \times 4 \text{ rydberg}$$

$$= 3 \text{ rydberg}$$

If $\lambda$ is the wavelength of radiation emitted, then

$$\frac{hc}{\lambda} = 3 \text{ rydberg, i.e., } \lambda = \frac{hc}{(3 \text{ rydberg})}$$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{3 \times 2.2 \times 10^{-18}}$$

$$= 301.4 \times 10^{-10} \text{ m} = 301.4 \text{ Å}$$

**(b)** Radius of first Bohr orbit, $r_1 = \frac{(\varepsilon_0 h^2 / \pi m e^2)}{Z}$

$$= \frac{\text{Radius of first Bohr orbit of hydrogen}}{Z} = \frac{5\times10^{-11}}{2}$$

$$= 2.5\times10^{-11}\ \text{m}$$

## EXAMPLE 6.4

A doubly ionized lithium atom is hydrogen-like with atomic number 3.

**(a)** Find the wavelength of the radiation required to excite the electron in $Li^{++}$ from the first to third Bohr orbit (ionization energy of hydrogen equals 13.6 eV).

**(b)** How many spectral lines are observed in the emission spectrum of the above excited system?

**Sol.** The energy of electron in $n^{th}$ orbit of hydrogen-like atoms is

$$E_n = -\frac{Z^2Rhc}{n^2}$$

Here, $Z = 3$

and $Rhc = 13.6$ eV

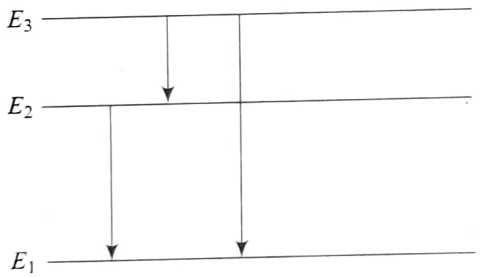
**(a)** The energy required to excite the electron in $Li^{++}$ from $n = 1$ to $n = 3$ is

$$E_3 - E_1 = -\frac{Z^2Rhc}{3^2} - \left(-\frac{Z^2Rhc}{1^2}\right)$$

$$= Z^2Rhc\left(1-\frac{1}{9}\right)$$

$$= \frac{8}{9}Z^2Rhc = \frac{8}{9}\times(3)^2\times13.6\ \text{eV}$$

$$= 8\times13.6\ \text{eV} = 8\times13.6\times1.6\times10^{-19}\ \text{J}$$




Therefore, the wavelength of incident radiation required for excitation is given by

$$\frac{hc}{\lambda} = E_3 - E_1$$

or $$\lambda = \frac{hc}{E_3 - E_1} = \frac{6.63\times10^{-34}\times3\times10^8}{8\times13.6\times1.6\times10^{-19}}$$

$$= 114.26\times10^{-10}\ \text{m} = 114.26\ \text{Å}$$

**(b)** The possible lines in the emission spectrum of this excited system are $\left[\frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3\right]$

Three in number, represented in figure.

## EXAMPLE 6.5

The energy of an electron in an excited hydrogen atom is –3.4 eV. Calculate the angular momentum of the electron according to Bohr's theory.

Given:
Rydberg's constant $R = 1.09737\times10^{-7}\ \text{m}^{-1}$, Planck's constant $h = 6.626176\times10^{-34}$ J-s, speed of light $c = 3\times10^8\ \text{m s}^{-1}$.

**Sol.** The energy of an electron in $n^{th}$ orbit of hydrogen atom is

$$E_n = -\frac{Rhc}{n^2}$$

$$Rhc = 1.09737\times10^7\times6.626176\times10^{-34}\times3\times10^8\ \text{J}$$

$$= \frac{1.09737\times10^7\times6.626176\times10^{-34}\times3\times10^8}{1.6\times10^{-19}}\ \text{eV}$$

$$= 13.6\ \text{eV}$$

Hence $$E_n = -\frac{13.6}{n^2}\text{eV} \quad \text{...(i)}$$

From Eq. (i), $-3.4\text{eV} = \frac{-13.6}{n^2}\text{eV}$

$$\therefore \quad n^2 = \frac{13.6}{3.4} = 4, \text{ i.e., } n = 2$$

According to Bohr's quantum condition,
Angular momentum,

$$mvr = n\frac{h}{2\pi} = 2\times\frac{6.626176\times10^{-34}}{2\times3.14} = 2.1102\times10^{-34}\ \text{J s}$$

## EXAMPLE 6.6

Light of wavelength 2000 Å falls on an aluminium surface. In aluminium, 4.2 eV of energy is required to remove an electron from its surface. What is the kinetic energy, in electron volt of **(a)** the fastest and **(b)** the slowest emitted photo-electrons. **(c)** What is the stopping potential? **(d)** What is the cut-off wavelength for aluminium? (Plack's constant $h = 6.6\times10^{-34}$ J-s and speed of light $c = 3\times10^8\ \text{m s}^{-1}$).

**Sol.** Energy corresponding to incident photon,

$$h\nu = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{2000\times10^{-10}}$$

$$= 9.9\times10^{-19}\ \text{J} = \frac{9.9\times10^{-19}}{1.6\times10^{-19}}\ \text{eV} = 6.2\ \text{eV}$$

**(a)** The kinetic energy of fastest electrons, $E_K = h\nu - W$

or $E_k = 6.2\ \text{eV} - 4.2\ \text{eV} = 2\ \text{eV}$

**(b)** The kinetic energy of slowest electrons is zero, since the emitted electrons have all possible energies from 0 to certain maximum value $E_k$.

**(c)** If $V_s$ is the stopping potential, then $E_k = eV_s$

or $$V_s = \frac{E_k}{e} = \frac{2\ \text{eV}}{e} = 2\ \text{V}$$

**(d)** If $\lambda_0$ is the cut-off wavelength for aluminium, then $W = (hc/\lambda_0)$

or $$\lambda_0 = (hc/W)$$

$$= \frac{6.6\times10^{-34}\times3\times10^8}{4.2\times1.6\times10^{-19}} = 3000\times10^{-10}\ \text{m}$$

$$= 3000\ \text{Å}$$

## EXAMPLE 6.7

Find the frequency of light which ejects electrons from a metal surface fully stopped by a retarding potential of 3 V. The photoelectric effect begins in this metal at a frequency of $6 \times 10^{15}$ Hz. Find the work function for this metal. (Given $h = 6.63 \times 10^{-34}$ J-s).

**Sol.** According to Einstein's photoelectric equation,

$$E_k = h\nu - W$$

If $V_s$ is retarding or stopping potential and $\nu_0$, the threshold frequency, then above equation becomes $eV_s = h\nu - h\nu_0$

or $h\nu = eV_s + h\nu_0$

or $\nu = \dfrac{eV_s}{h} + \nu_0$

Hence, $e = 1.6 \times 10^{-10}$ coulomb, $V_s = 3$ V, and $\nu_0 = 6 \times 10^{14}$ Hz. Therefore, required frequency

$$\nu = \frac{1.6 \times 10^{-10} \times 3}{6.63 \times 10^{-34}} + 6 \times 10^{14}$$

$$= 7.24 \times 10^{14} + 6 \times 10^{14} = 13.24 \times 10^{14} \text{ Hz}$$

$$= 1.324 \times 10^{15} \text{ Hz}$$

## EXAMPLE 6.8

A 40 W ultraviolet light source of wave-length 2480 Å illuminates a magnesium (Mg) surface placed 2 m away. Determine the number of photons emitted from the surface per second and the number incident on unit area of Mg surface per second. The photoelectric work function for Mg is 3.68 eV. Calculate the kinetic energy of the fastest electrons ejected from the surface. Determine the maximum wavelength for which the photoelectric effect can be observed with a Mg surface.

**Sol.** Energy of each photon, $\varepsilon = \dfrac{hc}{\lambda}$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2480 \times 10^{-10}} \text{ J} = 8.0 \times 10^{-19} \text{ J}$$

Number of photons emitted per second,

$$N = \frac{P}{\varepsilon} = \frac{40}{8.0 \times 10^{-19}} = 5.0 \times 10^{19} \text{ s}^{-1}$$

These photons spread in all directions over surface area $4\pi r^2$, therefore, the number of photons incident per unit area per second

$$N_i = \frac{N}{4\pi r^2} = \frac{5.0 \times 10^{19}}{4 \times 3.14 \times (2)^2} = 9.95 \times 10^{13} \text{ s}^{-1} \quad (\text{As } r = 2 \text{ m})$$

From Einstein's photoelectric equation, $E_k = h\nu - W$

$$\varepsilon = h\nu = \frac{hc}{\lambda} = 8.0 \times 10^{-19} \text{J} = \frac{8.0 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = 5.0 \text{ eV}$$

$\therefore \quad E_k = 5.0 \text{ eV} - 3.68 \text{ eV} = 1.32 \text{ eV}$

Threshold (or maximum) wavelength for photoelectrons emission,

$$\lambda_0 = \frac{hc}{W} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{3.68 \times 1.6 \times 10^{-19}} \text{ m} = 3.373 \times 10^{-7} \text{ m} = 3373 \text{ Å}$$

## EXAMPLE 6.9

Hydrogen atom in its ground state is excited by means of monochromic radiation of wavelength 975 Å. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV.

**Sol.** The energy of an electron in $n^{\text{th}}$ orbit of hydrogen ($Z = 1$) is given by

$$E_n = \frac{Rhc}{n^2}$$

$\therefore$ Ionization energy $= E_\infty - E_1 = Rhc = 13.6$ eV

$$\therefore \quad E_n = -\frac{13.6}{n^2} \text{ eV}$$

Therefore, energy of electron in ground state ($n = 1$),

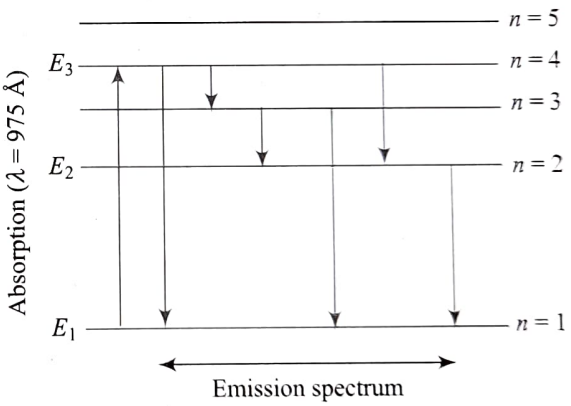
$$E_1 = -\frac{13.6}{1} = -13.6 \text{ eV}$$

Energy of electron in the first excited state ($n = 2$),

$$E_2 = -\frac{13.6}{2^2} = -\frac{13.6}{4} = -3.4 \text{ eV}$$

The energy of electron in the second excited state ($n = 3$),

$$E_3 = -\frac{13.6}{3^3} = \frac{13.6}{9} = -1.511 \text{ eV}$$




The energy of electron in the third excited state ($n = 4$),

$$E_4 = -\frac{13.6}{4^2} = -\frac{13.6}{16} = -0.85 \text{ eV}$$

Energy of incident photon of wavelength

$$\lambda = 975 \text{ Å} = 975 \times 10^{-10} \text{ m}$$

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10}} \text{ J}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{ eV} = 12.75 \text{ eV}$$

When the incident photon of this energy is absorbed by hydrogen atom, let its ground state electron occupy $(n-1)^{\text{th}}$ excited state or $n^{\text{th}}$ orbit. Then,

$$E = -\frac{Rhc}{n^2} - \left(-\frac{Rhc}{1^2}\right) = Rhc\left(1 - \frac{1}{n^2}\right)$$

i.e., $\quad 12.75 \text{ eV} = 13.6 \text{ eV}\left(1 - \dfrac{1}{n^2}\right)$

This gives $n = 4$.

That is the electron is excited to the third excited state. The emission spectrum will contain the transitions shown in figure. The longest wavelength emitted corresponds to transition $(4 \rightarrow 3)$ for which the energy difference is minimum.

i.e., $E_{min} = E_4 - E_3 = -0.85 - (-1.511)$ eV

$= 1.511 - 0.85 = 0.661$ eV $= 0.661 \times 1.6 \times 10^{-19}$ J

$$\therefore \quad \lambda_{max} = \frac{hc}{E_{min}} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.661 \times 1.6 \times 10^{-19}} = 18.807 \times 10^{-7} \text{ m}$$

$= 18807$ Å

## EXAMPLE 6.10

A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level $A$ and some atoms in a particular upper (excited) energy level $B$ and there are no atoms in any other energy level. The atoms of the gas make transitions to a higher energy level by absorbing monochromatic light of photon energy 2.7 eV, some have energy more and some have less than 2.7 eV.

**(a)** Find the principal quantum number of the initially excited level $B$.

**(b)** Find the ionization energy for the gas atoms.

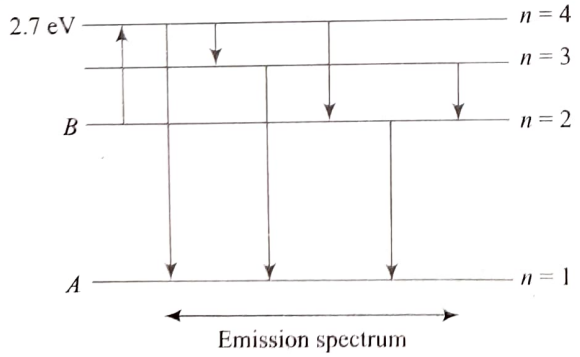
**(c)** Find the maximum and minimum energies of the emitted photons.

**Sol.** The energy levels of identical hydrogen-like atoms are given by

$$E_n \approx -\frac{B}{n^2}, \; B \text{ being a constant.}$$

**(a)** When a hydrogen-like atom absorbs energy 2.7 eV. This is only possible if transition of electron is from state $B$ to any higher energy state. Six radiations are emitted only if the final state is $n = 4$.

The energy of emitted radiation is also more than 2.7 eV, therefore, initial state cannot be $n = 1$. As energy of emitted radiation is also less than 2.7 eV. This shows that initial excited state cannot be $n = 3$. Hence, the only possibility is that initially excited state $B$ has $n = 2$.




That is, the principal quantum number of initially excited state $B$ is $n = 2$.

**(b)** Given $E_4 - E_2 = 2.7$ eV

$$\therefore \quad -\frac{B}{4^2} - \left(-\frac{B}{2^2}\right) = 2.7 \text{ eV}$$

Therefore, this gives $B = 14.4$ eV

The transition of electron should be from ground state $(n = 1)$ to $n = \infty$.

Ionization energy of identical hydrogen-like gas atoms,

$$\Delta E = E_\infty - E_1 = -\frac{B}{\infty^2} - \left(-\frac{B}{1^2}\right)$$

**(c)** Maximum energy of emitted photon is obtained when transition of electron is from $n = 4$ to $n = 1$.

$$\therefore \quad E_{max} = -\frac{B}{4^2} - \left(-\frac{B}{1^2}\right)$$

$$= B\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 14.4\left(1 - \frac{1}{16}\right) \text{eV} = 13.65 \text{ eV}$$

Minimum energy of emitted photon is obtained when electron jumps from $n = 4$ to $n = 3$.

i.e., $E_{min} = B\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 14.4\left(\frac{1}{9} - \frac{1}{16}\right) = 0.4$ eV

## EXAMPLE 6.11

The wavelength of the characteristic X-ray $K_\alpha$ line emitted by a hydrogen-like element is 0.32 Å. Calculate the wavelength of $K_\beta$ line emitted by the same element.

**Sol.** For hydrogen-like element $\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

For $K_\alpha$ line, $\frac{1}{\lambda_\alpha} = Z^2 R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ ...(i)

For $K_\beta$ line, $\frac{1}{\lambda_\beta} = Z^2 R\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$ ...(ii)

Dividing, $\frac{\lambda_\beta}{\lambda_\alpha} = \frac{\frac{1}{1^2} - \frac{1}{2^2}}{\frac{1}{1^2} - \frac{1}{3^2}} = \frac{1 - \frac{1}{4}}{1 - \frac{1}{9}} = \frac{3/4}{8/9} = \frac{3 \times 9}{4 \times 8} = \frac{27}{32}$

$$\therefore \quad \lambda_B = \frac{27}{32}\lambda_\alpha = \frac{27}{32} \times 0.32 \text{ Å} = 0.27 \text{ Å}$$

## EXAMPLE 6.12

Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find:

**(a)** The energy of the photons causing the photoelectric emission.

**(b)** The quantum numbers of the two levels involved in the emission of these photons.

**(c)** The change in the angular momentum of the electron in the hydrogen atom in the above transition.

**(d)** The recoil speed of the emitting atom assuming it to be at rest before the transition.

(Ionization potential of hydrogen is 13.6 eV)

**Sol.** $E_k = 0.73$ eV, $W = 1.82$ eV

Ionization energy of H atom = 13.6 eV

(a) $h\nu = W + E_k = 1.82 \text{ eV} + 0.73 \text{ eV} = 2.55 \text{ eV}$

(b) The electronic energy levels of H-atoms are given by

$$E_n = -\frac{Rhc}{n^2} = -\frac{13.6}{n^2} \text{ eV}$$

For $n = 1$, $E_1 = -13.6$ eV

For $n = 2$, $E_2 = -3.4$ eV

For $n = 3$, $E_3 = -1.51$ eV

For $n = 4$, $E_4 = -0.85$ eV

Clearly, $E_4 - E_2 = -0.85 \text{ eV} - (-3.4 \text{ eV}) = 2.55$ eV

i.e., Quantum numbers involved in the photon of energy 2.55 eV are 2 and 4. The transition is specified by $n_1 = 4 \rightarrow n_2 = 2$.

(c) The angular momentum of electron in H atom

$$J = n\frac{h}{2\pi}$$

For $n = 4$, $J_1 = 4\dfrac{h}{2\pi} = \dfrac{2h}{\pi}$

For $n = 2$, $J_2 = 2\dfrac{h}{2\pi} = \dfrac{h}{\pi}$

Therefore, change in angular momentum,

$$\Delta J = J_1 - J_2 = \frac{2h}{\pi} - \frac{h}{\pi} = \frac{h}{\pi}$$

(d) According to conservation of momentum, $\dfrac{h\nu}{c} + mv = 0$

$$\therefore \quad v = -\frac{h\nu}{cm} = -\frac{2.55}{(3\times10^8)\times(1.67\times10^{-27})}$$

$$= -\frac{2.55\times1.6\times10^{-19}}{3\times10^8\times1.67\times10^{-27}}$$

$$= -0.814 \text{ ms}^{-1}$$

Therefore, recoil speed of H atom = 0.814 ms$^{-1}$.

## EXAMPLE 6.13

Two hydrogen-like atoms $A$ and $B$ are of different masses and each atom contains equal numbers of protons and neutrons. The difference in the energies between the first Balmer lines emitted by $A$ and $B$, is 5.667 eV. When atoms $A$ and $B$, moving with the same velocity, strike a heavy target, they rebound with the same velocity. In the process, atom $B$ imparts twice the momentum to the target than that $A$ imparts. Identify the atoms $A$ and $B$.

**Sol.** Let $Z_A$ and $Z_B$ be the atomic numbers and $m_A$ and $m_B$ be the mass numbers of hydrogen-like atoms $A$ and $B$, respectively.

Energy of $n$th state of hydrogen-like atom is

$$E_n = -\frac{Z^2Rhc}{n^2} = -\frac{Z^2\times13.6}{n^2} \text{ eV}$$

Energy emitted for I line of Balmer series for atom $A$,

$$\Delta E_1 = -Z_A^2\times13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \text{eV}$$

Energy emitted for I line of Balmer series for atom $B$,

$$\Delta E_2 = Z_B^2\times13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \text{eV}$$

Given $\Delta E_1 - \Delta E_2 = 5.667$ eV, therefore

$$5.667 \text{ eV} = (Z_B^2 - Z_A^2)\times13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \text{eV}$$

$$\therefore \quad Z_B^2 - Z_A^2 = \frac{5.667\times36}{13.6\times5} = 3 \qquad \text{...(i)}$$

Let $u$ be the initial velocity of each from $A$ and $B$ and $m$ the mass of target.

If $v_1$ and $v_2$ are velocities of target after collision with $A$ and $B$, respectively, then according to principle of conservation of momentum for $A$,

$$m_Au = Mv_1 - m_Au$$

or $\quad Mv_1 = 2m_Au \qquad \text{...(ii)}$

Similarly, for $B$ $Mv_2 = 2m_Bu \qquad \text{...(iii)}$

Given, $Mv_2 = 2mv_1$, therefore, $2m_Bu = 2(2m_Au)$

This gives, $m_B = 2m_A \qquad \text{...(iv)}$

As number of protons and neutrons in each of $A$ and $B$ is same separately,

$\therefore \quad m_B = 2Z_B$ and $m_A = 2Z_A$

Substituting this in (iv), we get $2Z_B = 2(2Z_A)$, i.e., $Z_B = 2Z_A \qquad \text{...(v)}$

Solving Eq. (i) and Eq. (v), we get $Z_A = 1$ and $Z_B = 2$

i.e., atom $A$ contains 2 protons and 2 neutrons.

Hence, atom $B$ is singly ionized helium.

## EXAMPLE 6.14

**(a)** A stopping potential of 0.82 V is required to stop the emission of photoelectrons from the surface of a metal by light of wavelength 4000 Å. For light of wavelength 3000 Å, the stopping potential is 1.85 volt. Find the value of Planck's constant. [1 electron-volt (eV) = $1.6\times10^{-19}$ J.

**(b)** At stopping potential, if the wavelength of the incident light is kept fixed at 4000 Å, but the intensity of light increased two times, will photoelectric current be obtained? Give reasons for your answer.

**Sol.**

**(a)** $\dfrac{hc}{\lambda_1} = W + eV_1 \qquad \text{...(i)}$

$$\frac{hc}{\lambda_2} = W + eV_2 \qquad \text{... (ii)}$$

Subtracting $\dfrac{hc}{\lambda_2} - \dfrac{hc}{\lambda_1} = e(V_2 - V_1)$

Here, $c = 3\times10^8$ m s$^{-1}$,

$\lambda_1 = 4000$ Å $= 4\times10^{-7}$ m, $V_1 = 0.82$ V

$\lambda_2 = 3000$ Å $= 3\times10^{-7}$ m, $V_2 = 1.85$ V

$$\therefore \quad h\times3\times10^8\left(\frac{1}{3\times10^{-7}} - \frac{1}{4\times10^{-7}}\right)$$

$$= 1.6\times10^{-19}(1.85 - 0.82)$$

or $\quad h\times3\times10^8\dfrac{1}{12\times10^{-7}} = 1.6\times10^{-19}\times1.03$

Therefore, Planck's constant,

$$h = \frac{1.6\times10^{-19}\times1.03\times12\times10^{-7}}{3\times10^{8}} = 6.592\times10^{-34}\ \text{J-s}$$

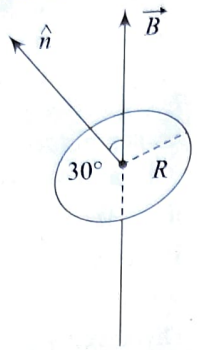
**(b)** No, because stopping potential does not depend on intensity of incident light.

## EXAMPLE 6.15

An electron in the ground state of hydrogen atom is revolving in anticlockwise direction in a circular orbit of radius $R$ (see figure).

**(a)** Obtain an expression for the orbital magnetic dipole moment of the electron.

**(b)** The atom is placed in a uniform magnetic induction $\vec{B}$ such that the plane normal to the electron orbit makes an angle of 30° with the magnetic induction. Find the torque experienced by the orbiting electron.




**Sol.**

**(a)** If $f$ is the frequency of orbital motion, then the orbital magnetic dipole moment of circulating electron

$$M = iA = feA \qquad ...(i)$$

According to Bohr's condition of stationary orbits,

$$mvR = n\frac{h}{2\pi}$$

For ground state, $n = 1$ and area $A = \pi R^2$

Also, $v = R\omega = R\,2\pi f$

$$\therefore \quad m(R\,2\pi f)R = 1\frac{h}{2\pi}$$

$$\therefore \quad f = \frac{h}{4\pi^2 mR^2}$$

$$\therefore \text{ From Eq. (i), } M = \left(\frac{h}{4\pi^2 mR^2}\right)e\pi R^2$$

$$\Rightarrow \quad M = \frac{eh}{4\pi m}\ \text{A-m}^2$$

**(b)** The direction of magnetic moment is along the normal to the plane of electron orbit, i.e., along $\hat{n}$.

Angle between $\vec{M}$ and $\vec{B}$ is 30°.

Therefore, Torque on orbiting electron,

$$\tau = MB\sin\theta = \left(\frac{eh}{4\pi m}\right)B\sin 30°$$

$$\text{i.e.,} \quad \tau = \frac{ehB}{8\pi m}\ \text{newton} \times \text{meter}$$

## EXAMPLE 6.16

An electron in a hydrogen-like atom is in an excited state. It has a total energy of –3.4 eV. Calculate **(a)** the kinetic energy, and **(b)** the de-Broglie wavelength of the electron.
($h = 6.63\times10^{-34}$ J-s)

**Sol.**

**(a)** Energy of electron in hydrogen-like atom,

$$E_n = -\frac{Z^2 Rhc}{n^2} = -3.4\ \text{eV}$$

Kinetic energy of electron in hydrogen-like atom is equal to negative of total energy,

$$\text{i.e.,} \quad E_k = -E_n = -(-3.4\ \text{eV}) = +3.4\ \text{eV}$$

**(b)** The de Broglie wavelength of electron,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times3.4\times1.6\times10^{-19}}} = 6.66\ \text{Å}$$

## EXAMPLE 6.17

Assume that the de-Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one-dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance between the atoms of the array is 2 Å. A similar standing wave is again formed if $d$ is increased to 2.5 Å but not for any intermediate value of $d$. Find the energy of the electrons in electron, volt, and the least value of $d$ for which the standing wave of the type described above can form.
($h = 6.63\times10^{-34}$ J-s)

**Sol.** For standing wave, the separation between consecutive nodes (or antinodes) is λ/2.

$$\therefore \quad d = \frac{n\lambda}{2}, \text{ where } n \text{ is integer.} \qquad ...(i)$$

For next node, $d' = (n+1)\frac{\lambda}{2}$

$$\therefore \quad d' - d = \frac{\lambda}{2}$$

$$\Rightarrow \quad \lambda = 2(d' - d)$$

Here, $d = 2$ Å, $d' = 2.5$ Å

$$\therefore \quad \lambda = 2(2.5\ \text{Å} - 2\ \text{Å}) = 1\ \text{Å}$$

Therefore, energy of electrons, $E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

Here $h = 6.63\times10^{-34}$ J-s

$m = 9.1\times10^{-31}$ kg

$\lambda = 1\ \text{Å} = 10^{-10}$ m

$$\therefore \quad E = \frac{(6.63\times10^{-34})^2}{2\times9.1\times10^{-31}\times(10^{-10})^2}\ \text{J}$$

$$= \frac{(6.63)^2}{2\times9.1}\times10^{-17}\ \text{joule} = 2.415\times10^{-17}\ \text{J}$$

$$= \frac{2.415\times10^{-17}}{1.6\times10^{-19}}\ \text{eV} \approx 151\ \text{eV}$$

For least value of $d$, $n = 1$. Therefore, from Eq. (i)

$$d_{min} = \frac{\lambda}{2} = \frac{1}{2}\ \text{Å} = 0.5\ \text{Å}$$

## EXAMPLE 6.18

A gas of hydrogen-like ion is prepared in such a way that the ions are only in the ground state and the first excited state. A monochromatic light of wavelength 1216 Å is absorbed by the ions. The ions are lifted to higher excited states and emit

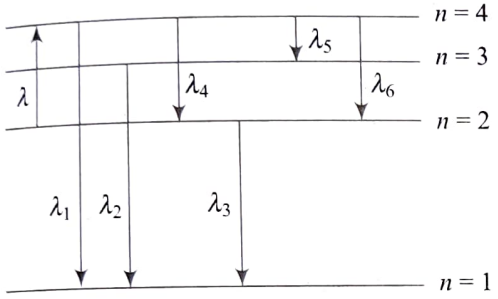
radiation of six wavelengths, some higher and some lower than the incident wavelength. Find the principle quantum number of the excited states. Identify the nuclear charge on the ions. Calculate the values of the maximum and minimum wavelengths.

**Sol.** The energy state of hydrogen-like atoms is given by

$$E_n = -\frac{B}{n^2}$$

where $B = Z^2Rhc$.

The emission of six wavelengths implies that the higher excited state corresponds to quantum number $n = 4$. As some wavelengths are lower and some wavelengths are higher, it implies that the initial excited state is $n = 2$. Thus, the principal quantum numbers of excited states are $n = 2, n = 3, n = 4$.




$$\therefore \quad \frac{hc}{\lambda} = B\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \quad \Rightarrow \quad \frac{hc}{\lambda} = \frac{3B}{16}$$

$$\therefore \quad B = \frac{16 \times 6.62 \times 10^{-34} \times 3 \times 10^8}{3 \times (1216 \times 10^{-10})} = 8.71 \times 10^{-18}\ \text{J}$$

If $Z$ is atomic number of nucleus, then $B = Z^2\,Rhc$.

$$\therefore \quad Z^2 = \frac{B}{Rhc} = \frac{8.71 \times 10^{-18}\ \text{J}}{13.6\ \text{eV}} = \frac{8.71 \times 10^{-18}}{13.6 \times 1.6 \times 10^{-19}} = 4$$

$$\therefore \quad Z = 2$$

The maximum wavelength of emitted radiation corresponds to transition $n = 4$ to $n = 3$ and is given by

$$\frac{hc}{\lambda_{max}} = B\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\Rightarrow \quad \lambda_{max} = \frac{144hc}{7B} = \frac{144 \times 6.62 \times 10^{-34} \times 3 \times 10^8}{7 \times 8.71 \times 10^{-18}} = 4.69 \times 10^{-7}\ \text{m}$$

The minimum wavelength of emitted radiation corresponds to transition $n = 4$ to $n = 1$ and is given by

$$\frac{hc}{\lambda_{min}} = B\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$$

$$\Rightarrow \quad \lambda_{min} = \frac{16hc}{15B} = \frac{16 \times 6.62 \times 10^{-34} \times 3 \times 10^8}{15 \times 8.71 \times 10^{-18}} = 0.24 \times 10^{-7}\ \text{m}$$

## EXAMPLE 6.19

Photoelectrons are emitted when 400 nm radiation is incident on a surface of work function 1.9 eV. These photoelectrons pass through a region containing $\alpha$ particles. A maximum energy electron combines with an $\alpha$ particles to form a $He^+$ ion, emitting a single photon in this process. $He^+$ ions thus formed are in their fourth excited state. Find the energies in eV of the photons, lying in the 2 to 4 eV range, that are likely to be emitted during and after the combination.
[Take $h = 4.14 \times 10^{-15}$ eV s]

**Sol.** From Einstein's photoelectric equation, maximum kinetic energy of emitted electrons

$$E_k = \frac{hc}{\lambda} - W$$

$$\text{Now, } E = \frac{hc}{\lambda} = \frac{6.624 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}}$$

$$= 4.968 \times 10^{-19}\ \text{J-s} = \frac{4.968 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.1\ \text{eV}$$

$$E_k = 3.1\ \text{eV} - 1.9\ \text{eV} = 1.2\ \text{eV}$$

$$e + \alpha \rightarrow He^+ + \text{Photon}$$

Energy of He atom in their fourth excited state ($n = 5$) is

$$E_n = -\frac{Z^2Rhc}{n^2} = -\frac{(2)^2 \times 13.6}{(5)^2} = -2.176\ \text{eV}$$

From conservation of energy, $1.2\ \text{eV} + 0 = -2.176\ \text{eV} + E_\gamma$

Energy of photon during combination, $E_\gamma = 1.2 + 2.176 = 3.376\ \text{eV}$

$$\text{Energy of helium ion, } E_n = -\frac{Z^2Rhc}{n^2} = -\frac{4 \times 13.6}{n^2}$$

$$\therefore E_n = -\frac{54.4}{n^2}\text{eV}, \ n = 1, 2, 3, \ldots$$

$$= -54.4\text{eV}, -13.6\text{eV}, -6.04\text{eV}, -3.4\text{eV}, -2.176\text{eV}, -1.51\text{eV}$$

Difference of energies lying between 2 and 4 eV is

$-3.4 + 6.04 = 2.64$ eV

$-2.176 + 6.04 = 3.86$ eV

Energies of photons emitted are 2.64 eV and 3.86 eV.

## EXAMPLE 6.20

When a beam of 1.6 eV photons of intensity 2.0 W $m^{-2}$ falls on a platinum surface of area $1.0 \times 10^{-4}$ $m^2$ and work function 5.6 eV, 0.53% of the incident photons eject photoelectrons. Find the number of photoelectrons emitted per second and their minimum and maximum energies (in eV). Take 1 eV = $1.6 \times 10^{-19}$ J.

**Sol.** Energy incident on surface per second

$$P = IA = 2.0 \times 1.0 \times 10^{-4} = 2 \times 10^{-4}\ \text{J}$$

Energy of each photon, $\varepsilon = 10.6\ \text{eV} = 10.6 \times 1.6 \times 10^{-19}$ J

$$\text{Number of photons incident on the surface} = \frac{2 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}}$$

$$\text{Number of photoelectrons emitted} = \frac{0.53}{100} \times \frac{2 \times 10^{-4}}{10.6 \times 1.6 \times 10^{-19}}$$

$$= 6.25 \times 10^{11}$$

According to Einstein's photoelectric equation, maximum KE of photoelectrons

$$E_k = \varepsilon - W = 10.6\ \text{eV} - 5.6\ \text{eV} = 5\ \text{eV}$$

Minimum kinetic energy of photoelectrons = zero.

## EXAMPLE 6.21

An electron in hydrogen-like atom makes a transition from $n$th orbit and emits radiation corresponding to Lyman series. If de-Broglie wavelength of electron in $n$th orbit is equal to the wavelength of radiation emitted, find the value of $n$. The atomic number of atom is 11.

**Sol.** If $\lambda$ is de-Broglie wavelength, then for $n^{th}$ stationary orbit

$$2\pi r_n = n\lambda, \text{ where } r_n \text{ is radius of } n^{th} \text{ orbit } r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2}$$

$$\therefore \quad 2\pi\left(\frac{\varepsilon_0 h^2 n^2}{\pi m Ze^2}\right) = n\lambda$$

$$\Rightarrow \quad \frac{1}{\lambda} = \frac{mZe^2}{2\varepsilon_0 h^2 n} \qquad ...(i)$$

For Lyman series of hydrogen like atom

$$\frac{1}{\lambda} = Z^2 R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad ...(ii)$$

From Eqs. (i) and (ii), $Z^2R\left(1-\frac{1}{n^2}\right) = \frac{mZe^2}{2\varepsilon_0 h^2 n}$

Rydberg constant, $R = \frac{me^4}{8\varepsilon_0^2 ch^3}$

$$\therefore \quad \frac{Z^2 me^4}{8\varepsilon_0^2 ch^3}\left(1-\frac{1}{n^2}\right) = \frac{mZe^2}{2\varepsilon_0 h^2 n}$$

$$\left(1-\frac{1}{n^2}\right) = \frac{4\varepsilon_0 ch}{ne^2 Z}$$

$$= \frac{4\times(8.85\times10^{-12})\times(3\times10^8)\times(6.62\times10^{-34})}{n\times(1.6\times10^{-19})^2\times11}$$

$$= \frac{25}{n}$$

$$\therefore \quad n^2 - 1 = 25n$$

or $\quad n^2 - 25n - 1 = 0$

$$n = \frac{25 \pm \sqrt{(-25)^2 + 4\times1\times1}}{2} \approx \frac{25 \pm \sqrt{625}}{2} \approx 25$$

As negative $n$ is not possible, $n \approx 25$.

## EXAMPLE 6.22

An X-ray tube operated at 40 kV emits a continuous X-ray spectrum with a short wavelength limit $\lambda_{min} = 0.310$ Å. Calculate Planck's constant.

**Sol.** We have, $\lambda_{min} = \frac{hc}{eV}$ or $h = \frac{eV\lambda_{min}}{c}$

Here, $e = 1.6\times10^{-19}$ coulomb, $V = 40$ kV $= 40\times10^3$ V

$\lambda_{min} = 0.310$ Å $= 0.310\times10^{-10}$ m

$c = 3\times10^8$ m s$^{-1}$

$$\therefore \quad h = \frac{(1.6\times10^{-19})\times(40\times10^3)\times0.310\times10^{-10}}{3\times10^8}$$

$$= 6.61\times10^{-34} \text{ J-s}$$

# Exercises

## Single Correct Answer Type

1. An electron jumps from the fourth orbit to the second orbit of hydrogen atom. Given: the Rydberg's constant $R = 10^5\ cm^{-1}$. The frequency, in Hz, of the emitted radiation will be

(1) $\frac{3}{16} \times 10^5$ (2) $\frac{3}{6} \times 10^{15}$

(3) $\frac{9}{16} \times 10^5$ (4) $\frac{9}{16} \times 10^{15}$

2. Given: mass number of gold = 197, density of gold = 19.7 g $cm^{-3}$, Avogadro's number = $6 \times 10^{23}$. The radius of the gold atom is approximately:

(1) $1.5 \times 10^{-8}$ m (2) $1.5 \times 10^{-9}$ m

(3) $1.5 \times 10^{-10}$ m (4) $1.5 \times 10^{-12}$ m

3. Transitions between three energy levels in a particular atom give rise to three spectral lines of wavelengths, in increasing magnitudes, $\lambda_1$, $\lambda_2$, and $\lambda_3$. Which one of the following equations correctly relates $\lambda_1$, $\lambda_2$, and $\lambda_3$?

(1) $\lambda_1 = \lambda_2 - \lambda_3$ (2) $\lambda_1 = \lambda_3 - \lambda_2$

(3) $\frac{1}{\lambda_1} = \frac{1}{\lambda_2} + \frac{1}{\lambda_3}$ (4) $\frac{1}{\lambda_1} = \frac{1}{\lambda_3} - \frac{1}{\lambda_2}$

4. The ratio of the speed of the electron in the first Bohr orbit of hydrogen and the speed of light is equal to (where $e$, $h$, and $c$ have their usual meanings in cgs system)

(1) $2\pi hc/e^2$ (2) $er^2h/2\pi c$

(3) $e^2c/2\pi h$ (4) $2\pi e^2/hc$

5. Suppose two deuterons must get as close as $10^{-14}$ m in order for the nuclear force to overcome the repulsive electrostatic force. The height of the electrostatic barrier is nearest to

(1) 0.14 MeV (2) 2.3 MeV

(3) 1.8 MeV (4) 0.56 MeV

6. An electron in H atom makes a transition from $n = 3$ to $n = 1$. The recoil momentum of H atom will be

(1) $6.45 \times 10^{-27}$ N s (2) $6.8 \times 10^{-27}$ N s

(3) $6.45 \times 10^{-24}$ N s (4) $6.8 \times 10^{-24}$ N s

7. A hydrogen-like atom emits radiations of frequency $2.7 \times 10^{15}$ Hz when it makes a transition from $n = 2$ to $n = 1$. The frequency emitted in a transition from $n = 3$ to $n = 1$ will be

(1) $1.8 \times 10^{15}$ Hz (2) $3.2 \times 10^{15}$ Hz

(3) $4.7 \times 10^{5}$ Hz (4) $6.9 \times 10^{15}$ Hz

8. The total energy of an electron in the ground state of hydrogen atom is −13.6 eV. The potential energy of an electron in the ground state of $Li^{2+}$ ion will be

(1) 122.4 eV (2) −122.4 eV

(3) 244.8 eV (4) −244.8 eV

9. The wavelength of radiation required to excite the electron from the first orbit to the third orbit in a doubly ionized lithium atom will be

(1) 134.25 Å (2) 125.5 Å

(3) 113.7 Å (4) 110 Å

10. The force acting on the electron in a hydrogen atom depends on the principal quantum number as

(1) $F \propto n^2$ (2) $F \propto \frac{1}{n^2}$

(3) $F \propto n^4$ (4) $F \propto \frac{1}{n^4}$

11. If potential energy between a proton and an electron is given by $|U| = ke^2/2R^3$, where $e$ is the charge of electron and $R$ is the radius of atom, then radius of Bohr's orbit is given by ($h$ = Planck's constant, $k$ = constant)

(1) $\frac{ke^2m}{h^2}$ (2) $\frac{6\pi^2}{n^2}\frac{ke^2m}{h^2}$

(3) $\frac{2\pi}{n}\frac{ke^2m}{h^2}$ (4) $\frac{4\pi^2ke^2m}{n^2h^2}$

12. How many revolutions does an electron complete in one second in the first orbit of hydrogen atom?

(1) $6.62 \times 10^{15}$ (2) 100

(3) 1000 (4) 1

13. The radius of hydrogen atom in the ground state is $5.3 \times 10^{-11}$ m. When struck by an electron, its radius is found to be $21.2 \times 10^{-11}$ m. The principal quantum number of the final state will be

(1) 1 (2) 2

(3) 3 (4) 4

14. The wavelength of the first line of Balmer series is 6563 Å. The Rydberg's constant is

(1) $1.09 \times 10^5\ m^{-1}$ (2) $1.09 \times 10^6\ m^{-1}$

(3) $1.097 \times 10^7\ m^{-1}$ (4) $1.09 \times 10^8\ m^{-1}$

15. A hydrogen atom and a $Li^{++}$ ion are both in the second excited state. If $l_H$ and $l_{Li}$ are their respective electronic angular momenta, and $E_H$ and $E_{Li}$ their respective energies, then

(1) $l_H < l_{Li}$ and $|E_H| > |E_{Li}|$ (2) $l_H = l_{Li}$ and $|E_H| < |E_{Li}|$

(3) $l_H = l_{Li}$ and $|E_H| > |E_{Li}|$ (4) $l_H < l_{Li}$ and $|E_H| < |E_{Li}|$

16. Imagine an atom made of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength (given in terms of the Rydberg constant $R$ for the hydrogen atom) equal to

(1) $\frac{9}{5R}$ (2) $\frac{36}{5R}$

(3) $\frac{18}{5R}$ (4) $\frac{4}{R}$

17. The electric potential energy between a proton and an electron is given by $U = U_0 \ln r/r_0$, where $r_0$ is a constant. Assuming Bohr's model to be applicable, write the variation of $r$ with $n$, $n$ being the principal quantum number.

(1) $r_n \propto n$ (2) $r_n \propto 1/n$
(3) $r_n \propto n^2$ (4) $r_n \propto 1/n^2$

**18.** The longest wavelength that a singly ionized helium atom in its ground state will absorb is
(1) 912 Å (2) 304 Å
(3) 606 Å (4) 1216 Å

**19.** The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where $n_1$ and $n_2$ are the principal quantum numbers of the two states. Assume the Bohr model as valid in this case. The frequency of the orbital motion of the electron in the initial state is 1/27 of that in the final state. The possible values of $n_1$ and $n_2$ are
(1) $n_1 = 6, n_2 = 3$ (2) $n_1 = 4, n_2 = 2$
(3) $n_1 = 8, n_2 = 1$ (4) $n_1 = 3, n_2 = 1$

**20.** A photon collides with a stationary hydrogen atom in the ground state inelastically. Energy of the colliding photon is 10.2 eV. Almost instantaneously, another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector?
(1) Two photons of energy 10.2 eV
(2) Two photons of energy 1.4 eV
(3) One photon of energy 10.2 eV and one electron of energy 1.4 eV
(4) One electron having kinetic energy nearly 11.6 eV

**21.** The ratio (in SI units) of magnetic dipole moment to that of the angular momentum of an electron of mass $m$ kg and charge $e$ coulomb in Bohr's orbit of hydrogen atom is
(1) $\frac{e}{2m}$ (2) $\frac{e}{m}$
(3) $\frac{2e}{m}$ (4) none of these

**22.** If an electron in $n = 3$ orbit of hydrogen atom jumps down to $n = 2$ orbit, the amount of energy released and the wavelength of radiation emitted are
(1) 0.85 eV, 6566 Å (2) 1.89 eV, 1240 Å
(3) 1.89 eV, 6566 Å (4) 1.5 eV, 6566 Å

**23.** In hydrogen spectrum, the shortest wavelength in Balmer series is the $\lambda$. The shortest wavelength in the Brackett series will be
(1) $2\lambda$ (2) $4\lambda$
(3) $9\lambda$ (4) $16\lambda$

**24.** If the first excitation potential of a hydrogen-like atom is $V$ electron volt, then the ionization energy of this atom will be
(1) $V$ electron volt
(2) $\frac{3V}{4}$ electron volt
(3) $\frac{4V}{3}$ electron volt
(4) cannot be calculated by given information

**25.** Two hydrogen atoms are in excited state with electrons residing in $n = 2$. The first one is moving toward left and emits a photon of energy $E_1$ toward right. The second one is moving toward right with the same speed and emits a photon of energy $E_2$ toward left. Taking recoil of nucleus into account, during the emission process
(1) $E_1 > E_2$ (2) $E_1 < E_2$
(3) $E_1 = E_2$ (4) information insufficient

**26.** In a hydrogen atom following Bohr's postulates, the product of linear momentum and angular momentum is proportional to $(n)^x$ where $n$ is the orbit number. Then $x$ is
(1) 0 (2) 2
(3) –2 (4) 1

**27.** The voltage applied to an X-ray tube is 18 kV. The maximum mass of photon emitted by the X-ray tube will be
(1) $2 \times 10^{-13}$ kg (2) $3.2 \times 10^{-36}$ kg
(3) $3.2 \times 10^{-32}$ kg (4) $9.1 \times 10^{-31}$ kg

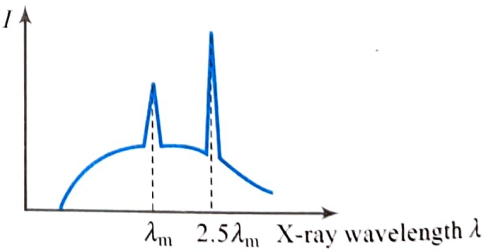
**28.** The wavelength of $K_\alpha$ $X$-rays of two metals $A$ and $B$ are $4/1875R$ and $1/675R$, respectively, where '$R$' is Rydberg's constant. The number of elements lying between $A$ and $B$ according to their atomic numbers is
(1) 3 (2) 6
(3) 5 (4) 4

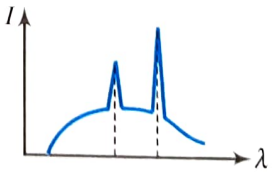
**29.** One of the lines in the emission spectrum of $Li^{2+}$ has the same wavelength as that of the second line of Balmer series in hydrogen spectrum. The electronic transition corresponding to this line is
(1) $n = 4 \rightarrow n = 2$ (2) $n = 8 \rightarrow n = 2$
(3) $n = 8 \rightarrow n = 4$ (4) $n = 12 \rightarrow n = 6$

**30.** The photon radiated from hydrogen corresponding to the second line of Lyman series is absorbed by a hydrogen-like atom $X$ in the second excited state. Then, the hydrogen-like atom $X$ makes a transition of $n$th orbit.
(1) $X = He^+, n = 4$ (2) $X = Li^{++}, n = 6$
(3) $X = He^+, n = 6$ (4) $X = Li^{++}, n = 9$

**31.** The element which has a $K_\alpha$ X-rays line of wavelength 1.8 Å is
($R = 1.1 \times 10^7$ m$^{-1}$, $b = 1$ and $\sqrt{5/33} = 0.39$)
(1) Co, $Z = 27$ (2) Fe, $Z = 26$
(3) Mn, $Z = 25$ (4) Ni, $Z = 28$

**32.** When an electron accelerated by potential difference $U$ is bombarded on a specific metal, the emitted X-ray spectrum obtained is shown in figure. If the potential difference is reduced to $U/3$, the correct spectrum is




(1)
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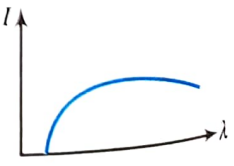
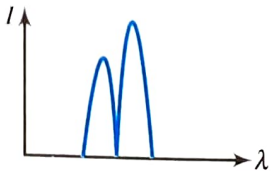
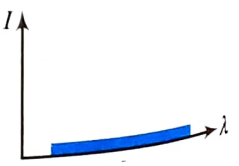



(3)



(4)


33. In a hydrogen atom, the electron makes a transition from $n=2$ to $n=1$. The magnetic field produced by the circulating electron at the nucleus
(1) decreases 16 times (2) increases 4 times
(3) decreases 4 times (4) increases 32 times

34. An electron in a hydrogen atom makes a transition from first excited state to ground state. The equivalent current due to circulating electron:
(1) increases 2 times
(2) increases 4 times
(3) increases 8 times
(4) remains the same

35. When the voltage applied to an X-ray tube increases from $V_1 = 10$ kV to $V_2 = 20$ kV, the wavelength interval between $K_\alpha$ line and cut-off wavelength of continuous spectrum increases by a factor of 3. Atomic number of the metallic target is
(1) 28 (2) 29
(3) 65 (4) 66

36. Monochromatic radiations of wavelength $\lambda$ are incident on a hydrogen sample in the ground state. Hydrogen atom absorbs the light and subsequently emits radiations of 10 different wavelengths. The value of $\lambda$ is nearly
(1) 203 nm (2) 95 nm
(3) 80 nm (4) 73 nm

37. The power of an X-ray tube is 16 W. If the potential difference applied across the tube is 5 kV, then the number of electrons striking the target per second is
(1) $8.4 \times 10^{16}$ (2) $5 \times 10^{17}$
(3) $2 \times 10^{16}$ (4) $2 \times 10^{19}$

38. A hydrogen-like atom (atomic number $Z$) is in a higher excited state of quantum number $n$. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.2 eV and 17.0 eV, respectively. Alternatively, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV, respectively. The values of $n$ and $Z$ are, respectively,
(1) 6 and 6 (2) 3 and 3
(3) 6 and 3 (4) 3 and 6

39. Hydrogen atoms in a sample are excited to $n = 5$ state and it is found that photons of all possible wavelengths are present in the emission spectra. The minimum number of hydrogen atoms in the sample would be
(1) 5 (2) 6
(3) 10 (4) infinite

40. A sample of hydrogen atoms is in excited state (all the atoms). The photons emitted from this sample are made to pass through a filter through which light having wavelength greater than 800 nm can only pass. Only one type of photons are found to pass the filter. The sample's excited state initially is [Take $hc = 1240$ eV-nm, ground state energy of hydrogen atom $= -13.6$ eV.]
(1) fifth excited state (2) fourth excited state
(3) third excited state (4) second excited state

41. Mark out the correct statement regarding X-rays.
(1) When fast moving electrons strike the metal target, they enter the metal target and in a very short time span come to rest, and thus an accelerated charged electron produces electromagnetic waves (X-rays).
(2) Characteristic X-rays are produced due to transition of an electron from higher energy levels to vacant lower energy levels.
(3) X-rays spectrum is a discrete spectra just like hydrogen spectra.
(4) Both (1) and (2) are correct.

42. An electron collides with a hydrogen atom in its ground state and excites it to $n = 3$. The energy given to hydrogen atom in this inelastic collision is [Neglect the recoiling of hydrogen atom]
(1) 10.2 eV (2) 12.1 eV
(3) 12.5 eV (4) None of these

43. Electrons in a hydrogen-like atom ($Z = 3$) make transitions from fourth excited state to third excited state and from third to second excited state. The resulting radiations are incident on a metal plate to eject photoelectrons. The stopping potential for photoelectrons ejected by the shorter wavelength is 3.95 V. The stopping potential for the photoelectrons ejected by the longer wavelength is
(1) 2.0 V (2) 0.75 V
(3) 0.6 V (4) none of the above

44. In which of the following transitions will the wavelength be minimum?
(1) $n_1 = 5$ to $n_2 = 4$ (2) $n_1 = 4$ to $n_2 = 3$
(3) $n_1 = 3$ to $n_2 = 2$ (4) $n_1 = 2$ to $n_2 = 1$

45. A neutron having kinetic energy 5 eV is incident on a hydrogen atom in its ground state. The collision
(1) must be elastic
(2) must be completely inelastic
(3) may be partially elastic
(4) information is insufficient

46. If the average life time of an excited state of hydrogen is of the order of $10^{-8}$ s, then the number of revolutions an electron will make when it is in $n = 2$ state before coming to ground state will be [Take $a_0 = 0.53$ Å and all standard data if required]
(1) $10^7$ (2) $8 \times 10^6$
(3) $2 \times 10^5$ (4) none of these

47. An electron of energy 11.2 eV undergoes an inelastic collision with a hydrogen atom in its ground state. [Neglect recoiling of atom as $m_H >> m_e$]. Then in this case
(1) the outgoing electron has energy 11.2 eV
(2) the entire energy is absorbed by the H atom and the electron stops
(3) 10.2 eV of the incident electron energy is absorbed by the H atom and the electron would come out with 1.0 eV energy
(4) none of these

**48.** The recoil speed of hydrogen atom after it emits a photon in going from $n = 2$ state to $n = 1$ state is nearly [Take $R_\infty = 1.1 \times 10^7\ m^{-1}$ and $h = 6.63 \times 10^{-34}$ J s]

(1) 1.5 m s$^{-1}$ (2) 3.3 m s$^{-1}$

(3) 4.5 m s$^{-1}$ (4) 6.6 m s$^{-1}$

**49.** A beam of 13.0 eV electrons is used to bombard gaseous hydrogen. The series obtained in emission spectra is/are

(1) Lyman series (2) Balmer series

(3) Brackett series (4) All of these

**50.** The wavelength of the spectral line that corresponds to a transition in hydrogen atom from $n = 10$ to ground state would be [In which part of electromagnetic spectrum this line lies?]

(1) 92.25 nm, ultraviolet (2) 92.25 nm, infrared

(3) 86.95 nm, ultraviolet (4) 97.65 nm, ultraviolet

**51.** The angular momentum of an electron in hydrogen atom is $4h/2\pi$. Kinetic energy of this electron is

(1) 4.35 eV (2) 1.51 eV

(3) 0.85 eV (4) 13.6 eV

**52.** Difference between $n$th and $(n + 1)$th Bohr's radius of hydrogen atom is equal to $(n - 1)$th Bohr's radius. The value of $n$ is

(1) 1 (2) 2

(3) 3 (4) 4

**53.** $K_\alpha$ wavelength emitted by an atom of atomic number $Z = 11$ is $\lambda$. The atomic number for an atom that emits $K_\alpha$ radiation with wavelength $4\lambda$ is

(1) 6 (2) 4

(3) 11 (4) 44

**54.** A proton of mass $m$ moving with a speed $v_0$ approches a stationary proton that is free to move. Assume impact parameter to be zero, i.e., head-on collision. How close will the incident proton go to other proton?

(1) $\dfrac{e^3}{\pi \varepsilon_0 m^2 v_0}$ (2) $\dfrac{e^3}{\pi \varepsilon_0 m v_0}$

(3) $\dfrac{e^2}{\pi \varepsilon_0 m v_0^2}$ (4) None of these

**55.** If an X-ray tube operates at the voltage of 10 kV, find the ratio of the de Broglie wavelength of the incident electrons to the shortest wavelength of X-rays produced. The specific charge of electron is $1.8 \times 10^{11}$ C kg.

(1) 1 (2) 0.1

(3) 1.8 (4) 1.2

**56.** The potential difference across the Coolidge tube is 20 kV and 10 mA current flows through the voltage supply. Only 0.5% of the energy carried by the electrons striking the target is converted into X-rays. The power carried by the X-ray beam is $P$. Then

(1) $P = 0.1$ W (2) $P = 1$ W

(3) $P = 2$ W (4) $P = 10$ W

**57.** Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. $K$ shell electrons of tungsten have –72.5 keV energy. X-rays emitted by the tube contain only

(1) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of –0.155 Å

(2) a continuous X-ray spectrum (Bremsstrahlung) with all wavelengths

(3) a continuous X-ray spectrum of tungsten

(4) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of 0.155 Å and the characteristic X-ray spectrum of tungsten

**58.** Consider a hypothetical annihilation of a stationary electron with a stationary positron. What is the wavelength of the resulting radiation?

(1) $\lambda = \dfrac{h}{m_0 c}$ (2) $\lambda = \dfrac{2h}{m_0 c^2}$

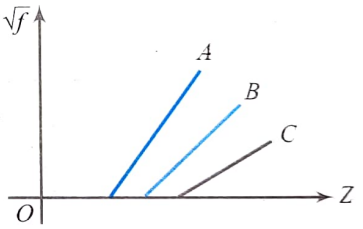
(3) $\lambda = \dfrac{h}{2m_0 c^2}$ (4) None of these

**59.** If 10,000 V are applied across an X-ray tube, find the ratio of wavelength of the incident electrons and the shortest wavelength of X-ray coming out of the X-ray tube, given $e/m$ of electron $= 1.8 \times 10^{11}$ C kg$^{-1}$

(1) 1:10 (2) 10:1

(3) 5:1 (4) 1:5

**60.** Figure shows Moseley's plot between $\sqrt{f}$ and $Z$, where $f$ is the frequency and $Z$ is the atomic number. Three lines $A$, $B$, and $C$ shown in the graph may represent




(1) $K_\alpha$, $K_\beta$, and $K_\gamma$ lines, respectively

(2) $K_\gamma$, $K_\beta$, and $K_\alpha$ lines, respectively

(3) $K_\alpha$, $L_\alpha$, and $M_\alpha$ lines, respectively

(4) Nothing

**61.** An electron in the ground state of hydrogen has an angular momentum $L_1$ and an electron in the first excited state of lithium has an angular momentum $L_2$. Then

(1) $L_1 = L_2$ (2) $L_1 = 4L_2$

(3) $L_2 = 2L_1$ (4) $L_1 = 2L_2$

**62.** The approximate value of quantum number $n$ for the circular orbit of hydrogen of 0.0001 mm in diameter is

(1) 1000 (2) 60

(3) 10000 (4) 31

**63.** If elements of quantum number greater than $n$ were not allowed, the number of possible elements in nature would have been

(1) $\dfrac{1}{2}n(n+1)$ (2) $\left\{\dfrac{n(n+1)}{2}\right\}^2$

(3) $\dfrac{1}{6}n(n+1)(2n+1)$ (4) $\dfrac{1}{3}n(n+1)(2n+1)$

64. The velocity of an electron in the first orbit of H atom is $v$. The velocity of an electron in the second orbit of $He^+$ is
(1) $2v$ (2) $v$
(3) $\frac{v}{2}$ (4) $\frac{v}{4}$

65. The recoil speed of a hydrogen atom after it emits a photon in going from $n = 5$ state to $n = 1$ state is
(1) 4.718 m $s^{-1}$ (2) 7.418 m $s^{-1}$
(3) 4.178 m $s^{-1}$ (4) 7.148 m $s^{-1}$

66. If a hydrogen atom emits a photon of energy 12.1 eV, its orbital angular momentum changes by $\Delta L$. Then, $\Delta L$ equals
(1) $1.05 \times 10^{-34}$ J s (2) $2.11 \times 10^{-34}$ J s
(3) $3.16 \times 10^{-34}$ J s (4) $4.22 \times 10^{-34}$

67. An electron with kinetic energy $E$ eV collides with a hydrogen atom in the ground state. The collision is observed to be elastic for
(1) $0 < E < \infty$ (2) $0 < E < 10.2$ eV
(3) $0 < E < 13.6$ eV (4) $0 < E < 3.4$ eV

68. The wavelength $K_\alpha$ of X-rays produced by the X-ray tube is 0.76 Å. The atomic number of the node material of the tube is
(1) 30 (2) 40
(3) 50 (4) 60

69. Let the potential energy of a hydrogen atom in the ground state be zero. Then, its energy in the first excited state will be
(1) 10.2 eV (2) 13.6 eV
(3) 23.8 eV (4) 27.2 eV

70. When photons of wavelength $\lambda_1$ are incident on an isolated sphere suspended by an insulated thread, the corresponding stopping potential is found to be $V$. When photons of wavelength $\lambda_2$ are used, the corresponding stopping potential was thrice the above value. If light of wavelength $\lambda_3$ is used, calculate the stopping potential for this case.
(1) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{1}{\lambda_1}\right]$ (2) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$
(3) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$ (4) $\frac{hc}{e}\left[\frac{1}{\lambda_3}-\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$

71. The ionization energy of the ionized sodium atom $Na^{10+}$ is
(1) 13.6 eV (2) $13.6 \times 11$ eV
(3) (13.6/11) eV (4) $13.6 \times (11^2)$ eV

72. The shortest wavelength produced in an X-ray tube operating at 0.5 million volt is
(1) dependent on the target element
(2) about $2.5 \times 10^{-12}$ m
(3) double of the shortest wavelength produced at 1 million volt
(4) dependent only on the target material

73. If the $K_\alpha$ radiation of Mo ($Z = 42$) has a wavelength of 0.71 Å find the wavelength of the corresponding radiation of Cu ($Z = 29$)
(1) 1 Å (2) 2 Å
(3) 1.52 Å (4) 1.25 Å

74. A beam of electron accelerated by a large potential difference $V$ is made to strike a metal target to produce X-rays. For which of the following values of $V$, will the resulting X-rays have the lowest minimum wavelength?
(1) 10 kV (2) 20 kV
(3) 30 kV (4) 40 kV

75. The minimum kinetic energy required for ionization of a hydrogen atom is $E_1$ in case electron is collided with hydrogen atom. It is $E_2$ if the hydrogen ion is collided and $E_3$ when helium ion is collided. Then,
(1) $E_1 = E_2 = E_3$ (2) $E_1 > E_2 > E_3$
(3) $E_1 < E_2 < E_3$ (4) $E_1 > E_3 > E_2$

76. The ratio between total acceleration of the electron in singly ionized helium atom and hydrogen atom (both in ground state) is
(1) 1 (2) 8
(3) 4 (4) 16

77. The shortest wavelength of the Brackett series of a hydrogen-like atom (atomic number = $Z$) is the same as the shortest wavelength of the Balmer series of hydrogen atom. The value of $Z$ is
(1) 2 (2) 3
(3) 4 (4) 6

78. A neutron moving with a speed $v$ makes a head-on collision with a hydrogen atom in ground state kept at rest. The minimum kinetic energy of the neutron for which inelastic collision will take place is (assume that mass of proton is nearly equal to the mass of neutron)
(1) 10.2 eV (2) 20.4 eV
(3) 12.1 eV (4) 16.8 eV

79. Angular momentum ($L$) and radius ($r$) of a hydrogen atom are related as
(1) $Lr$ = constant (2) $Lr^2$ = constant
(3) $Lr^4$ = constant (4) none of these

80. In hydrogen and hydrogen-like atoms, the ratio of $E_{4n} - E_{2n}$ and $E_{2n} - E_n$ varies with atomic number $Z$ and principal quantum number $n$ as
(1) $\frac{Z^2}{n^2}$ (2) $\frac{Z^4}{n^4}$
(3) $\frac{Z}{n}$ (4) none of these

81. According to Bohr's theory of hydrogen atom, the product of the binding energy of the electron in the $n^{th}$ orbit and its radius in the $n^{th}$ orbit
(1) is proportional to $n^2$
(2) is inversely proportional to $n^3$
(3) has a constant value of 10.2 eV-Å
(4) has a constant value of 7.2 eV-Å

82. In X-ray tube, when the accelerating voltage $V$ is halved, the difference between the wavelength of $K_\alpha$ line and minimum wavelength of continuous X-ray spectrum
(1) remains constant
(2) becomes more than two times

(3) becomes half
(4) becomes less than two times

**83.** An electron is in an excited state in a hydrogen like atom. It has a total energy of –3.4 eV. The kinetic energy of electron is $E$ and its de Broglie wavelength is $\lambda$
(1) $E = 6.8$ eV; $\lambda = 6.6 \times 10^{-10}$ m
(2) $E = 3.4$ eV; $\lambda = 6.6 \times 10^{-10}$ m
(3) $E = 3.4$ eV; $\lambda = 6.6 \times 10^{-11}$ m
(4) $E = 6.8$ eV; $\lambda = 6.6 \times 10^{-11}$ m

**84.** The circumference of the second Bohr orbit of electron in hydrogen atom is 600 nm. The potential difference that must be applied between the plates so that the electrons have the de Broglie wavelength corresponding in this circumference is
(1) $10^{-5}$ V
(2) $\frac{5}{3} \times 10^{-5}$ V
(3) $5 \times 10^{-5}$ V
(4) $3 \times 10^{-5}$ V

**85.** X-rays emitted from a copper target and a molybdenum target are found to contain a line of wavelength 22.85 nm attributed to the $K_\alpha$ line of an impurity element. The $K_\alpha$ lines of copper ($Z = 29$) and molybdenum ($Z = 42$) have wavelengths 15.42 nm and 7.12 nm, respectively. The atomic number of the impurity element is
(1) 22
(2) 23
(3) 24
(4) 25

**86.** An electron in a Bohr orbit of hydrogen atom with the quantum number $n_2$ has an angular momentum $4.2176 \times 10^{-34}$ kg m$^2$ s$^{-1}$. If the electron drops from this level to the next lower level, the wavelength of this line is
(1) 18 nm
(2) 187.6 pm
(3) 1876 Å
(4) $1.876 \times 10^4$ Å

**87.** In the Bohr model of a $\pi$-mesic atom, a $\pi$-meson of mass $m_\pi$ and of the same charge as the electron is in a circular orbit of radius $r$ about the nucleus with an orbital angular momentum $h/2\pi$. If the radius of a nucleus of atomic number $Z$ is given by $R = 1.6 \times 10^{-15} Z^{\frac{1}{3}}$ m, then the limit on $Z$ for which ($\varepsilon_0 h^2/\pi m e^2 = 0.53$ Å and $m_\pi/m_e = 264$) $\pi$-mesic atoms might exist is
(1) $< 105$
(2) $> 105$
(3) $< 37$
(4) $> 37$

**88.** In the spectrum of singly ionized helium, the wavelength of a line observed is almost the same as the first line of Balmer series of hydrogen. It is due to transition of electron
(1) from $n_1 = 6$ to $n_2 = 4$
(2) from $n_1 = 5$ to $n_2 = 3$
(3) from $n_1 = 4$ to $n_2 = 2$
(4) from $n_1 = 3$ to $n_2 = 2$

**89.** A $K_\alpha$ X-ray emitted from a sample has an energy of 7.46 keV. Of which element is the sample made?
(1) Calcium (Ca, $Z = 20$)
(2) Cobalt (Co, $Z = 27$)
(3) Cadmium (Cd, $Z = 48$)
(4) Nickel (Ni, $Z = 28$)

**90.** If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be
(1) 60
(2) 32
(3) 4
(4) 64

**91.** Consider the spectral line resulting from the transition $n = 2 \rightarrow n = 1$ in the atoms and ions given below. The shortest wavelength is produced by
(1) hydrogen atom
(2) deuterium atom
(3) singly ionized helium
(4) doubly ionized lithium

**92.** As per the Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom ($Z = 3$) is
(1) 1.51
(2) 13.6
(3) 40.8
(4) 122.4

**93.** If the atom $_{100}Fm^{257}$ follows the Bohr model and the radius of $_{100}Fm^{257}$ is $n$ times the Bohr radius, then find $n$.
(1) 100
(2) 200
(3) 4
(4) 1/4

**94.** Electrons with de Broglie wavelength $X$ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is
(1) $\lambda_0 = \frac{2mc\lambda^2}{h}$
(2) $\lambda_0 = \frac{2h}{mc}$
(3) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$
(4) $\lambda_0 = \lambda$

## Multiple Correct Answers Type

**1.** Hydrogen atoms absorb radiation of wavelength $\lambda_0$ and consequently emit radiations of 6 different wavelengths of which two wavelengths are shorter than $\lambda_0$. Then,
(1) the final excited state of the atoms is $n = 4$
(2) the initial state of the atoms may be $n = 2$
(3) the initial state of the atoms may be $n = 3$
(4) there are three transitions belonging to Lyman series

**2.** Energy liberated in the de-excitation of hydrogen atom from the third level to the first level falls on a photo-cathode. Later when the same photo-cathode is exposed to a spectrum of some unknown hydrogen-like gas, excited to the second energy level, it is found that the de Broglie wavelength of the fastest photoelectrons now ejected has decreased by a factor of 3. For this new gas, difference of energies of the second Lyman line and the first Balmer line is found to be 3 times the ionization potential of the hydrogen atom. Select the correct statement(s).
(1) The gas is lithium.
(2) The gas is helium.
(3) The work function of photo-cathode is 8.5 eV.
(4) The work function of photo-cathode is 5.5 eV.

**3.** According to Bohr's theory of hydrogen atom, for the electron in the $n$th permissible orbit
(1) linear momentum $\propto \frac{1}{n}$
(2) radius of orbit $\propto n$
(3) kinetic energy $\propto \frac{1}{n^2}$
(4) angular momentum $\propto n$

**4.** When a hydrogen atom is excited from ground state to first excited state, then
(1) its kinetic energy increases by 20 eV
(2) its kinetic energy decreases by 10.2 eV
(3) its potential energy increases by 20.4 eV
(4) its angular momentum increases by $1.05 \times 10^{-34}$ J s

5. In an X-ray tube, the voltage applied is 20 kV. The energy required to remove an electron from $L$ shell is 19.9 keV. In the X-rays emitted by the tube,
(1) minimum wavelength will be 62.1 nm.
(2) energy of the characteristic X-rays will be equal to or less than 19.9 keV.
(3) $L_\alpha$ X-ray may be emitted.
(4) $L_\alpha$ X-ray will have energy 19.9 keV.

6. Suppose the potential energy between an electron and a proton at a distance $r$ is given by $Ke^2/3r^3$. Application of Bohr's theory to hydrogen atom in this case shows that
(1) energy in the $n$th orbit is proportional to $n^6$
(2) energy is proportional to $m^{-3}$ ($m$ = mass of electron)
(3) energy in the $n$th orbit is proportional to $n^{-2}$
(4) energy is proportional to $m^3$ ($m$ = mass of electron)

7. Let $A_n$ be the area enclosed by the $n$th orbit in a hydrogen atom. The graph of $\ln(A_n/A_1)$ against $\ln(n)$
(1) will pass through origin
(2) will be a straight line with slope 4
(3) will be a monotonically increasing nonlinear curve
(4) will be a circle

8. Mark out the correct statement(s).
(1) Line spectra contain information about atoms only.
(2) Line spectra contain information about both atoms and molecules.
(3) Band spectra contain information about both atoms and molecules.
(4) Band spectra contain information about molecules only.

9. A hydrogen atom having kinetic energy $E$ collides with a stationary hydrogen atom. Assume all motions are taking place along the line of motion of the moving hydrogen atom. For this situation, mark out the correct statement(s).
(1) For $E \geq 20.4$ eV only, collision would be elastic.
(2) For $E \geq 20.4$ eV only, collision would be inelastic.
(3) For $E = 2.4$ eV, collision would be perfectly inelastic.
(4) For $E = 18$ eV, the KE of initially moving hydrogen atom after collision is zero.

10. In Bohr's model of hydrogen atom,
(1) the radius of $n$th orbit is proportional to $n^2$
(2) the total energy of electron in $n$th orbit is proportional to $n$
(3) the angular momentum of the electron in an orbit is an integral multiple of $h/2\pi$
(4) the magnitude of the potential energy of an electron in any orbit is greater than its kinetic energy

11. Which of the following are in the ascending order of wavelength?
(1) $H_\alpha$, $H_\beta$, $H_\gamma$, ... lines in the Balmer series of hydrogen atom.
(2) Lyman limit, Balmer limit, and Paschen limit in the hydrogen spectrum
(3) Violet, blue, yellow, and red colors in solar spectrum.
(4) None of the above.

12. Continuous spectrum is produced by
(1) incandescent electric bulb (2) sun
(3) hydrogen molecules (4) sodium vapor lamp

13. A gas of monoatomic hydrogen is bombarded with a stream of electrons that have been accelerated from rest through a potential difference of 12.75 V. In the emission spectrum, one can observe lines of
(1) Lyman series (2) Balmer series
(3) Paschen series (4) Pfund series

14. If the potential energy of the electron in the first allowed orbit in hydrogen atom is $E$, its
(1) ionization potential is $-E/2$
(2) kinetic energy is $-E/2$
(3) total energy is $E/2$
(4) none of these

15. According to Bohr's theory of the hydrogen atom, for the electron in the $n$th allowed orbit
(1) the linear momentum is proportional to $(1/n)$
(2) the radius is proportional to $n$
(3) the kinetic energy is proportional to $(1/n^2)$
(4) the angular momentum is proportional to $n$

16. Whenever a hydrogen atom emits a photon in the Balmer series,
(1) it may emit another photon in the Balmer series
(2) it must emit another photon in the Lyman series
(3) the second photon, if emitted, will have a wavelength of about 122 nm
(4) it may emit a second photon, but the wavelength of this photon cannot be predicted

17. Let $\lambda_\alpha$, $\lambda_\beta$, and $\lambda'_\alpha$ denote the wavelengths of the X-rays of the $K_\alpha$, $K_\beta$, and $L_\alpha$ lines in the characteristic X-rays for a metal. Then,
(1) $\lambda'_\alpha > \lambda_\alpha > \lambda_\beta$ (2) $\lambda'_\alpha > \lambda_\beta > \lambda_\alpha$
(3) $\frac{1}{\lambda_\beta} = \frac{1}{\lambda_\alpha} + \frac{1}{\lambda'_\alpha}$ (4) $\frac{1}{\lambda_\alpha} = \frac{1}{\lambda_\beta} + \frac{1}{\lambda'_\alpha}$

18. Which of the following statements are correct for an X-ray tube?
(1) On increasing potential difference between the filament and target, photon flux of X-rays increases.
(2) On increasing potential difference between the filament and target, frequency of X-rays increases.
(3) On increasing filament current, cut-off wavelength increases.
(4) On increasing filament current, intensity of X-rays increases.

19. Which of the following statements about hydrogen spectrum are correct?
(1) All the lines of Lyman series lie in ultraviolet region.
(2) All the lines of Balmer series lie in visible region.
(3) All the lines of Paschen series lie in infrared region.
(4) None of these.

**20.** Which of the following statements are true?

(1) The shortest wavelength of X-rays emitted from an X-ray tube depends on the current in the tube.

(2) Characteristic X-ray spectra is simple as compared to optical spectra.

(3) X-rays cannot be diffracted by means of an ordinary grating.

(4) There exists a sharp limit on the short wavelength side for each continuous X-ray spectrum.

**21.** An X-ray tube is operated at 6.6 kV. In the continuous spectrum of the emitted X-rays, which of the following frequencies will be missing?

(1) $10^{18}$ Hz (2) $1.5 \times 10^{18}$ Hz

(3) $2 \times 10^{18}$ Hz (4) $2.5 \times 10^{18}$ Hz

**22.** Which of the following statements are correct?

(1) If angular momentum of the Earth due to its motion around the Sun were quantized according to Bohr's relation $L = nh/2\pi$, then the quantum number $n$ would be of the order of $10^{74}$.

(2) If elements with principal quantum number > 4 were not allowed in nature, then the number of possible elements would be 64.

(3) Rydberg's constant varies with mass number of the element.

(4) The ratio of the wave number of $K_\alpha$ line of Balmer series for hydrogen and that of $K_\alpha$ line of Balmer series for singly ionized helium is exactly 4.

**23.** According to Einstein's theory of relativity, mass can be converted into energy and vice-versa. The lightest elementary particle, taken to be the electron, has a mass equivalent to 0.51 MeV of energy. Then, we can say that

(1) the minimum amount of energy available through conversion of mass into energy is 1.2 MeV

(2) the least energy of a $\gamma$-ray photon that can be converted into mass is 1.02 MeV

(3) whereas the minimum energy released by conversion of mass into energy is 1.02 MeV, it is only a $\gamma$-ray photon of energy 0.51 MeV and above that can be converted into mass

(4) whereas the minimum energy released by conversion of mass into energy is 0.51 MeV, it is only a $\gamma$-ray photon of energy 1.02 MeV and above that can be converted into mass

**24.** X-ray from a tube with a target $A$ of atomic number $Z$ shows strong $K$ lines for target $A$ and weak $K$ lines for impurities. The wavelength of $K_\alpha$ lines is $\lambda_z$ for target $A$ and $\lambda_1$ and $\lambda_2$ for two impurities.

$$\frac{\lambda_z}{\lambda_1} = 4 \quad \text{and} \quad \frac{\lambda_z}{\lambda_2} = \frac{1}{4}$$

Assuming the screening constant of $K_\alpha$ lines to be unity, select the correct statement(s).

(1) The atomic number of first impurity is $2z - 1$.

(2) The atomic number of first impurity is $2z + 1$.

(3) The atomic number of second impurity is $\frac{(z+1)}{2}$.

(4) The atomic number of second impurity is $\frac{z}{2} + 1$.

**25.** In Bohr's model of the hydrogen atom, let $R$, $V$, $T$, and $E$ represent the radius of the orbit, speed of the electron, time period of revolution of electron, and the total energy of the electron, respectively. The quantities proportional to the quantum number $n$ are

(1) $VR$ (2) $RE$

(3) $\frac{T}{R}$ (4) $\frac{V}{E}$

**26.** An X-ray tube is operating at 50 kV and 20 mA. The target material of the tube has mass of 1 kg and specific heat 495 J $kg^{-1}\,°C^{-1}$. One percent of applied electric power is converted into X-rays and the remaining energy goes into heating the target. Then,

(1) a suitable target material must have high melting temperature

(2) a suitable target material must have low thermal conductivity

(3) the average rate of rise of temperature of the target would be 2°C $s^{-1}$

(4) the minimum wavelength of X-rays emitted is about $0.25 \times 10^{-10}$ m

**27.** For a certain metal, the $K$ absorption edge is at 0.172 Å. The wavelength of $K_\alpha$, $K_\beta$, and $K_\gamma$ lines of $K$ series are 0.210 Å, 0.192 Å, and 0.180 Å, respectively. The energies of $K$, $L$, and $M$ orbits are $E_K$, $E_L$ and $E_M$, respectively. Then

(1) $E_K = -13.04$ keV (2) $E_L = -7.52$ keV

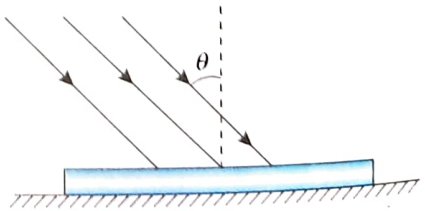
(3) $E_M = -3.21$ keV (4) $E_K = 13.04$ keV

**28.** The third line of the Balmer series spectrum of a hydrogen-like ion of atomic number $Z$ equals to 108.5 nm. The binding energy of the electron in the ground state of these ions is $E_B$. Then

(1) $Z = 2$ (2) $E_B = 54.4$ eV

(3) $Z = 3$ (4) $E_B = 122.4$ eV

**29.** A hydrogen like atom is in higher excited state of quantum number $n$. This excited atom can make transition to the ground state by successively emitting two photon of energies 28.3 eV and 302.2 eV respectively, when this element is used in X-ray experiment $K_\alpha$ peak occurs at $\lambda = 7.64$ nm ($R = 1.09 \times 10^7\ m^{-1}$)




(1) Atomic number of atom $Z = 5$

(2) The value of $n = 6$

(3) $n$ can't be calculated until state of atom after first emission is known.

(4) Atom is in 3rd excited state after emitting first photon

30. Consider an atom made up of a proton and a hypothetical particle of triple the mass of electron but having same charge as the electron. Apply the Bohr model and consider all possible transitions of this hypothetical particle from the second excited state to the lower states. The possible wavelengths emitted is (are) (given in terms of the Rydberg constant $R$ for the hydrogen atom)

(1) $\frac{8}{5R}$ (2) $\frac{3}{8R}$

(3) $\frac{4}{9R}$ (4) $\frac{12}{5R}$

31. The mass number of a nucleus is

(1) always less than its atomic number

(2) always more than its atomic number

(3) sometimes equal to its atomic number

(4) sometimes more than and sometimes equal to its atomic number

32. The electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where $n_1$ and $n_2$ are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times that in the final state. The possible values of $n_1$ and $n_2$ are

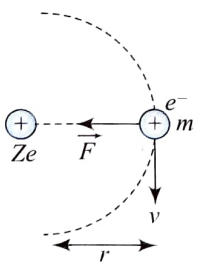
(1) $n_1 = 4, n_2 = 2$ (2) $n_1 = 8, n_2 = 2$

(3) $n_1 = 8, n_2 = 1$ (4) $n_1 = 6, n_2 = 3$

## Linked Comprehension Type

**For Problems 1–3**

Hydrogen is the simplest atom of nature. There is one proton in its nucleus and an electron moves around the nucleus in a circular orbit. According to Niels Bohr, this electron moves in a stationary orbit. When this electron is in the stationary orbit, it emits no electromagnetic radiation. The angular momentum of the electron is quantized, i.e., $mvr = (nh/2\pi)$, where $m$ = mass of the electron, $v$ = velocity of the electron in the orbit, $r$ = radius of the orbit, and $n = 1, 2, 3, \ldots$ When transition takes place from $K$th orbit to $J$th orbit, energy photon is emitted. If the wavelength of the emitted photon is $\lambda$, we find that $\frac{1}{\lambda} = R\left[\frac{1}{J^2} - \frac{1}{K^2}\right]$, where $R$ is Rydberg's constant.




On a different planet, the hydrogen atom's structure was somewhat different from ours. The angular momentum of electron was $P = 2n(h/2\pi)$, i.e., an even multiple of $(h/2\pi)$.

Answer the following questions regarding the other planet based on above passage:

1. The minimum permissible radius of the orbit will be

(1) $\frac{2\varepsilon_0 h^2}{m\pi e^2}$ (2) $\frac{4\varepsilon_0 h^2}{m\pi e^2}$

(3) $\frac{\varepsilon_0 h^2}{m\pi e^2}$ (4) $\frac{\varepsilon_0 h^2}{2m\pi e^2}$

2. In our world, the velocity of electron is $v_0$ when the hydrogen atom is in the ground state. The velocity of electron in this state on the other planet should be

(1) $v_0$ (2) $v_0/2$

(3) $v_0/4$ (4) $v_0/8$

3. In our world, the ionization potential energy of a hydrogen atom is 13.6 eV. On the other planet, this ionization potential energy will be

(1) 13.6 eV (2) 3.4 eV

(3) 1.5 eV (4) 0.85 eV

**For Problems 4–6**

The electrons in a H-atom kept at rest, jumps from the $m$th shell to the $n$th shell $(m > n)$. Suppose instead of emitting electromagnetic wave, the energy released is converted into kinetic energy of the atom. Assume Bohr's model and conservation of angular momentum are valid. Now, answer the following questions.

4. What principle is violated here?

(1) Laws of motion (2) Energy conservation

(3) Nothing is violated (4) Cannot be decided

5. Calculate the angular velocity of the atom about the nucleus if $I$ is the moment of inertia.

(1) $\frac{(m+n)}{6.28}\frac{h}{I}$ (2) $\frac{(m+n)}{1.57}\frac{h}{I}$

(3) $\frac{(m-n)}{6.28}\frac{h}{I}$ (4) $\frac{(m-n)}{1.57}\frac{h}{I}$

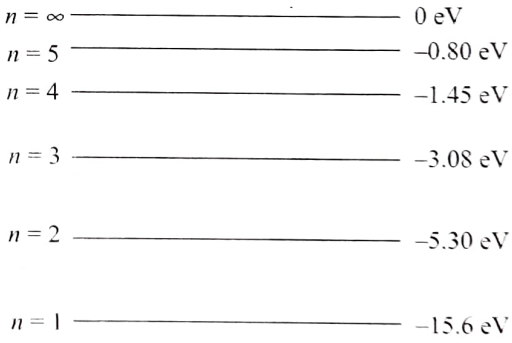
6. If the above comprehension be true, what is not valid here?

(1) $F = ma$ (2) $\tau = I\alpha$

(3) $F = dp/dt$ (4) All of them

**For Problems 7–12**

The energy levels of a hypothetical one electron atom are shown in figure.




7. Find the ionization potential of the atom.

(1) 11.2 eV (2) 13.5 eV

(3) 15.6 eV (4) 12.6 eV

8. Find the short wavelength limit of the series terminating at $n = 2$.

(1) 3256 Å (2) 2339 Å

(3) 2509 Å (4) 3494 Å

9. Find the excitation potential for the state $n = 3$.

(1) 14.64 eV (2) 9.93 eV

(3) 12.52 eV (4) 10.04 eV

10. Find the wave number of the photon emitted for the transition $n = 3$ to $n = 1$

(1) $2.23 \times 10^7\ m^{-1}$ (2) $1.009 \times 10^7\ m^{-1}$

(3) $3.005 \times 10^6\ m^{-1}$ (4) $0.432 \times 10^6\ m^{-1}$

**11.** If an electron with initial kinetic energy 6 eV is to interact with this hypothetical atom, what minimum energy will this electron carry after interaction?

(1) 2 eV (2) 3 eV
(3) 6 eV (4) 0 eV

**12.** The initial kinetic energy of an electron is 11 eV and it interacts with the above said hypothetical one electron atom, the minimum energy carried by the electron after interaction is

(1) 0.7 eV (2) 0.3 eV
(3) 0.9 eV (4) 1 eV

**For Problems 13–14**

The electron in a hydrogen atom at rest makes a transition from $n = 2$ energy state to the $n = 1$ ground state.

**13.** Find the energy (eV) of the emitted photon.

(1) 5.8 eV (2) 8.3 eV
(3) 10.2 eV (4) 12.7 eV

**14.** Assume that all of the energy corresponding to transition from $n = 2$ to $n = 1$ is carried off by the photon. By setting the momentum of the system (atom + photon) equal to zero after the emission and assuming that the recoil energy of the atom is smaller compared with the $n = 2$ to $n = 1$ energy level separation, find the energy of the recoiling hydrogen atom.

(1) $2.75 \times 10^{-7}$ eV (2) $5.54 \times 10^{-8}$ eV
(3) $8.11 \times 10^{-8}$ eV (4) $10.36 \times 10^{-7}$ eV

**For Problems 15–16**

Consider a hypothetical hydrogen-like atom. The wavelength, in Å, for the spherical lines for transitions from $n = p$ to $n = 1$ are given by

$$\lambda = \frac{1500p^2}{p^2 - 1} \quad \text{where} \quad p = 1, 2, 3, 4, \ldots$$

**15.** Find the wavelength of the most energetic photons in this series.

(1) 1800 Å (2) 1500 Å
(3) 1300 Å (4) 1650 Å

**16.** What is the ionization potential of this element?

(1) 3.96 V (2) 9.23 V
(3) 6.34 V (4) 8.28 V

**For Problems 17–20**

A sample of hydrogen gas in its ground state is irradiated with photons of 10.2 eV energies. The radiation from the above sample is used to irradiate two other samples of excited ionized $He^+$ and excited ionized $Li^{2+}$, respectively. Both the ionized samples absorb the incident radiation.

**17.** How many spectral lines are obtained in the spectra of $Li^{2+}$?

(1) 10 (2) 15
(3) 20 (4) 17

**18.** Which is the smallest wavelength that will be observed in spectra of $He^+$ ion?

(1) 24.4 nm (2) 28.8 nm
(3) 22.2 nm (4) 30.6 nm

**19.** How many spectral lines are observed in spectra of $He^+$ ion?

(1) 2 (2) 4
(3) 6 (4) 8

**20.** Which is the smallest wavelength observed in the spectra of $Li^{2+}$?

(1) 8.6 nm (2) 10.4 nm
(3) 12.8 nm (4) 14.6 nm

**For Problems 21–22**

An electron and a proton are separated by a distance $r$ so that the potential energy between them is $u = k \log r$, where $k$ is a constant.

**21.** In such an atom, radius of $n$th Bohr's orbit is

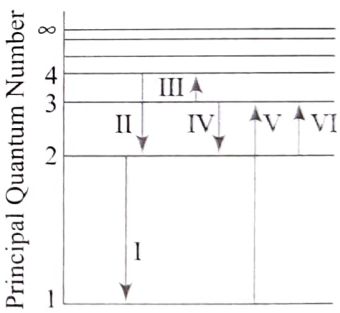
(1) $\frac{2nh}{\pi\sqrt{mk}}$ (2) $\frac{nh}{2\pi\sqrt{2mk}}$
(3) $\frac{nh}{2\pi\sqrt{mk}}$ (4) $\frac{nh}{4\pi\sqrt{mk}}$

**22.** The expression for various energy levels of the above said hypothetical atom is

(1) $\frac{k}{2}\left[1 + \log\frac{n^2h^2}{4\pi^2mk}\right]$ (2) $2k\left[2 + \log\frac{n^2h^2}{4\pi^2mk}\right]$
(3) $k\left[2 + \log\frac{n^2h^2}{4\pi^2mk}\right]$ (4) $\frac{k}{2}\left[1 + \log\frac{n^2h^2}{2\pi^2mk}\right]$

**For Problems 23–25**

Pertain to the following statement and figure.




The figure above shows an energy level diagram of the hydrogen atom. Several transitions are marked as I, II, III, ... The diagram is only indicative and not to scale.

**23.** In which transition is a Balmer series photon absorbed?

(1) II (2) III
(3) IV (4) VI

**24.** The wavelength of the radiation involved in transition II is

(1) 291 nm (2) 364 nm
(3) 487 nm (4) 652 nm

**25.** Which transition will occur when a hydrogen atom is irradiated with radiation of wavelength 103 nm?

(1) I (2) II
(3) IV (4) V

**For Problems 26–28**

A certain species of ionized atoms produces emission line spectrum according to the Bohr model. A group of lines in the spectrum is forming a series in which the shortest wavelength is

22.79 nm and the longest wavelength is 41.02 nm. The atomic number of atom is $Z$.

Based on above information, answer the following questions:

**26.** The value of $Z$ is

(1) 2 (2) 3

(3) 4 (4) 5

**27.** The series belongs to

(1) Lyman (2) Balmer

(3) Paschen (4) Brackett

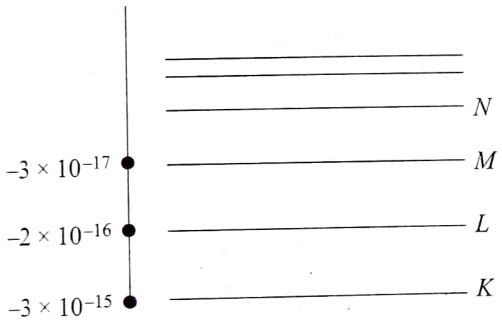
**28.** The next to longest wavelength in the series of lines is

(1) 35.62 nm (2) 30.47 nm

(3) 25.68 nm (4) 12.64 nm

**For Problems 29–31**

Simplified model of electron energy levels for a certain atom is shown in figure. The atom is bombarded with fast moving electrons. The impact of one of these electrons can cause the removal of electron from $K$-level, thus, creating a vacancy in the $K$-level. This vacancy in $K$-level is filled by an electron from $L$-level and the energy released in this transition can either appear as electromagnetic waves or may all be used to knock out an electron from $M$-level of the atom.




Energy in joule

Based on the above information, answer the following questions:

**29.** The minimum potential difference through which bombarding electron beam must be accelerated from rest to cause the ejection of electron from $K$-level is

(1) 18750 V (2) 400 kV

(3) 2.16 kV (4) 21.6 kV

**30.** Wavelength of the electromagnetic waves emitted due to transition from $L$ to $K$-level is

(1) $8 \times 10^{-10}$ m (2) $7.104 \times 10^{-11}$ m

(3) 22.46 nm (4) $142.6 \times 10^{-8}$ m

**31.** The KE of the emitted electron from $M$-level is

(1) $260 \times 10^{-17}$ J (2) $2 \times 10^{-18}$ J

(3) $240 \times 10^{-17}$ J (4) $277 \times 10^{-17}$ J

**For Problems 32–34**

A monochromatic beam of light having photon energy 12.5 eV is incident on a sample $A$ of atomic hydrogen gas in which all atoms are in the ground state. The emission spectra obtained from this sample is incident on another sample $B$ of atomic hydrogen gas in which all atoms are in the first excited state.

Based on the above information, answer the following questions:

**32.** The atoms of sample $A$ after passing of light through it

(1) may be in the first excited state

(2) may be in the second excited state

(3) may be in both first and second excited states

(4) none of the above

**33.** The emission spectra of sample $A$

(1) must have 3 lines (2) must have 2 lines

(3) may have 2 lines (4) it is not formed

**34.** The atoms of sample $B$

(1) will ionize when emission spectra of $A$ is incident on $B$

(2) may ionize when emission spectra of $A$ is incident on $B$

(3) will be excited to some higher energy state but won't ionize

(4) none of the above

**For Problems 35–37**

An electron orbits a stationary nucleus of charge $+Ze$, where $Z$ is a constant and $e$ is the magnitude of electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit.

**35.** The value of $Z$ is

(1) 5 (2) 4

(3) 3 (4) 2

**36.** The radius of the first Bohr orbit is

(1) $0.529 \times 10^{-10}$ m (2) $0.106 \times 10^{-10}$ m

(3) $0.318 \times 10^{-10}$ m (4) none of these

**37.** Angular momentum of electron in the first Bohr orbit is

(1) $1.05 \times 10^{-13}$ J s (2) $2.10 \times 10^{-13}$ J s

(3) $3.15 \times 10^{-13}$ J s (4) Can be (1) or (2)

**For Problems 38–39**

When high energetic electron beam, (i.e., cathode rays) strike the heavier metal, then X-rays are produced. Spectrum of X-rays are classified into two categories: (i) continuous spectrum, and (ii) characteristic spectrum. The wavelength of continuous spectrum depends only on the potential difference across the electrode. But wavelength of characteristic spectrum depends on the atomic number ($Z$).

**38.** The production of characteristic X-ray is due to the

(1) continuous acceleration of incident electrons toward the nucleus

(2) continuous retardation of incident electrons toward the nucleus

(3) electron transitions between inner shells of the target atom

(4) electron transitions between outer shells of the target atom

**39.** The production of continuous X-ray is due to the

(1) acceleration of incident electrons by the nucleus of the target atom

(2) electron transitions between inner shells of the target atom

(3) electron transitions between outer shells of the target atom

(4) annihilation of the mass of incident electrons

**For Problems 40–42**

Light from a discharge tube containing hydrogen atoms falls on the surface of a plate of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV.

**40.** The energy of the photons causing the photoelectric emission is

(1) 2.55 eV (2) 0.73 eV

(3) 1.82 eV (4) Information insufficient

**41.** The quantum numbers of two levels involved in the emission of these photons is

(1) $4 \rightarrow 2$ (2) $3 \rightarrow 1$

(3) $3 \rightarrow 2$ (4) $4 \rightarrow 3$

**42.** The change in the angular momentum of the electron in the hydrogen atom in above transition is

(1) $\frac{2h}{\pi}$ (2) $\frac{h}{2\pi}$

(3) $\frac{h}{\pi}$ (4) $\frac{h}{4\pi}$

**For Problems 43–45**

The electron in a $Li^{++}$ ion is in the $n$th shell, $n$ being very large. One of the $K$-electrons in another metallic atom has been knocked out. The second metal has four orbits. Now, we take two samples—one of $Li^{++}$ ions and the other of the second metallic ions. Suppose the probability of electronic transition from higher to lower energy levels is directly proportional to the energy difference between the two shells. Take $hc = 1224$ eV nm, where $h$ is Planck's constant and $c$ the velocity of light in vacuum. It is found that major electromagnetic waves emitted from the two samples are identical. Now, answer the following questions:

**43.** What is the X-ray having least intensity emitted by the second sample?

(1) $K_\alpha$ (2) $L_\alpha$

(3) $M_\alpha$ (4) data insufficient

**44.** What is the major X-ray emitted by the sample?

(1) $K_\alpha$ (2) $K_\beta$

(3) $K_\gamma$ (4) $K_\delta$

**45.** The wavelength of this major X-ray is

(1) 0.90 Å (2) 1.0 Å

(3) 1.1 Å (4) none of these

**For Problems 46–47**

Two hydrogen-like atoms $A$ and $B$ are of different and each atom has ratio of neutron to proton equal to unity. The difference in the energies between the first Lyman lines emitted by $A$ and $B$ is 81.6 eV. When the atoms $A$ and $B$ moving with the same velocity strike separately a heavy target, they rebound with half of the speed before collision. However, in this process atom $B$ imparts the target a momentum which is three times the momentum imparted to target by atom $A$.

**46.** Atom $A$ is

(1) ${}_1^1H$ (2) ${}_1^2H$

(3) ${}_3^6Li$ (4) ${}_2^4Li$

**47.** The difference in the energies between the first Balmer lines emitted by $A$ and $B$ is

(1) 12.1 eV (2) 13.6 eV

(3) 14.3 eV (4) 15.1 eV

**For Problems 48–49**

1.8 g of hydrogen is excited by irradiation. The study of spectra indicated that 27% of the atoms are in the first excited state, 15% of the atoms in the second excited state, and the rest in the ground state. The ground state ionization potential energy of hydrogen atom is $21.4 \times 10^{-12}$ ergs.

**48.** The number of atoms present in the second excited state is

(1) $1.61 \times 10^{23}$ (2) $0.805 \times 10^{23}$

(3) $2.92 \times 10^{23}$ (4) $1.46 \times 10^{23}$

**49.** The total amount of energy that would be evolved when all the atoms return to the ground state is

(1) 782 kJ (2) 978.7 kJ

(3) $19.63 \times 10^{11}$ erg (4) $97.87 \times 10^{11}$ erg

**For Problems 50–52**

In a hydrogen like atom in which every atom is in a particular excited state. Now a stream of photons of energy $\frac{64}{225}E_0$ bombarded into it and is absorbed by the hydrogen like atom and subsequently its emission spectrum shows 10 different lines in which some lines have energy less than $\frac{64}{225}$ and some lines have energy more than $\frac{64}{225}E_0$ and some lines have energy equal to $\frac{64}{225}E_0$, (where $E_0$ is ionization energy of hydrogen atom in ground state).

**50.** The final quantum number of electron in the atom is

(1) 2 (2) 3

(3) 4 (4) 5

**51.** The initial quantum number of electron in the atom is

(1) 2 (2) 3

(3) 4 (4) 5

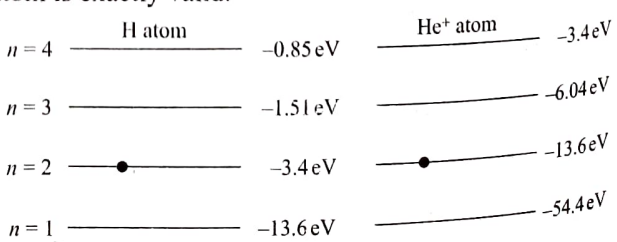
**52.** Atomic number of the atom is

(1) 2 (2) 3

(3) 4 (4) 5

**For Problems 53–55**

In a mixture of H-$He^+$ gas ($He^+$ is singly ionized He atom), H atoms and $He^+$ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to $He^+$ ions (by collisions). Assume that the Bohr model of atom is exactly valid.



53. The quantum number $n$ of the state finally populated in $He^+$ ions is
(1) 2 (2) 3
(3) 4 (4) 5

54. The wavelength of light emitted in the visible region by $He^+$ ions after collisions with H atoms is
(1) $6.5 \times 10^{-7}$ m (2) $5.6 \times 10^{-7}$ m
(3) $4.8 \times 10^{-7}$ m (4) $4.0 \times 10^{-7}$ m

55. The ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of $He^+$ ion is
(1) $\frac{1}{4}$ (2) $\frac{1}{2}$
(3) 1 (4) 2

## Matrix Match Type

1. In each situation of Column I, a physical quantity related to orbiting electron in hydrogen-like atom is given. The terms "$Z$" and "$n$" given in Column II have usual meaning in Bohr's theory. Match the quantities in Column I with the terms they depend on it Column II

| Column I | Column II |
|---|---|
| i. Frequency of orbiting electron | a. is directly proportional to $Z^2$ |
| ii. Angular momentum of orbiting electron | b. is directly proportional to $n$ |
| iii. Magnetic moment of orbiting electron | c. is inversely proportional to $n^3$ |
| iv. The average current due to orbiting of electron | d. is independent of $Z$ |

2. Take the usual meanings of the symbols to match the following:

| Column I | Column II |
|---|---|
| i. Average kinetic energy of photoelectrons | a. zero |
| ii. Minimum kinetic energy of photoelectrons | b. $hc/\lambda$ |
| iii. Maximum wavelength of continuous X-rays | c. $hc/eV$ |
| iv. Minimum wavelength of continuous X-rays | d. not predictable |

3. Match the following:

| Column I | Column II |
|---|---|
| i. The voltage applied to X-ray tube is increased | a. Average KE of the electrons decreases |
| ii. In photoelectric effect, work function of the target is increased | b. Average KE of the electrons increases |
| iii. Stopping potential decreases | c. Cut-off wavelength decreased |
| iv. Wavelength of $K_\alpha$ X-ray increased | d. Atomic number of target material decreases |

4. Match the following:

| Column I | Column II |
|---|---|
| i. Characteristic X-ray | a. Inverse process of photoelectric effect |
| ii. X-ray production | b. High potential difference |
| iii. Cut-off wavelength | c. Moseley's law |
| iv. Continuous X-ray | d. Emission of radiations |

5.

| Column I | Column II |
|---|---|
| i. $K_\alpha$ photon of aluminium | a. will be most energetic among the four |
| ii. $K_\beta$ photon of aluminium | b. will be least energetic among the four |
| iii. $K_\alpha$ photon from sodium | c. will be more energetic than the $K_\alpha$ photon of lithium |
| iv. $K_\beta$ photon of beryllium | d. constant speed |

6. Match the entries of Column I with the entries of Column II:

| Column I | Column II |
|---|---|
| i. Emission spectra | a. Discrete |
| ii. Absorption spectra | b. Continuous |
| iii. X-ray spectra | c. Electronic transition |
| iv. Thermal radiation spectra | d. Quantum theory of electromagnetic radiation |

7.

| Column I | Column II |
|---|---|
| i. Radius of orbit depends on principal quantum number as | a. increase |
| ii. Due to orbital motion of electron, magnetic field arises at the center of nucleus is proportional to principal quantum number as | b. decrease |
| iii. If electron is going from lower energy level to higher energy level, then velocity of electron will | c. proportional to $\frac{1}{n^2}$ |
| iv. If electron is going from lower energy level to energy level, then total energy of electron will | d. proportional to $n^2$ |
| | e. proportional to $\frac{1}{n^5}$ |

8.

| Column I | Column II |
|---|---|
| i. Radius of orbit is related with atomic number ($Z$) | a. is proportional to $Z$ |
| ii. Current associated due to orbital motion of electron with atomic number ($Z$) | b. is inversely proportional to $Z$ |
| iii. Magnetic field at the center due to orbital motion of electron related with $Z$ | c. is proportional to $Z^2$ |
| iv. Velocity of an electron related with atomic number ($Z$) | d. is proportional to $Z^3$ |

9. The spectral lines of hydrogen-like atom fall within the wavelength range from 950 Å to 1350 Å. Then, match the following.

| Column I | Column II |
|---|---|
| i. If it is atomic hydrogen atom and energy $E = -0.85$ eV | a. $\lambda = 1212$ Å and it corresponds to transition from 2 to 1 |
| ii. If it is atomic hydrogen atom and energy $E = -3.4$ eV | b. $\lambda = 134$ Å and it corresponds to transition from 2 to 1 |
| iii. If it is doubly ionized lithium atom, then | c. $\lambda = 303$ Å and it corresponds to transition from 2 to 1 |
| iv. If it is singly ionized helium atom, then | d. $\lambda = 970$ Å and it corresponds to transition from 4 to 1 |

## Numerical Value Type

1. Find recoil speed (approximately in m $s^{-1}$) when a hydrogen atom emits a photon during the transition from $n = 5$ to $n = 1$.
2. An atom of atomic number $Z = 11$ emits $K_\alpha$ wavelength which is $\lambda$. Find the atomic number for an atom that emits $K_\alpha$ radiation with wavelength $4\lambda$ (an integer).
3. An electron in $n$th excited state in a hydrogen atom comes down to first excited state by emitting ten different wavelengths. Find value of $n$ (an integer).
4. The shortest wavelength of the Brackett series of a hydrogen-like atom (atomic number $Z$) is the same as the shortest wavelength of the Balmer series of hydrogen atom. Find the value of $Z$.
5. Heat at the rate of 200 W is produced in an X-ray tube operating at 20 kV. Find the current in the circuit. Assume that only a small fraction of the kinetic energy of electrons is converted into X-rays. (in $\times 10^{-2}$A)
6. In the spectrum of singly ionized helium, the wavelength of a line observed is almost the same as the first line of Balmer series of hydrogen. It is due to transition of electron from $n_1 = 6$ to $n_2 =$ "*". What is the value of "*"?
7. A Bohr hydrogen atom undergoes a transition $n = 5$ to $n = 4$ and emits a photon of frequency $f$. Frequency of circular motion of electron in $n = 4$ orbit is $f_4$. The ratio $f/f_4$ is found to be $18/5m$. State the value of $m$.
8. The average lifetime for the $n = 3$ excited state of a hydrogen-like atom is $4.8 \times 10^{-8}$ s and that for the $n = 2$ state is $12.8 \times 10^{-8}$ s. The ratio of the average number of revolutions made in the $n = 2$ state to the average number of revolutions made in the $n = 3$ state before any transitions can take place from these states is.
9. An electron in hydrogen atom jumps from $n_1$ state to $n_2$ state, where $n_1$ and $n_2$ represent the quantum number of two states. The time period of revolution of electron in initial state is 8 times that in final state. Then the ratio of $n_1$ and $n_2$ is
10. Find the quantum number '$n$' corresponding to exciting state of $He^+$ ion if on transition to the ground state that ion emits two photons in succession with wavelength 1026.7 Å and 304 Å. ($R = 1.096 \times 10^7$/m)
11. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. Find the smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer in nm).
12. The peak emission from a black body at a certain temperature occurs at a wavelength of 9000 Å. On increasing its temperature, the total radiation emitted is increased 81 times. At the initial temperature when the peak radiation from the black body is incident on a metal surface, it does not cause any photoemission from the surface. After the increase of temperature, the peak radiation from the black body caused photoemission. To bring these photoelectrons to rest, a potential equivalent to the excitation energy between $n = 2$ and $n = 3$ Bohr levels of hydrogen atoms is required. Find the work function of the metal (in eV).
13. $K_\alpha$ wavelength emitted by an atom, of atomic number $Z = 11$ is $\lambda$. Find the atomic number for an atom that emits $K_\alpha$ radiation with wavelength $4\lambda$.
14. In a hydrogen-like atom, an electron is orbiting in an orbit having quantum number $n$. Its frequency of revolution is found to be $13.2 \times 10^{15}$ Hz. Energy required to move this electron from the atom to the above orbit is 54.4 eV. In a time of 7 nanosecond the electron jumps back to orbit having quantum number $n/2$. If $\tau$ be the average torque acted on the electron during the above process, then find $\tau \times 10^{-34}$ in Nm. (Given: $h/\lambda = 2.1 \times 10^{-34}$ J-s, frequency of revolution of electron in the ground state of H atom $v_0 = 6.6 \times 10^{15}$ Hz and ionization energy of H atom $E_0 = 13.6$ eV)
15. A hydrogen-like gas is kept in a chamber having a slit of width $d = 0.01$ mm. The atoms of the gas are continuously excited to a certain energy state. The excited electrons make transitions to lower levels; from the initial excited state to the second excited state and then from the second excited state to the ground state. In the process of de-excitation, photons are emitted and come out of the container through a slit. The intensity of the photons is observed on a screen placed

parallel to the plane of the slit. The ratio of the angular width of the central maximum corresponding to the two transitions is 25/2. The angular width of the central maximum due to first transition is $6.4 \times 10^{-2}$ radian. Find the atomic number of the gas .

16. A gas containing hydrogen-like ions, with atomic number $Z$, emits photons in transition $n+2 \rightarrow n$, where $n = Z$. These photons fall on a metallic plate and eject electrons having minimum de Broglie wavelength $\lambda$ of 5 Å. Find the value of $Z$, if the work function of metal is 4.2 eV

17. A hydrogen atom (mass = $1.66 \times 10^{-27}$ kg, ionization potential = 13.6 eV), moving with a velocity $6.24 \times 10^4$ m s$^{-1}$ makes a completely inelastic head-on collision with another stationary hydrogen atom. Both atoms are in the ground state before collision. Up to what state ($n$ = ?) either one atom may be excited?

18. Radiation from hydrogen gas excited to first excited state is used for illuminating certain photoelectric plate. When the radiation from some unknown hydrogen-like gas excited to the same level is used to expose the same plate, it is found that the de Broglie wavelength of the fastest photoelectron has decreased 2.3 times. It is given that the energy corresponding to the longest wavelength of the Lyman series of the unknown gas is 3 times the ionization energy of the hydrogen gas (13.6 eV). Find the work function of photoelectric plate in eV. [Take $(2.3)^2 = 5.25$.]

19. When the voltage applied to an X-ray tube is increased from 10 kV to 20 kV, the wavelength interval between the $K_\alpha$ line and the short wave cut off of the continuous X-ray spectrum increases by a factor 3. Find the atomic number of the element of which the tube anti cathode is made. (Rydberg's constant = $10^7$ m$^{-1}$).

# Archives

## JEE MAIN

### Single Correct Answer Type

1. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from

(1) $2 \rightarrow 1$ (2) $3 \rightarrow 2$

(3) $4 \rightarrow 2$ (4) $5 \rightarrow 4$ **(AIEEE 2009)**

2. Energy required for the electron excitation in $Li^{++}$ from the first to the third Bohr orbit is

(1) 12.1 eV (2) 36.3 eV

(3) 108.8 eV (4) 122.4 eV

**(AIEEE 2011)**

3. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be

(1) 2 (2) 3

(3) 5 (4) 6 **(AIEEE 2012)**

4. A diatomic molecule is made of two masses $m_1$ and $m_2$ which are separated by a distance $r$. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by

(1) $\dfrac{n^2h^2}{2(m_1+m_2)r^2}$ (2) $\dfrac{2n^2h^2}{(m_1+m_2)r^2}$

(3) $\dfrac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$ (4) $\dfrac{(m_1+m_2)^2n^2h^2}{2m_1^2m_2^2r^2}$

**(AIEEE 2012)**

5. In a hydrogen-like atom an electron makes transition from an energy level with quantum number $n$ to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to

(1) $\dfrac{1}{n^2}$ (2) $\dfrac{1}{n^{3/2}}$

(3) $\dfrac{1}{n^3}$ (4) $\dfrac{1}{n}$ **(JEE Main 2015)**

6. The radiation corresponding to $3 \times 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of $3 \times 10^{-4}$ T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to

(1) 0.8 eV (2) 1.6 eV

(3) 1.8 eV (4) 1.1 eV

**(JEE Main 2015)**

7. As an electron makes a transition from an excited state to the ground state of a hydrogen-like atom/ion

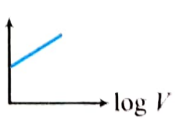
(1) its kinetic energy increases but potential energy and total energy decrease

(2) kinetic energy, potential energy and total energy decrease

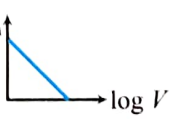
(3) kinetic energy decreases, potential energy increases but total energy remains same

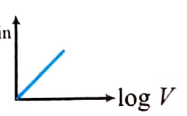
(4) kinetic energy and total energy decrease but potential energy increases **(JEE Main 2015)**

8. An electron beam is accelerated by a potential difference $V$ to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If $\lambda_{min}$ is the smallest possible wavelength of X-ray in the spectrum, the variation of $\log \lambda_{min}$ with $\log V$ is correctly represented in

(1) 

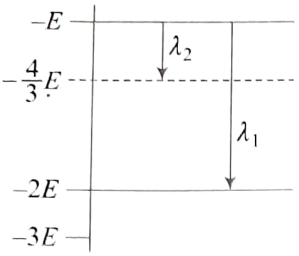

(2)


(3) 


(4) 


**(JEE Main 2017)**

9. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by




(1) $r=\frac{3}{4}$ (2) $r=\frac{1}{3}$

(3) $r=\frac{4}{3}$ (4) $r=\frac{2}{3}$

**(JEE Main 2017)**

**10.** An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let $\lambda_n$, $\lambda_g$ be the de Broglie wavelength of the electron in the $n^{th}$ state and the ground state respectively. Let $\Lambda_n$ be the wavelength of the emitted photon in the transition from the $n^{th}$ state to the ground state. For large $n$, ($A$, $B$ are constants)

(1) $\Lambda_n^2 \approx \lambda$ (2) $\Lambda_n \approx A+\frac{B}{\lambda_n^2}$

(3) $\Lambda_n \approx A+B\lambda_n$ (4) $\Lambda_n^2 \approx A+B\lambda_n^2$

**(JEE Main 2018)**

**11.** If the series limit frequency of the Lyman series is $\nu_L$, then the series limit frequency of the $P$ fund series is

(1) $\frac{\nu_L}{25}$ (2) $25\ \nu_L$

(3) $16\ \nu_L$ (4) $\frac{\nu_L}{16}$ **(JEE Main 2018)**

# JEE ADVANCED

## Single Correct Answer Type

**1.** The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly ionized helium atom is

(1) 1215 Å (2) 1640 Å

(3) 2430 Å (4) 4687 Å

**(IIT-JEE 2011)**

## Multiple Correct Answers Type

**1.** Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge $Ze$ are defined by their principal quantum number $n$, where $n >> 1$. Which of the following statement(s) is (are) true?

(1) Relative change in the radii of two consecutive orbitals does not depend on Z

(2) Relative change in the radii of two consecutive oribitals varies as $1/n$

(3) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(4) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

**(JEE Advanced 2016)**

## Linked Comprehension Type

**For Problems 1–3**

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition.

**(IIT-JEE 2010)**

**1.** A diatomic molecule has moment of inertia $I$. By Bohr's quantization condition, its rotational energy in the $n$th level ($n = 0$ is not allowed) is

(1) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (2) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(3) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (4) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

**2.** It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $4/\pi \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to (Take $h = 2\pi \times 10^{-34}$ J s)

(1) $2.76 \times 10^{-46}$ kg m$^2$

(2) $1.87 \times 10^{-46}$ kg m$^2$

(3) $4.67 \times 10^{-47}$ kg m$^2$

(4) $1.17 \times 10^{-47}$ kg m$^2$

**3.** In a CO molecule, the distance between $C$ (mass = 12 a.m.u) and O (mass = 16 a.m.u.), where 1 a.m.u. = $5/3 \times 10^{-27}$ kg, is close to

(1) $2.4 \times 10^{-10}$ m (2) $1.9 \times 10^{-10}$ m

(3) $1.3 \times 10^{-10}$ m (4) $4.4 \times 10^{-11}$ m

## Numerical Value Type

**1.** A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as −13.6 eV, the number of lines present in the emission spectrum is

**(JEE Advanced 2016)**

**2.** An electron in a hydrogen atom undergoes a transition from an orbit with quantum number $n_i$ to another with quantum number $n_f$. $V_i$ and $V_f$ are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$, then the smallest possible $n_f$ is _____.

**(JEE Advanced 2017)**

**3.** Consider a hydrogen-like ionized atom with atomic number $Z$ with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy 74.8 eV higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of $Z$ is..... .

**(JEE Advanced 2018)**

# Answers Key

## EXERCISES

### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| 1. (4) | 2. (3) | 3. (3) | 4. (4) | 5. (1) |
| 6. (1) | 7. (2) | 8. (4) | 9. (3) | 10. (4) |
| 11. (2) | 12. (1) | 13. (2) | 14. (3) | 15. (2) |
| 16. (3) | 17. (1) | 18. (2) | 19. (4) | 20. (4) |
| 21. (1) | 22. (3) | 23. (2) | 24. (3) | 25. (2) |
| 26. (1) | 27. (3) | 28. (4) | 29. (4) | 30. (4) |
| 31. (1) | 32. (2) | 33. (4) | 34. (3) | 35. (2) |
| 36. (2) | 37. (3) | 38. (3) | 39. (2) | 40. (3) |
| 41. (2) | 42. (2) | 43. (2) | 44. (4) | 45. (1) |
| 46. (2) | 47. (3) | 48. (2) | 49. (4) | 50. (1) |
| 51. (3) | 52. (4) | 53. (1) | 54. (3) | 55. (2) |
| 56. (2) | 57. (4) | 58. (1) | 59. (1) | 60. (4) |
| 61. (3) | 62. (4) | 63. (4) | 64. (2) | 65. (3) |
| 66. (2) | 67. (2) | 68. (2) | 69. (3) | 70. (2) |
| 71. (4) | 72. (3) | 73. (2) | 74. (4) | 75. (3) |
| 76. (2) | 77. (1) | 78. (2) | 79. (4) | 80. (4) |
| 81. (4) | 82. (4) | 83. (2) | 84. (2) | 85. (3) |
| 86. (4) | 87. (3) | 88. (1) | 89. (4) | 90. (1) |
| 91. (4) | 92. (4) | 93. (4) | 94. (1) | |

### Multiple Correct Answers Type

| | | |
|---|---|---|
| 1. (1),(2),(4) | 2. (2),(3) | 3. (1),(3),(4) |
| 4. (2),(3),(4) | 5. (1),(2),(3) | 6. (1),(2) |
| 7. (1),(2) | 8. (2),(4) | 9. (2),(3),(4) |
| 10. (1),(3),(4) | 11. (2),(3) | 12. (1),(2) |
| 13. (1),(2),(3) | 14. (1),(2),(3) | 15. (1),(3),(4) |
| 16. (2),(3) | 17. (1),(3) | 18. (2),(4) |
| 19. (1),(3) | 20. (2),(3),(4) | 21. (3),(4) |
| 22. (1),(3) | 23. (1),(2) | 24. (1),(3) |
| 25. (1),(3),(4) | 26. (1),(3) | 27. (1),(2),(3) |
| 28. (1),(2) | 29. (1),(2),(4) | 30. (2),(3),(4) |
| 31. (3),(4) | 32. (1),(4) | |

### Linked Comprehension Type

| | | | | |
|---|---|---|---|---|
| 1. (2) | 2. (2) | 3. (2) | 4. (1) | 5. (3) |
| 6. (4) | 7. (3) | 8. (2) | 9. (3) | 10. (2) |
| 11. (3) | 12. (1) | 13. (3) | 14. (2) | 15. (2) |
| 16. (4) | 17. (2) | 18. (1) | 19. (3) | 20. (2) |
| 21. (3) | 22. (1) | 23. (4) | 24. (3) | 25. (4) |
| 26. (3) | 27. (2) | 28. (2) | 29. (1) | 30. (2) |
| 31. (4) | 32. (4) | 33. (4) | 34. (4) | 35. (1) |
| 36. (2) | 37. (1) | 38. (3) | 39. (1) | 40. (1) |
| 41. (1) | 42. (3) | 43. (1) | 44. (3) | 45. (4) |
| 46. (2) | 47. (4) | 48. (3) | 49. (1) | 50. (4) |
| 51. (2) | 52. (1) | 53. (3) | 54. (3) | 55. (1) |

### Matrix Match Type

1. i. → a., c.; ii. → b., d.; iii. → b., d.; iv. → b., c.
2. i. → d.; ii. → a.; iii. → d.; iv. → c.
3. i. → b., c.; ii. → a., c.; iii. → a., c.; iv. → d.
4. i. → c.; ii. → a., b., c., d.; iii. → b.; iv. → d.
5. i. → c., d.; ii. → b., c., d.; iii. → c., d.; iv. → b., c., d.
6. i. → b., c., d.; ii. → a., b., c., d.; iii. → b., c., d.; iv. → b., c., d.
7. i. → d.; ii. → e.; iii. → b.; iv. → a.
8. i. → b.; ii. → c.; iii. → d.; iv. → a.
9. i. → d.; ii. → a.; iii. → b.; iv. → c.

### Numerical Value Type

| | | | | |
|---|---|---|---|---|
| 1. (4) | 2. (6) | 3. (6) | 4. (2) | 5. (1) |
| 6. (4) | 7. (5) | 8. (9) | 9. (2) | 10. (6) |
| 11. (823) | 12. (2.25) | 13. (6) | 14. (15) | 15. (2) |
| 16. (2) | 17. (2) | 18. (3) | 19. (30) | |

## ARCHIVES

### JEE Main

#### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| 1. (4) | 2. (3) | 3. (4) | 4. (3) | 5. (3) |
| 6. (4) | 7. (1) | 8. (3) | 9. (2) | 10. (2) |
| 11. (1) | | | | |

### JEE Advanced

#### Single Correct Answer Type

1. (2)

#### Multiple Correct Answers Type

1. (1),(2),(4)

#### Linked Comprehension Type

| | | |
|---|---|---|
| 1. (4) | 2. (1) | 3. (3) |

#### Numerical Value Type

| | | |
|---|---|---|
| 1. (6) | 2. (5) | 3. (3) |

# 7 Nuclear Physics

## NUCLEAR STRUCTURE

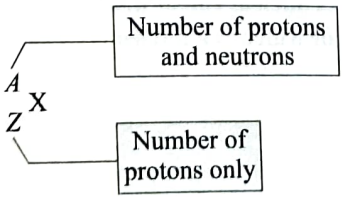
Nuclear physics is the branch of physics which deals with the study of nucleus. Atoms consist of electrons in orbit about a central nucleus. The electron orbits are quantum mechanical in nature and have interesting characteristics. Little has been said about the nucleus, however. Since the nucleus is interesting in its own right, we now consider it in greater detail.

The nucleus of an atom consists of neutrons and protons, collectively referred to as nucleons. The neutron, discovered in 1932 by the English physicist James Chadwick (1891–1974), carries no electrical charge and has a mass slightly larger than that of a proton.

The number of protons in the nucleus is different in different elements and is given by the atomic number $Z$. In an electrically neutral atom, the number of nuclear protons equals the number of electrons in orbit around the nucleus. The number of neutrons in the nucleus is $N$. The total number of protons and neutrons is referred to as the atomic mass number $A$ because the total nuclear mass is approximately equal to $A$ times the mass of a single nucleon:

$$\begin{matrix}A\\ \text{[Number of proton and neutrons (atomic mass number or nucleon number)]}\end{matrix} = \begin{matrix}A\\ \text{[Number of protons (atomic number)]}\end{matrix} + \begin{matrix}N\\ \text{[Number of neutrons]}\end{matrix} \quad ...(i)$$

Sometimes $A$ is also called the nucleon number. A shorthand notation is often used to specify $Z$ and $A$ along with the chemical symbol for the element. For instance, the nuclei of all naturally occurring aluminum atoms have $A = 27$, and the atomic number for aluminum is $Z = 13$. In shorthand notation, then, the aluminium nucleus is specified as ${}^{27}_{13}\text{Al}$. The number of neutrons in an aluminium nucleus is $N = A - Z = 14$. In general, for an element whose chemical symbol is X, the symbol for the nucleus is shown in figure.




For a proton the symbol is ${}^{1}_{1}\text{H}$, since the proton is the nucleus of a hydrogen atom. A neutron is denoted by ${}^{1}_{0}n$. In the case of an electron we use ${}^{0}_{-1}e$, where $A = 0$ because an electron is not composed of protons or neutrons and $Z = -1$ because the electron has a negative charge.

The number of protons in the nucleus characterizes the species of the atom—the element to which the atom belongs. Electrons can be added to or removed from an atom to form an ion, but this does not change its species. For example, removing two electrons from calcium results in calcium ion $Ca^{2+}$, but its species is still calcium. The number of electrons can vary by ionization, but the proton number cannot vary without changing an atom of a different element, the chemical symbol implies the proton number.

Each nuclear species with a given $Z$ and $A$ is called a nuclide. Each $Z$ characterizes a chemical element. Nucleus mass is roughly the sum of its constituent proton and neutron masses. The nuclear charge is $Z$ (atomic number) times the charge of a proton ($e = 1.6 \times 10^{-19}$ C).

The chemical properties of an atom are determined by its electron configuration. The number of electrons and protons are equal in number in a neutral atom; the chemical properties are essentially determined by $Z$. The dependence of the chemical properties on $N$ is negligible.

### ILLUSTRATION 7.1

How many electrons, protons, and neutrons are there in 12 g of ${}_{6}C^{12}$ and in 14 g of ${}_{6}C^{14}$?

**Sol.** Mass number (or atomic weight) of ${}_{6}C^{12}$ is 12.

Therefore, the number of atoms in 12 g of ${}_{6}C^{12}$ is

Avogadro number $= 6 \times 10^{23}$

The number of electrons in 12 g of ${}_{6}C^{12}$ is

$6 \times 6 \times 10^{23} = 36 \times 10^{23}$

The number of protons in 12 g of ${}_{6}C^{12}$ is $36 \times 10^{23}$.

The number of neutrons in 12 g of ${}_{6}C^{12}$ is

$(A - Z) \times 6 \times 10^{23} = (12 - 6) \times 6 \times 10^{23} = 36 \times 10^{23}$

Similarly, number of electrons in 14 g of ${}_{6}C^{14}$ is $36 \times 10^{23}$.

Number of protons in 14 g of ${}_{6}C^{14}$ is $36 \times 10^{23}$.

Number of neutrons in 14 g of ${}_{6}C^{14} = (A - Z) \times 6 \times 10^{23}$

$= (14 - 6) \times 6 \times 10^{23}$

$= 48 \times 10^{23}$

## ATOMIC MASS NUMBER

It is the nearest integer value of mass represented in amu (atomic mass unit). Atomic masses refer to the masses of neutral atoms. Thus, an atomic mass always includes the mass of its $Z$ electrons. The atomic mass of an atom is measured relative to the mass of an atom of the neutral carbon-12 isotope (the nucleus plus six electrons). Atomic masses are measured in atomic mass units, which are denoted by the symbol "u". Atomic mass units are defined in terms of the mass of the isotope ${}^{12}C$, whose atomic mass is defined to be exactly 12 u.

$$1 \text{ amu} = \frac{1}{12} \times [\text{mass of one atom of } {}_6C^{12} \text{ atom at rest and in ground state}]$$
$$= 1.6603 \times 10^{-27} \text{ kg}$$

### Properties of particles in the atom

| Particle | Electric charge (C) | Mass | |
|---|---|---|---|
| | | Kilogram (kg) | Atomic mass units (u) |
| Electron | $-1.60 \times 10^{-19}$ | $9.109390 \times 10^{-31}$ | $5.485799 \times 10^{-4}$ |
| Proton | $+1.60 \times 10^{-19}$ | $1.672623 \times 10^{-27}$ | 1.007276 |
| Neutron | 0 | $1.674929 \times 10^{-27}$ | 1.008665 |

## SOME DEFINITIONS

**Isotopes:** The nuclei having the same number of protons but different number of neutrons are called isotopes.

Isotopes of carbon: ${}^{6}_{11}C$, ${}^{12}_{6}C$, ${}^{13}_{6}C$, ${}^{14}_{6}C$

Isotopes of hydrogen: ${}^{1}_{1}H$, ${}^{2}_{1}H$, ${}^{3}_{1}H$

The isotopes $^1H$ is called ordinary hydrogen or simply hydrogen; $^2H$ is called deuterium. Deuterium, sometimes known as heavy hydrogen, can combine with oxygen to form heavy water ($D_2O$). The third isotope of hydrogen, $^3H$, is called tritium, which is unstable.

**Isotones:** Nuclei with the same neutron number $N$ but different atomic number $Z$ are called isotones.

${}^{14}_{6}C$, ${}^{15}_{7}N$, ${}^{16}_{8}C$, ${}^{17}_{9}F$

**Isobars:** The nuclei with the same mass number but different atomic number are called isobars.

${}^{16}_{6}C$, ${}^{16}_{7}N$, ${}^{16}_{8}O$, ${}^{16}_{9}F$

## SIZE OF NUCLEI

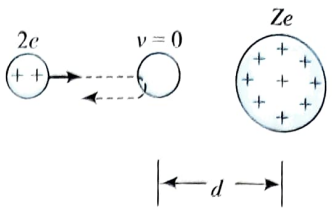
The size and structure of nuclei were first investigated in the scattering experiments of Rutherford. Using the principle of conservation of energy, Rutherford found an expression for how close an alpha particle moving directly toward the nucleus can come to the nucleus before being turned around by Coulomb repulsion.

In such a head-on collision, the kinetic energy of the incoming alpha particle must be converted completely to electrical potential energy when the particle stops at the point of closest approach and turns around (refer figure). If we equate the initial kinetic energy of the alpha particle to the maximum electrical potential energy of the system (alpha particle + target nucleus), we have

$$\frac{1}{2}mv^2 = k_e\frac{q_1q_2}{r} = k_e\frac{(2e)(Ze)}{d}$$

where $d$ is the distance of closest approach. Solving for $d$, we get

$$d = \frac{4k_eZe^2}{mv^2}$$




**An alpha particle on a head-on collision with a nucleus of charge Ze. Because of the Coulomb repulsion between charges of the same sign, the alpha particle approaches to a distance $d$ from the target nucleus called the distance of closest approach**

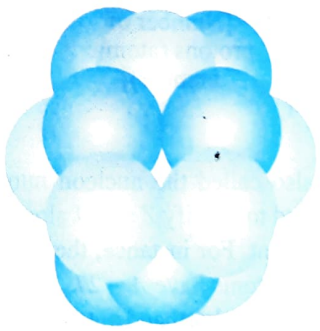
From this expression, Rutherford found that alpha particles approached to within $3.2 \times 10^{-14}$ m of a nucleus when the foil was made of gold. Thus, the radius of the gold nucleus must be less than this value. For silver atoms, the distance of closest approach was $2 \times 10^{-14}$ m. From these results, Rutherford concluded that the positive charge in an atom is concentrated in a small sphere, which he called the nucleus, whose radius is no greater than about $10^{-14}$ m. Because such small lengths are common in nuclear physics, a convenient unit of length is the femtometer (fm), sometimes called the Fermi, defined as

$$1 \text{ fm} = 10^{-15} \text{ m}$$

Since the time of Rutherford's scattering experiments, a couple of other experiments have shown that most nuclei are approximately spherical and have an average radius given by

$$r = r_0A^{1/3} \quad \ldots(i)$$

where $A$ is the total number of nucleons and $r_0$ is a constant equal to $1.2 \times 10^{-15}$ m. Because the volume of a sphere is proportional to the cube of its radius, it follows from Eq. (i) that the volume of a nucleus (assumed to be spherical) is directly proportional to $A$, the total number of nucleons. This suggests that all nuclei have nearly the same density. Nucleons combine to form a nucleus as though they were tightly packed spheres (see figure).



**A nucleus can be modelled as a cluster of tightly packed spheres, each of which is a nucleon**

### ILLUSTRATION 7.2

**(a)** Find an approximate expression for the mass of a nucleus of mass number $A$.

**(b)** Find an expression for the volume of this nucleus in terms of the mass number.

**(c)** Find a numerical value for its density.

**Sol.**

**(a)** The mass of the proton is approximately equal to that of neutron. Thus, if the mass of one of these particles is $m$, the mass of the nucleus is approximately $Am$.

(b) Assuming the nucleus is spherical and using Eq. (i), we find that the volume is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r_0^3 A$$

(c) The nuclear density can be found as follows:

$$\rho_n = \frac{\text{mass}}{\text{volume}} = \frac{Am}{\frac{4}{3}\pi r_0^3 A} = \frac{3m}{4\pi r_0^3}$$

The fact that $A$ cancels out of our expression for nuclear density proves our statement above that all nuclei have roughly the same density. Taking $r_0 = 1.2 \times 10^{-15}$ m and $m = 1.67 \times 10^{-27}$ kg, we find that

$$\rho_n = \frac{3(1.67 \times 10^{-27}\,\text{kg})}{4\pi(1.2 \times 10^{-15}\,\text{m})^3} = 2.3 \times 10^{17}\ \text{kg m}^{-3}$$

Note that the nuclear density is about $2.3 \times 10^{14}$ times greater than the density of water ($1 \times 10^3$ kg m$^{-3}$).

**ILLUSTRATION 7.3**

Calculate the radius of $^{70}$Ge.

**Sol.** We have,

$$R = R_0 A^{1/3} = (1.2\ \text{fm})\,(70)^{1/3} = (1.2\ \text{fm})\,(4.12) = 4.94\ \text{fm}$$

**ILLUSTRATION 7.4**

The most common kind of iron nucleus has a mass number of 56. Find the radius, approximate mass, and approximate density of the nucleus.

**Sol.** We use two key ideas. The radius and mass of a nucleus depend on the mass number $A$ and density is mass divided by volume.

We use Eq. (i) to determine the radius of the nucleus. The mass of the nucleus in atomic mass units is approximately equal to the mass number. The radius is

$$R = R_0 A^{1/3} = (1.2 \times 10^{-15}\,\text{m})(56)^{1/3} = 4.6 \times 10^{-15}\,\text{m} = 4.6\ \text{fm}$$

Since $A = 56$, the mass of the nucleus is approximately 56 u, or $m \approx (56)\,(1.66 \times 10^{-27}\,\text{kg}) = 9.3 \times 10^{-26}$ kg. The volume is

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3} \times 3.14 \times (4.6 \times 10^{-15})^3 \approx 4.1 \times 10^{-13}\ \text{m}^3$$

And the density $\rho$ is approximately

$$\rho = \frac{m}{V} \approx \frac{9.3 \times 10^{-26}\,\text{kg}}{4.1 \times 10^{-43}\,\text{m}^3} = 2.3 \times 10^{17}\ \text{kg m}^{-3}$$

The density of solid iron is about 700 kg m$^{-3}$, so we see that the iron nucleus is more than $10^{13}$ times as dense as the bulk material. Densities of this magnitude are also found in neutron stars, which are similar to gigantic nuclei made almost entirely of neutrons. A 1 cm cube of material with this density would have a mass of $2.3 \times 10^{11}$ kg or 230 million metric tons.

**ILLUSTRATION 7.5**

Calculate the electric potential energy of interaction due to the electric repulsion between two nuclei of $^{12}$C when they "touch" each other at the surface.

**Sol.** The radius of a $^{12}$C nucleus is

$$R = R_0 A^{1/3} = (1.2\ \text{fm})\,(12)^{1/3} = 2.75\ \text{fm}$$

The separation between the centers of the nuclei is $2R = 5.50$ fm. The potential energy of the pair is

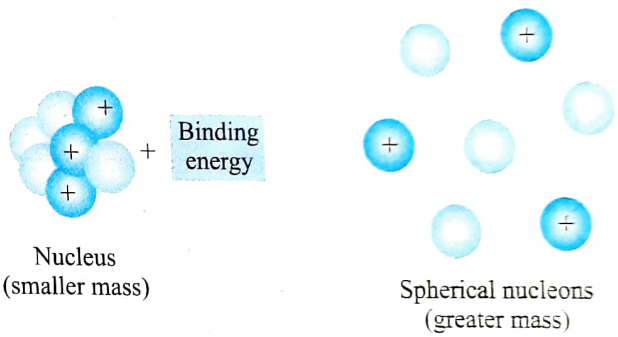
$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r} = (9 \times 10^9\ \text{N m}^2\ \text{C}^{-2})\ \frac{(6 \times 1.6 \times 10^{-19}\,\text{C})^2}{5.50 \times 10^{-15}\,\text{m}}$$

$$= 1.50 \times 10^{-12}\ \text{J} = 9.39\ \text{MeV}$$

## NUCLEAR BINDING ENERGY

We know that in a stable nucleus, because of strong nuclear force of attraction, the nucleons are held tightly together in a small volume. As the system is stable, we can relatively say that the total potential energy of the system is negative and to separate all the nucleons from each other, some energy must be supplied to break the nucleus. The more stable the nucleus is, the greater is the energy needed to break it apart. This required energy is called binding energy of the nucleus.

The mass of each element atom is less than the sum of masses of its constituent particles.




For example, let us discuss the $^4_2$He atom. We compare the mass of atom to that of its constituents. To calculate the mass of the components, we can either add the masses of two protons, two neutrons, and two electrons or we can add the masses of two H atoms and two neutrons. Using the values from table, we have

$$2m_H + 2m_H = 2(1.007825) + 2(1.008665) = 4.0329804 \quad \ldots(i)$$

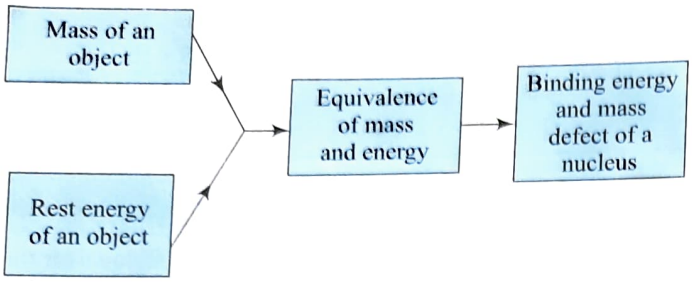
Now, we can see that in the experimental mass of $^4_2$He atom is 4.0026034 which is less than the value given in Eq. (i), the sum of masses of constituent particles. Similarly, this can be verified for all the other elements. Also, the reason for this can be explained by Eq. (ii) which is a basic nuclear reaction for formation of a nuclei $^A_Z$X. In this reaction, we can see that when $Z$ protons and $A - Z$ neutrons fuse together to form a nucleus $^A_Z$X, some amount of energy must be released.

$$Zm_p - (A + Z)m_n \rightarrow M_X + \text{Energy} \quad \ldots(ii)$$

If we think from where this energy comes, we can simply say by Einstein's mass–energy relationship, some amount of mass from independent nucleons is converted into energy and released when nucleons bind with each other to form a stable nucleus. This is the energy that hold the nucleons together in a nucleus, we call this binding energy of nucleus. So, when this energy is supplied to a nucleus, it splits into its constituent particles.

The binding energy used to disassemble the nucleus appears as extra mass of the separated nucleons. In other words, the sum of the individual masses of the separated protons and

neutrons is greater by an amount $\Delta m$ than the mass of the stable nucleus. The difference in mass $\Delta m$ is known as the mass defect of the nucleus.




For the reaction given in Eq. (ii), if we calculate the difference in masses of nucleons and that of nucleon X, then it is given as

$$\Delta m = Zm_p + (A - Z)m_n - M_X \quad ...(iii)$$

This difference in masses of independent nucleons and mass of nucleus is called mass defect of the nuclear reaction. Using mass defect $\Delta m$, we can find the energy released in a nuclear reaction, e.g., the binding energy of the above nucleus X can be given as

$$\Delta E_{BE} = \Delta mc^2 \quad ...(iv)$$

In a similar way, we can find the binding energy for any nucleus in nature for a known composition.

### MASS DEFECT

It has been observed that there is a difference between expected mass and actual mass of a nucleus.

$$M_{expected} = Zm_p + (A - Z)m_n$$
$$M_{observed} = M_{atom} - Zm_e$$

It is found that $M_{observed} < M_{expected}$

Hence, mass defect is defined as

Mass defect, $\Delta m = M_{expected} - M_{observed}$

$$\Rightarrow \quad \Delta m = [Zm_p + (A - Z)m_n] - [M_{atom} - Zm_e]$$

## MASS–ENERGY EQUIVALENCE

We have discussed in previous section that using mass defect of a nuclear reaction, we can find the energy released in the nuclear process. We know generally nuclear masses are given in atomic mass unit where

$$1 \text{ amu} = 1.656 \times 10^{-27} \text{ kg}$$

If in a reaction 1 amu mass is converted into energy, then using Einstein's mass–energy relationship the amount of energy released is

$$\Delta E' = \Delta mc^2 = (1.656 \times 10^{-27})(3 \times 10^8)^2 \text{ J}$$

$$= \frac{(1.656 \times 10^{-27})(3 \times 10^8)^2}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 931.5 \times 10^6 \text{ eV} = 931.5 \text{ MeV}$$

Thus, we can say that 1 amu mass is equivalent to 931.5 MeV energy.

### ILLUSTRATION 7.6

The most abundant isotope of helium has a ${}_2^4\text{H}$ nucleus whose mass is $6.6447 \times 10^{-27}$ kg. For this nucleus, find **(a)** the mass defect and **(b)** the binding energy.

Given: Mass of the electron: $m_e = 5.485799 \times 10^{-4}$ u, mass of the proton: $m_p = 1.007276$ u and mass of the neutron: $m_n = 1.008\,665$ u.

**Sol.** The symbol ${}_2^4\text{He}$ indicates that the helium nucleus contains $Z = 2$ protons and $N = 4 - 2 = 2$ neutrons. To obtain the mass defect $\Delta m$, we first determine the sum of the individual masses of the separated protons and neutrons. Then, we subtract from this sum the mass of the nucleus.

**(a)** We find that the sum of the individual masses of the nucleons is

$$\underbrace{2(1.6726 \times 10^{-27} \text{ kg})}_{\text{Two protons}} + \underbrace{2(1.6749 \times 10^{-27} \text{ kg})}_{\text{Two neutrons}}$$

$$= 6.6950 \times 10^{-27} \text{ kg}$$

This value is greater than the mass of the intact He nucleus, and the mass defect is

$$\Delta m = 6.6950 \times 10^{-27} \text{ kg} - 6.6447 \times 10^{-27} \text{ kg}$$
$$= 0.0503 \times 10^{-27} \text{ kg}$$

**(b)** According to Eq. (iv), the binding energy is given by

$$\Delta E_{BE} = (\Delta m)c^2 = (0.0503 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m s}^{-1})^2$$
$$= 4.53 \times 10^{-12} \text{ J}$$

Usually, binding energies are expressed in energy units of electron volt instead of joule ($1 \text{ eV} = 1.60 \times 10^{-19}$ J).

$$\therefore \quad \text{Binding energy} = (4.53 \times 10^{-12} \text{ J})\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right)$$

$$= 2.83 \times 10^7 \text{ eV} = 28.3 \text{ MeV}$$

In this result, one million electron volt is denoted by the unit MeV. The value of 28.3 MeV is more than two million times greater than the energy required to remove an orbital electron from an atom.

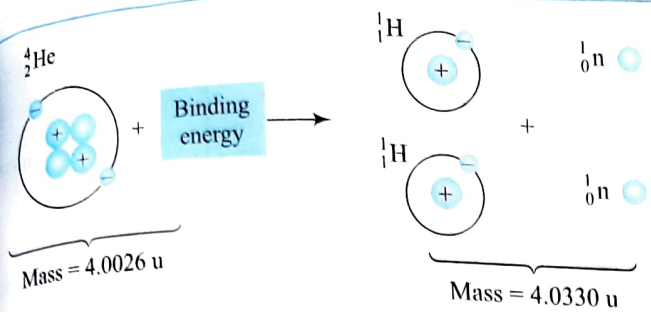
### ILLUSTRATION 7.7

The atomic mass of ${}_2^4\text{He}$ is 4.0026 u and the atomic mass of ${}_1^1\text{H}$ is 1.0078 u. Using atomic mass units instead of kilograms, obtain the binding energy of the ${}_2^4\text{He}$ nucleus.

**Sol.** To determine the binding energy, we calculate the mass defect in atomic mass units and then use the fact that one atomic mass unit is equivalent to 931.5 MeV of energy. The mass of 4.0026 u for ${}_2^4\text{He}$ includes the mass of the two electrons in the neutral helium atom. To calculate the mass defect, we must subtract 4.0026 u from the sum of the individual masses of the nucleons, including the mass of the electrons. As the figure illustrates, the electron mass will be included if the masses of two hydrogen atoms are used in the calculation instead of the masses of two protons. The mass of a hydrogen atom is given as 1.0078 u, and the mass of a neutron is given in table as 1.0087 u.

The sum of the individual masses is

$$\underbrace{2(1.0078 \text{ u})}_{\substack{\text{Two hydrogen} \\ \text{atoms}}} + \underbrace{2(1.0087 \text{ u})}_{\text{Two neutrons}} = 4.0330 \text{ u}$$




The mass defect is $\Delta m = 4.0330\text{ u} - 4.0026\text{ u} = 0.0304\text{ u}$. Since 1 u is equivalent to 931.5 MeV, the binding energy is 28.3 MeV.

**ILLUSTRATION 7.8**

The nucleus of the deuterium atom, called the deuteron, consists of a proton and a neutron. Calculate the deuteron's binding energy, given atomic mass, i.e., the mass of a deuterium nucleus plus an electron is measured to be 2.014102 u.

**Sol.** We know that the proton and neutron masses are

$$m_p = 1.007825\text{ u}, \quad m_n = 1.008665\text{ u}$$

Note that the masses used for the proton and neutron in this example are actually those of the neutral atoms. We are able to use atomic masses for these calculations because the electron masses cancel out. That is, when $m_d$ containing one electron mass is subtracted from $(m_p + m_n)$ containing one electron mass, the electron masses cancel out. We have,

$$m_p + m_n = 2.016490\text{ u}$$

To calculate the mass difference, we subtract the deuteron mass from this value.

$$\therefore \quad \Delta m = (m_p + m_n) - m_d$$
$$= 2.016490\text{ u} - 2.014102\text{ u} = 0.002388\text{ u}$$

Because 1 u corresponds to an equivalent energy of 931.494 MeV (i.e., $1\text{ u}.c^2 = 931.494$ MeV), the mass difference corresponds to the binding energy

$$E_b = (0.002388\text{ u})\,(931.494\text{ MeV/u}) = 2.224\text{ MeV}$$

This result tells us that to separate a deuteron into a proton and a neutron, it is necessary to add 2.224 MeV of energy to the deuteron to overcome the attractive nuclear force between the proton and neutron. One way of supplying the deuteron with this energy is by bombarding it with energetic particles.

If the binding energy of a nucleus were zero, the nucleus would separate into its constituent protons and neutrons without the addition of any energy, i.e., it would spontaneously break apart.

## BINDING ENERGY PER NUCLEON

If we wish to see how nuclear binding energy varies from nucleus to nucleus, it is necessary to compare the binding energy per nucleon basis. The binding energy per nucleon for a given element can be given as binding energy divided by the nucleon number $A$ as

$$(\text{BE})_\text{N} = \frac{\Delta mc^2}{A} \qquad \ldots(\text{i})$$

This value $(\text{BE})_\text{N}$ in Eq. (i) gives the criterion of stability among different elements. We can define binding energy per nucleon theoretically as the amount of energy needed to remove a nucleon from the nucleus of an element. For example, let us consider two elements X and Y with mass numbers $A_\text{X}$ and $A_\text{Y}$ $(A_\text{X} > A_\text{Y})$ and binding energies $\Delta E_\text{X}$ and $\Delta E_\text{Y}$ such that $\Delta E_\text{X} > \Delta E_\text{Y}$. Here, one can say that as for element X biding energy $\Delta E_x$ is more as compared to that for element Y, nucleus X is more stable than nucleus Y. But if we find the $(\text{BE})_\text{N}$ values for both elements, it is given as

$$(\text{BE}_\text{N})_\text{X} = \frac{\Delta E_\text{X}}{A_\text{X}}\,;\ (\text{BE}_\text{N})_\text{Y} = \frac{\Delta E_\text{Y}}{A_\text{Y}}$$

These values are such that $(\text{BE}_\text{N})_\text{X} < (\text{BE}_\text{N})_\text{Y}$ which implies that to remove a nucleon from element X requires less energy than from element Y. This implies that for the nucleus of Y it is difficult to remove one nucleon from its nucleus, thus structure of Y is more stable than X.

To understand this in a better way, let us consider an example of $^{56}$Fe and $^{209}$Bi. Their binding energies are given as

$$\Delta E_\text{Fe} = 492.8\text{ MeV};\ \Delta E_\text{Bi} = 1640\text{ MeV}$$

From the above values, it seems that to break the nucleus of Bi more energy is required, hence it is more stable than that of Fe. But we should not ignore the bigger size of bismuth nucleus. If we find biding energy per nucleon for both of these nuclei, we get

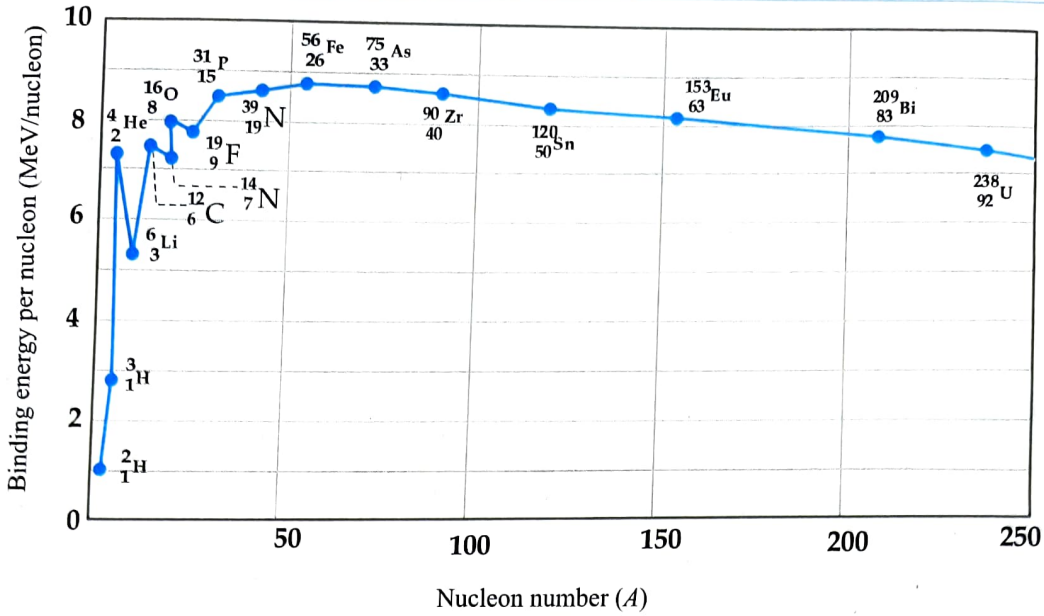
$$(\text{BE}_\text{N})_\text{Fe} = \frac{492.8}{56} = 8.8\text{ MeV};\ (\text{BE}_\text{N})_\text{Bi} = \frac{1640}{209} = 7.84\text{ MeV}$$

Now, we can see that removal of one electron from Fe nucleus is more difficult as compared to that from Bi nucleus. So, Fe nuclei are more stable than Bi. Thus, to judge or compare the stability of different nuclei, we compare the binding energy per nucleon and not the nuclear binding energy.

To see how the nuclear binding energy varies from nucleus to nucleus, it is necessary to compare the binding energy for each nucleus on a per-nucleon basis. Figure shows a graph in which the binding energy divided by the nucleon number $A$ is plotted against the nucleon number itself. In the graph, the peak for the $^4_2$He isotope of helium indicates that the $^4_2$He nucleus is particularly stable. The binding energy per nucleon increases rapidly for nuclei with small masses and reaches a maximum of approximately 8.7 MeV/nucleon for a nucleon number of about $A = 60$. For greater nucleon numbers, the binding energy per nucleon decreases gradually. Eventually, the binding energy per nucleon decreases enough so there is insufficient binding energy to hold the nucleus together. Nuclei more massive than the $^{209}_{83}$Bi nucleus of bismuth are unstable and hence radioactive.

### VARIATION OF BINDING ENERGY PER NUCLEON WITH MASS NUMBER

The binding energy per nucleon is a characteristic property of elements. The graph in figure shows the variation of binding energy per nucleon with mass number for all the elements of periodic table. In the graph, we can see that the binding energy per nucleon increases rapidly for nuclei with small masses and reaches a maximum of approximately 8.8 MeV/nucleon for iron $\left(^{56}_{26}\text{Fe}\right)$. That is, nuclei with mass




**A plot of binding energy per nucleon versus the nucleon number *A***

numbers greater or less than 60 are not as strongly bound as those near the middle of the periodic table. As we shall see later, this fact allows energy to be released in fission and fusion reactions. The curve is slowly varying for $A > 40$, which suggests that the nuclear force saturates. In other words, a particular nucleon can interact with only a limited number of other nucleons, which can be viewed as the "nearest neighbors" in the close-packed structure illustrated in figure.

For greater nucleon numbers, the binding energy per nucleon decreases gradually. Later, the binding energy per nucleon decreases enough so there is insufficient binding energy to hold the nucleon together in the nucleus. It is observed that nuclei with $A > 209$ are unstable and hence radioactive.

In the figure, in the beginning there are some fluctuations in the graph. We can see that binding energies per nucleon for $^{4}He$, $^{12}C$, and $^{16}O$ are relatively higher as compared to their neighboring elements or these elements have nuclei which are relatively more stable than their neighbors. This is because of the existence of nuclear energy levels in the nucleus. Each nuclear energy level can contain two neutrons of opposite spins or two protons of opposite spins. Energy levels in nuclei are filled in sequence, just as energy levels in atoms, to achieve configurations of minimum energy and therefore maximum stability. Similar to the case of atomic orbitals, here also the configuration of opposite spins in nucleons with pairs in nuclear energy levels are more stable. This concept can be used to explain the reason of more stability of $^{4}He$, $^{12}C$, and $^{16}O$ compared to their neighboring elements.

**Note:** If binding energy per nucleon is more for a nucleus, then it is more stable. For example, if

$$\left(\frac{BE_1}{A_1}\right) > \left(\frac{BE_2}{A_2}\right)$$

then nucleus 1 would be more stable.

**Important observations from the graph:**

1. The maximum binding energy per nucleon is 8.8 MeV and it is for $Fe^{56}$. Thus, iron nucleus is the most stable and the most tightly bound.
2. The fact that (BE/$A$) varies by less than 10 percent above $A = 10$ suggests that each nucleon in the nucleus interacts only with the neighbors, independently of the total number of nucleons present in the nucleus.
3. A small decrease after $A = 56$ is due to the destabilizing effect of the long-range repulsive Coulomb force.
4. Not only the Coulomb force reduces the BE, it also shifts the neutron–proton ratio in heavy nuclei toward the neutron excess, by an amount that will increase with $A$.

### ILLUSTRATION 7.9

Calculate the binding energy for nucleon of $_{6}{}^{12}C$ nucleus, if mass of proton $m_p = 1.0078$ u, mass of neutron $m_n = 1.0087$ u, mass of $C_{12}$, $m_C = 12.0000$ u; and 1 u = 931.4 MeV.

**Sol.** In the nucleus of carbon, there are 6 protons and 6 neutrons.

Now, $6m_p = 6(1.0078) = 6.0468$ u

$6m_n = 6(1.0087) = 6.05224$ u

Total mass $= 6m_p + 6m_n = 12.0990$ u

$m_C = 12.0000$ u

Mass defect, $\Delta m = 0.0990$ u

BE $= \Delta m \times 931.4 = (0.0990)\ 931.4 = 92.2$ MeV

### ILLUSTRATION 7.10

Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data:

$m_p = 1.007825$ u, $m = 1.008665$ u

$m(^{56}_{26}Fe) = 55.934939$ u , $m(^{209}_{83}Bi) = 208.980388$ u

Which nucleus has greater binding energy per nucleon?

**Sol.** In $^{56}_{26}Fe$ nucleus, there are 26 protons and $(56 - 26) = 30$ neutrons.

Mass defect $= 26 \times m_p + 30\, m_n - m(^{56}_{26}Fe)$

$= 26\ (1.007825\ u) + 30\ (1.008665\ u) - 55.934939\ u$

$= 56.463400\ u - 55.934939\ u = 0.528461\ u$

Binding energy = $0.528461 \times 931.5$ MeV = 492.26 MeV
(as 1 u = 931.5 MeV)

Binding energy per nucleon = $\frac{492.26\,\text{MeV}}{56} = 8.79\,\text{MeV}$

In $^{209}_{83}$Bi nucleus, there are 83 protons and (209–83) = 126 neutrons.

Mass defect = $83 \times m_p + 126 \times m_n - m(^{209}_{83}\text{Bi})$
= 83 (1.007825 u) + 126 (1.008665 u) – 208.980388 u
= 1.760872 u

Binding energy = $1.760872 \times 931.5$ MeV = 164.3 MeV

Binding energy per nucleon = $\frac{1640.3}{209} = 7.85\,\text{MeV}$

Thus, $^{54}_{26}$Fe has greater binding energy per nucleon than $^{209}_{83}$Bi.

### ILLUSTRATION 7.11

Find the binding energy of an $\alpha$-particle from the following data.
Mass of the helium nucleus = 4.001265 amu
Mass of proton = 1.007277 amu
Mass of neutron = 1.00866 amu (take 1 amu = 931.4813 MeV)

**Sol.** Mass of two protons is $2 \times 1.007277 = 2.014554$ amu
Mass of two neutrons is $2 \times 1.008666$ amu = 2.017332 amu
Total initial mass of two protons and two neutrons is
2.041554 + 2.017332 = 4.031866 amu
Mass defect, $\Delta m = 4.031816 - 4.001265$ amu = 0.030621 amu
Now, binding energy of $\alpha$-particle in MeV is given as
$\Delta E_n = \Delta m \times 931.2$ MeV $= 0.030621 \times 931.2 = 28.5142$ MeV
Binding energy per nucleon = 28.5142/4 = 7.12855 MeV

### ILLUSTRATION 7.12

The binding energies per nucleon for deuteron ($_1H^2$) and helium ($_2He^4$) are 1.1 MeV and 7.0 MeV, respectively. Calculate the energy released when two deuterons fuse to form a helium nucleus ($_2He^4$).

**Sol.** The equation of fusion is represented as
$_1H^2 + {}_1H^2 \rightarrow {}_2He^4 + \text{Energy}$
The binding energy per nucleon of deuteron is 1.1 MeV
Therefore, the net initial binding energy = $4 \times 1.1 = 4.4$ MeV
The binding energy per nucleon of $_2He^4$ is 7.0 MeV.
Therefore, the net final binding energy is $4 \times 7.0 = 28.0$ MeV
Hence, energy released = 28.0 MeV – 4.4 MeV = 23.6 MeV

## Q-VALUES

We have just examined some nuclear reactions for which mass numbers and atomic numbers must be balanced in the equations. We shall now consider the energy involved in these reactions, because energy is another important quantity that must be conserved.

Let us illustrate this procedure by analyzing the following nuclear reaction:

$$^2_1\text{H} + {}^{14}_7\text{N} \rightarrow {}^{12}_6\text{C} + {}^4_2\text{He} \qquad \text{...(i)}$$

The total mass on the left side of the equation is the sum of the mass of $^2_1$H (2.014102 u) and the mass of $^{14}_7$N (14.003074 u), which equals 16.017176 u. Similarly, the mass on the right side of the equation is the sum of the mass of $^{12}_6$C (12.000000 u) plus the mass of $^4_2$He (4.002602 u), for a total of 16.002602 u. Thus, the total mass before the reaction is greater than the total mass after the reaction. The mass difference in this reaction is greater than the total mass after the reaction. The mass difference in this reaction is equal to 16.017176 u – 16.002602 u = 0.014574 u. This "lost" mass is converted to the kinetic energy of the nuclei present after the reaction. In energy units, 0.0145774 u is equivalent to 13.576 MeV of kinetic energy carried away by the carbon and helium nuclei.

The energy required to balance the equation is called the $Q$ value of the reaction. In Eq. (i), the $Q$ value is 13.576 MeV. Nuclear reactions in which there is a release of energy—that is, positive $Q$ values—are said to be exothermic reactions.

The energy balance sheet is not complete; however, we must also consider the kinetic energy of the incident particle before the collision. As an example, let us assume that the deuteron in Eq. (i) has a kinetic energy of 5 MeV. Adding this to our $Q$ value, we find that the carbon and helium nuclei have a total kinetic energy of 18.576 MeV following the reaction. Now, consider the reaction

$$^4_2\text{He} + {}^{17}_7\text{N} \rightarrow {}^{17}_8\text{O} + {}^1_1\text{H} \qquad \text{...(ii)}$$

Before the reaction, the total mass is the sum of the masses of the alpha particle and the nitrogen nucleus: 4.002602 u + 14.003074 u = 18.005676 u. After the reaction, the total mass is the sum of the masses of the oxygen nucleus and the proton: 16.999133 u + 1.07825 u = 18.006958 u. In this case, the total mass deficit is 0.001282 u, equivalent to an energy deficit of 1.194 MeV. This deficit is expressed by the negative $Q$ value of the reaction, –1.194 MeV. Reactions with negative $Q$ values are called endothermic reactions. Such reactions will not take place unless the incoming particle has at least enough kinetic energy to overcome the energy deficit.

At first it might appear that the reaction in Eq. (ii) could take place if the incoming alpha particle had a kinetic energy of 1.194 MeV. In practice, however, the alpha particle must have more energy than this. If it had an energy of only 1.194 MeV, energy would be conserved but careful analysis would show that momentum was not. This can easily be understood by recognizing that the incoming alpha particle has some momentum before the reaction. However, if its kinetic energy were only 1.194 MeV, the products (oxygen and a proton) would be created with zero kinetic energy and thus zero momentum. It can be shown that, in order to conserve both energy and momentum, the incoming particle must have a minimum kinetic energy given by

$$\text{KE}_{\text{min}} = \left(1 + \frac{m}{M}\right)|Q| \qquad \text{...(iii)}$$

where $m$ is the mass of the incident particle, $M$ is the mass of the target, and the absolute value of the $Q$ value is used. For the reaction given by Eq. (ii), we find

$$\text{KE}_{\text{min}} = \left(1 + \frac{4.002\,62}{14.003\,074}\right)|-1.194\ \text{MeV}|$$
$$= 1.535\ \text{MeV}$$

This minimum value of the kinetic energy of the incoming particle is called the threshold energy. The nuclear reaction shown in Eq. (ii) will not occur if the incoming alpha particle has a kinetic energy of less than 1.535 MeV, but can occur if its kinetic energy is equal to or greater than 1.535 MeV.

**ILLUSTRATION 7.13**

Find the minimum kinetic energy of an α-particle to cause the reaction $^{14}N(\alpha, p)^{17}O$. The masses of $^{14}N$, $^{4}He$, $^{1}H$ and $^{17}O$ are respectively 14.00307 u, 4.00260 u, 1.007.83 u and 16.99913 u.

**Sol.** Since, the masses are given in atomic mass units, it is easiest to proceed by finding the mass difference between reactants and products in the same units and then multiplying by 931.5 MeV/u. Thus, we have

$$Q=(14.00307\,u+4.00260u-1.00783u-16.99913u)\left(931.5\frac{MeV}{u}\right)$$
$$=-1.20\,MeV$$

$Q$ value is negative. It means reaction is endothermic. So, the minimum kinetic energy of α-particle to initiate this reaction would be

$$K_{min}=|Q|\left(\frac{m_\alpha}{m_N}+1\right)=(1.20)\left(\frac{4.00260}{14.00307}+1\right)=1.54\,MeV$$

**ILLUSTRATION 7.14**

Show that $^{230}_{92}U$ does not decay by emitting a neutron or proton. Given:

$M(^{229}_{92}U) = 230.033927$ amu, $M(^{230}_{92}U) = 229.033496$ amu,

$M(^{229}_{91}Pa) = 229.032089$ amu, $m(n) = 1.008665$ amu,

$m(p) = 1.007825$ amu.

**Sol.** The corresponding decay equations would be

(a) $^{230}_{92}U \rightarrow n + {}^{229}_{92}U$ (b) $^{230}_{92}U \rightarrow n + {}^{229}_{91}U$

In the first equation, the energy released can be given by

$Q = [230.033927 - 229.033496 - 1.008665] \times 931.2 = -7.7$ MeV

Because $Q < 0$, neutron decay is not possible spontaneously.

Similarly, in the second equation, the energy released can be given as

$Q = [230.033927 - 229.032089 - 1.007825] \times 931.5 = -5.6$ MeV

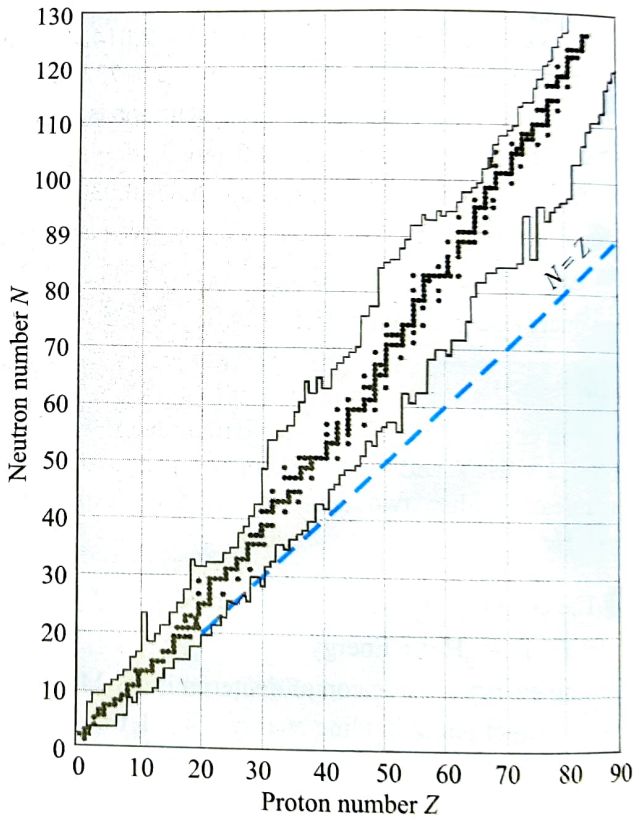
As $Q < 0$, proton decay is also not possible spontaneously.

## NUCLEAR STABILITY

Given that the nucleus consists of a closely packed collection of protons and neutrons, you might be surprised that it can exist. The very large repulsive electrostatic forces between protons should cause the nucleus to fly apart. However, nuclei are stable because of the presence of another, short-range (about 2 fm) force, the **nuclear force**. This is an attractive force that acts between all nuclear particles. The protons attract each other via the nuclear force, and at the same time they repel each other through the Coulomb force. The attractive nuclear force also acts between pairs of neutrons and between neutrons and protons.

The nuclear attractive force is stronger than the Coulomb repulsive force within the nucleus (at short ranges). If this were not the case, stable nuclei would not exist. Moreover, the strong nuclear force is nearly independent of charge. In other words, the nuclear forces associated with the proton–proton, proton–neutron, and neutron–neutron interactions are approximately the same.

There are about 260 stable nuclei; hundreds of others have been observed but are unstable. A plot of $N$ versus $Z$ for a number of stable nuclei is given in figure. Note that light nuclei are most stable if they contain equal number of protons and neutrons—that is, if $N = Z$—but heavy nuclei are more stable if $N > Z$. This can be partially understood by recognizing that, as the number of protons increases, the strength of the Coulomb force increases, which tends to break the nucleus apart. As a result, more neutrons are needed to keep the nucleus stable, because neutrons experience only the attractive nuclear forces. In effect, the additional neutrons "dilute" the nuclear charge density. Eventually, when $Z = 83$, the repulsive forces between protons cannot be compensated by the addition of more neutrons. Elements that contain more than 83 protons do not have stable nuclei, but decay or disintegrate into other particles in various amounts of time. The masses of several stable particles are given in table.




**CONCEPT APPLICATION EXERCISE 7.1**

1. How many electrons, protons, and neutrons are there in a nucleus of atomic number 11 and mass number 24?
2. Calculate the average binding energy per nucleon of $^{93}_{41}Nb$ having mass 92.906 u.
3. Protons and neutrons exist together in an extremely small space within the nucleus. How is this possible when protons repel each other?
4. Following data are available about three nuclei $P$, $Q$, and $R$. Arrange them in decreasing order of stability.

| | P | Q | R |
|---|---|---|---|
| Atomic mass number ($A$) | 10 | 5 | 6 |
| Binding energy (MeV) | 100 | 60 | 66 |

5. A nucelus has binding energy of 100 MeV. It further releases 10 MeV energy. Find the new binding energy of the nucleus.
6. A nuclear reaction is given as $A + B \rightarrow C + D$ Binding energies of $A$, $B$, $C$, and $D$ are given as $B_1$, $B_2$, $B_3$ and $B_4$. Find the energy released in the reaction.
7. Calculate the binding energy of an alpha particle from the following data:

Mass of $^1_1$H atom = 1.007826 u

Mass of neutron = 1.008665 u

Mass of $^4_2$He atom = 4.00260 u

(take 1 u = 931 MeV/$c^2$)

8. Find the binding energy of $^{56}_{26}$Fe. Atomic mass of $^{56}$Fe is 55.9349 u and that of $^1$H is 1.00783 u. Mass of neutron is 1.00867 u.
9. Use Avogadro's number to show that the atomic mass unit is 1 u = $1.66 \times 10^{-27}$ kg.
10. Take a sample of lead and oxygen. They contain different atoms and the density of solid lead is much greater than that of gaseous oxygen. Decide whether the density of the nucleus in a lead atom is greater than, approximately equal to, or less than that in an oxygen atom.
11. Show that the nuclide $^8$Be has a positive binding energy but is unstable with respect to decay into two alpha particles, where masses of neutron, $^1$H, and $^8$Be are 1.008665 u, 1.007825 u, and 8.005305 u, respectively.

**ANSWERS**

1. 11.11.13 **2.** 8.66 MeV/nucleon **5.** 110 MeV
6. $(B_3 + B_4) - (B_1 + B_2)$ **7.** 28.3 MeV
8. 492 MeV **10.** approximate equal to

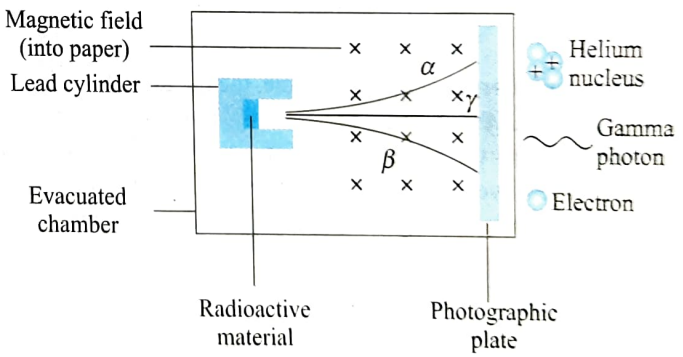
## RADIOACTIVITY

We have discussed that inside a nucleus electrostatic attraction is counterbalanced by short-range strong nuclear forces and the nucleus becomes stable. Despite the forces being balanced, many nuclides are unstable because of nuclear size or the limited range of nuclear forces. Due to slight imbalance in small sized nuclides, these nuclides spontaneously disintegrate into other nuclides. This phenomenon of spontaneous disintegration is called radioactivity. Further, in this chapter, we will discuss the aspects of radioactivity by which unstable nuclides disintegrate to achieve stability.

When an unstable or radioactive nucleus disintegrates spontaneously, certain kinds of particles and/or high-energy photons are released. These particles and photons are collectively called "rays". Three kinds of rays are produced by naturally occurring radioactivity: $\alpha$-rays, $\beta$-rays, and $\gamma$-rays. They are named according to the first three letters of the Greek alphabet, alpha ($\alpha$), beta ($\beta$), and gamma ($\gamma$), to indicate the extent of their ability to penetrate matter. $\alpha$-rays are the least penetrating, being blocked by a thin (≈0.01 mm) sheet of lead, while $\beta$-rays penetrate lead to a much greater distance (≈0.1 mm). $\gamma$-rays are the most penetrating and can pass through an appreciable thickness (≈100 mm) of lead.

The nuclear disintegration process that produces $\alpha$-, $\beta$-, and $\gamma$-rays must obey the laws of physics; these laws are called conservation laws because each of them deals with a property (such as mass/energy, electric charge, linear momentum, and angular momentum) that is conserved or does not change during a process. To these conservation laws, we now add a fifth, the conservation of nucleon number. In all radioactive decay processes, it has been observed that the number of nucleons present before the decay is equal to the number of nucleons after the decay. Therefore, the number of nucleons is conserved during a nuclear disintegration. As applied to the disintegration of a nucleus, the conservation laws require that the energy, electric charge, linear momentum, angular momentum, and nucleon number that a nucleus possesses must remain unchanged when it disintegrates into nuclear fragments and accompanying $\alpha$-, $\beta$-, or $\gamma$-rays.

The three types of radioactivity that occur naturally can be observed in a relatively simple experiment. A piece of radioactive material is placed at the bottom of a narrow hole in a lead cylinder. The cylinder is located within an evacuated chamber, as figure illustrates. A magnetic field is directed perpendicular to the plane of the paper, and a photographic plate is positioned to the right of the hole. Three nods of spots appear on the developed plate, which are associated with the radioactivity of the nuclei in the material. Since moving particles are deflected by a magnetic field only when they are electrically charged, this experiment reveals that two types of radioactivity ($\alpha$- and $\beta$-rays, as it turns out) consist of charged particles, while the third type ($\gamma$-rays) does not.



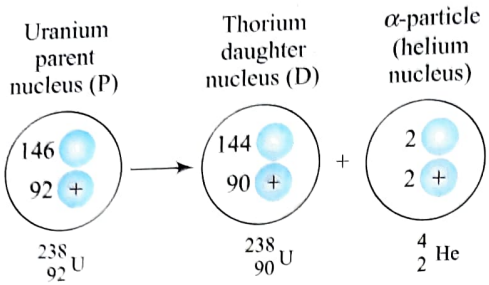

In figure, $\alpha$- and $\beta$-ray are deflected by a magnetic field and, therefore, consist of moving charged $\gamma$-rays are not deflected by a magnetic field and, consequently, must be uncharged.

### $\alpha$-DECAY

When a nucleus disintegrates and produces $\alpha$-rays, it is said to undergo $\alpha$-decay. Experimental evidence shows that $\alpha$-rays consist of positively charged particles, each one being the $^4_2$He nucleus of helium. Thus, an $\alpha$-particle has a charge of $+2e$ and a nucleon number of $A = 4$. Since the grouping of 2 protons and 2 neutrons in a $^4_2$He nucleus is particularly stable, it is not surprising that an $\alpha$-particle can be ejected as a unit from a more massive unstable nucleus.

Figure shows the disintegration process as an example of $\alpha$-decay.

$$\underset{\text{Parent nucleus (uranium)}}{{}^{238}_{92}\text{U}} \rightarrow \underset{\text{Daughter nucleus (thorium)}}{{}^{234}_{90}\text{Th}} + \underset{\text{$\alpha$-particle (helium nucleus)}}{{}^{4}_{2}\text{He}}$$




The original nucleus is referred to as the parent nucleus (P), and the nucleus remaining after disintegration is called the daughter nucleus (D). Upon emission of an $\alpha$-particle, the uranium ${}^{238}_{92}\text{U}$ parent is converted into the ${}^{234}_{90}\text{Th}$ daughter, which is an isotope of thorium. The parent and daughter nuclei are different, so $\alpha$-decay converts one element into another, a process known as transmutation.

Electric charge is conserved during $\alpha$-decay. In figure, for instance, 90 of the 92 protons in the uranium nucleus end up in the thorium nucleus, and the remaining two protons are carried off by the $\alpha$-particle. The total number of 92, however, is the same before and after disintegration. $\alpha$-decay also conserves the number of nucleons, for the number is the same before (238) and after (234 + 4) disintegration. Consistent with the conservation of electric charge and nucleon number, the general form for $\alpha$-decay is

**$\alpha$-Decay:** $\underset{\text{Parent nucleus}}{{}^{A}_{Z}\text{P}} \rightarrow \underset{\text{Daughter nucleus}}{{}^{A-4}_{Z-2}\text{D}} \rightarrow \underset{\text{$\alpha$-particle (helium nucleus)}}{{}^{4}_{2}\text{He}}$

When a nucleus releases an $\alpha$-particle, the nucleus also releases energy. In fact, the energy released by radioactive decay is responsible, in part, for keeping the interior of the earth hot and, in some places, even molten. The following example shows how the conservation of mass/energy can be used to determine the amount of energy released in $\alpha$-decay.

**$\alpha$-Decay:**

$$_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4} + Q$$

$Q$ value: It is defined as the energy released during the decay process.

$Q$ value = rest mass energy of reactants – rest mass energy of products.

This energy is available in the form of increase in KE of the products. Let us consider the following:

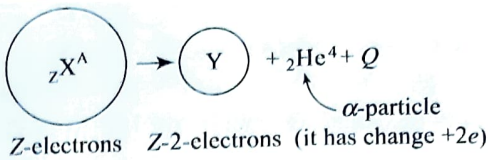
$M_X$ = mass of atom $_{Z}X^{A}$

$M_Y$ = mass of atom $_{Z-2}Y^{A-4}$

$M_{He}$ = mass of atom $_{2}He^{4}$

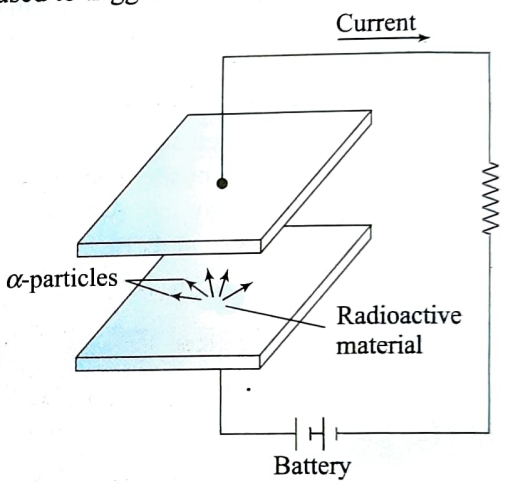
$$Q \text{ value} = [(M_X - Zm_e) - \{(M_Y - (Z-2)m_e) - (M_{He} - 2m_e)\}]c^2$$
$$= [M_X - M_Y - M_{He}]c^2$$

Considering actual number of electrons in $\alpha$-decay




$$Q \text{ value} = [M_X - (M_Y + 2m_e) - (M_{He} - 2m_e)]c^2$$
$$= [M_X - M_Y - M_{He}]c^2$$

One widely used application of $\alpha$-decay is in smoke detectors. Figure illustrates how a smoke detector operates. Two small and parallel metal plates are separated by a distance of about one centimetre. A tiny amount of radioactive material at the center of one of the plates emits $\alpha$-particles, which collide with air molecules. During the collisions, the air molecules are ionized to form positive and negative ions. The voltage from a battery causes one plate to be positive and the other negative, so that each plate attracts ions of opposite charge. As a result there is a current in the circuit attached to the plates. The presence of smoke particles between the plates reduces the current, since the ions that collide with a smoke particle are usually neutralized. The drop in current that smoke particles cause is used to trigger an alarm.




**A smoke detector**

**ILLUSTRATION 7.15**

The atomic mass of uranium ${}^{238}_{92}\text{U}$ is 238.0508 u, that of thorium ${}^{234}_{90}\text{Th}$ is 234.0436 u and that of an alpha particle ${}^{4}_{2}\text{He}$ is 4.0026 u. Determine the energy released when $\alpha$-decay converts ${}^{238}_{92}\text{U}$ into ${}^{234}_{90}\text{Th}$.

**Sol.** Since energy is released during the decay, the combined mass of the ${}^{234}_{90}\text{Th}$ daughter nucleus and the $\alpha$-particle is less than the mass of the ${}^{238}_{92}\text{U}$ parent nucleus. The difference in mass is equivalent to the energy released. We will determine the difference in mass in atomic mass units and then use the fact that 1 u is equivalent to 931.5 MeV.

The decay and the masses are shown below:

$$\underset{238.0508\text{ u}}{{}^{238}_{92}\text{U}} \rightarrow \underbrace{\underset{234.0436\text{ u}}{{}^{234}_{90}\text{Th}} + \underset{4.0026\text{ u}}{{}^{4}_{2}\text{He}}}_{238.0462\text{ u}}$$

The decrease in mass is 238.0508 u – 238.0462 u = 0.0046 u. As usual, the masses are atomic masses and include the mass

of the orbital electrons. But this causes no error here because the same total number of electrons is included for $^{238}_{92}U$, on the one hand, and for $^{234}_{90}Th$ plus $^{4}_{2}He$, on the other. Since 1 u is equivalent to 931.5 MeV, the released energy is 4.3 MeV.

When $\alpha$-decay occurs as discussed in Illustration 7.15, the energy released appears as kinetic energy of the recoiling $^{234}_{90}Th$ nucleus and the $\alpha$-particle, except for a small portion carried away as a $\gamma$-ray. Illustration 7.16 discusses how the $^{234}_{90}Th$ nucleus and the $\alpha$-particle share the released energy.

**ILLUSTRATION 7.16**

Refer to Illustration 7.15, the energy released by the $\alpha$-decay of $^{238}_{92}U$ is found to be 4.3 MeV. Since this energy is carried away as kinetic energy of the recoiling $^{234}_{90}Th$ nucleus and the $\alpha$-particle, it follows that $KE_{Th} + KE_{\alpha} = 4.3$ MeV. However, $KE_{Th}$ and $KE_{\alpha}$ are not equal. Which particle carries away more kinetic energy, the $^{234}_{90}Th$ nucleus or the $\alpha$-particle?

**Sol.** Kinetic energy depends on the mass $m$ and speed $v$ of a particle, since $KE = \frac{1}{2}mv^2$. The $^{234}_{90}Th$ nucleus has a much greater mass than the $\alpha$-particle, and since the kinetic energy is proportional to the mass, it is tempting to conclude that the $^{234}_{90}Th$ nucleus has greater kinetic energy. This conclusion is not correct, however, since it does not take into account the fact that the $^{234}_{90}Th$ nucleus and the $\alpha$-particle have different speeds after the decay. In fact, we expect the thorium nucleus to recoil with smaller speed precisely because it has the greater mass. The decaying of $^{238}_{92}U$ is like a father and his young daughter on ice skates, pushing off against one another. The more massive father recoils with much less speed than the daughter. We can use the principle of conservation of linear momentum to verify our explanation.

As we know, the conservation principle states that the total linear momentum of an isolated system remains constant. An isolated system is one for which the vector sum of the external forces acting on the system is zero, and the decaying $^{238}_{92}U$ nucleus fits this description. It is stationary initially, and since momentum is mass times velocity, its initial momentum is zero. In its final form, the system consists of the $^{234}_{90}Th$ nucleus and the $\alpha$-particle and has a final total momentum of $m_{Th}v_{Th} + m_{\alpha}v_{\alpha}$. According to momentum conservation, the initial and final values of the total momentum of the system must be the same, so that $m_{Th}v_{Th} + m_{\alpha}v_{\alpha} = 0$. Solving this equation for the velocity of the thorium nucleus, we find that $v_{Th} = -m_{\alpha}v_{\alpha}/m_{Th}$. Since $m_{Th}$ is much greater than $m_{\alpha}$, we can see that the speed of the thorium nucleus is less than the speed of the $\alpha$-particle. Moreover, the kinetic energy depends on the square of the speed and only the first power of the mass. As a result of its much greater speed, the $\alpha$-particle has the greater kinetic energy.

**ILLUSTRATION 7.17**

Find the $Q$-value and the kinetic energy of the emitted $\alpha$-particle in the $\alpha$-decay of (a) $^{226}_{88}Ra$ and (b) $^{220}_{86}Rn$.

Given: $m(^{226}_{88}Ra) = 226.02540\,u$

$m(^{222}_{86}Rn) = 222.01750\,u$

$m(^{216}_{84}Po) = 216.00189\,u$

**Sol.**

**(a)** As $^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He + Q$

$$Q = [m(^{226}_{88}Ra) - m(^{222}_{86}Rn) - m(^{4}_{2}He)]$$
$$= [\,226.02540 - 222.01750 - 4.00260]u = 0.0053\ u$$
$$= 0.0053 \times 931.5\ MeV = 4.94\ MeV$$

KE of the $\alpha$-particle

$$K_{\alpha} = \frac{(A-4)}{A} \times Q = \frac{(226-4)}{226} \times 4.94\,MeV = 4.85\,MeV$$

**(b)** Proceeding as in (a),

$Q = 0.00688\ u = 0.00688 \times 931.5\ MeV = 6.41\ MeV$

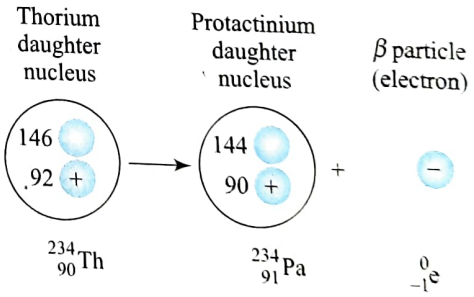
KE of the $\alpha$-particle,

$$K_{\alpha} = \frac{(A-4)}{A}Q = \left(\frac{220-4}{220}\right) \times 6.41\,MeV = 6.29\,MeV$$

## $\beta$-DECAY

The $\beta$-rays are deflected by the magnetic field in a direction opposite to that of the positively charged $\alpha$-rays. Consequently, these $\beta$-rays, which are the most common kind, consist of negatively charged particles or $\beta$-particles. Experiment shows that $\beta$-particles are electrons. When a radioactive nucleus undergoes $\beta$-decay, the daughter nucleus has the same number of nucleons as the parent nucleus, but the atomic number is changed by 1.

As an illustration of $\beta$-decay, consider the thorium $^{234}_{90}Th$ nucleus, which decays by emitting a $\beta$-particle, as shown in figure. $\beta$-decay occurs when a neutron in an unstable parent nucleus decay into a proton and an electron, the electron being emitted as the $\beta$-particle. In the process, the parent nucleus is transformed into the daughter nucleus.




$\beta^-$ decay, like $\alpha$-decay, causes a transmutation of one element into another. In this case, thorium $^{234}_{90}Th$ is converted into protactinium $^{234}_{91}Pa$. The law of conservation of charge is obeyed, since the net number of positive charges is the same before (90) and after (91 – 1) the $\beta^-$ emission. The law of conservation of nucleon number is obeyed, since the nucleon number remains at $A = 234$.

### $\beta^-$ Decay

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + {}^{0}_{-1}e$$

Parent nucleus — Daughter nucleus — $\beta^-$ particle (electron)

The electron emitted in $\beta^-$ decay does not actually exist within the parent nucleus and is not one of the orbital electrons. Instead, the electron is created when a neutron decays into a proton and

an electron; when this occurs, the proton number of the parent nucleus increases from $Z$ to $Z+1$ and the nucleon number remains unchanged. The electron is usually fast-moving and escapes from the atom, leaving behind a positively charged atom.

### $\beta^+$ Decay

A second kind of $\beta$-decay sometimes occurs. In this process, the particle emitted by the nucleus is a positron rather than an electron. A positron, also called a $\beta^+$ particle, has the same mass as an electron but carries a charge of $+e$ instead of $-e$. The disintegration process for $\beta^+$ decay is

$$\underset{\text{Parent nucleus}}{{}^{A}_{Z}\text{P}} \rightarrow \underset{\text{Daughter nucleus}}{{}^{A}_{Z-1}\text{P}} \rightarrow \underset{\beta^+\text{particle (positron)}}{{}^{0}_{1}\text{e}}$$

The emitted positron does not exist within the nucleus but, rather, is created when a nuclear proton is transformed into a neutron. In the process, the proton number of the parent nucleus decreases from $Z$ to $Z-1$, and the nucleon number remains the same. As with $\beta^-$ decay, the laws of conservation of charge and nucleon number are obeyed, and there is a transmutation of one element into another.

In the following illustration, we will discuss how energy is released during $\beta^-$ decay, just as it is during $\alpha$-decay, and that the conservation of mass/energy applies.

**ILLUSTRATION 7.18**

The atomic mass of thorium ${}^{234}_{90}\text{Th}$ is 234.04359 u, while that of protactinium ${}^{234}_{91}\text{Pa}$ is 234.04330 u. Find the energy released when $\beta^-$ decay changes ${}^{234}_{90}\text{Th}$ into ${}^{234}_{91}\text{Pa}$.

**Sol.** To find the energy released, we follow the usual procedure of determining how much the mass has decreased because of the decay and then calculating the equivalent energy. The decay and the masses are shown below:

$$\underbrace{{}^{234}_{90}\text{Th}}_{234.04359\text{ u}} \rightarrow \underbrace{{}^{234}_{91}\text{Pa} + {}^{0}_{-1}\text{e}}_{234.04330\text{ u}}$$

When the ${}^{234}_{90}\text{Th}$ nucleus of a thorium atom is converted into a ${}^{234}_{91}\text{Pa}$ nucleus, the number of orbital electrons remains the same, so the resulting protactinium atom is missing one orbital electron. However, the given mass includes all 91 electrons of a neutral protactinium atom. In effect, then, the value of 234.04330 u for ${}^{234}_{91}\text{Pa}$ already includes the mass of the $\beta^-$ particle. The mass decrease that accompanies the $\beta^-$ decay is 234.04359 u − 234.04330 u = 0.00029 u. The equivalent energy (1 u = 931.5 MeV) is 0.27 MeV. This is the maximum kinetic energy that the emitted electron can have.

**ILLUSTRATION 7.19**

Consider the beta decay

$$^{198}\text{Au} \rightarrow {}^{198}\text{Hg}^* + \beta^- + \bar{\nu}$$

where ${}^{198}\text{Hg}^*$ represents a mercury nucleus in an excited s tate at energy 1.088 MeV above the ground state. What can be the maximum kinetic energy of the electron emitted? The atomic mass of ${}^{198}\text{Au}$ is 197.968233 u and that of ${}^{198}\text{Hg}$ is 197.966760 u.

**Sol.** If the product nucleus ${}^{198}\text{Hg}$ is formed in its ground state, the kinetic energy available to the electron and the antineutrino is

$$Q = [m({}^{198}\text{Au}) - m({}^{198}\text{Hg})]c^2$$

As ${}^{198}\text{Hg}^*$ has energy 1.088 MeV more than ${}^{198}\text{Hg}$ in ground state, the kinetic energy actually available is

$$Q = [m({}^{198}\text{Au}) - m({}^{198}\text{Hg})]c^2 - 1.088\text{ MeV}$$

$$= (197.968233\text{ u} - 197.966760\text{ u})\left(931\frac{\text{MeV}}{\text{u}}\right) - 1.088\text{ MeV}$$

$$= 1.3686\text{ MeV} - 1.088\text{ MeV} = 0.2806\text{ MeV}$$

This is also the maximum possible kinetic energy of the electron emitted.

**ILLUSTRATION 7.20**

Calculate the $Q$ value in the following decays:

**(a)** ${}^{19}\text{O} \rightarrow {}^{19}\text{F} + e^- + \bar{\nu}$ **(b)** ${}^{25}\text{Al} \rightarrow {}^{25}\text{Mg} + e^+ + \nu$

The atomic masses needed are as follows:

| ${}^{19}\text{O}$ | ${}^{19}\text{F}$ | ${}^{25}\text{Al}$ | ${}^{25}\text{Mg}$ |
|---|---|---|---|
| 19.003576 u | 18.998403 u | 24.990432 u | 24.985839 u |

**Sol.**

**(a)** The $Q$ value of $\beta^-$-decay is

$$Q = [m({}^{19}\text{O}) - m({}^{19}\text{F})]c^2$$

$$= [19.003576\text{ u} - 18.998403\text{ u}]\ (931\text{ MeV/u})$$

$$= 4.816\text{ MeV}$$

**(b)** The $Q$ value of $\beta^+$-decay is

$$Q = [m({}^{25}\text{Al}) - m({}^{25}\text{Mg}) - 2m_e]c^2$$

$$= \left[24.99032\text{ u} - 24.985839\text{ u} - 2\times 0.511\frac{\text{MeV}}{c^2}\right]c^2$$

$$= (0.004593\text{ u})\ (931\text{ MeV/u}) - 1.022\text{ MeV}$$

$$= 4.276\text{ MeV} - 1.022\text{ MeV} = 3.254\text{ MeV}$$

**ILLUSTRATION 7.21**

Find the energy liberated in the beta decay of ${}^{14}_{6}\text{C}$ to ${}^{14}_{7}\text{N}$ as represented by equation: ${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^- + \bar{\nu}$

Equation refers to nuclei. Adding six electrons to both sides gives

$${}^{14}_{6}\text{C atom} \rightarrow {}^{14}_{7}\text{N atom}$$

**Sol.** We know that ${}^{14}_{6}\text{C}$ has a mass of 14.003242 u and ${}^{14}_{7}\text{N}$ has a mass of 14.003074 u. Here, the mass difference between the initial and final states is

$$\Delta m = 14.003242\text{ u} - 14.003074\text{ u} = 0.000168\text{ u}$$

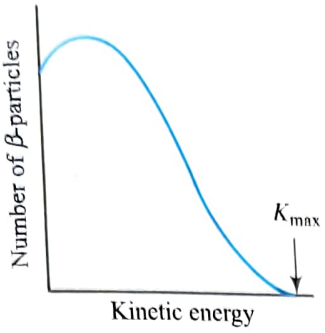
This corresponds to an energy release of

$$E = (0.000168\text{ u})(931.494\text{ MeV/u}) = 0.156\text{ MeV}$$

### Neutrino

From illustrations discussed in previous section, we see that the energy released in the beta decay of ${}^{14}\text{C}$ is approximately 0.16 MeV. As with alpha decay, we expect the electron to carry away virtually all of this as kinetic energy because apparently it is the lightest particle produced in the decay. However, as figure shows, only a small number of electrons have this maximum kinetic energy, represented as $K_{max}$ on the graph; most of the

electrons emitted have kinetic energies lower than this predicted value. If the daughter nucleus and the electron are not carrying away this liberated energy, then the energy conservation requirement leads to the question, what accounts for the missing energy? As an additional complication, further analysis of beta decay shows that the principles of conservation of both angular momentum and linear momentum appear to be violated.




When a $\beta$-particle is emitted by a radioactive nucleus, energy is simultaneously released. Experimentally, however, it is found that most $\beta$-particles do not have enough kinetic energy to account for all the energy released. If a $\beta$-particle carries away only part of the energy, where does the remainder go? In 1930, Pauli proposed that a third particle must be present to carry away the "missing" energy and to conserve momentum. Enrico Fermi later developed a complete theory of beta decay and named this particle the neutrino ("little neutral one") because it had to be electrically neutral and have little or no mass. Although it eluded detection for many years, the neutrino ($\nu$) was finally detected experimentally in 1956. For instance, the $\beta$-decay of thorium $^{234}_{90}$Th is more correctly written as

$$^{234}_{90}\text{Th} \rightarrow {}^{234}_{91}\text{Pa} + {}^{0}_{-1}\text{e} + \bar{\nu}$$

The bar above the $\nu$ is included because the neutrino emitted in this particular decay process is an antimatter neutrino or antineutrino. A normal neutrino ($\nu$ without the bar) is emitted when $\beta^+$ decay occurs.

The neutrino has zero electrical charge and is extremely difficult to detect because it interacts very weakly with matter. The neutrino has the following properties:

1. Zero electric charge.
2. A mass much smaller than that of the electron, but probably not zero. (Recent experiments suggest that the neutrino definitely has mass, but the value is uncertain—perhaps less than 1 eV/$c^2$.)
3. A spin of $\frac{1}{2}$.
4. Very weak interaction with matter, making it quite difficult to detect.

Thus, with the introduction of the neutrino, we are now able to represent the beta decay process of equation in its correct form:

$$^{14}_{6}\text{C} = {}^{14}_{7}\text{N} + e^- + \bar{\nu} \qquad \text{...(i)}$$

where the bar in the symbol $\bar{\nu}$ indicates an antineutrino. To explain what an antineutrino is, let us first consider the following decay:

$$^{12}_{7}\text{N} = {}^{12}_{6}\text{C} + e^+ + \nu \qquad \text{...(ii)}$$

Here, we see that when $^{12}$N decays into $^{12}$C, a particle is produced that is identical to the electron except that it has a positive charge of $+e$. This particles is called a positron. Because it is like the electron in all respects except charge, the positron is said to be the antiparticle to the electron. We shall discuss antiparticles further in the chapter. For now, suffice it to say that in beta decay, an electron and an antineutrino are emitted or a positron and a neutrino are emitted.

**ILLUSTRATION 7.22**

Find whether $\alpha$-decay or any of the $\beta$-decay are allowed for $^{226}_{89}$Ac.

**Sol.** Our first step will be to write the reaction, then find the disintegration energy $Q$. If $Q > 0$, the decay is allowed.

$\alpha$-decay: $^{226}_{89}\text{Ac} \rightarrow {}^{222}_{87}\text{Fr} + \alpha$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{222}_{87}\text{Fr}) - M(^{4}_{2}\text{He})]c^2$$
$$= 5.50 \text{ MeV} \qquad (\alpha \text{ decay is allowed})$$

$\beta^-$ decay: $^{226}_{89}\text{Ac} \rightarrow {}^{226}_{90}\text{Th} + \beta^- + \bar{\nu}$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{90}\text{Th})]c^2$$
$$= 1.12 \text{ MeV} \qquad (\beta^- \text{ decay is allowed})$$

$\beta^+$ decay: $^{226}_{89}\text{Ac} \rightarrow {}^{226}_{88}\text{Ra} + \beta^+ + \bar{\nu}$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{88}\text{Ra}) - 2m_e]c^2$$
$$= -0.38 \text{ MeV} \qquad (\beta^+ \text{ decay is not allowed})$$

Electron capture: $^{226}_{89}\text{Ac} + e^- \rightarrow {}^{226}_{88}\text{Ra} + \bar{\nu}$

$$Q = [M(^{226}_{89}\text{Ac}) - M(^{226}_{88}\text{Ra})]c^2$$
$$= 0.64 \text{ MeV (Electron capture is allowed)}$$

**ILLUSTRATION 7.23**

The nucleus $^{23}_{10}$Ne decays by $\beta^-$-emission. Write down the $\beta^-$ decay equation and determine the maximum kinetic energy of the electrons emitted from the following data:

$$m(^{23}_{10}\text{Ne}) = 22.994466\text{u}\,,\ m(^{23}_{11}\text{Na}) = 22.989770\text{u}$$

**Sol.** The equation representing the $\beta^-$-emission of $^{23}_{10}$Ne is as follows: $^{23}_{10}\text{Ne} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{-1}e + \bar{\nu} + Q$

Ignoring the rest mass of antineutrino( $\bar{\nu}$ ),

$$Q = [m_N(^{23}_{10}\text{Ne}) - m_N(^{23}_{11}\text{Na}) - m_e]c^2$$

If the mass defect due to electronic binding energy is ignored. We can replace the nuclear masses ($m_N$) by the respective atomic masses ($m$). That is,

$$Q = [\{m(^{23}_{10}\text{Ne}) - 10m_e\} - \{m(^{23}_{11}\text{Na}) - 11m_e\}]c^2$$

or $$Q = [m(^{23}_{10}\text{Ne}) - m(^{23}_{11}\text{Na})]c^2$$
$$= [22.994466\text{ u} - 22.989770\text{ u}]\,c^2$$

or $$Q = (0.004696\text{ u}) \times 931.5 \text{ MeV} = 4.37 \text{ MeV}$$
(as 1 u $c^2$ = 931.5 MeV)

This energy is shared, mainly by $e^-$ and $\bar{\nu}$ as $^{23}_{10}$Ne is far heavier than both these particles. Further, the energy carried by the electron is maximum (i.e., 4.37 MeV) and this happens when the neutrino carries no energy.

## $\gamma$-DECAY

Very often a nucleus that undergoes radioactive decay is left in an excited energy state. The nucleus can then undergo a second

decay to a lower energy state, perhaps to the ground state, by emitting one or more high-energy photons. The process is very similar to the emission of light by an atom. An atom emits radiation to release some extra energy when an electron "jumps" from a state of higher energy to a state of lower energy. Likewise, the nucleus uses essentially the same method to release any extra energy it may have following a decay or some other nuclear event. In neutrons in the nucleus *s* they move from a higher energy level to a lower level. The photons emitted in such a de-excitation process are called gamma rays, which have very high energy relative to the energy of visible light.

A nucleus may reach an excited state as the result of a violent collision with another particle. However, it is more common for a nucleus to be in an excited state as a result of alpha or beta decay. The following sequence of events represents a typical situation in which gamma decay occurs:

$$^{12}_{5}B = {}^{12}_{6}C^{*} + e^{-} + \bar{\nu} \quad ...(i)$$

$$^{12}_{6}C^{*} = {}^{12}_{6}C + \gamma \quad ...(ii)$$

Equation (i) represents beta decay in which $^{12}B$ decays to $^{12}C^{*}$ where the asterisk indicates that the carbon nucleus is left in an excited state following the decay. The excited carbon nucleus then decays to the ground state by emitting a gamma ray, as indicated by Eq. (ii). Note that gamma emission does not result in any change in either *Z* or *A*.

**ILLUSTRATION 7.24**

What is the wavelength of the 0.186 MeV $\gamma$-ray photon emitted by radium $^{226}_{88}Ra$?

**Sol.** The photon energy is the difference between two nuclear energy levels. Equation $E_i - E_f = hf$ gives the relation between the energy level separation $\Delta E$ and the frequency $f$ of the photon as $\Delta E = hf$. Since $f\lambda = c$, the wavelength of the photon is $\lambda = hc/\Delta E$.

First, we must convert the photon energy into joule:

$$\Delta E = (0.186 \times 10^{6}\,\text{eV})\left(\frac{1.60 \times 10^{-19}}{1\,\text{eV}}\right) = 2.98 \times 10^{-14}\,\text{J}$$

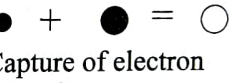
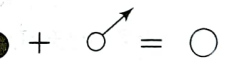
The wavelength of the photon is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63 \times 10^{-34}\,\text{Js})(3.00 \times 10^{8}\,\text{m s}^{-1})}{2.98 \times 10^{-14}\,\text{J}} = 6.67 \times 10^{-12}\,\text{m}$$

## STABILITY AND NEUTRON–PROTON RATIO (*n/p*)

Stable nuclei have the ratio *n/p* (neutron/proton) either equal to 1 or more than 1. The ratio is nearly 1 for light nuclei upto calcium $\left(^{40}_{20}Ca\right)$. It increases up to 1.6 for heavy nuclei. The following table shows *n/p* ratio for some stable nuclei.

| Isotope | *n* | *p* | *n/p* |
|---|---|---|---|
| $^{12}_{6}C$ | 6 | 6 | 1 |
| $^{14}_{7}N$ | 7 | 7 | 1 |
| $^{16}_{8}O$ | 8 | 8 | 1 |
| $^{20}_{10}Ne$ | 10 | 10 | 1 |
| $^{40}_{20}Ca$ | 20 | 20 | 1 |
| $^{64}_{30}Zn$ | 34 | 30 | 1.13 |
| $^{90}_{40}Zr$ | 50 | 40 | 1.40 |
| $^{202}_{80}Hg$ | 122 | 80 | 1.53 |

| | Decay event | Reason for instability |
|---|---|---|
| Gamma decay | Emission of gamma ray reduces energy of nucleus | Nucleus has excess energy |
| Alpha decay | Emission of alpha particle reduces size of nucleus | Nucleus too large |
| Beta decay | Emission of electron by neutron in nucleus changes the neutron to a proton | Nucleus has too many neutrons relative to number of protons |
| Electron capture |  Capture of electron by proton in nucleus changes the proton to a neutron | Nucleus has too many protons relative to number of neutrons |
| Positron emission |  Emission of positron by proton in nucleus changes the proton to a neutron | Nucleus has too many protons relative to number of neutrons |

● Proton (charge = +e) ● Electron (charge = e)

○ Neutron (charge = 0) ○ Positron (charge = +e)

1. When *n/p* ratio is higher than that required for stability, $\beta$-emission takes place. It decrease *n/p* ratio, e.g., when $^{14}_{6}C$ decays to $^{14}_{7}N$, and *n/p* ratio decreases from 1.33 to 1.

$$^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta^{-} + \bar{\nu}$$

2. When *n/p* ratio is less than that required for stability, the nuclei can either emit $\alpha$-particle, emit $\beta^{+}$ or undergo K-electron capture.

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

*n/p* 146/92 144/90

$$^{13}_{7}N \rightarrow {}^{13}_{6}C + \beta^{+} + \nu$$

n/p 6/7 7/6

$$^{194}_{79}N + {}^{0}_{-1}e \rightarrow {}^{194}_{78}P$$

n/p 115/79 116/78

## CONCEPT APPLICATION EXERCISE 7.2

1. A uranium nucleus (atomic number 92, mass number 231) emits an $\alpha$-particle and the resultant nucleus emits a $\beta$-particle. What are the atomic and mass numbers of the final nucleus?
2. A radioactive nucleus undergoes a series of decay according to the scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72, respectively, what are these number for $A_4$?

3. A nucleus, absorbing a neutron, emits an electron to go over to neptunium which on further emitting an electron goes over to plutonium. How would you represent the resulting plutonium?
4. A uranium nucleus U-238 of atomic number 92 emits two $\alpha$-particles and two $\beta$-particles and transforms into a thorium nucleus. What is the mass number and atomic number of the thorium nucleus so produced?
5. How many electrons, protons, and neutrons are there in 12 g of $_6C^{12}$ and in 14 g of $_6C^{14}$?
6. Determine the product of the reaction: $^{7}_{3}Li + {}^{4}_{2}He \rightarrow ? + n$
7. How many alpha and beta particles are emitted when uranium ($_{92}U^{238}$) decays to lead $_{82}Pb^{206}$?
8. An $\alpha$-particle strikes an aluminium atom $^{27}_{13}Al$. As a result, an unknown nucleus $^{A}_{Z}X$ and a neutron are formed.

$$^{2}_{4}He + {}^{27}_{13}Al \rightarrow {}^{A}_{Z}X + {}^{1}_{0}n$$

Identify the nucleus produced, indicating its atomic number $Z$ (the number of protons) and its mass number $A$ (the number of nucleons).

9. A neutron is observed to strike an $^{16}_{8}O$ nucleus and a deuteron is given off. What is the nucleus that result?
10. A $^{238}_{92}U$ undergoes alpha decay. What is the resulting daughter nucleus?
11. Thorium $^{238}_{90}Th$ produces a daughter nucleus that is radioactive. The daughter, in turn, produces its own radioactive daughter and so on. This process continues until bismuth $^{812}_{83}Bi$ is reached. What are the total number $N_\alpha$ of $\alpha$-particles and the total number $N_\beta$ of $\beta$-particles that are generated in this series of radioactive decays?

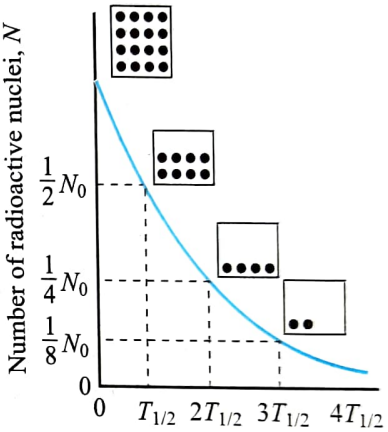
### ANSWERS

1. 91, 234 **2.** 69 **3.** $^{239}_{94}Pu$ **4.** 230, 90 **6.** $^{10}_{5}B$

7. 8, 6 **8.** $^{30}_{15}P$ **9.** $^{15}_{7}N$ **10.** $^{234}_{90}Th$ **11.** 4,1

# RADIOACTIVE DECAY AND ACTIVITY

The question of which radioactive nucleus in a group of nuclei disintegrates at a given instant is decided like the drawing of numbers in a state lottery; individual disintegrations occur randomly. As time passes, the number $N$ of parent nuclei decreases, as figure shows. This graph of $N$ versus time indicates that the decrease occurs in a smooth fashion, with $N$ approaching zero after enough time has passed. To help describe the graph, it is useful to define the half-life $T_{1/2}$ of a radioactive isotope as the time required for one-half of the nuclei present to disintegrate. For example, radium $^{226}_{88}Ra$ has a half-life of 1600 years, for it takes this amount of time for one-half of a given quantity of this isotope to disintegrate. In another 1600 years, one-half of the remaining radium atoms will disintegrate, leaving only one-fourth of the original number intact. In the figure, the number of nuclei present at time $t = 0$ s is $N = N_0$, while the number present at $t = T_{1/2}$ is $N = \frac{1}{2}N_0$. The number present at $t = 2T_{1/2}$ is $N = \frac{1}{4}N_0$ and so forth. The value of the half-life depends on the nature of the radioactive nucleus.




**The half-life $T_{1/2}$ of a radioactive decay is the time in which one-half of the radioactive nuclei disintegrate.**

# MEASUREMENT OF RADIOACTIVITY

Radioactivity of an element is measured in terms of "activity". The activity of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atoms disintegrate. If $N$ are the number of nuclei present in a radioactive sample at an instant, then activity of this sample is given as

$$A_c = -\frac{dN}{dt} \qquad ...(i)$$

Here, $dN/dt$ is negative as with time the number of elements always decreases due to disintegration; due to negative sign, $A_c$ is always taken positive. The activity of a substance is measured in terms of "dps" or disintegrations per second. The SI unit of activity is named after Becquerel, defined as

1 becquerel = 1 Bq = 1 disintegration/second

Generally, activities of radioactive samples in nature are very high. That is why bequeral being a very small unit, in normal practice, more often MBq or GBq is used. For the same traditional units, Curie (Ci) and Rutherford (Rd) are also used. These are defined as

1 Ci = $3.7 \times 10^{10}$ disintegrations $s^{-1}$

1 Rd = $10^6$ disintegrations $s^{-1}$

Curie was originally defined as roughly the activity of 1 g of $^{226}_{88}Ra$. Similarly, it was observed that 1 kg of ordinary potassium has an activity of about 1 mCi ($10^{-3}$ Ci) because in ordinary potassium small proportion of radioisotope $^{40}_{19}K$ is also present.

## FUNDAMENTAL LAWS OF RADIOACTIVITY

On the basis of experiments performed by Rutherford and Soddy, some conclusions were made for behaviour radioactive elements and the properties of radioactivity. These conclusions are summarized as fundamental laws of radioactivity. These are discussed as follows.

1. Radioactivity is purely a nuclear process. It is not concerned in any manner with the electrons orbiting the nucleus.
2. As radioactivity is a nuclear process, it is independent from any chemical property of the element. As we have discussed that radioactive property of an element is only the process concerned with nucleus of the element, it does not affect the electronic configuration of the element. If this element takes part in a chemical reaction, the product formed will also have the radioactive property in the same fraction by which the radioactive atom is present in the molecule of product.
3. As radioactivity is a random process, its study is only possible by laws of probability. In a group of several radioactive atoms, which one will disintegrate first is just a matter of chance.
4. As radioactivity is a random process, the disintegration density throughout the volume of a radioactive element remains constant. If an element X decays to a daughter nuclide Y, then in a given volume of the element, all portions of volume will have same ratio of number of atoms of Y to that of X. Thus, homogenity is maintained.

Thus, due to randomness, the amount of disintegrations per unit volume per second (called disintegration density) remains approximately constant in the whole volume of the substance.

## RADIOACTIVE DECAY LAW

This law relates the activity of a substance with the number of active or undecayed atoms present in a group of radioactive atoms at an instant of time. This law is stated as follows.

"The activity of a radioactive element at any instant is directly proportional to the number of undecayed active atoms (parent atoms) present at that instant."

Let us consider that at $t = 0$, there are $N_0$ parent atoms in a substance and after a time $t$, $N$ atoms are left undecayed. This implies that in the duration from $t = 0$ to $t = t$, $N_0 - N$ atoms are decayed to their daughter element. If in further time from $t = t$ to $t = t + dt$, $dN$ more atoms will decay then at time $t = t$, we can say that the activity of the element is given as

$$A_c = \left|\frac{dN}{dt}\right| \qquad ...(i)$$

Now, according to radioactive decay law, we have

$$\left|\frac{dN}{dt}\right| \propto N \quad \text{or} \quad A_c = \left|\frac{dN}{dt}\right| = \lambda N \qquad ...(ii)$$

Here, $\lambda$ is the proportionality constant, called decay constant for the decay process. The value of decay constant differs for different elements. From Eq. (ii), we can see that if $\lambda$ is high the element will have high value of activity and if $\lambda$ is less, the activity will be relativity less. Thus, we can say that the decay constant for a radioactive element gives a relative criteria of its stability as well as rate of reaction. If the value of $\lambda$ for an element is more, it is more active or relatively less stable and if for an element $\lambda$ is less, it is more stable. From Eq. (ii), we can also write

$$\lambda = \frac{\left|\frac{dN}{dt}\right|}{N} \qquad ...(iii)$$

Thus, decay constant of a process can be given as "activity per atom" (as given in Eq. (iii)). This shows that for a given radioactive element the activity per atom always remains constant whereas we have already discussed that with time the overall activity of a substance decreases with time as the number of parent elements continuously decreases with time.

Now, from Eq. (iii), we can write

$$\frac{dN}{dt} = -\lambda N \qquad ...(iv)$$

Here, negative sign shows that $\left|\frac{dN}{dt}\right|$, the rate at which the activity elements are disintegrating, is negative or number of active elements are decreasing with time. Now, we have from Eq. (iv),

$$\frac{dN}{N} = -\lambda dt$$

Decay constant is different for different radioactive substances, but it does not depend on the amount of substance and time. SI unit of $\lambda$ is $s^{-1}$.

If $\lambda_1 > \lambda_2$, then the first substance is more radioactive (less stable) than the second one.

For the case, if A decays to B with decay constant $\lambda$

$$\begin{array}{lccc} & A & \xrightarrow{\lambda} & B \\ t = 0 & N_0 & & 0 \end{array}$$

where $N_0$ = number of active nuclei of $A$ at $t = 0$.

$$\begin{array}{lccc} t = t & N & & N' \end{array}$$

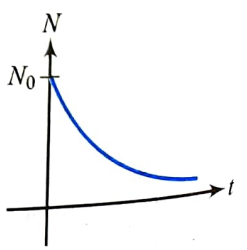
where $N$ = Number of active nuclei of $A$ at $t = t$.

Integrating the above expression within time limits from $t = 0$ to $t = t$, we have

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_0^t \lambda dt \quad \text{or} \quad [\ln N]_{N_0}^{N} = -\lambda t$$

$$\text{or} \quad \ln\left(\frac{N}{N_0}\right) = -\lambda t \quad \text{or} \quad N = N_0 e^{-\lambda t} \qquad ...(v)$$

Equation (v) gives the number of active parent atoms $N$ present at time $t$ in the mixture. This equation is called radioactive decay equation.




From Eq. (v), we can have

$$\lambda N = \lambda N_0 e^{-\lambda t}$$

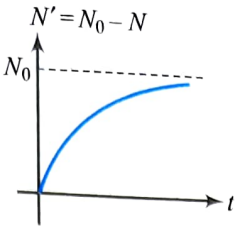
$$\text{or} \quad A_c = A_{c_0} e^{-\lambda t} \qquad ...(vi)$$

Here, $A_{c_0} = \lambda N_0$ is the initial activity of the substance at $t=0$. Equation (vi) is another form of radioactive decay equation. This equation can be used to find the activity of a radioactive substance at any instant of time.

Number of nuclei decayed (i.e., the number of nuclei of $B$ formed),

$$N' = N_0 - N = N_0 - N_0 e^{-\lambda t}$$

or $$N' = N_0(1 - e^{-\lambda t})$$




## ACTIVITY

The activity of a radioactive sample is the number of disintegrations per second that occur. Each time a disintegration occurs, the number $N$ of radioactive nuclei decreases. As a result, the activity can be obtained by dividing $\Delta N$ (the change in the number of nuclei) by $\Delta t$ (the time interval during which the change take place); the average activity over the time interval $\Delta t$ is the magnitude of $\Delta N/\Delta t$. Since the decay of any individual nucleus is completely random, the number of disintegrations per second that occur in a sample is proportional to the number of radioactive nuclei present, so that

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

where $\lambda$ is a proportionality constant referred to as the decay constant. The minus sign is present in this equation because each disintegration decreases the number $N$ of nuclei originally present.

The SI unit for activity is the becquerel (Bq); one becquerel equals one disintegration per second. Activity is also measured in terms of a unit called the curie (Ci), in honour of Marie Curie (1867–1934) and Pierre Curie (1859–1906), the discoverers of radium and polonium. Historically, the curie was chosen as a unit because it is roughly the activity of one gram of pure radium. In terms of becquerels,

$$1\ \text{Ci} = 3.70 \times 10^{10}\ \text{Bq}$$

The activity of the radium put into the dial of a watch to make it glow in the dark is about $4 \times 10^4$ Bq, and the activity used in radiation therapy for cancer treatment is approximately a billion times greater, or $4 \times 10^{13}$ Bq.

## HALF-LIFE TIME

In the previous section, we have discussed that during decay of a radioactive sample, the amount of radionuclide fall off exponentially with time. Every radioactive sample has a characteristic half-life. Half-life time is defined as the time duration in which half of the total number of nuclei will decay or be left undecayed. Say, for example, at any instant if we look into the quantity of a sample of radioactive element, it is observed that after every 3 hour some half-lives are only a millionth of a second for highly active elements and some less active elements have half-life in billions of years.

In some nuclear power plants, a major problem is disposal of the radioactive wastes since some of the nuclide present in waste have long half-lives.

For a radioactive element in a sample if at $t = 0$, $N_0$ nuclei are present of active parent element and during observation after $t = T$, $N_0/2$ are left, then this duration $T$ can be taken as half-life of this element. From radioactive decay equation, we have

$$N = N_0 e^{-\lambda t} \qquad ...(i)$$

Here, at $t = T$, $N = N_0/2$. Thus, we have in Eq. (i)

$$\frac{N_0}{2} = N_0 e^{-\lambda t} \quad \text{or} \quad \ln\left(\frac{1}{2}\right) = -\lambda T$$

or $$\ln(2) = \lambda T$$

or $$T = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda} \qquad ...(ii)$$

Number of nuclei present after $n$ half-lives, i.e., after a time $t = nT_{1/2}$,

$$N = N_0 e^{-\lambda t} = N_0 e^{-\lambda n T_{1/2}} = N_0 e^{-\lambda n \frac{\ln 2}{\lambda}}$$

$$= N_0 e^{\ln 2^{(-n)}} = N_0 (2)^{-n} = N_0 (1/2)^n = \frac{N_0}{2^n}$$

$\left[ n = \dfrac{t}{T_{1/2}}. \text{ It may be a fraction, need not to be an integer.} \right]$

or $$N_0 \xrightarrow[\text{half life}]{\text{after 1st}} \frac{N_0}{2} \xrightarrow{2} N_0\left(\frac{1}{2}\right)^2 \xrightarrow{3} N_0\left(\frac{1}{2}\right)^3$$

$$\cdots \xrightarrow{n} N_0\left(\frac{1}{2}\right)^n$$

### ILLUSTRATION 7.25

A radioactive sample has $6.0 \times 10^{18}$ active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives?

**Sol.** In one half-life, the number of active nuclei reduces to half the original number. Thus, in two half-lives the number is reduced to $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)$ of the original number. The number of remaining active nuclei is, therefore,

$$N_{\text{active}} = 6.0 \times 10^{18} \times \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = 1.5 \times 10^{18}$$

### ILLUSTRATION 7.26

The half-life of a radioactive nuclide is 20 h. What fraction of original activity will remain after 40 h?

**Sol.** 40 h means 2 half-lives. Thus,

$$A = \frac{A_0}{2^2} = \frac{A_0}{4} \quad \text{or} \quad \frac{A}{A_0} = \frac{1}{4}$$

So, one-fourth of the original activity will remain after 40 h.

**Specific activity:** The activity per unit mass is called specific activity.

### ALTERNATIVE FORM OF DECAY EQUATION IN TERMS OF HALF-LIFE TIME

Let a radioactive element X decays to a daughter nucleus Y with a decay process nuclear reaction written as

$$X \xrightarrow{\lambda} Y$$

If initially $N_0$ nuclei of element X are present, then after time $t$ number of nuclei present in the sample is given by decay equation given as

$$N = N_0 e^{-\lambda t} \quad ...(i)$$

If we rearrange the equation, we have $\ln \frac{N}{N_0} = -\lambda T$

We know that half-life of a substance is defined as $T = \frac{\ln(2)}{\lambda}$

or we can write $\lambda = \frac{\ln(2)}{T}$

Therefore, we have $\ln\left(\frac{N}{N_0}\right) = -\frac{\ln(2)}{T} t$ or $\ln\left(\frac{N}{N_0}\right) = \ln(2)^{-t/T}$

Taking antilog on both sides, we get $\frac{N}{N_0} = (2)^{-t/T}$ or $N = N_0(2)^{-t/T}$

This equation is an alternate form of decay equation useful for numerical applications.

### ILLUSTRATION 7.27

Suppose $3.0 \times 10^7$ radon atoms are trapped in a basement at a given time. The basement is sealed against further entry of the gas. The half-life of radon is 3.83 days. How many radon atoms remain after 31 days?

**Sol.** During each half-life, the number of radon atoms is reduced by a factor of two. Thus, we determine the number of half-lives in a period of 31 days and reduce the number of radon atoms by a factor of two for each one.

In a period of 31 days, there are 31 days/3.83 days = 8.1 half-lives. In 8 half-lives, the number of radon atoms is reduced by a factor of $2^8 = 256$. Ignoring the difference between 8 and 8.1 half-lives, we find that the number of atoms remaining is $3.0 \times 10^7/256 = 1.2 \times 10^5$.

### ILLUSTRATION 7.28

*In the above illustration, suppose there are $3.0 \times 10^7$ radon atoms ($T_{1/2} = 3.83$ days or $3.31 \times 10^5$ s) trapped in a basement. **(a)** How many radon atoms remain after 31 days? **(b)** Find the activity **(i)** just after the basement is sealed against further entry of radon and **(ii)** 31 days later.*

**Sol.** *The number $N$ of radon atoms remaining after a time $t$ is given by $N = N_0 e^{-\lambda t}$, where $N_0 = 3.0 \times 10^7$ is the original number of atoms when $t = 0$ s and $\lambda$ is the decay constant. The decay constant is related to the half-life $T_{1/2}$ of the radon atoms by $\lambda = 0.693/T_{1/2}$. The activity can be obtained from the equation $\Delta N/\Delta t = -\lambda N$.*

**(a)** *The decay constant is*

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.83 \text{ days}} = 0.181 \text{ days}^{-1} = 0.181 \text{ days}^{-1} \quad ...(i)$$

*and the number $N$ of radon atoms remaining after 31 days is*

$$N = N_0 e^{-\lambda t} = (3.0 \times 10^7) \exp\{-(0.181 \text{days}^{-1})(31 \text{days})\}$$
$$= 1.1 \times 10^5 \quad ...(ii)$$

*This value is slightly different from that found in Illustration 7.27 because there we ignored the difference between 8.0 and 8.1 half-lives.*

**(b)** **(i)** The activity can be obtained from equation $\Delta N/\Delta t = -\lambda N$, provided the decay constant is expressed in reciprocal seconds: $\lambda = 0.693/(3.31 \times 10^5 \text{ s}) = 2.09 \times 10^{-6} \text{ s}^{-1}$. Thus, the number of disintegrations per second is

$$\frac{\Delta N}{\Delta t} = -\lambda N = -(2.09 \times 10^{-6} \text{ s}^{-1})(3.0 \times 10^7)$$
$$= -63 \text{ disintegrations s}^{-1}$$

The activity is the magnitude of $\Delta N/\Delta t$, so initially

Activity = 63 Bq

**(ii)** From part (a), the number of radioactive nuclei remaining at the end of 31 days is $N = 1.1 \times 10^5$, and reasoning similar to that in part (b) reveals that

Activity = 0.23 Bq

SI unit of activity is becquerel (Bq) which is same as 1 dps (disintegrations per second).

The popular unit of activity is curie which is defined as

1 curie = $3.7 \times 10^{10}$ dps

(which is activity of 1 g radium)

### ILLUSTRATION 7.29

The decay constant for the radioactive nuclide $^{64}Cu$ is $1.516 \times 10^{-5} \text{ s}^{-1}$. Find the activity of a sample containing 1 mg of $^{64}Cu$. Atomic weight of copper = 63.5 g mol$^{-1}$. Neglect the mass difference between the given radioisotope and normal copper.

**Sol.** 63.5 g of copper has $6 \times 10^{23}$ atoms. Thus, the number of atoms in 1 mg of Cu is

$$N = \frac{6 \times 10^{23} \times 1 \text{ µg}}{63.5 \text{ g}} = 9.45 \times 10^{15}$$

$$\text{Activity} = \lambda N$$
$$= (1.516 \times 10^{-5} \text{ s}^{-1}) \times (9.45 \times 10^{15})$$
$$= 1.43 \times 10^{11} \text{ disintegrations s}^{-1}$$
$$= \frac{1.43 \times 10^{11}}{3.7 \times 10^{10}} \text{ Ci} = 3.86 \text{ Ci}$$

Activity after $n$ half-lives is $A_0/2^n$.

## AVERAGE LIFE

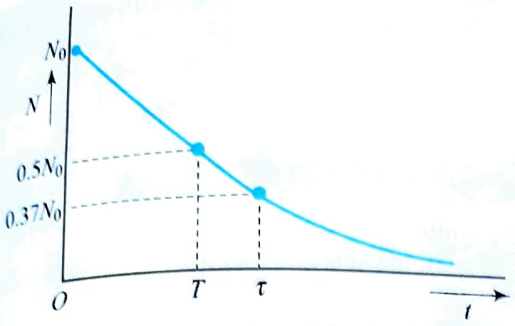
$$T_{avg} = \frac{\text{Sum of ages of all the nuclei}}{N_0} = \frac{\int_0^\infty N_0 e^{-\lambda t} dt}{N_0} = \frac{1}{\lambda}$$

**Important Points:**

- Time constant or average life is more than the half-life, i.e.,

$$\tau = T_{av} > T$$

- The fractional radioactive decay in one mean life (or time constant) is more than that in half life, as shown in figure.




- The number of undecayed nuclei present after $n$ mean life is

$$N = (0.37)^n N_0 = \left(\frac{1}{e}\right)^n N_0$$

- If a nuclide can decay simultaneously by two different processes which have decay constants $\lambda_1$ and $\lambda_2$, then the effective decay constant of the nuclide is

$$\lambda = \lambda_1 + \lambda_2 ; N = N_0 e^{-(\lambda_1 + \lambda_2)t}$$

**ILLUSTRATION 7.30**

The half-life of $^{198}$Au is 2.7 days. Calculate **(a)** the decay constant, **(b)** the average-life, and **(c)** the activity of 1.00 mg of $^{198}$Au. Take atomic weight of $^{198}$Au to be 198 g mol$^{-1}$.

**Sol.**

(a) The half-life and the decay constant are related as

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

or $$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{2.7 \text{ days}} = \frac{0.693}{2.7 \times 24 \times 3600 \text{ s}}$$

$$= 2.9 \times 10^{-6} \text{ s}^{-1}$$

(b) The average-life is $t_{av} = \frac{1}{\lambda} = 3.9$ days.

(c) The activity is $A = \lambda N$. Now, 198 g of $^{198}$Au has $6 \times 10^{23}$ atoms. The number of atoms in 1.00 mg of $^{198}$Au is

$$N = 6 \times 10^{23} \times \frac{1.0 \text{ mg}}{198 \text{ g}} = 3.03 \times 10^{18}$$

Thus, $A = \lambda N = (2.9 \times 10^{-6} \text{ s}^{-1})(3.03 \times 10^{18})$

$= 8.8 \times 10^{12}$ disintegrations s$^{-1}$

$$= \frac{8.8 \times 10^{12}}{3.7 \times 10^{10}} \text{ Ci} = 240 \text{ Ci}$$

**ILLUSTRATION 7.31**

A 50.0 g sample of carbon is taken from the pelvis bone of a skeleton and is found to have a $^{14}$C decay rate of 200.0 decays min$^{-1}$. It is known that carbon from a living organism has a decay rate of decays/min$^{-1}$ g$^{-1}$ and that $^{14}$C has a half-life of 5730 year = $3.01 \times 10^9$ min. Find the age of the skeleton.

**Sol.** Let us start with the equation $N = N_0 e^{-\lambda t}$ and multiply both sides by $\lambda$ to get $\lambda N = \lambda N_0 e^{-\lambda t}$ which is equivalent to

$$R = R_0 e^{-\lambda t} \quad \text{or} \quad \frac{R}{R_0} = e^{-\lambda t}$$

where $R$ is the present activity and $R_0$ was the activity when the skeleton was a part of a living organism. We can solve for the time by taking the natural log of both sides of this equation.

$$\ln\left(\frac{R}{R_0}\right) = \ln(e^{-\lambda t}) = -\lambda t$$

$$t = -\frac{\ln\left(\frac{R}{R_0}\right)}{\lambda}$$

Because we are given the decay rate and mass of the sample, we can find $R_0$ as

$$R_0 = \left(15.0 \frac{\text{decays}}{\text{min g}}\right)(50.0 \text{ g}) = 750 \frac{\text{decays}}{\text{min}}$$

The decay constant is found as

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.01 \times 10^9 \text{ min}} = 2.30 \times 10^{-10} \text{ min}^{-1}$$

**ILLUSTRATION 7.32**

**(a)** Determine the number of carbon $^{14}_{6}$C atoms present for every gram of carbon $^{12}_{6}$C in a living organism. **(b)** Find **(i)** the decay constant and **(ii)** the activity of this sample.

**Sol.** The total number of carbon $^{12}_{6}$C atoms in one gram of carbon $^{12}_{6}$C is equal to the corresponding number of moles times Avogadro's number. Since there is only one $^{14}_{6}$C atom for every $8.3 \times 10^{11}$ atoms of $^{12}_{6}$C, the number of $^{14}_{6}$C atoms is equal to the total number of $^{12}_{6}$C atoms divided by $8.3 \times 10^{11}$. The decay constant $\lambda$ for $^{14}_{6}$C is $\lambda = 0.693/T_{1/2}$, where $T_{1/2}$ is the half-life. The activity is equal to the magnitude of $\Delta N/\Delta t$, which is the decay constant times the number of $^{14}_{6}$C atoms present.

**(a)** One gram of carbon $^{12}_{6}$C (atomic mass = 12 u) is equivalent to 1.0/1.2 mol. Since Avogadro's number is $6.02 \times 10^{23}$ atoms mol$^{-1}$ and since there is one $^{14}_{6}$C atom for every $8.3 \times 10^{11}$ atoms of $^{12}_{6}$C, the number of $^{14}_{6}$C atoms for every 1.0 g of carbon $^{12}_{6}$C is

$$\left(\frac{1.0}{2} \text{mol}\right)\left(6.02 \times 10^{23} \frac{\text{atoms}}{\text{mol}}\right)\left(\frac{1}{8.3 \times 10^{11}}\right)$$

$$= 6.0 \times 10^{10} \text{ atoms}$$

**(b)** **(i)** Since the half-life of $^{14}_{6}$C is 5730 years ($1.81 \times 10^{11}$ s), the decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.81 \times 10^{11} \text{s}} = 3.83 \times 10^{-12} \text{ s}^{-1}$$

**(ii)** The activity is the magnitude of $\Delta N/\Delta t$ or $\lambda N$. Thus, we find activity of $^{14}_{6}$C for every 1.0 g of carbon $^{12}_{6}$C in a living organism = $\lambda N = (3.83 \times 10^{-12} \text{ s}^{-1})(6.0 \times 10^{10}$ atoms) = 0.23 Bq.

An organism that lived thousands of years ago presumably had an activity of about 0.23 Bq per gram of carbon. When the organism died, the activity began decreasing. From a sample of the remains, the current activity per gram of carbon can be

measured and compared to the value of 0.23 Bq to determine the time that has transpired since death.

**ILLUSTRATION 7.33**

At a given instant, there are 25% undecayed radioactive nuclei in a sample. After 10 s, the number of undecayed nuclei reduces to 12.5%. Calculate **(a)** mean life of the nuclei and **(b)** the time in which the number of decayed nuclei will further reduce to 6.25% of the reduced number.

**Sol.** Half-life of radioactive sample, i.e., the time in which number of undecayed nuclei becomes half ($T$) is 10 s.

**(a)** Mean life, $\tau = \dfrac{T}{\log_e 2} = \dfrac{10}{0.693}\text{s} = 1.443 \times 10 = 14.43\text{ s}$

**(b)** The reduced number further reduces to 6.25% in $n$ half-lives given by

$$\frac{N}{100} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{6.25}{100} = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \frac{1}{16} = \left(\frac{1}{2}\right)^n \Rightarrow \left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n \Rightarrow n = 4$$

Time, $t = 4T = 4 \times 10 = 40\text{ s}$

**ILLUSTRATION 7.34**

'$m_1$' g of non-radioactive isotopes ${}_zX^A$ are mixed with '$m_2$' g of the radioactive isotope ${}_zX^{A'}$. How much will the specific activity decrease? Half-life of ${}_zX^{A'} = T$. Take $N_A$ as Avagadro number.

**Sol.** Before mixing the two, the specific activity $a_1$ is given by

$$a_1 = \frac{dN}{m_2 dt} = \frac{\lambda N}{m_2} = \frac{\log_e 2}{T} \times \frac{N_A m_2}{A'} \times \frac{1}{m_2} = \frac{\log_e 2}{T} \times \frac{N_A}{A'}$$

After mixing, the activity $a_2$ is given by

$$a_2 = \frac{dN}{(m_1 + m_2)dt} = \frac{\log_e 2}{T} \frac{N_A m_2}{A'(m_1 + m_2)}$$

$$\therefore \text{ Change in activity} = a_1 - a_2 = \frac{(\log_e 2) N_A m_1}{TA'(m_1 + m_2)}$$

**ILLUSTRATION 7.35**

A stable nuclei $C$ is formed from radioactive nuclei $A$ and $B$ with decay constants of $\lambda_1$ and $\lambda_2$, respectively. Initially, the number of nuclei of $A$ is $N_0$ and that of $B$ is zero. Nuclei $B$ are produced at a constant rate of $P$. Find the number of nuclei of $C$ after time $t$.

**Sol.**

$$\frac{dN_B}{dt} = P - \lambda_2 N_B$$

$$\int_0^{N_2} \frac{dN_B}{P - \lambda_2 N_B} = \int_0^t dt$$

$$\ln\left(\frac{P - \lambda_2 N_B}{P}\right) = -\lambda_2 t$$

$$N_B = P(1 - e^{-\lambda_2 t})$$

The number of nuclei of A after time $t$ is $N_A = N_0 e^{-\lambda t}$.

Thus, $\dfrac{dN_C}{dt} = \lambda_1 N_A + \lambda_2 N_B = +\lambda_1 N_0 e^{-\lambda t} + P(1 - e^{-\lambda_2 t})$

$$N_C = N_0(1 - e^{-\lambda_1 t}) + P\left(t + \frac{e^{-\lambda_2 t} - 1}{\lambda_2}\right)$$

**ILLUSTRATION 7.36**

Natural uranium is a mixture of three isotopes ${}_{92}U^{234}$, ${}_{92}U^{235}$ and ${}_{92}U^{238}$ with the shares of negligible 0.0006%, 0.71%, and 99.28%, respectively. The half-life of the three isotopes are $2.5 \times 10^5$ years, $7.1 \times 10^8$ years, and $4.5 \times 10^9$ years, respectively. Determine the share of radioactivity of each isotope into the total activity of the natural uranium.

**Sol.** Let $A_1$, $A_2$ and $A_3$ be activities of ${}_{92}U^{234}$, ${}_{92}U^{235}$ and ${}_{92}U^{238}$, respectively. Total activity, $A = A_1 + A_2 + A_3$

Share of ${}_{92}U^{234}$ is $\dfrac{A_1}{A} = \dfrac{\lambda_1 N_1}{\lambda_1 N_1 + \lambda_2 N_2 + \lambda_3 N_3}$

Let $m$ be the total mass of natural uranium. Then,

$$m_1 = \frac{0.006}{100} m\,,\ m_2 = \frac{0.71}{100} m\,,\ m_3 = \frac{99.28}{100} m$$

Now, $N_1 = \dfrac{m_1}{M_1}, N_2 = \dfrac{m_2}{M_2}, N_3 = \dfrac{m_3}{M_3}$

where $M_1$, $M_2$ and $M_3$ are atomic weights.

$$\therefore \quad \frac{A_1}{A} = \frac{\left(\dfrac{m_1}{M_1}\right)\dfrac{1}{T_1}}{\dfrac{m_1}{M_1 T_1} + \dfrac{m_2}{M_2 T_2} + \dfrac{m_3}{M_3 T_3}} = 0.99 \approx 9.9\%$$

$$\frac{A_2}{A} = 0.039 = 3.9\%$$

$$\frac{A_3}{A} = 0.864 = 86.4\%$$

## RADIOACTIVE DATING

One important application of radioactivity is the determination of the age of archeological or geological samples. If an object contains radioactive nuclei when it is formed, then the decay of these nuclei marks the passage of time like a clock. If the half-life is known, a measurement of the number of nuclei present today relative to the number present initially can give the age of the sample. A more accurate way is to determine the present number of radioactive nuclei with the aid of a mass spectrometer.

The present activity of a sample can be measured, but how is it possible to know what the original activity was, perhaps thousands of years ago? Radioactive dating methods entail certain assumptions that make it possible to estimate the original activity. For instance, the radiocarbon technique utilizes the ${}^{14}_{6}C$ isotope of carbon, which undergoes $\beta^-$ decay with a half-life of 5730 years. This isotope is present in the earth's atmosphere at an equilibrium concentration of about one atom for every $8.3 \times 10^{11}$ atoms of normal carbon ${}^{12}_{6}C$. It is often assumed that this value has remained constant over the years because ${}^{14}_{6}C$ is created when cosmic rays interact with the earth's upper atmosphere, a

production method that offsets the loss via $\beta^-$ decay. Moreover, nearly all living organisms ingest the equilibrium concentration of $^{14}C$. However, once an organism dies, metabolism no longer sustains the input of $^{14}_{6}C$, and $\beta^-$ decay causes half of the $^{14}_{6}C$ nuclei to disintegrate every 5730 years. Illustration 5.24 illustrates how to determine the $^{14}_{6}C$ activity of one gram of carbon in a living organism.

**ILLUSTRATION 7.37**

The number of $^{238}U$ atoms in an ancient rock equals the number of $^{206}Pb$ atoms. The half-life of decay of $^{238}U$ is $4.5 \times 10^9$ years. Estimate the age of the rock assuming that all the $^{206}Pb$ atoms are formed from the decay of $^{238}U$.

**Sol.** Since the number of $^{206}Pb$ atoms equals the number of $^{238}U$ atoms, half of the original $^{238}U$ atoms have decayed. It takes one half-life to decay half of the active nuclei. Thus, the sample is $4.5 \times 10^9$ years old.

**ILLUSTRATION 7.38**

A bottle of red wine is thought to have been sealed about 5 years ago. The wine contains a number of different kinds of atoms, including carbon, oxygen, and hydrogen. Each of these has a radioactive isotope. The radio-active isotope of carbon is the familiar $^{14}_{6}C$, with a half-life of 5730 years. The radioactive isotope of oxygen is $^{15}_{8}O$ and has a half-life of 122.2 s. The radioactive isotope of hydrogen is $^{3}_{1}H$ and is called tritium; its half-life is 12.33 years. The activity of each of these isotopes is known at the time the bottle was sealed. However, only one of the isotopes is useful for determining the age of the wine accurately. Which is it?

**Sol.** In a dating method that measures the activity of a radioactive isotope, the age of the sample is related to the change in the activity during the time period in question. Here, the expected age is about 5 years. This period is only a small fraction of the 5730-year half-life of $^{14}_{6}C$. As a result, relatively few of the $^{14}_{6}C$ nuclei would decay during the wine's life, and the measured activity would change little from its initial value. To obtain an accurate age from such a small change would require prohibitively precise measurements. Nor is the $^{15}_{8}O$ isotope very useful. The difficulty is its relatively short half-life of 122.2 s. During a 5-year period, so many half-lives of 122.2 s would occur that the activity would decrease to a vanishingly small level. It would not be even possible to measure it. The only remaining option is the tritium isotope of hydrogen. The expected age of 5 years is long enough relative to the half-life of 12.33 years that a measurable change in activity will occur, but not so long that the activity will have completely vanished for all practical purposes.

**ILLUSTRATION 7.39**

The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilization from the known half-life (= 5730 years) of $C^{14}$.

**Sol.** Let $t$ be the age of the Indus-Valley civilization.

The rate of decay, i.e., $R$ is given by

$$R \propto N \quad \text{or} \quad R = \lambda N$$

As $N = N_0 e^{-\lambda t}, R = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$

where $R_0 = \lambda N_0$ is the initial rate of decay.

Thus, $$\frac{R}{R_0} = e^{-\lambda t} \qquad ...(i)$$

We are given that $$\frac{R}{R_0} = \frac{9}{15} = \frac{3}{5} \qquad ...(ii)$$

From Eqs. (i) and (ii), $e^{-\lambda t} = \frac{3}{5}$ or $e^{\lambda t} = \frac{5}{3}$

or $$\lambda t = \ln\left(\frac{5}{3}\right) \text{ or } t = \frac{\ln(5/3)}{\lambda}$$

or $$t = \frac{\ln(5/3)}{\ln 2} \times T_{1/2} = \frac{\log(5/3)}{\log 2} \times 5730 \text{ yr} \quad \left(\text{as } T_{1/2} = \frac{\ln 2}{\lambda}\right)$$

or $$t = \frac{0.2219}{0.3010} \times 5730 \text{ yr} = 4224 \text{ yr}$$

## NATURAL RADIOACTIVITY: RADIOACTIVE DECAY SERIES

Radioactive nuclei are generally classified into two groups: (1) unstable nuclei found in nature, which give rise to what is called natural radioactivity and (2) nuclei produced in the laboratory through nuclear reactions, which exhibit artificial radioactivity.

Three series of naturally occurring radioactive nuclei exist (as shown in the following table). Each series starts with a specific long-lived radioactive isotope whose half-life exceeds that of any of its descendants. The fourth series in the table begins with $^{237}Np$, a transuranic element (on having an atomic number greater than that of uranium) not found in nature. This element has a half-life of "only" $2.14 \times 10^6$ years.

**Four radioactive series**

| Series | Starting isotope | Half-life (year) | Stable end product |
|---|---|---|---|
| Uranium | $^{238}_{92}U$ | $4.47 \times 10^9$ | $^{206}_{82}U$ |
| Actinium | $^{235}_{92}U$ | $7.04 \times 10^8$ | $^{207}_{82}Pb$ |
| Thorium | $^{232}_{90}U$ | $1.41 \times 10^{10}$ | $^{208}_{82}Pb$ |
| Neptunium | $^{237}_{93}U$ | $2.14 \times 10^6$ | $^{209}_{83}Pb$ |

The two uranium series are somewhat more complex than the $^{232}Th$ series. Also, several other naturally occurring radioactive elements that would otherwise have disappeared long ago. For example, because the solar system is about $5 \times 10^9$ years old, the supply of $^{226}Ra$ (whose half-life is only 1600 years) would have been depleted by radioactive decay long ago if it were not for the decay series that starts with $^{238}U$, with a half-life of $4.47 \times 10^9$ years. Figure shows the radioactive decay series that begins with uranium $^{238}_{92}U$ and ends with lead $^{206}_{82}Pb$. Half-lives are given in seconds (s), minutes (min), hours (hr),

days (d), or years (yr). The inset in the upper left in figure identifies the type of decay that each nucleus undergoes.

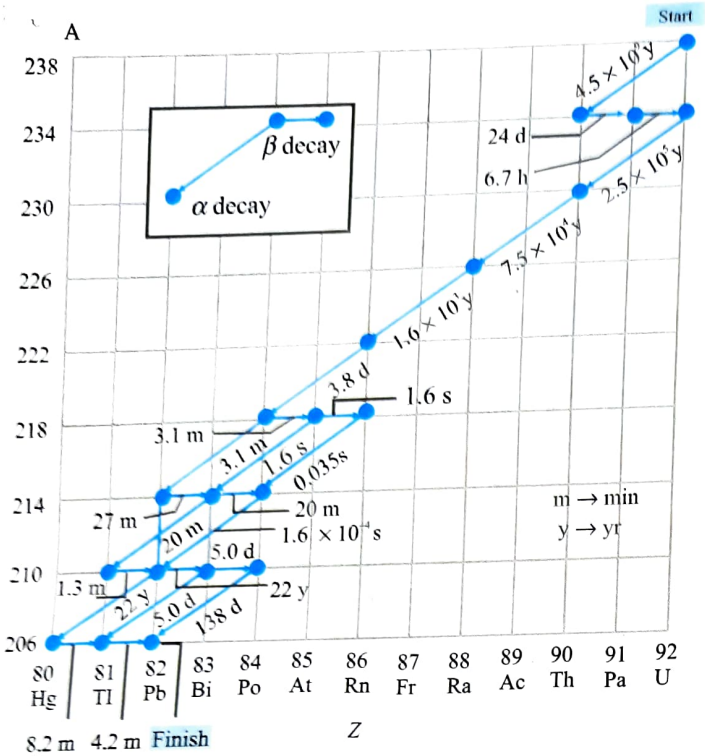



**ILLUSTRATION 7.40**

A radioactive nucleus undergoes a series of decay according to the scheme

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72, respectively, what are these numbers for $A_4$?

**Sol.** The successive process may be expressed as

$${}_{72}A^{180} \xrightarrow{\alpha} {}_{70}A_1^{176} \xrightarrow{\beta^-} {}_{71}A_2^{176} \xrightarrow{\alpha} {}_{69}A_3^{172} \xrightarrow{\gamma} {}_{69}A_4^{172}$$

## RADIOACTIVE EQUILIBRIUM

In a radioactive series, if we talk about an intermediate element, it is produced due to the decay of its previous element and it decays to the next element of the series. In Eq. (i), an intermediate element J decays to K with a decay constant $\lambda_1$ and K decays to L with decay constant $\lambda_2$.

$$\underset{N_1}{J} \xrightarrow{\lambda_1} \underset{N_2}{K} \xrightarrow{\lambda_2} L \qquad ...(i)$$

If at an instant, $N_1$ nuclei of J are present, its disintegration rate can be given as

$$R_1 = \lambda_1 N_1 \qquad ...(ii)$$

As J decays to K, the above relation in Eq. (ii) also gives the formation rate of nuclei of K. If at this instant $N_2$ nuclei of K are present, its decay rate can be given as

$$R_2 = \lambda_2 N_2 \qquad ...(iii)$$

If at some instant the production rate and decay rate of the element K become equal, then the amount of K appears to be a constant as the number of nuclei of K produced per second are equal to the number of nuclei of K disintegrating per second. This situation for the intermediate element K is called radioactive equilibrium. We can also state that his equilibrium is a dynamic equilibrium in which the amount of K element appears to be a constant along with the process of its continuous formation by decaying element J and its continuous disintegration to element L. Thus, for an element, condition of radioactive equilibrium is

Rate of formation = Rate of disintegration

Here, for element K to be in radioactive equilibrium, we have

$$\lambda_1 N_1 = \lambda_2 N_2 \qquad ...(iv)$$

## SIMULTANEOUS DECAY MODES OF RADIOACTIVE ELEMENT

We know that due to radioactive disintegration a radio nuclide transforms into its daughter nucleus. Depending on the nuclear structure and its unstability, a parent nucleus may undergo either $\alpha$ or $\beta$ emission. Sometimes a parent nucleus may undergo both types of emission with the probabilities of $\alpha$-decay or $\beta$-decay. The amount of daughter nuclide produced by $\alpha$- and $\beta$-decays will be in the probability ratio of $\alpha$- and $\beta$-decays.

Now, we analyze the decay of such elements which disintegrate with two or more decay modes simultaneously. If an element decays to different daughter nuclei with different decay constants $\lambda_1$, $\lambda_2$, $\lambda_3$, etc., for each decay mode, then the effective decay constant of the parent nuclei can be given as

$$\lambda_{\text{eff}} = \lambda_1 + \lambda_2 + \cdots$$

Similarly, for a radioactive element with decay constant $\lambda$ which decays by both $\alpha$- and $\beta$-decays given that the probability for an $\alpha$-emission is $P_1$ and that for $\beta$-emission is $P_2$, the decay constant of the element can be split for individual decay modes. Like in this case the decay constants for $\alpha$- and $\beta$-decays separately can be given as

$$\lambda_\alpha = P_1\lambda$$
$$\lambda_\beta = P_2\lambda$$

## ACCUMULATION OF A RADIOACTIVE ELEMENT IN RADIOACTIVE SERIES

In a radioactive series, we have discussed that each element decays into its daughter nuclei until a stable element appears. Consider a radioactive series shown below

$$A_1 \xrightarrow{\lambda_1} A_2 \xrightarrow{\lambda_2} A_3 \xrightarrow{\lambda_3} \cdots$$

To analyze mathematically the above series, we assume initially at $t = 0$, $N_0$ atoms of parent element $A_1$ are present which decay to the element $A_2$ with a decay constant $\lambda_1$. Thus, after time $t$, number of undecayed nuclei of $A_1$ present at a time instant $t$ can be given by decay law as

$$N_1 = N_0 e^{-\lambda_1 t} \qquad ...(i)$$

Due to disintegration of $A_1$, nuclei of $A_2$ are formed and these start decaying with a decay constant $\lambda_2$ to another element $A_3$. Let at an instant $t$, $N_2$ undecayed nuclei of $A_2$ be present. Then, the decay rate of $A_2$ at this instant can be given as

$$\text{Decay rate of } A_2 = \lambda_2 N_2 \qquad ...(ii)$$

Due to disintegration of $A_1$, $A_2$ is produced. Thus, the production rate of nuclei of $A_2$ will be the decay rate of nuclei of $A_1$. Hence, production rate of $A_2$ at this instant can be given as

$$\text{Production rate of } A_2 = \lambda_1 N_1 \qquad ...(iii)$$

Now, in a further time $dt$, if $dN_2$ nuclei of element $A_2$ are accumulated, then the accumulation rate of nuclei of element $A_2$ can be given as

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_0 e^{-\lambda_1 t} \quad \text{...(iv)}$$

or

Equation (iv) is a simple linear differential equation which on solving gives the number of nuclei of element $A_2$ as a function of time $t$. On solving the equation, we get

$$N_2 = \frac{\lambda_1 N_0}{(\lambda_1 - \lambda_2)}(e^{-\lambda_2 t} - e^{-\lambda_1 t}) \quad \text{...(v)}$$

Here, we can see that in the beginning as $N_2 = 0$. Due to disintegration of $A_1$, $A_2$ is being formed and as the amount of $A_1$ is decreased and that of $A_2$ is increased, decay rate of $A_2$ increases and that of $A_1$ decreases. After a time when both decay rates become equal, the element $A_2$ will be said to be in radioactive equilibrium when the yield of the radionuclide $A_2$ is maximum.

Let us take some examples to understand the above phenomenon better.

### ILLUSTRATION 7.41

Suppose the daughter nucleus in a nuclear decay is itself radioactive. Let $\lambda_p$ and $\lambda_d$ be the decay constants of the parent and the daughter nuclei. Also, let $N_p$ and $N_d$ be the number of parent and daughter nuclei at time $t$. Find the condition for which the number of daughter nuclei becomes constant.

**Sol.** The number of parent nuclei decaying in a short time interval $t$ to $t + dt$ is $\lambda_p N_p dt$. The number of daughter nuclei decaying during the same time interval is $\lambda_d N_d dt$. The number of daughter nuclei will be constant if

$$\lambda_p N_p dt = \lambda_d N_d dt \quad \text{or} \quad \lambda_p N_p = \lambda_d N_d$$

### ILLUSTRATION 7.42

A radioactive nucleus can decay by two different processes. The half-life for the first process is $t_1$ and that for the second process is $t_2$. Show that the effective half-life $t$ of the nucleus is given by

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

**Sol.** The decay constant for the first process is $\lambda_1 = \frac{\ln 2}{t_1}$ and for the second process is $\lambda_2 = \frac{\ln 2}{t_1}$. The probability that an active nucleus decays by the first process in a time interval $dt$ is $\lambda_1 dt$. Similarly, the probability that it decays by the second process is $\lambda_2 dt$. The probability that it either decays by the first process or by the second process is $\lambda_1 dt + \lambda_2 dt$. If the effective decay constant is $\lambda$, this probability is also equal to $\lambda dt$. Thus,

$$\lambda dt = \lambda_1 dt + \lambda_2 dt$$

or

$$\lambda = \lambda_1 + \lambda_2$$

or

$$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$$

### ILLUSTRATION 7.43

A factory produces a radioactive substance A at a constant rate $R$ which decays with a decay constant $\lambda$ to form a stable substance. Find **(a)** the number of nuclei of A and **(b)** number of nuclei of B, at any time $t$ assuming the production of A starts at $t = 0$. **(c)** Also, find out the maximum number of nuclei of "A" present at any time during its formation.

**Sol.**

**(a)** Factory $\xrightarrow[\text{const. rate}]{R}$ A $\xrightarrow[\text{decay}]{\lambda}$ B

Let $N$ be the number of nuclei of A at any time $t$.

$$\therefore \quad \frac{dN}{dt} = R - \lambda N \qquad \int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt$$

On solving, we will get $N = R/\lambda(1 - e^{-\lambda t})$

**(b)** Number of nuclei of B at any time $t$,

$$N_B = Rt - N_A = Rt - R/\lambda(1 - e^{-\lambda t}) = R/\lambda\,(\lambda t - 1 + e^{-\lambda t})$$

**(c)** Maximum number of nuclei of "A" present at any time during its formation $= R/\lambda$.

### ILLUSTRATION 7.44

Nuclei of radioactive element A are being produced at a constant rate $\alpha$. The element has a decay constant $\lambda$. At time $t = 0$, there are $N_0$ nuclei of the element.

**(a)** Calculate the number $N$ of nuclei of A at time $t$.

**(b)** If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life time of $A$ and also the limiting value of $N$ at $t \to \infty$.

**Sol.**

**(a)** The rate of formation of radioactive nuclei is $\alpha$.

Rate of decay of radioactive nuclei is $\lambda N$. Therefore,

$$\frac{dN}{dt} = \alpha - \lambda N$$

$$\Rightarrow \quad \frac{dN}{\alpha - \lambda N} = dt$$

Integrating, $\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + A$ ...(i)

where $A$ is constant of integration.

At $t = 0$, $N = N_0$

$$\therefore \quad \frac{\log_e(\alpha - \lambda N_0)}{-\lambda} = A$$

Therefore, Eq. (i) gives

$$\frac{\log_e(\alpha - \lambda N)}{-\lambda} = t + \frac{\log_e(\alpha - \lambda N)}{-\lambda}$$

$$\Rightarrow \quad \log_e\left(\frac{\alpha - \lambda N}{\alpha - \lambda N_0}\right) = -\lambda t$$

$$\Rightarrow \quad \frac{\alpha - \lambda N}{\alpha - \lambda N_0} = e^{-\lambda t}$$

$$\Rightarrow \quad N = \frac{\alpha}{\lambda}(1 - e^{-\lambda t}) + N_0 e^{-\lambda t} \quad \text{...(ii)}$$

**(b)** Given, $a = 2N_0\lambda$

$$t = T_{1/2} = \frac{0.693}{\lambda}$$

$$\therefore \quad N = \frac{2N_0\lambda}{\lambda}(1 - e^{-0.693}) + N_0 e^{-0.693}$$

$$= N_0(2 - e^{-0.693}) = N_0(2 - 0.5)$$

$$\therefore \quad N = 1.5\,N_0$$

When $t \to \infty$, Eq. (ii) gives $N = \frac{\alpha}{\lambda}$

## ILLUSTRATION 7.45

The mean lives of a radioactive substance are 1620 years and 405 years for $\alpha$-emission and $\beta$-emission, respectively. Find out the time during which three-fourth of a sample will decay if it is decaying both by $\alpha$-emission and $\beta$-emission simultaneously.

**Sol.** Radioactivity is the statistical phenomenon. In statistics the important theorem is that the probability of a composite event is equal to the product of probabilities of individual and independent events. Let $\lambda_\alpha$ and $\lambda_\beta$ be the decay constant for $\alpha$-and $\beta$-emission, respectively.

According to theorem, the decay constant for composite event is $\lambda_\alpha + \lambda_\beta$.

If $T$ is half-life of composite event and $T_\alpha$ and $T_\beta$ are the half-lives of $\alpha$- and $\beta$-emissions, then $\lambda = \frac{0.693}{T}$.

$$\frac{1}{T} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} \quad \text{or} \quad \frac{1}{\tau} = \frac{1}{\tau_\alpha} + \frac{1}{\tau_\beta}$$

This gives $\tau = \frac{\tau_\alpha \tau_\beta}{\tau_\alpha + \tau_\beta} = \frac{1620 \times 405}{1620 + 405} = 324$ years

Let $t$ be the time in which the given sample decays three-fourth. Therefore, the fraction of sample undecayed in time $t$ is 1/4. That is,

$$\frac{N}{N_0} = \frac{1}{4}$$

From the relation $N = N_0 e^{-\lambda t}$, we have $\frac{N}{N_0} = e^{-\lambda t}$

or $\log_e \frac{N}{N_0} = -\lambda t$

$$\Rightarrow \quad t = \frac{1}{\lambda}\log_e \frac{N_0}{N} = \tau \log_e \frac{N_0}{N} \qquad \left(\because \lambda = \frac{1}{\tau}\right)$$

$$= 2.3026\,\tau \log_{10} \frac{N_0}{N}$$

$$= 2.3026 \times 324 \log_{10} 4$$

$$= 2.3026 \times 324 \times 0.6031$$

Required time, $t = 449.94$ years

## ILLUSTRATION 7.46

Find the half-life of uranium, given that $3.32 \times 10^{-7}$ g of radium is found per gram of uranium in old minerals. The atomic weights of uranium and radium are 238 and 226 and half-life of radium is 1600 years (Avogadro number is $6.023 \times 10^{23}$/g-atom).

**Sol.** In very old minerals, the amount of an element is constant. This implies that the element exists in radioactive equilibrium. Thus, here we can use

$$\lambda_U N_U = \lambda_R N_R$$

or $$\frac{N_U}{T_U} = \frac{N_R}{T_R}$$

or $$T_U = \frac{N_U}{N_R} \times T_R$$

or $$T_U = \frac{m_U A_R}{m_R A_U} \times T_R$$

or $$T_U = \frac{1 \times 226}{3.32 \times 10^{-7} \times 238} \times 1600 \text{ years} = 4.7 \times 10^9 \text{ years}$$

## CONCEPT APPLICATION EXERCISE 7.3

1. The activity of a sample is $R_1$ at time $t_1$ and $R_2$ at time $t_2$. The mean life of the sample is $\tau$. What is the number of nuclei that have disintegrated in time interval $(t_1 - t_2)$?
2. The half-life of cobalt–60 is 5.25 yrs. After how long does its activity reduce to about one eight of its original value?
3. A nucleus of $U_{x_1}$ has a half-life of 24.1 days. How long a sample of $U_{x_1}$ take to change to 90% of it to $U_{x_2}$ ?
4. Small amount of a radioactive substance (half-life = 10 days) is spread inside a room and consequently the level of radiation becomes 50 times the level for normal occupancy of the room. After how many days will the room be safe for occupation?
5. Find the half-life of $^{238}U$ if 1 g of it emits $1.24 \times 10^4$ $\alpha$-particles per second. Avogadro's number $= 6.023 \times 10^{23}$.
6. The half-life of radon is 3.8 days. After how many days will only one-twentieth of radon sample be left over?
7. A radioactive sample has mass $m$, decay constant $\lambda$ and molecular weight $M$. If the Avogadro number is $N_A$, then
   **(a)** find the initial number of nuclei present;
   **(b)** find the number of decayed nuclei after a time $t$;
   **(c)** find the activity of the sample after a time $t$.
8. Calculate the time taken to decay 100 percent of a radioactive sample in terms of **(a)** half-life $T$ and **(b)** mean-life $T_{av}$.
9. The activity of a sample of radioactive material is $A_1$ at time $t_1$ and $A_2$ at time $t_2$ $(t_2 > t_1)$. Obtain an expression for its mean-life.
10. Determine the average $^{14}C$ activity in decays per minute per gram of natural carbon found in living organisms if the concentration of $^{14}C$ relative to that of $^{12}C$ is $1.4 \times 10^{-12}$ and half-life of $^{14}C$ is $T_{1/2} = 57.30$ years.
11. A radio nuclide $A_1$ with decay constant $\lambda_1$ transforms into a radio nuclide $A_2$ with decay constant $\lambda_2$. Assuming that at the initial moment, the preparation contained only the ratio nuclide $A_1$.

(a) Find the equation describing accumulation of radio nuclide $A_2$ with time.
(b) Find the time interval after which the activity of radio nuclide $A_2$ reaches its maximum value.

12. The isotope $^{14}_{6}C$ is radioactive and has a half-life of 5730 years. If you start with a sample of 1000 carbon-14 nuclei, how many will still be around in 17,190 years?
13. The half-life of the radioactive nucleus $^{226}_{86}Ra$ is $1.6 \times 10^3$ yr. If a sample contains $3.0 \times 10^{16}$ such nuclei, determine the activity at this time.

### ANSWERS

1. $(R_1 - R_2)\tau$ 2. 15.75 yr
3. 80 days 4. 56.45 days
5. $4.5 \times 10^9$ yr 6. 16.45 days
7. (a) $N_A \frac{m}{M}$ (b) $\frac{mN_A(1-e^{-\lambda t})}{M}$ (c) $\frac{mN_A\lambda e^{-\lambda t}}{M}$
8. (a) $\frac{T}{0.693}\ln\left|\frac{1}{1-\eta}\right|$ (b) $T_{av}\ln\left|\frac{1}{1-\eta}\right|$ 9. $T = \frac{t_2 - t_1}{\ln|A_1/A_2|}$
10. 16 decay $g^{-1}$ $min^{-1}$
11. (a) $N_2(t) = \frac{\lambda_1 N_0}{\lambda_2 - \lambda_1}(e^{-\lambda_1 t} - e^{-\lambda_2 t})$ (b) $\frac{\ln(\lambda_2/\lambda_1)}{\lambda_2 - \lambda_1}$
12. ~500 nuclei 13. 11.1 $\mu$Ci

## NUCLEAR REACTIONS

It is possible to change the structure of nuclei by bombarding them with energetic particles. Such changes are called nuclear reactions. Rutherford was the first to observe nuclear reactions, using naturally occurring radioactive source for the bombarding particles. He found that protons were released when alpha particles were allowed to collide with nitrogen atoms. The process can be represented symbolically as

$$^{4}_{2}He + ^{14}_{7}N \rightarrow X + ^{1}_{1}H \quad ...(i)$$

This equation says that an alpha particle ($^{4}_{2}He$) strikes a nitrogen nucleus and produces are an unknown product nucleus (X) and a proton ($^{1}_{1}H$). Balancing atomic numbers and mass numbers, as we did for radioactive decay, enables us to conclude that the unkown is characterized as $^{17}_{8}X$. Because the element with atomic number 8 is oxygen, we see that the reaction is

$$^{4}_{2}He + ^{14}_{7}N \rightarrow ^{17}_{8}O + ^{1}_{1}H \quad ...(ii)$$

This nuclear reaction starts with two stable isotopes, helium and nitrogen, and produces two different stable isotopes, hydrogen and oxygen.

### KINEMATICS OF NUCLEAR REACTIONS

When two particles approach each other, their mutual interaction changes their motion. Exchange of momentum and energy takes place between particles.

When the final particles or system are the same as initial ones, the term scattering is used. The collision is said to be a reaction if final particles are not identical to initial ones, e.g., in a collision between an electron and a proton, the final product may be a hydrogen atom.

Since only internal force is involved in a collision, both the momentum and total energy are conserved. From conservation of momentum,

$$\underbrace{P_1' + P_2'}_{\text{after}} = \underbrace{P_1 + P_2}_{\text{before}} \quad ...(i)$$

The kinetic energies of particles before and after collision are given by

$$E_K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} \quad \text{(before)} \quad ...(ii)$$

$$E_K' = \frac{1}{2}m_1'v_1'^2 + \frac{1}{2}m_2'v_2'^2 = \frac{p_1'^2}{2m_1'} + \frac{p_2'^2}{2m_2'} \quad \text{(after)} \quad ...(iii)$$

The internal energy of the particles may be different after collision. Hence,

$$\underbrace{E_K' + E_{int}'}_{\text{after}} = \underbrace{E_K + E_{int}}_{\text{before}}$$

If the internal energy changes in the collision, the kinetic energy also changes.

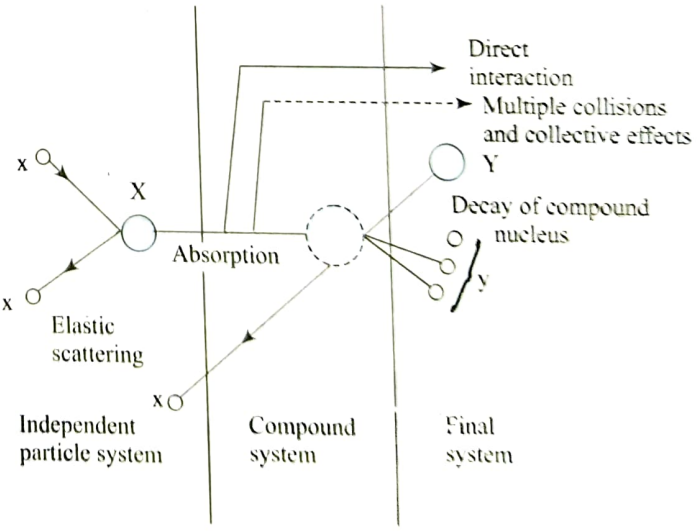
The $Q$ of the reaction or collision is defined by

$$Q = E_K' - E_K = U_{int} - U_{int}'$$
$$= (U_{int})_{before} - (U_{int})_{after} = -\Delta U_{int}$$
$$= \left(\frac{p_1'^2}{2m_1'} + \frac{p_2'^2}{2m_2'}\right) - \left(\frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}\right) \quad ...(iv)$$

When $Q = 0$, there is no change in kinetic energy or internal potential energy and the collision is elastic.

When $Q < 0$, there is a decrease in kinetic energy with a corresponding increase in internal energy of the particles. Such collision is called endothermic. When $Q > 0$, there is an increase in kinetic energy at the expense of internal energy. This is an exothermic reaction.

Figure shows the possible stages of nuclear reactions. Reflection of the incident particles as described by Rutherford's theory is called elastic scattering.




In a direct interaction, the incident particles (which have high energy and can penetrate deeper into the nucleus) interact with a single nucleon in the nucleus so that the nucleon leaves the nucleus. If the nucleon does not leave the nucleus but interacts with several other nucleons, then the nucleus is raised to an excited state called a compound nucleus. The energy carried by

the incident particle is shared by many nucleons. The compound nucleus can emit a particle identical to the incident particle and with the same kinetic energy or emit photons or other particles. The decay of the compound nucleus is a statistical process. It does not depend on the process of formation or external conditions applied, similar to radioactivity. The emission of particles or photons cannot be predicted.

A nuclear reaction can occur only if certain conservation laws are followed. These are:

1. Conservation of mass number $A$.
2. Conservation of charge.
3. Conservation of energy, linear momentum, and angular momentum.

## ENERGY CONSERVATION IN NUCLEAR REACTIONS

Consider a general reaction of particle x that is incident on nucleus X resulting in nucleus Y and particle y. The reaction may be written as

$$x + X \rightarrow Y + y + Q$$

Sometimes the reaction is written as X(x, y)Y, e.g., consider the reaction $^7$Li (p, $\alpha$) $^4$He which can be written as

$$^1_1H + ^7_3Li \rightarrow ^4_2He + ^4_2He$$

We apply conservation of energy to a reaction to compute total kinetic energy released or absorbed in the reaction. Here, we assume that target particle is at rest, $K_x$, $K_y$, $K_Y$ represent kinetic energyies of bombarding particle x and reaction products y and Y, respectively.

From conservation of energy,

$$m_xc^2 + K_x + m_Xc^2 = m_yc^2 + K_y + m_Yc^2 + K_Y$$

The total energy released in the reaction ($Q$) is equal to the difference in kinetic energies of the final particles and of initial particles. So,

$$Q = K_Y + K_y - K_x = (m_x + m_X - m_y - m_Y)c^2$$

which is the energy released in the reaction and is called the $Q$ value of the reaction.

In exothermic reactions, energy is released by a nuclear reaction. In an exothermic reaction, the total mass of the initial particles is greater than that of the final particles and the $Q$ value is positive. In these reactions, some mass is converted to energy.

In endothermic reactions, energy is absorbed in a nuclear reaction. If the total mass of the initial particles is less than that of the final particles, the $Q$ value is negative and some minimum input kinetic energy from the bombarding particles is required. For example,

$$n + ^1H \rightarrow ^2H + y + 2.22 \text{ MeV} \quad \text{(exothermic)}$$

$$y + ^2H \rightarrow ^1H + (n) - 2.22 \text{ MeV} \quad \text{(endothermic)}$$

The minimum kinetic energy needed to initiate a nuclear reaction is called threshold energy. An endothermic reaction cannot take place unless certain threshold energy is supplied to the system. The threshold energy $K_{th}$ must not only supply $|Q|$ but also some kinetic energy to the products to conserve momentum. In the center of mass frame, the total momentum is zero; the threshold energy is just $|Q|$.

But for reactions occurring with nucleus X at rest relative to the laboratory (called the laboratory frame), the incident particle X must have energy greater than $|Q|$ because by conservation of momentum, the kinetic energies of y and Y cannot be zero. Consider a particle x, of mass $m$, incident on X, of mass $M$. Both particles have momenta of equal magnitude in the center of mass frame.

The total kinetic energy is $E_{CM} = \frac{p^2}{2m} + \frac{p^2}{2M} = \frac{1}{2}p^2\left(\frac{m+M}{mM}\right)$

CM system Lab system

We can transform the velocity of incident and target particles to the lab frame by adding $V$ to each velocity so that $M$ is at rest and $m$ has velocity $v + V$. The momentum of $m$ in the lab frame is then

$$P_{lab} = m(v + V) = mv\left(1 + \frac{m}{M}\right) = p\left(\frac{m+M}{M}\right)$$

And its energy is

$$E_{lab} = \frac{p_{lab}^2}{2m} = \frac{p^2}{2m}\left(\frac{M+m}{M}\right)^2 = \frac{M+m}{M}E_{CM}$$

Hence, the threshold energy for an endothermic reaction in the lab frame is

$$E = \frac{m+M}{M}|Q|$$

### ILLUSTRATION 7.47

Find the $Q$ value of the reaction p + $^7$Li $\rightarrow$ $^4$He + $^4$He. Determine whether the reaction is exothermic or endothermic. The atomic masses of $^1$H, $^4$He, and $^7$Li are 1.007825 u, 4.002603 u, and 7.016004 u, respectively.

**Sol.** The total mass of the initial particles,

$$m_i = 1.007825 + 7.016004 = 8.023829 \text{ u}$$

and the total mass of the final particles

$$m_f = 2 \times 4.002603 = 8.005206 \text{ u}$$

Difference between initial and final mass of particles

$$\Delta m = m_i - m_f = 8.023829 - 8.0065206 = 0.018623 \text{ u}$$

This mass is converted into energy and the reaction is exothermic. The $Q$ value is positive and given by

$$Q = (\Delta m)c^2 = 0.018623 \times 931.5 = 17.35 \text{ MeV}$$

### ILLUSTRATION 7.48

Consider a collision between two particles one of which is at rest and the other strikes it head on with momentum $p_1$. Calculate the energy of reaction $Q$ in terms of the kinetic energy of the particles before and after they collide.

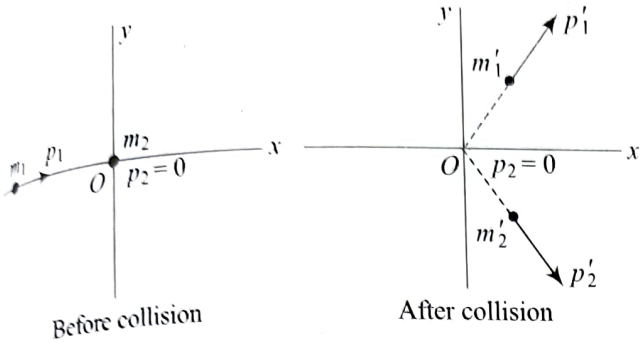
**Sol.** Initially, $m_1$ has a momentum $p_1$ and $m_2$ is at rest ($p_2 = 0$) in the lab frame. The masses of the particles after the collision are $m_1'$ and $m_2'$.

The conservation of momentum gives

$$p_1' + p_2' = p_1 \text{ or } p_2' = p_1 - p_1' \qquad \text{...(i)}$$

Squaring this equation, we have

$$p_2'^2 = (p_1 - p_1')^2 = p_1^2 + p_1'^2 - 2p_1p_1'$$
$$= p_1^2 + p_1'^2 - 2p_1p_1' \cos\theta$$




Before collision After collision

We have

$$Q = \frac{p_1'^2}{2m_1'} + \frac{p_2'^2}{2m_2'} - \frac{p_1^2}{2m_1} + \frac{p_1'^2}{2m_1'}$$
$$+ \frac{1}{2m_2'}(p_1^2 + p_1'^2 - 2p_1p_1'\cos\theta) - \frac{p_1^2}{2m_1}$$

or $$Q = \frac{1}{2}\left(\frac{1}{m_1'} + \frac{1}{m_2'}\right)p_1'^2 + \frac{1}{2}\left(\frac{1}{m_2'} - \frac{1}{m_1}\right)p_1^2 - \frac{p_1p_1'}{m_2'}\cos\theta$$

...(ii)

Note that the kinetic energy of a particle can be expressed in terms of momentum of particles as $E_k = \frac{p^2}{2m}$.

Now, we can express the above results as

$$Q = E_{k,1}'\left(1 - \frac{m_1}{m_2'}\right) = -\frac{2(m_1m_1'E_{k,1}E_{k,1}')^{1/2}}{m_2'}\cos\theta \qquad \text{...(iii)}$$

The equation derived above is called $Q$ equation. It is applied in analysis of nuclear collisions.

## NUCLEAR FISSION

Nuclear fission occurs when a heavy nucleus, such as $^{235}U$, splits, or fissions, into two smaller nuclei. In such a reaction, the total mass of the products is less than the original mass of the heavy nucleus.

Nuclear fission was first observed in 1939 by Otto Hahn and Fritz Strassman, following some basic studies by Fermi. After bombarding uranium ($Z = 92$) with neutrons, Hahn and Strassman discovered among the reaction products two medium-mass elements, barium and lanthanum. Shortly thereafter, Lisa Meithner and Otto Frisch explained what had happened. The uranium nucleus had split into two nearly equal fragments after absorbing a neutron. Such an occurrence was of considerable interest to physicists attempting to understand the nucleus. But it was to have even more far-reaching consequences. Measurements showed that about 200 MeV of energy is released in each fission event, and this fact was to affect the course of human history.

The fission of $^{235}U$ by slow (low-energy) neutrons can be represented by the equation

$$^1_0n + ^{235}_{92}U \rightarrow ^{236}_{92}U^* \rightarrow X + Y + \text{neutrons} \qquad \text{...(i)}$$

where $^{236}U^*$ is an intermediate state that lasts only for about $10^{-12}$ s before splitting into X and Y. The resulting nuclei, X and Y, are called fission fragments. Many combinations of X and Y satisfy the requirements of conservation of energy and charge. In the fission of uranium, there are about 90 different daughter nuclei that can be formed. The process also results in the production of several (typically two or three) neutrons per fission event. On the average, 2.47 neutrons are released per event.

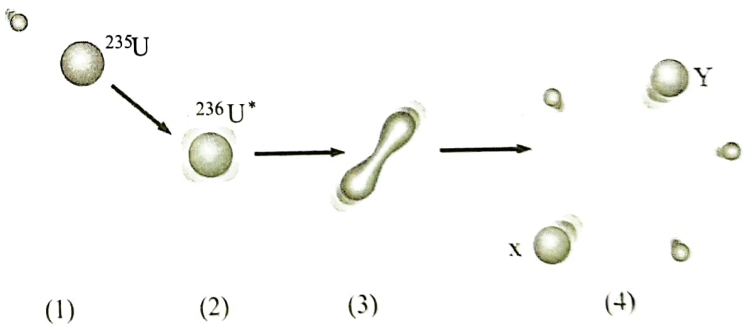
A typical reaction of this type is

$$^1_0n + ^{235}_{92}U \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^1_0n \qquad \text{...(ii)}$$

The fission fragments, barium and krypton, and the released neutrons have a great deal of kinetic energy following the fission event.

The breakup of the uranium nucleus can be equated to the case of a drop of water when excess energy is added to it. All of the atoms in the drop have energy, but not enough to break up the drop. However, if enough energy is added to set the drop vibrating, it will undergo elongation and compression until the amplitude of vibration becomes large enough to cause the drop to break apart. In the uranium nucleus, a similar process occurs (see figure). The sequence of events is as follows:

1. $^{235}U$ nucleus captures a thermal (slow-moving) neutron.
2. This capture results in the formation of $^{236}U^*$, and the excess energy of this nucleus causes it to undergo violent oscillations.
3. The $^{236}U^*$ nucleus becomes highly elongated, and the force of repulsion between protons in the two halves of the dumbbell shape tends to increase the distortion.
4. The nucleus splits into two fragments, emitting several neutrons in the process.




Let us estimate the disintegration energy, $Q$, released in a typical fission process. From figure, we see that the binding energy per nucleon is about 7.2 MeV for heavy nuclei (those having a mass number of approximately 240) and about 8.2 MeV for nuclei of intermediate mass. This means that the nucleons in the fission fragments are more tightly bound and, therefore, have less mass than the nucleons in the original heavy nucleus. This decrease in mass per nucleon appears as released energy when fission occurs. The amount of energy released is (8.2–7.2) MeV per nucleon. Assuming a total of 240 nucleons, we find that the energy released per fission event is

$Q$ = (240 nucleons) (8.2 MeV/nucleon – 7.2 MeV/nucleon)
= 240 MeV

This is a very large amount of energy relative to the amount released in chemical processes.

In nuclear fission, heavy nuclei of $A$ above 200 break up into two or more fragments of comparable masses. The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use ${}_{92}U^{236}$ as the fission material. The technique is to hit a uranium sample by slow-moving neutrons (kinetic energy ≈ 0.04 eV, also called thermal neutrons). A ${}_{92}U^{235}$ nucleus has large probability of absorbing a slow neutron and forming ${}_{92}U^{236}$ nucleus. This nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have

$${}_{92}U^{236} \rightarrow {}_{53}I^{137} + {}_{39}Y^{97} + 2n$$

$${}_{92}U^{236} \rightarrow {}_{53}I^{140} + {}_{36}Y^{94} + 2n$$

and a number of other combinations.

1. On an average, 2.5 neutrons are emitted in each fission event.
2. Mass lost per reaction ≈ 0.2 amu
3. In nuclear fission, the total BE increases and excess energy is released.
4. In each fission event, about 200 MeV of energy is released, a large part of which appears in the form of kinetic energies of the two fragments. Neutrons take away about 5 MeV. For example,

$${}^{235}_{92}U + {}_0n^1 \rightarrow {}^{236}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}_0n^1 + \text{energy}$$

$$Q \text{ value} = [(M_U - 92m_e + m_n) - \{(M_{Ba} - 56\,m_e) + (M_{Kr} - 36m_e) + 3m_n\}]c^2$$

$$= [(M_U + m_n) - (M_{Ba} + M_{Kr} + 3m_n)]c^2$$

5. A very important and interesting feature of neutron-induced fission is the chain reaction. For working of nuclear reactor, refer your text book.

### ILLUSTRATION 7.49

Two other possible ways by which $^{235}U$ can undergo fission when bombarded with a neutron are (1) by the release of $^{140}Xe$ and $^{94}Sr$ as fission fragments and (2) by the release of $^{132}Sn$ and $^{101}Mo$ as fission fragments. In each case, neutrons are also released. Find the number of neutrons released in each of these events.

**Sol.** By balancing mass numbers and atomic numbers, we find that these reactions can be written as

$${}^1_0n + {}^{235}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2{}^1_0n$$

$${}^1_0n + {}^{235}_{92}U \rightarrow {}^{132}_{50}Sn + {}^{101}_{42}Mo + 3{}^1_0n$$

Thus, two neutrons are released in the first event and three in the second.

### ILLUSTRATION 7.50

Calculate the total energy released if 1.0 kg of $^{235}U$ undergoes fission, taking the disintegration energy per event to be $Q = 208$ MeV (a more accurate value than the estimate given previously).

**Sol.** We need to know the number of nuclei in 1.00 kg of uranium. Because $A = 235$, the number of nuclei is

$$N = \left(\frac{6.02 \times 10^{23} \text{ nuclei mol}^{-1}}{235 \text{ g mol}^{-1}}\right)(1.00 \times 10^3 \text{ g})$$

$$= 2.56 \times 10^{24} \text{ nuclei}$$

Hence, the disintegration energy is

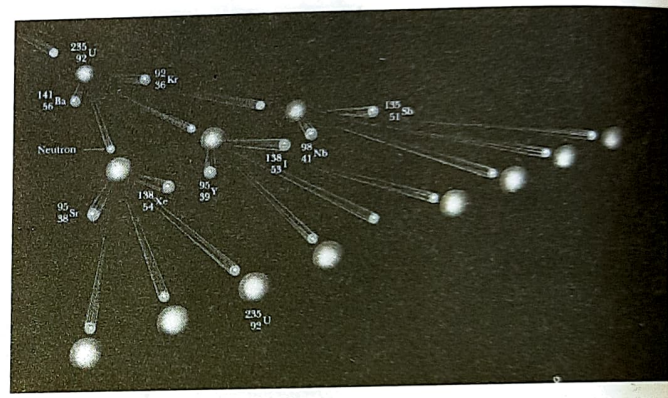
$$E = nQ = (2.56 \times 10^{24} \text{ nuclei})\left(208 \frac{\text{MeV}}{\text{nucleus}}\right)$$

$$= 5.32 \times 10^{26} \text{ MeV}$$

Because 1 MeV is equivalent to $4.45 \times 10^{-20}$ kWh, $E = 2.37 \times 10^7$ kWh. This is enough energy to keep a 100 W light bulb burning for about 30000 years.

## NUCLEAR REACTORS

We have seen that neutrons are emitted when $^{235}U$ undergoes fission. These neutrons can in turn trigger other nuclei to undergo fission, with the possibility of a chain reaction (see figure). Calculations show that if the chain reaction is not controlled (that is, if it does not proceed slowly), it can result in a violent explosion, with the release of an enormous amount of energy. This, of course, was the principle behind the first nuclear bomb, an uncontrolled fission reaction.




A nuclear reactor is a system designed to maintain what is called a self-sustained chain reaction. This important process was first achieved in 1942 by a group led by Fermi at the University of Chicago, with natural uranium as the fuel. Most reactors in operation today also use uranium as fuel. Natural uranium contains only about 0.7% of the $^{235}U$ isotope, with the remaining 99.3% being the $^{238}U$ isotope. This is important to the operation of a reactor because $^{238}U$ almost never undergoes fission. Instead, it tends to absorb neutrons, producing neptunium and plutonium. For this reason, reactor fuels must be artificially enriched so that percentage of the $^{235}U$ isotope can be increased.

Earlier, we mentioned that an average of about 2.5 neutrons are emitted in each fission event of $^{235}U$. In order to achieve a self-sustained chain reaction, one of these neutrons must be captured by another $^{235}U$ nucleus and cause it to undergo fission. A useful parameter for describing the level of reactor operation is the reproduction constant $K$, defined as the average number of neutrons from each fission event that will cause another event. As we have seen, $K$ can have a maximum value of 2.5 in the fission of uranium. However, in practice $K$ is less than this because of several factors, which we shall discuss in a moment.

In a self-sustained chain reaction, $K = 1$ and the reactor is said to be critical. When $K$ is less than unity, the reactor is subcritical and the reaction dies out. When $K$ is greater than unity, the reactor is said to be supercritical, and a runaway reaction occurs. In a nuclear reactor used to furnish power to a utility company, it is necessary to maintain a $K$ value close to unity.

### NEUTRON LEAKAGE

In any reactor, a fraction of the neutrons produced in fission will leak out of the core before inducing other fission events. If the fraction leaking out is too large, the reactor will not operate. The percentage lost is large if the reactor is very small because leakage is a function of the ratio of surface area to volume. Therefore, a critical requirement of reactor design is choosing the correct surface area to volume ratio so that a sustained reaction can be achieved.

### REGULATING NEUTRON ENERGIES

The neutrons released in fission events are very energetic, with kinetic energies of about 2 meV. It is found that slow neutrons are far more likely than fast neutrons to produce fission events in $^{235}U$. $^{238}U$ does not absorb slow neutrons. Therefore, in order for the chain reaction to continue, the neutrons must be slowed down. This is accomplished by surrounding the fuel with a moderator substance.

To understand how neutrons are slowed down, consider a collision between a light object and a very massive one. In such an event, the light object rebounds from the collision with most of its original kinetic energy. However, if the collision is between objects with nearly the same masses, the incoming projectile transfers a large percentage of its kinetic energy to the target. In the first nuclear reactor ever constructed, Fermi placed bricks of graphite (carbon) between the fuel elements. Carbon nuclei are about 12 times more massive than neutrons, but after about 100 collisions with carbon nuclei, a neutron is slowed sufficiently to increase its likelihood of fission with $^{235}U$. In this design, the carbon is the moderator; most modern reactors use heavy water ($D_2O$) as the moderator.

### NEUTRON CAPTURE

In the process of being slowed down, the neutrons may be captured by nuclei that do not undergo fission. The most common event of this type is neutron capture by $^{238}U$. The probability of neutron capture by $^{238}U$ is very high when the neutrons have high kinetic energies and very low when they have low kinetic energies. Thus, the slowing down of the neutrons by the moderator serves the dual purpose of making them available for reaction with $^{235}U$ and decreasing their chances of being captured by $^{238}U$.

### ILLUSTRATION 7.51

Suppose, we think of fission of a $^{56}_{26}Fe$ nucleus into two equal fragments, $^{28}_{13}Al$. Is the fission energetically possible? Argue by working out $Q$ of the process. Given $m(^{56}_{26}Fe) = 55.93494\,u$ and $m(^{28}_{13}Al) = 27.98191\,u$.

**Sol.** The fission of Fe-56 into two fragments of $^{28}_{13}Al$ with energy released $Q$ can be written as $^{56}_{26}Fe \rightarrow {}^{28}_{13}Al + {}^{28}_{13}Al + Q$

$$Q = \left[m\left(^{56}_{26}Fe\right) - 2m\left(^{28}_{13}Al\right)\right]c^2$$
$$= [55.93494 - 2\times 27.98191]\times 931.5\,MeV$$
$$= -0.02888\times 931.5 = -26.90\ MeV$$

As the Q-value is negative, the fission is not possible energetically.

### ILLUSTRATION 7.52

The fission properties of $^{239}_{94}Pu$ are very similar to those of $^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission?

**Sol.** Number of atoms in 1 kg of pure $^{239}_{84}Pu$

$$= \left(\frac{6.023\times 10^{23}}{239}\right)\times 1000 = 2.52\times 10^{24}$$

Energy released in the fission $= (2.52 \times 10^{24}) \times 180$
$= 4.536 \times 10^{26}$ MeV

### ILLUSTRATION 7.53

A 1000 MW fission reactor consumes half of its fuel in 5.00 yr. How much $^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of $^{235}_{92}U$ and this nuclei is consumed only by the fission process.

**Sol.** In the fission of one nucleus of 1 kg of $^{235}_{92}U$, energy generated is 200 MeV.

Energy generated in the fission of 1 kg of $^{235}_{92}U$

$$= (6.023\times 10^{23})\left(\frac{1000\,g}{235\,g}\right)(200\times 1.6\times 10^{-13}\,J)$$
$$= 8.2\times 10^{13}\ J/kg$$

Time for which the reactor actually operates

$$= \left(\frac{8}{100}\right)\times 5yr = 4yr = 4\times(3.154\times 10^7\,s) = 12.62\times 10^7\,s$$

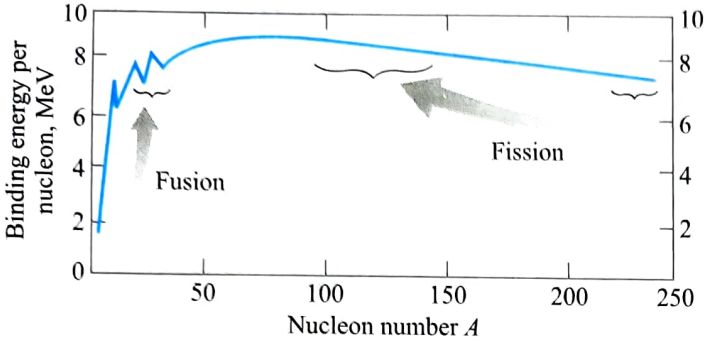
Energy generated by the reactor in 5y = power × time
$= (1000 \times 10^6\ W)\,(12.62 \times 10^7\ s) = 12.62 \times 10^{16}\ J$

$$\text{Fuel } (U^{235}) \text{ consumed in 5 years} = \frac{12.62\times 10^{16}\,J}{8.2\times 10^{13}\,J/kg} = 1.5\times 10^3\ kg$$

## NUCLEAR FUSION

Figure shows that the binding energy for lighter nuclei (those having a mass number lower than 20) is much smaller than the binding energy for heavier nuclei. This suggests a possible process that is the reverse of fission. When two light nuclei combine to form a heavier nucleus, the process is called nuclear fussion. Because the mass of the final nucleus is less than the masses of the original nuclei, there is a loss of mass accompanied

by a release of energy. Although fusion power plants have not yet been developed, a worldwide effort is under way to harness the energy from fusion reactions in the laboratory. Later, we shall discuss the possibilities and advantages of this process for generating electric power.




## FUSION IN THE SUN

All stars generate their energy through fusion process. About 90% of the stars, including the Sun, fuse hydrogen, whereas some older stars fuse helium or other heavier elements. Stars are born in regions of space containing vast clouds of dust and gas. Recent mathematical models of these clouds indicate that star formation is triggered by shock waves passing through a cloud. These shock waves are similar to sonic booms and are produced by events such as the explosion of a nearby star, called a supernova explosion. The shock wave compresses certain regions of the cloud, causing these regions to collapse under their own gravity. As the gas falls inward toward the center, the atoms gain speed, which causes the temperature of the gas to rise. Two conditions must be met before fusion reactions in the star can sustain its energy needs.

1. The temperature must be high enough (about $10^7$ K for hydrogen) to allow the kinetic energy of the positively charged hydrogen nuclei to overcome their mutual Coulomb repulsion as they collide.
2. The density of nuclei must be high enough to ensure a high rate of collision.

When fusion reactions occur at the core of a star, the energy liberated eventually becomes sufficient to prevent further collapse of the star under its own gravity. The star then continues to live out the remainder of its life under a balance between the inward force of gravity pulling it toward collapse and the outward force due to thermal effects and radiation pressure.

The proton–proton cycle is a series of three nuclear reactions that are believed to be the stages in the liberation of energy in the Sun and other stars rich in hydrogen. An overall view of the proton–proton cycle is that four protons combine to form an alpha particle and two positrons, with the release of 25 MeV of energy in the process.

The specific steps in the proton–proton cycle are

$$_1^1H + {}_1^1H \rightarrow {}_1^2H + e^+ + \nu$$

$$_1^1H + {}_1^2H \rightarrow {}_2^3He + \gamma$$

This second reaction is followed by either hydrogen–helium fusion or helium–helium fusion:

$$_1^1H + {}_2^3He \rightarrow {}_2^4He + e^+ + \nu$$

or

$$_2^3He + {}_2^3He \rightarrow {}_2^4He + {}_1^1H + {}_1^1H$$

The energy liberated is carried primarily by gamma rays, positrons, and neutrinos, as can be seen from the reactions. The gamma rays are soon absorbed by the dense gas, thus, raising its temperature. The positrons combine with electrons to produce gamma rays, which in turn are also absorbed by the gas within a few centimeters. The neutrinos, however, almost never interact with matter; hence, they escape from the star, carrying about 2% of the generated energy with them. These energy-liberating fusion reactions are called thermonuclear fusion reactions. The hydrogen (fusion) bomb, first exploded in 1952, is an example of an uncontrolled thermonuclear fusion reaction.

**Note:**

- Some unstable light nuclei of A below 20, fuse together, the BE per nucleon increases and hence the excess energy is released. The easiest thermonuclear reaction that can be handled on earth is the fusion of two deuterons (D–D reaction) or fusion of a deuteron with a triton (D–T reaction).

$$_1H^2 + {}_1H^2 \rightarrow {}_2He^3 + n + 3.3 \text{ MeV (D–D)}$$

$$Q\text{-value} = [2(M_D - m_e) - \{(M_{He\text{-}3} - 2m_e) + m_n\}]c^2$$

$$= [2M_D - (M_{He\text{-}3} + m_n)]c^2$$

$$_1H^2 + {}_1H^2 \rightarrow {}_1H^3 + {}_1H^1 + 4.0 \text{ MeV (D–D)}$$

$$Q\text{-value} = [2(M_D - m_e) - \{(M_T - m_e) + (M_H - m_e)\}]c^2$$

$$= [2M_D - (M_T + M_H)]c^2$$

$$_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + n + 17.6 \text{ MeV (D–D)}$$

$$Q\text{-value} = [\{(M_D - m_e) + (M_T - m_e)\} - \{(M_{He\text{-}4} - 2m_e) + m_n\}]c^2$$

$$= [(M_D + M_T) - (M_{He\text{-}4} + m_n)]c^2$$

In case of fission and fusion, $\Delta m = \Delta m_{atom} = \Delta m_{nucleus}$.

- These reactions take place at ultra-high temperature ($\cong 10^7$–$10^9$). At high pressure, it can take place at low temperature also. For these reactions to take place, nuclei should be brought up to 1 fermi distance which requires very high kinetic energy.
- Energy released in fusion exceeds the energy liberated in the fission of heavy nuclei.

## FUSION REACTORS

The enormous amount of energy released in fusion reactions suggests the possibility of harnessing this energy for useful purposes on earth. A great deal of effort is focused on developing a sustained and controllable thermonuclear reactor—a fusion power reactor. Controlled fusion is often called the ultimate energy source because of the vast availability of its fuel source, i.e., water. An additional advantage of fusion reactors is that comparatively few radioactive by products are formed. As noted in Eq. (iii), the end product of the fusion of hydrogen nuclei is safe, nonradioactive helium. Unfortunately, a thermonuclear

reactor that can deliver a net power output over a reasonable time interval is not yet a reality, and many difficulties must be solved before a successful device is constructed.

We have seen that the Sun's energy is based, in part, on a set of reactions in which ordinary hydrogen is converted to helium. Unfortunately, the proton–proton interaction is not suitable for use in a fusion reactor because the event requires very high pressures and densities. The process works in the Sun only because of the extremely high density of protons in the Sun's interior. In fact, even at the densities and temperature that exist at the center of the Sun, the average proton takes 14 billion years to react!

The fusion reactions that appear most promising in the construction of a fusion power reactor involve deuterium and tritium, which are isotopes of hydrogen. These reactions are

$$^2_1H + ^2_1H \rightarrow ^3_2He + ^1_0n, \qquad Q = 3.27 \text{ MeV}$$

$$^2_1H + ^2_1H \rightarrow ^3_1H + ^1_1H, \qquad Q = 4.03 \text{ MeV}$$

$$^2_1H + ^3_1H \rightarrow ^4_2He + ^1_0n, \qquad Q = 17.59 \text{ MeV}$$

where the $Q$ values refer to the amount of energy released per reaction. As noted earlier, deuterium is available in almost unlimited quantities form our lakes and oceans and is very inexpensive to extract. Tritium, however, is radioactive ($T_{1/2}$ = 12.3 yr) and undergoes beta decay to $^3$He. For this reason, tritium does not occur naturally to any great extent and must be artificially produced.

One of the major problems in obtaining energy from nuclear fusion is the fact that the Coulomb repulsion force between two charged nuclei must be overcome before they can fuse. The fundamental challenge is to give the two nuclei enough kinetic energy to overcome this repulsive force. This can be accomplished by heating the fuel to extremely high temperatures (about $10^8$ K, far greater than the interior temperature of the Sun). As you might expect, such high temperatures are not easy to obtain in a laboratory or a power plant. At these high temperatures, the atoms are ionized and the system consists of a collection of electrons and nuclei, commonly referred to as a plasma.

### ILLUSTRATION 7.54

On disintegration of one atom of $^{235}$U, the amount of energy obtained is 200 MeV. The power obtained in a reactor is 1000 kilo watt. How many atoms are disintegrated per second in the reactor? What is the decay in mass per hour?

**Sol.** Power produced in the reactor is

$$P = 1000 \text{ kW} = 1000 \times 10^3 \text{ W} = 10^6 \text{ J s}^{-1}$$

$$= \frac{10^6}{1.6 \times 10^{-19}} \text{eV s}^{-1} = 6.25 \times 10^{18} \text{ MeV s}^{-1}$$

As in each disintegration 200 MeV energy is released, number of atoms disintegrated per second are

$$N = \frac{6.25 \times 10^{18}}{200} = 3.125 \times 10^{16} \text{ s}^{-1}$$

Energy released per second is $10^6$ J.

Energy released per hour = $10^6 \times 60 \times 60$ J

Thus, mass decay per hour can be given as

$$\Delta m = \frac{\Delta E}{c^2} \qquad \text{(Einstein's mass–energy formula)}$$

$$= \frac{10^6 \times 60 \times 60 \text{ J}}{(3 \times 10^8 \text{ m s}^{-1})^2} = 4 \times 10^{-8} \text{ kg} = 4 \times 10^{-5} \text{ g}$$

### ILLUSTRATION 7.55

A deuterium reaction that occurs in an experimental fusion reactor is in two stages:

(i) Two deuterium ($^2_1$D) nuclei fuse together to form a tritium nucleus, with a proton as a by product written as D(D, p)T.

(ii) A tritium nucleus fuses with another deuterium nucleus to form a helium $^4_2$He nucleus with neutron as a by-product, written as T (D, n) $^4_2$He.

Compute: **(a)** the energy released in each of the two stages, **(b)** the energy released in the combined reaction per deuterium. **(c)** What percentage of the mass energy of the initial deuterium is released. Given,

$^2_1$D = 2.014102 amu; $^3_1$T = 3.016049; $^4_2$He = 4.002603 amu; $^1_1$H = 1.007825 amu; $^1_0$n = 1.00665 amu

**Sol.**

**(a)** (i) The reaction involved in the process is

$$^2_1D + ^2_1D \rightarrow ^3_1T + ^1_1p + E_1$$

Mass defect of the above reaction is

$$\Delta m = [2(2.014102) - (3.016049 + 1.007825)]$$
$$= 0.00433 \text{ amu}$$

Energy released in the process is

$$E_1 = 0.00433 \times 931.2 \text{ MeV} = 4.033 \text{ MeV}$$

(ii) The reaction involved in the process is

$$^3_1T + ^2_1D \rightarrow ^4_2He + ^1_0n + E_2$$

Mass defect of above reaction is

$$\Delta m = [(3.016049 + 2.014102) - (4.002603 + 1.008665)]$$
$$= 0.01888 \text{ amu}$$

Energy released in the process is

$$E_2 = 0.01888 \times 931 \text{ MeV} = 17.585 \text{ MeV}$$

Total energy released in both processes is

$$17.585 + 4.033 = 21.618 \text{ MeV}$$

**(b)** Energy released per deuterium atom is

$$E_1 = \frac{21.618}{3} = 7.206 \text{ MeV}$$

**(c)** % of rest mass of $^2_1$D released

$$f = \frac{7.206}{2.014102 \times 931} = 0.385\%$$

### ILLUSTRATION 7.56

In the process of nuclear fission of 1 g uranium, the mass lost is 0.92 mg. The efficiency of power house run by the fission reactor is 10%. To obtain 400 megawatt power from the power house, how much uranium will be required per hour? ($c = 3 \times 10^8$ m s$^{-1}$)

**Sol.** Power to be obtained from power house = 400 megawatt
In this case, energy obtained per hour

$= 400 \text{ megawatt} \times 1 \text{ hour}$

$= (400 \times 10^6 \text{ W}) \times 3600 \text{ s}$

$= 144 \times 10^{10} \text{ J}$

Here, only 10% of output is utilized. In order to obtain $144 \times 10^{10}$ joule of useful energy, the output energy from the power house is given by

$$E = \frac{(144 \times 10^{10}) \times 100}{10} = 144 \times 10^{11} \text{ J}$$

Let this energy is obtained from a mass loss of $\Delta m$ kg. Then,

$$(\Delta m)c^2 = 144 \times 10^{11} \text{ J}$$

or $$\Delta m = \frac{144 \times 10^{11}}{(3 \times 10^8)^2} = 16 \times 10^{-5} \text{ kg} = 0.16 \text{ g}$$

Since 0.92 mg ($= 0.92 \times 10^{-3}$ g) mass is lost in 1 g uranium, hence, for a mass loss of 0.16 g, the uranium required is given by

$$\Delta m' = \frac{1 \times 0.16}{0.92 \times 10^{-3}} = 174 \text{ g}$$

Thus, to run the power house, 174 g uranium is required per hour.

### ILLUSTRATION 7.57

A nuclear reactor using $^{235}U$ generates 250 MW of electrical power. The efficiency of the reactor (i.e., efficiency of conversion of thermal energy into electrical energy) is 25%. What is the amount of $^{235}U$ used in the reactor per year? The thermal energy released per fission of $^{235}U$ is 200 MeV.

**Sol.** Rate of electrical energy generation is 250 MW = $250 \times 10^6$ W (or J $s^{-1}$). So, electrical energy generation is 250 MW = $250 \times 10^6$ W (or J $s^{-1}$). Therefore, electrical energy generated in 1 year is

$$(250 \times 10^6 \text{ J s}^{-1}) \times (365 \times 24 \times 60 \times 60 \text{ s}) = 7.884 \times 10^{15} \text{ J}$$

Thermal energy from fission of one $^{235}U$ nucleus is 200 MeV $= 200 \times 1.6 \times 10^{-13} = 3.2 \times 10^{-11}$ J. Since the efficiency is 25%, the electrical energy obtained from the fission of one $^{235}U$ nucleus is

$$E_1 = 3.2 \times 10^{-11} \times \frac{25}{100} = 8.0 \times 10^{-12} \text{ J}$$

Therefore, the number of fissions of $^{235}U$ required in one year will be

$$N = \frac{7.884 \times 10^{15}}{8.0 \times 10^{-12}} = 9.855 \times 10^{26}$$

Number of moles of $^{235}U$ required per year is

$$n = \frac{9.855 \times 10^{26}}{6.023 \times 10^{23}} = 1.636 \times 10^3$$

Therefore, mass of $^{235}U$ required per year is

$$m = 1.636 \times 10^3 \times 235 = 3.844 \times 10^5 \text{ g} = 384.4 \text{ kg}$$

### ILLUSTRATION 7.58

What is the power output of a $_{92}U^{235}$ reactor if it takes 30 days to use up 2 kg of fuel, and if each fission gives 185 MeV of usable energy?

**Sol.** 235 amu of uranium gives 185 MeV energy. Therefore, the energy given by 1 amu of $_{92}U^{235}$

$$= \frac{185}{2.35} \text{ MeV} = \frac{185}{235} \times 1.6 \times 10^{-13} \text{ J}$$

But 1 amu $= 1.66 \times 10^{-27}$ kg. Therefore, energy released by $1.66 \times 10^{-27}$ kg of $_{92}U^{235}$

$$= \frac{185 \times 1.6 \times 10^{-13}}{235}$$

Hence, energy released by 2 kg of $_{92}U^{235}$,

$$W = \frac{185 \times 1.6 \times 10^{-13} \times 2}{235 \times 1.66 \times 10^{-27}} = 1.517 \times 10^{14} \text{ J}$$

Therefore, power output of reactor $= \frac{W}{t} = \frac{1.517 \times 10^{14}}{(30 \text{ days})}$

$$P = \frac{1.517 \times 10^{14}}{30 \times 24 \times 60 \times 60} = 5.85 \times 10^7 \text{ W}$$

### ILLUSTRATION 7.59

**(a)** A nuclear explosion is designed to deliver 1 MW of heat energy, how many fission events must be required in a second to attain this power level. **(b)** If this explosion is designed with a nuclear fuel consisting of uranium 235 to run a reactor at this power level for one year, then calculate the amount of fuel needed. You can assume that the amount of energy released per fission event is 200 MeV.

**Sol.**

**(a)** Energy per fission, $\varepsilon = 200$ MeV

$= 200 \times 1.6 \times 10^{-13} \text{ J} = 3.2 \times 10^{-11} \text{ J}$

Power delivered, $P = 1 \text{ MW} = 10^6 \text{ J s}^{-1}$

If $t$ is the time, then energy delivered $= Pt$

$\therefore$ Number of fissions $= \frac{Pt}{\varepsilon} = \frac{10^6 \times t}{3.2 \times 10^{-11}}$

If $t = 1$ s, number of fissions per second is

$$\frac{10^6}{3.2 \times 10^{-11}} = 3.125 \times 10^{16}$$

**(b)** Total energy required for 1 year $= Pt$

$= 10^6 \times 3.15 \times 10^7 \text{ J} = 3.15 \times 10^{13} \text{ J}$

Hence, number of fissions required is

$$\frac{3.15 \times 10^{13}}{3.2 \times 10^{-11}} = 9.85 \times 10^{23}$$

Mass of uranium required is

$$9.85 \times 10^{23} \times \frac{235}{6.02 \times 10^{23}} \text{ g} = 384.5 \text{ g}$$

## CONCEPT APPLICATION EXERCISE 7.4

1. If 200 MeV energy is released in the fission of a single nucleus $_{92}^{235}U$, how many fissions must occur per second to produce a power of 1 kW?
2. It is proposed to use nuclear fission reaction:

$$_1^2H + _1^2H \longrightarrow _2^4He$$

in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with a 25% efficiency in the reactor, how many grams of deuterium will be needed per day? The masses of $_1^2H$ and $_2^4He$ are 2.0141 u and 4.0026 u respectively.

3. Fission of one atom of $_{92}^{235}U$ releases 200 MeV of energy. What mass of $_{92}^{235}U$ would be released in a reactor to supply one million kW power per year?
4. What is the power output of a $_{92}^{235}U$ reactor if it takes 30 days to use up 2 kg of fuel and if each fission gives 185 MeV of usable energy?
5. Assuming that in a star, three alpha particles join in a single reaction to form $_6^{12}C$ nucleus, calculate the energy released in this reaction. Given mass of $_2^4He = 4.002604$ u and that of $_6^{12}C = 12.000000$ u.
6. In a nuclear reactor, fission is produced in 1 g for $U^{235}$ (235.0439 u) in 24 hours by slow neutrons (1.0087 u). Assume that $_{35}Kr^{92}$ (91.8973 u) and $_{56}Ba^{141}$ (140.9139 amu) are produced in all reactions and no energy is lost.
   (i) Write the complete reaction **(ii)** Calculate the total energy produced in kilowatt hour. Given 1u = 931 MeV.
7. It is proposed to use the nuclear fusion reaction : $_1H^2 + _1H^2 \rightarrow _2He^4$ in a nuclear reactor of 200 MW rating. If the energy from above reaction is used with at 25% efficiency in the reactor, how many grams of deuterium will be needed per day. (Mass of $_1H^2$ is 2.0141 u and mass of $_2He^4$ is 4.0026 u)
8. How long can an electric lamp of 100 W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$_1^2H + _1^2H \rightarrow _2^3He + n + 3.27\,MeV$$

9. Estimate the minimum amount of $_{92}^{235}U$ that needs to undergo fission in order to run a 1000 MW power reactor per year of continuous operation. Assume an efficiency of about 33 percent.
10. The $_{88}^{226}Ra$ nucleus undergoes $\alpha$-decay to $_{88}^{226}Rn$. Calculate the amount of energy liberated in this decay. Take the mass of $_{88}^{226}Ra$ to be 226.025 402 u, that of $_{88}^{226}Rn$ to be 222.017571 u, and that of $_2^4He$ to be 4.002602 u.
11. Calculate the energy released when three alpha particles combine to form a $^{12}C$ nucleus. The atomic mass of $_2^4He$ is 4.002603 u.

### ANSWERS

**1.** $3.125 \times 10^{13}$ **2.** 120.3 g **3.** 384.5 kg
**4.** 58.54 MW **5.** 7.277 MeV
**6.** (i) $_{92}U^{235} + _0n^1 \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3_0n^1$
(ii) $2.28 \times 10^4$ kWh
**7.** 120. 83 g/day **8.** $5 \times 10^4$ yr
**9.** 10 tons **10.** 4.87 MeV **11.** 7.27 MeV

## Solved Examples

### EXAMPLE 7.1

A radionuclide with disintegration constant $\lambda$ is produced in a reactor at a constant rate $\alpha$ ($= 2\lambda$) nuclei per second. At $t = 0$, there are no nuclei present in the reactor. During each decay, energy $E_0$ is released. 20% of this energy is utilized in increasing the temperature of water. Find the increase in temperature of mass $m$ of water in time $t = T_{1/2}$ if specific heat of water is $s$. Assume that there is no loss of energy through water surface.

(Given $E_0$ = 100 times the energy is required to raise the temperature of mass $m$ of water from 0°C to 100°C)

**Sol.** Rate of formation of nuclei at time $t$ is $\frac{dN}{dt} = \alpha - \lambda N$

where $N$ = number of nuclei at that time

$$N = \frac{\alpha}{\lambda}(1 - e^{-\lambda t})$$

Number of nuclei disintegrated in time $t$,

$$N_d = \alpha t - -(1 - e^{-\lambda t})$$

Therefore, energy released till time $t$,

$$E = E_0 N_d = E_0\left[\alpha t - \frac{\alpha}{\lambda}(1 - e^{-\lambda t})\right]$$

20% of this energy is used for heating water.

Hence, $$0.2E_0\left[\alpha t - \frac{\alpha}{\lambda}(1 - e^{-\lambda t})\right] = ms\Delta\theta$$

$$\Rightarrow \quad \Delta\theta = \frac{0.2\,E_0\left[\alpha t - \frac{\alpha}{\lambda}(1 - e^{-\lambda t})\right]}{ms}$$

At $t = T_{1/2}$,

$$\Delta\theta = \frac{0.2\,E_0\left[2\lambda T_{1/2} - \frac{2\lambda}{\lambda}\left(1 - \frac{1}{2}\right)\right]}{ms} = \frac{0.2\,E_0[0.386]}{ms} = 772\ K$$

### EXAMPLE 7.2

Find the $Q$ value of the reaction $N^{14} + \alpha \rightarrow O^{17} + p$. The masses of $N^{14}$, $He^4$, p, and $O^{17}$ are, respectively, 14.00307 u, 4.00260 u, 1.00783 u, and 16.99913 u. Find the total kinetic energy of the products if the striking $\alpha$-particle has the minimum kinetic energy required to initiate the reaction.

**Sol.** $N^{14} + \alpha \rightarrow O^{17} + \text{proton}$

$Q$ value $= (14.00307 + 4.00260 - 1.00783 - 16.99913)\,931.5$

$= -1.20$ MeV

Let $m$ and $M$ be the mass of $\alpha$-particle and nitrogen nucleus, respectively, and let minimum KE of $\alpha$-particle be $\frac{1}{2}mu^2$.

From energy equation, $\frac{1}{2}mu^2 = |Q| + \text{minimum KE of system}$

$$\therefore \quad \frac{1}{2}mu^2 = |Q| + \frac{1}{2}(m+M)\left[\frac{mu}{m+M}\right]^2$$

$$\frac{1}{2}mu^2\left(\frac{M}{m+M}\right) = |Q|$$

$$\frac{1}{2}mu^2 = |Q|\left(\frac{m+M}{M}\right)$$

$$\text{KE of products} = \frac{1}{2}(m+M)\left[\frac{mu}{m+M}\right]^2 = \frac{1}{2}mu^2\left(\frac{m}{m+M}\right)$$

$$= |Q|\frac{m}{M} = 1.2 \times \frac{4}{14} = 0.34 \text{ MeV}$$

## EXAMPLE 7.3

The nucleus $^{23}$Ne decays by $\beta$-emission into the nucleus $^{23}$Na. Write down the $\beta$-decay equation and determine the maximum kinetic energy of the electrons emitted. Given, $m(^{23}_{10}\text{Ne}) = 22.994466$ amu and $m(^{23}_{11}\text{Na}) = 22.989770$ amu. Ignore the mass of antineutrino $(\bar{\nu})$.

**Sol.** The $\beta$-decay of $^{23}$Ne is represented by the equation

$$^{23}_{10}\text{Ne} \rightarrow {}^{23}_{11}\text{Na} + e^- + \bar{\nu} + Q$$

Mass defect for the above reaction is

$$\Delta m = \left[(^{23}_{10}\text{Ne}) - M(^{23}_{11}\text{Na})\right]$$

$$= [22.994466 - 22.989770] \text{ amu} = 0.004696 \text{ amu}$$

Now, $Q$ value of the above reaction is given as

$$Q = \Delta m \times 931.5 \text{ MeV} = 4.374 \text{ MeV}$$

The energy $Q$ released is shared by $^{23}$Na nucleus and the electron–antineutrino pair. Since $^{23}$Na is massive, most of the kinetic energy is carried by the e–$\bar{\nu}$ pair. The electron will carry the maximum energy if the antineutrino carries zero energy. Thus, the maximum energy of the electrons emitted is 4.374 MeV.

## EXAMPLE 7.4

A radioactive source in the form of a metal sphere of diameter $10^{-3}$ m emits $\beta$-particles at a constant rate of $6.25 \times 10^{10}$ particles per second. If the source is electrically insulated, how long will it take for its potential to rise by 1.0 V, assuming that 80% of the emitted $\beta$-particles escape the source?

**Sol.** Let $t$ be the time for potential of metal sphere to rise by 1.0 V. Then, up to this time $\beta$-particles emitted from sphere are

$$N = (6.25 \times 10^{10}) \times t$$

Number of $\beta$-particles escaped in this time are

$$N_e = (80/100) \times (6.25 \times 10^{10})t = 5 \times 10^{10}\, t$$

Thus, charge acquired by the sphere in $t$ s

$$Q = (5 \times 10^{10}t) \times (1.6 \times 10^{-19}) = 8 \times 10^{-9}t \text{ coulomb}$$

(emission of $\beta$-particle leads to a charge $e$ on the metal sphere)

The capacitance $C$ of a metal sphere is given by

$$C = 4\pi\varepsilon_0 \times r = \left(\frac{1}{9 \times 10^9}\right) \times \left(\frac{10^{-3}}{2}\right) = \frac{10^{-12}}{18} \text{ farad}$$

We know that $Q = C \times V$ (here, $V = 1$ volt)

or $$(8 \times 10^{-9})t = \left(\frac{10^{-12}}{18}\right) \times 1$$

Solving it for $t$, we get $t = 6.95$ μs.

## EXAMPLE 7.5

Show that $^{55}_{26}$Fe may electron capture, but not $\beta^+$ decay. Masses given are $M(^{55}_{26}\text{Fe}) = 54.938298$ amu, $M(^{55}_{25}\text{Mn}) = 54.938050$ amu, $m_e = 0.000549$ amu.

**Sol.** The two possible reactions for electron capture and $\beta^+$ decay by $^{55}_{26}$Fe are

Electron capture: $^{55}_{26}\text{Fe} + e \rightarrow {}^{55}_{25}\text{Mn} + \nu$ ...(i)

$\beta^+$ decay: $^{55}_{26}\text{Fe} \rightarrow {}^{55}_{25}\text{Mn} + \beta^+ + \nu$ ...(ii)

First, we will determine the disintegration energy $Q$ of Eqs. (i) and (ii).

$M(^{55}_{26}\text{Fe}) = 54.938298$ amu

$M(^{55}_{25}\text{Mn}) = 54.938050$ amu and $m_e = 0.000549$ amu

For $\beta^+$ decay, the $Q$ value of the reaction is

$$Q = [54.938298 - 54.938050 - 2 \times 0.000549] \times 931.5 \text{ MeV}$$

$$= -0.79 \text{ MeV}$$

Negative value of $Q$ implies that positron decay is not possible spontaneously. For electron capture, the $Q$ value of reaction is

$$Q = [54.938298 - 54.938050] \times 931.5 = 0.23 \text{ MeV}$$

Positive value of $Q$ implies that electron capture is possible in this case.

## EXAMPLE 7.6

There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutron is 700 s, what fraction of neutrons will decay before they travel a distance of 10 km.

**Sol.** Given that kinetic energy of neutrons is

$$\frac{1}{2}mv^2 = 0.0327 \times (1.6 \times 10^{-19}) \text{ J}$$

or $$v^2 = \frac{2 \times 0.0327 \times (1.6 \times 10^{-19})}{1.675 \times 10^{-27}} = 625 \times 10^4$$

or $$v = 2500 \text{ m s}^{-1}$$

Time to travel a distance of 10 km is $\frac{10^4 \text{ m}}{2500 \text{ m s}^{-1}} = 4$ s

After 4 s, number of neutrons left can be given as $N = N_0 2^{-n}$

where $n = \frac{t}{T}$ = no. of half-lives. Here, $n = \frac{4}{700} = \frac{1}{175}$

$$\frac{N}{N_0} = 2^{-1/175} = 0.996 \quad \text{or} \quad N = 0.996N_0$$

Thus, fraction of neutrons decayed is

$$f = \frac{N_0 - N}{N_0} = \frac{0.004N_0}{N_0} = 0.004$$

## EXAMPLE 7.7

Nuclei of a radioactive element A are being produced at a constant rate $\alpha$. The element has a decay constant $\lambda$. At time $t = 0$, there are $N_0$ nuclei of the element.

(a) Calculate the number $N$ of nuclei of A at time $t$.

(b) If $\alpha = 2N_0\lambda$, calculate the number of nuclei of A after one half-life of A, and also the limiting value of $N$ as $t \to \infty$.

**Sol.**

(a) At $t = 0$, $N = N_0$

Rate of decay $= -\lambda N$, and rate of formation $= \alpha$. Thus, accumulation rate of element is

$$\frac{dN}{dt} = \alpha - \lambda N$$

or $$\frac{dN}{\alpha - \lambda N} = dt$$

Integrating this expression, we get $\log_e\left[\frac{\alpha - \lambda N}{\alpha - \lambda N_0}\right] = -\lambda t$

or $$\frac{\alpha - \lambda N}{\alpha - \lambda N_0} = e^{-\lambda t}$$

or $$\alpha - \lambda N = (\alpha - \lambda N_0)e^{-\lambda t}$$

or $$N = \frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}]$$

(b) If $\alpha = 2N_0\lambda$,

$$N = \frac{1}{\lambda}[2N_0\lambda - (2N_0\lambda - N_0\lambda)e^{-\lambda t}] = N_0(2 - e^{-\lambda t})$$

At the time of half-life, $T = 0.693/\lambda$. So,

$$N = N_0[2 - e^{0.693}] - \frac{3N_0}{2}$$

Limiting value of $N$ (as $t \to \infty$) is $N = N_0[2 - e^{-\infty}] = 2N_0$

## EXAMPLE 7.8

Calculate the ground state $Q$ value of the induced fission reaction in the equation

$$n + {}^{235}_{92}\text{U} \to {}^{236}_{92}\text{U}^* \to {}^{99}_{40}\text{Zr} + {}^{134}_{52}\text{Te} + 3n$$

if the neutron is thermal. A thermal neutron is in thermal equilibrium with its environment; it has an average kinetic energy given by (3/2) $kT$. Given:

$m(\text{n}) = 1.0087$ amu, $M(^{235}\text{U}) = 235.0439$ amu,

$M(^{99}\text{Zr}) = 98.916$ amu, $M(^{134}\text{Te}) = 133.9115$ amu

**Sol.** Kinetic energy of a thermal neutron can be neglected; even for a temperature of $10^6$ K, the thermal energy is only 130 eV. The $Q$ value of the above reaction is given by the equation

$$Q = \Delta m \times 931.2 \text{ MeV}$$

Here, $\Delta m$ is the mass defect of the reaction, given by

$$\Delta m = [M(^{235}\text{U}) + m(\text{n})] - [M(^{99}\text{Zr}) + M(^{134}\text{Te}) + 3m(\text{n})]$$
$$= [235.0439 - 98.9165 - 133.9115 - 2(1.0087)] \text{ amu}$$
$$= 0.1985 \text{ amu}$$

Thus, energy released is $Q = 0.1985 \times 931.2$. Here, $\Delta m$ is the mass defect of the reaction, given by

$$\Delta m = [M(^{235}\text{U}) + m(\text{n})] - [M(^{99}\text{Zr}) + M(^{134}\text{Te}) + 3m(\text{n})]$$

Thus, energy released is $Q = 0.1985 \times 931.2 = 184.84$ MeV

Even with a thermal neutron of negligible kinetic energy, a tremendous amount of energy is released.

## EXAMPLE 7.9

The nuclear reaction $n + {}^{10}_{5}\text{B} \to {}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$ is observed to occur even when very slow-moving neutrons ($M_n = 1.0087$ amu) strike a boron atom at rest. For a particular reaction in which $K_n = 0$, the helium ($M_{He} = 4.0026$ amu) is observed to have a speed of $9.30 \times 10^6$ m s$^{-1}$. Determine **(a)** the kinetic energy of the lithium ($M_{Li} = 7.0160$ amu) and **(b)** the $Q$ value of the reaction.

**Sol.** Since the neutron and boron are both initially at rest, the total momentum before the reaction is zero and afterward also it is zero. Therefore, $M_{Li}v_{Li} = M_{He}v_{He}$.

**(a)** We solve this for $v_{Li}$ and substitute it into the equation for kinetic energy. We can use classical kinetic energy with little error, rather than relativistic formulas, because $v_{He} = 9.30 \times 10^6$ m s$^{-1}$ is not close to the speed of light $c$ and $v_{Li}$ will be even less since $M_{Li} > M_{He}$. Thus, we can write

$$K_{Li} = \frac{1}{2}M_{Li}v_{Li}^2 = \frac{1}{2}M_{Li}\left(\frac{M_{He}v_{He}}{M_{Li}}\right)^2 = \frac{M_{He}^2v_{He}^2}{2M_{Li}}$$

We put in numbers, changing the mass in u to kg and recalling that $1.60 \times 10^{-13}$ J = 1 MeV

$$K_{Li} = \frac{(4.0026)^2(1.66 \times 10^{-27})(9.30 \times 10^6)^2}{2(7.0160)(1.66 \times 10^{-27})}$$
$$= 1.64 \times 10^{-13} \text{ J} = 1.02 \text{ MeV}$$

**(b)** We are given the data $K_\alpha = K_X = 0$, so $Q = K_{Li} + K_{He}$, where

$$K_{He} = \frac{1}{2}M_{He}v_{He}^2$$
$$= \frac{1}{2}(4.0026)(1.66 \times 10^{-27})(9.30 \times 10^6)^2$$
$$= 2.87 \times 10^{-13} \text{ J} = 1.80 \text{ MeV}$$

Hence, $Q = 1.02$ MeV + 1.80 MeV = 2.82 MeV

## EXAMPLE 7.10

A count rate meter is used to measure the activity of a given sample. At one instant, the meter shows 4750 counts per minute. Five minutes later, it shows 2700 counts per minute. Find **(a)** the decay constant and **(b)** the half-life of the sample. ($\log_{10} 1.760 = 0.2455$)

**Sol.** We have, $N = N_0e^{-\lambda t}$ ...(i)

The activity of the sample is given by

$$\frac{dN}{dt} = -\lambda N_0e^{-\lambda t} = -\lambda N$$ ...(ii)

i.e., activity is proportional to the number of undecayed nuclei.

At $t = 0$, $\left(\frac{dN}{dt}\right)_{t=0} = -\lambda N$

At $t = 5$ min, $\left(\frac{dN}{dt}\right)_{t=5\,\text{min}} = -\lambda N$

$$\therefore \quad \frac{N_0}{N} = \frac{\left(\frac{dN}{dt}\right)_{t=0}}{\left(\frac{dN}{dt}\right)_{t=5\,\text{min}}} = \frac{4750}{2700} = 1.760$$

**(a)** From Eq. (i), we have $\frac{N}{N_0} = e^{-\lambda t}$

or $\frac{N_0}{N} = e^{\lambda t}$ or $\log_e \frac{N_0}{N} = \lambda t$

$$\Rightarrow \quad \lambda = \frac{1}{t}\log_e \frac{N_0}{N} = \frac{2.3026}{t} \times \log_{10}\left(\frac{N_0}{N}\right) \qquad \text{...(iii)}$$

Substituting the given values, we have

$$\lambda = \frac{2.3026}{5} \times 0.2455 = 0.113 \text{ min}^{-1}$$

**(b)** Half-life of sample, $T = \frac{0.6931}{\lambda} = \frac{0.6931}{0.113} = 6.1$ min

## EXAMPLE 7.11

There is a stream of neutrons with a kinetic energy of 0.0327 eV. If the half-life of neutrons is 700 s, what fraction of neutrons will decay before they travel a distance of 10 m? Given mass of neutron = $1.676 \times 10^{-27}$ kg.

**Sol.** The velocity $v$ of neutron is given by $\frac{1}{2}mv^2 = 0.0327$ eV

or $\frac{1}{2} \times 1.675 \times 10^{-27} v^2 = 0.0327 \times 1.6 \times 10^{-19}$ J

$$\therefore \quad v^2 = \frac{2 \times 0.0327 \times 1.6 \times 10^{-19}}{1.675 \times 10^{-27}}$$

or $v = 2.5 \times 10^3$ m s$^{-1}$

Time taken by neutron to travel a distance of 10 m is

$$t = \frac{\text{disance}}{\text{velocity}} = \frac{10}{2.5 \times 10^3} = 4 \times 10^{-3} \text{ s}$$

and half-life $T = 700$ s.

The fraction of neutrons decayed in time $t$ is

$$\frac{N}{N_0} = e^{-\lambda t} = \left(\frac{1}{2}\right)^{t/T} = \left(\frac{1}{2}\right)^{4\times10^{-3}/700} = \left(\frac{1}{2}\right)^{5.7 \times 10^{-6}}$$

$$= 0.999952$$

Fraction of neutrons decayed is

$$1 - 0.999952 = 0.000048 = 4.8 \times 10^{-5}.$$

## EXAMPLE 7.12

Some amount of a radioactive substance (half-life = 10 days) is spread inside a room and consequently the level of radiation becomes 50 times the permissible level for normal occupancy of the room. After how many days will the room be safe for occupation?

**Sol.** We have, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$

Given, half-life $T = 10$ days.

$$\frac{N}{N_0} = \frac{1}{50} \quad \Rightarrow \quad \frac{1}{50} = \left(\frac{1}{2}\right)^{t/T}$$

Taking log, $\log 1 - \log 50 = \frac{t}{T}(\log 1 - \log 2)$

$$\Rightarrow \quad 0 - \log \frac{100}{2} = \frac{t}{T}(0 - \log 2)$$

$$\Rightarrow \quad \log 100 - \log 2 = \frac{t}{T}(0 - \log 2)$$

or $\log 100 - \log 2 = \frac{t}{T}\log 2$

As $\log_{10} 2 = 0.3010$, we have $2 - 0.3010 = \frac{t}{T} \times 0.3010$

or $t = \frac{1.6990}{0.3010}T = \frac{1699}{301} \times (10 \text{ days}) = 56.45$ days

## EXAMPLE 7.13

A small quantity of solution containing $^{24}$Na radionuclide (half-life 15 h) of activity 1.0 microcurie is injected into the blood of a person. A sample of the blood of volume 1 cm$^3$ taken after 5 h shows an activity of 296 disintegrations min$^{-1}$. Determine the total volume of blood in the body of the person. Assume that the radioactivity solution mixes uniformly in the blood of the person. (1 curie = $3.7 \times 10^{10}$ disintegrations s$^{-1}$.)

**Sol.** Initial activity of $^{24}$Na,

$$A = \frac{dN}{dt} = 1.0\ \mu\text{Ci} = 1.0 \times 10^{-6} \times 3.7 \times 10^{10}$$

$$= 3.7 \times 10^4 \text{ disintegrations s}^{-1}$$

Half-life, $T = 15$ h $= 15 \times 3600$ s

Initial activity, $A = \frac{dN}{dt} = \lambda N_0$

$$3.7 \times 10^4 = \frac{0.693}{15 \times 3600} N_0$$

$$\therefore \quad N_0 = \frac{3.7 \times 10^4 \times 15 \times 3600}{0.693} = 2.883 \times 10^9$$

Let the number of radioactive nuclei present after 5 h be $N'$ in 1 cm$^3$ of sample of blood. Then,

$$\frac{dN}{dt} = \lambda N' \quad \Rightarrow \quad \frac{296}{60} = \frac{0.693}{15 \times 3600} N'$$

$$\therefore \quad N' = \frac{296 \times 15 \times 3600}{60 \times 0.693} = 3.844 \times 10^5$$

If $N_0'$ is initial number of radioactive nuclei in 1 cm$^3$ of sample, then

$$\frac{N'}{N_0'} = \left(\frac{1}{2}\right)^{t/T}$$

$$N_0' = (2)^{t/T} N' = (2)^{5/15} N' = (2)^{1/3} \times 3.844 \times 10^5$$

$\therefore y = (2)^{1/3}$.

$\log y = \frac{1}{3}\log 2 = \frac{1}{3} \times 0.3010 = 0.1003$

$\therefore y = \text{Antilog } 0.1003 = 1.2598$

$N_0' = 1.2598 \times 3.844 \times 10^5 = 4.843 \times 10^5$

Volume of blood $= \frac{N_0}{N_0'} = \frac{2.883 \times 10^9}{4.843 \times 10^5} = 0.595 \times 10^4 \text{ cm}^3 = 5.95 \text{ L}$

## EXAMPLE 7.14

In an ore containing uranium, the ratio of $U^{238}$ to $Pb^{206}$ is 3. Calculate the age of the ore, assuming that all the lead present in the ore is the final stable product of $U^{238}$. Take the half-life of $U^{238}$ to be $4.5 \times 10^9$ years.

**Sol.** Given, $\frac{M_U}{M_{Pb}} = 3$

Let the initial mass of uranium be $M_0$.

Final mass of uranium after time $t$, $M = \frac{3}{4}M_0$

According to law of disintegration,

$$\frac{M}{M_0} = \left(\frac{1}{2}\right)^{t/T} \Rightarrow \frac{M_0}{M} = (2)^{t/T}$$

$$\log_{10}\frac{M_0}{M} = \frac{t}{T}\log_{10}2$$

$$t = T\frac{\log_{10}\frac{M_0}{M}}{\log_{10}2} = \frac{4\log_{10}\frac{4}{3}}{\log_{10}2} = T\frac{\log_{10}1.3333}{\log_{10}2}$$

$$= 4.5 \times 10^9\left(\frac{0.1249}{0.3010}\right) = 1.867 \times 10^9 \text{ years}$$

## EXAMPLE 7.15

The element curium $^{218}_{96}Cm$ has a mean-life of $10^{13}$ s. Its primary decay modes are spontaneous fission and $\alpha$-decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV energy. The masses involved in $\alpha$-decay are as follows: $^{218}_{98}Cm$ = 248.07220 u, $^{244}_{94}Pu$ = 244.064100 u, and $^{4}_{2}He$ = 4.002603 u.

Calculate the power output from a sample of $10^{20}$ Cm atoms. (1 u = 931 MeV $c^{-2}$)

**Sol.** Activity (or rate) of fission,

$$A = \frac{dN}{dt} = \lambda N = \frac{1}{\tau}N = \frac{10^{20}}{10^{13}} = 10^7$$

As probability of fission is 8% only, therefore, actual rate of fission

$$= \frac{8}{100} \times 10^7 = 8 \times 10^5 \text{ s}^{-1}$$

Energy released per fission = 200 MeV

Power output of fission $= 8 \times 10^5 \times 200 \text{ MeV s}^{-1}$

$= 16 \times 10^7 \text{ MeV s}^{-1}$

Probability of $\alpha$-particle decay is 92%. Therefore, rate of decay for $\alpha$-particle is

$$\frac{92}{100} \times 10^7 = 92 \times 10^5 \text{ s}^{-1}$$

For $\alpha$-decay, the equation is $^{248}_{96}Cm \rightarrow {}^{244}_{94}Pu + {}^{4}_{2}He$

(248.072220) (244.064100) (4.002603)

Mass of decay products is 244.064100 + 4.002603 = 248.06673.

Mass defect, $\Delta m$ = 248.072220 − 248.066703 = 0.005517 u

Energy released per $\alpha$-decay is

$92 \times 10^5 \times 5.1363 \text{ MeV s}^{-1} = 4.725 \times 10^7 \text{ MeV s}^{-1}$

Total power output $= 16 \times 10^7 + 4.725 \times 10^7$

$= 20.725 \times 10^7 \text{ MeV s}^{-1}$

$= 20.725 \times 10^7 \times 1.6 \times 10^{-13} \text{ J s}^{-1}$

$= 33.16 \times 10^{-6} \text{ W} = 33.16 \ \mu\text{W}$

## EXAMPLE 7.16

In a nuclear reactor, $^{235}U$ undergoes fission liberating 200 MeV of energy. The reactor has a 10% efficiency and produces 1000 MW power. If the reactor is to function for 10 years, find the total mass of uranium required.

**Sol.** Total energy produced by the reactor in time $t$ = 10 years,

$E = 1000 \times 10^6 \times 10 \times 3.15 \times 10^7 \text{ J} = 3.15 \times 10^{17} \text{ J}$

$$\text{Efficiency} = \frac{\text{output energy}}{\text{input energy}}$$

$$\Rightarrow \text{Input energy caused by fission} = \frac{\text{output energy}}{\text{efficiency}}$$

$$= \frac{3.15 \times 10^{17}}{(10/100)} = 3.15 \times 10^{18} \text{ J}$$

Energy produced by 1 fission of $^{235}U$ = 200 MeV

$= 200 \times 1.6 \times 10^{-13} \text{ J}$

$= 3.2 \times 10^{-11} \text{ J}$

$$\text{Therefore, Number of fissions required} = \frac{\text{Total energy}}{\text{Energy per fission}}$$

$$= \frac{3.15 \times 10^{18}}{3.2 \times 10^{-11}}$$

$$\approx 9.8 \times 10^{28}$$

Hence, mass of uranium required is given by

$$m = \frac{N}{N_a} \times 235 \text{ kg} = \frac{9.8 \times 10^{28}}{6.02 \times 10^{26}} = 38.2 \times 10^3 \text{ kg}$$

## EXAMPLE 7.17

A nucleus at rest undergoes $\alpha$-decay emitting an $\alpha$-particle of de Broglie wavelength $\lambda = 5.76 \times 10^{-15}$ m. If the mass of the daughter nucleus is 223.610 amu and that of the $\alpha$-particle is 4.002 amu, determine the total kinetic energy in the final state. Hence, obtain the mass of parent nucleus in amu (1 amu = 931.470 MeV $c^{-2}$)

**Sol.** de Broglie wavelength of $\alpha$-particle, $\lambda_\alpha = 5.76 \times 10^{-15}$ m

de Broglie wavelength, $\lambda = \frac{h}{p}$

Therefore, momentum of $\alpha$-particle,

$$p_\alpha = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{5.76\times10^{-15}} = 1.15\times10^{-19}\ \text{kg m s}^{-1}$$

If $p_d$ is momentum of daughter nucleus, then from conservation of linear momentum,

$$p_d = -p_\alpha = -1.15\times10^{-19}\ \text{kg m s}^{-1}$$

Hence, total kinetic energy of final state,

$$E = E_\alpha + E_d = \frac{p_\alpha^2}{2m_\alpha} + \frac{p_d^2}{2m_d}$$

$$= \frac{p_\alpha^2}{2}\left(\frac{1}{m_\alpha}+\frac{1}{m_d}\right) = \frac{p_\alpha^2(m_d+m_\alpha)}{2m_\alpha m_d}$$

$$1\ \text{amu} = 931.470\ \text{MeV/c}^2 = \frac{931.470\times1.6\times10^{-13}}{(3\times10^8)^2}\ \text{kg}$$

$$= 1.66\times10^{-27}\ \text{kg}$$

Therefore, total kinetic energy of final state

$$= \frac{(1.15\times10^{-19})^2\times(223.610+4.002)\ \text{amu}}{2\times(4.002\ \text{amu})\times(223.610\ \text{amu})}$$

$$= \frac{(1.15\times10^{-19})^2\times227.612}{2\times4.002\times223.610\times1.66\times10^{-27}}\ \text{J}$$

$$= \frac{(1.15)^2\times227.612}{2\times4.002\times223.610\times1.66}\times10^{-11}\ \text{J}$$

$$= 10^{-12}\ \text{J} = \frac{10^{-12}}{1.6\times10^{-13}}\ \text{MeV} = 6.25\ \text{MeV}$$

Mass of parent nucleus $= m_d + m_\alpha - (\text{BE})$

$$= \left(223.610+4.002-\frac{6.25}{931.47}\right)\text{amu}$$

$$= (227.612 - 0.007)\ \text{amu} = 227.605\ \text{amu}$$

## EXAMPLE 7.18

A radioactive element decays by $\beta$-emission. A detector records $n$ beta particles in 2 s and in next 2 s, it records $0.75n$ beta particles. Find mean life correct to nearest whole number. Given ln 2 = 0.6931, ln 3 = 1.0986.

**Sol.** Let $N_0$ be initial number of nuclei at time $t = 0$.

The number of undecayed nuclei in time $t$ is $N = N_0e^{-\lambda t}$

The number of nuclei decayed in time $t$ is

$$n = N_0 - N = N_0 - N_0e^{-\lambda t} \quad ...(i)$$

The number of undecayed nuclei in next time $t$,

$$N' = Ne^{-\lambda t} = (N_0e^{-\lambda t})\,e^{-\lambda t} = N_0e^{-2\lambda t} \quad ...(ii)$$

Number of decayed nuclei in next time $t$,

$$0.75n = N - N' = N_0e^{-\lambda t} - N_0e^{-2\lambda t}$$

$$= N_0e^{-\lambda t}(1-e^{-\lambda t}) \quad ...(iii)$$

Dividing Eq. (ii) by Eq. (i), we get $0.75 = e^{-\lambda t} \Rightarrow \frac{4}{3} = e^{\lambda t}$

Taking natural logarithm, $\ln\frac{4}{3} = \lambda t \Rightarrow \lambda = \frac{\ln\frac{4}{3}}{t}$

Given $t = 2$ s,

$$\lambda = \frac{\ln4-\ln3}{2} = \frac{2\ln2-\ln3}{2} = \frac{2\times0.6931-1.0986}{2}$$

$$= 0.1438\ \text{s}^{-1}$$

Mean life, $\tau = \frac{1}{\lambda} = \frac{1}{0.1438} = 7$ s (whole number)

## EXAMPLE 7.19

A rock is $1.5\times10^9$ years old. The rock contains $^{238}$U which disintegrates to form $^{206}$Pb. Assume that there was no $^{206}$Pb in the rock initially and it is the only stable product formed by the decay. Calculate the ratio of number of nuclei of $^{238}$U to that of $^{206}$Pb in the rock. Half-life of $^{238}$U is $4.5\times10^9$ years. ($2^{1/3} = 1.259$)

**Sol.** $^{238}$U atoms decay to form $^{206}$Pb atoms. Let $N_0$ be initial number of U atoms. After time $t$, let $N_U$ be the number of U atoms left. Then

$$\frac{N_U}{N_0} = \left(\frac{1}{2}\right)^n, \text{ where } n \text{ is number of half-lives}$$

$$n = \frac{t}{T} = \frac{1.5\times10^9\ \text{years}}{4.5\times10^9\ \text{years}} = \frac{1}{3}$$

$$\therefore \quad N_U = \left(\frac{1}{2}\right)^{1/3}N_0 \quad ...(i)$$

Number of Pb atoms, $N_{Pb} = N_0 - N_U$

Hence, required ratio is

$$R = \frac{N_U}{N_{Pb}} = \frac{1}{2^{1/3}-1} = \frac{1}{1.259-1} = \frac{1}{0.259} = 3.86$$

## EXAMPLE 7.20

What is the minimum photon energy required to remove the least bound neutron of $^{40}_{20}$Ca and $^{40}_{18}$Ar?

The necessary atomic masses (in $u$) are given below:

$M$ ($^{40}$Ca) = 39.962591 u, $M$ ($^{39}$Ca) = 38.970719 u, $M(^{40}\text{Ar}) = 39.962383$ u, $M(^{39}\text{Ar}) = 38.964314$ u, $m_n = 1.008665$ u

**Sol.** Total mass (neutron plus product nucleus) after the removal is expected to be more than the initial nucleus mass.

This mass difference when expressed in energy units (MeV) is the binding energy of the least bound neutron.

The mass after break-up,

$M(^{39}_{30}\text{Ca}) = 38.970719$ u

$m_n = 1.08665$ u

Total mass = 39.979384 u

The mass difference is

$$\Delta m = [M(^{39}\text{Ca}) + m_n] - M(^{40}\text{Ca})$$

$$= (39.979384 - 39.962591) = 0.016793\ \text{u}$$

$$= (0.016793)\times(931.5)\ \text{MeV} = 15.64\ \text{MeV}$$

So, the binding energy of least bound neutron is 15.64 MeV.

Similarly, for $^{39}_{20}$Ar,

$$\Delta m = [M(^{39}_{20}\text{Ar}) + m_n] - M(^{40}_{18}\text{Ar})]$$

$= (38.964314 + 1.008665) - 39.962383$ u
$= 0.010596$ u $= (0.010596) \times (931.5)$ MeV
$= 9.870$ MeV

The inert elements are relatively unreactive because their outer shells of electrons are full. Large energies are involved in gaining or losing electrons which is therefore unlikely. An analogous behaviour takes place in the nucleus. Experimental evidence does indicate the existence of 'closed nuclear shell' when the number of protons or neutrons is 2, 8, 20, 28, 50, 82 or 126, although the concepts of individual nucleon 'orbits' and filled shells inside the nucleus is hard to believe.

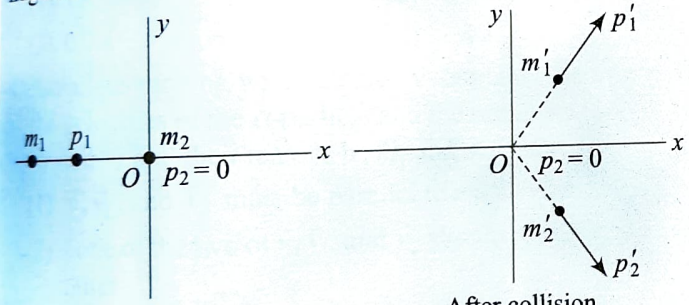
The above mentioned number of protons or neutron are called magic numbers. The elements with magic number of protons have unusually high number of stable isotope. For example, aluminium with 13 protons has just one stable isotope $^{27}Al$, but tin with $Z = 50$ (a magic number) has 10 stable isotopes ranging from $N = 62$ to $N = 74$, whereas neighboring indium with $Z = 49$ and antimony with $Z = 51$ have only two.

Thus, the magic numbers are associated with extra binding energies, implying higher stability.

Thus, in the given example it takes about 6 MeV more energy to remove a paired neutron from a pair that completes a closed nuclear shell (a magic number) than from a pair that does not complete a shell.

## EXAMPLE 7.21

Consider a body at rest in the L-frame, which explodes into two fragments of masses $m_1$ and $m_2$. Calculate energies of the fragments of the body.




Before collision After collision

**Sol.** Since the body is initially at rest, its total momentum is zero. As a result of the explosion, the two fragments will separate in opposite directions with momenta $p_1$ and $p_2$ such that $p_1' + p_2' = 0$; magnitude of momentum will be same, i.e., $p_1' = p_2'$.

On substituting,

$$E_k' = \frac{p_1'^2}{2m_1} + \frac{p_2'^2}{2m_2} = \frac{1}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)p_1'$$

$$E_k = 0$$

In Eq. (iv) of the section "kinematics of reaction", we get

$$\frac{1}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)p_1'^2 = Q$$

On rearranging this equation, we get

$$p_1' = \left(\frac{2m_1m_2}{m_1 + m_2}Q\right)^{1/2} = (2\mu_{12}Q)^{1/2}$$

where $\mu_{12}$ is the reduced mass of the system. The kinetic energies of the fragments are $\frac{p_1'^2}{2m_1}$ and $\frac{p_2'^2}{2m_2}$ with $p_1' = p_2'$. Hence,

$$E_{k,1}' = \frac{p_1'^2}{2m_1} = \frac{m_1Q}{m_1 + m_2} \quad \text{and} \quad E_{k,2}' = \frac{p_2'^2}{2m_2} = \frac{m_2Q}{m_1 + m_2}$$

**Important Points:**

- The kinetic energy of the fragments are inversely proportional to their masses.
- This analysis applies to the fission of the nucleus into two fragments, to the dissociation of disatomic molecule or to the disintegration of the nucleus.
- When there are three fragments, several solutions are possible. There are three unknown momenta with two equations: of conservation of energy and of momentum. When two particles are observed in a reaction and the energy and momentum are not conserved, it is suspected that a third particle is present.

  It is possible that the particle is unobserved because it has no electric charge. Missing momentum and energy are assigned to this hypothetical particle, in order to conserve energy and momentum. Neutron and the neutrino were discovered as a result of such an experiment.

## EXAMPLE 7.22

**(a)** Find the energy needed to remove a neutron from the nucleus of the calcium isotope $^{42}_{20}Ca$.

**(b)** Find the energy needed to remove a proton from this nucleus.

**(c)** Why are these energies different?

Mass of $^{42}_{20}Ca = 40.962278$ u, mass of proton $= 1.007825$ u.

**Sol.**

**(a)** $^{41}_{20}Ca$ nucleus is formed after removing a neutron from $^{42}_{20}Ca$. The mass of $^{41}_{20}Ca$ plus the mass of a free neutron equals 40.962278 u

Mass of neutron $= 40.962278$ u $+ 1.008665$ u $= 41.970943$ u

Difference between $^{42}_{20}Ca$ plus the mass of a free neutron and the mass of $^{42}_{20}Ca$ is 0.012321 u.

So, the binding energy of the missing neutron is
(0.012321 u) (931.49 MeV/u) = 11.5 MeV

**(b)** When a proton is removed from $^{42}_{20}Ca$, the resulting nucleus is the postassium isotope $^{41}_{19}K$. On a similar pattern as above, the binding energy for the missing proton can be calculated; result is 10.27 MeV.

**(c)** Neutron and proton have different energies because only attractive nuclear forces act on the neutron whereas the proton was also acted upon by repulsive electric force that decreases its binding energy.

## EXAMPLE 7.23

Write the decay equations and expressions for the disintegration energy $Q$ of the following decay: $\beta^-$ decay, $\beta^+$ decay, electron capture.

**Sol.** In all the cases discussed here, we neglect any neutrino mass.

$\beta^-$ decay: $M_{nucl}({}^{A}_{Z}\mathrm{X}) = M_{nucl}({}^{A}_{Z+1}\mathrm{D}) + m_e + Q/c^2$

where $M_{nucl}$ indicates the nuclear mass.

In order to convert it to atomic mass, we add $Zm_e$ on both sides:

$$M_{nucl}({}^{A}_{Z}\mathrm{X}) + Zm_e = M_{nucl}({}^{A}_{Z+1}\mathrm{D}) + (Z+1)m_e + Q/c^2$$

Since atomic binding energies are less than nuclear binding energies, they are neglected on the two sides of the equation.

$Q$ for this equation $= [M({}^{A}_{Z}\mathrm{X}) - M({}^{A}_{Z+1}\mathrm{D})]\,c^2$

$\beta^+$ decay: $M_{nucl}({}^{A}_{Z}\mathrm{X}) = M_{nucl}({}^{A}_{Z-1}\mathrm{D}) + m_e + Q/c^2$

In this case, only $(Z-1)m$ is needed for the daughter atomic mass which gives us a remaining mass of $2m_e$.

$Q$ for this equation $= [M({}^{A}_{Z}\mathrm{X}) - ({}^{A}_{Z-1}\mathrm{D}) - 2m_e]c^2$

We add $(Z-1)m_e$ to each side above and obtain

$$M_{nucl}({}^{A}_{Z}\mathrm{X}) + Zm_e = M_{nucl}({}^{A}_{Z-1}\mathrm{D}) + (Z-1)m_e + Q/c^2$$

In this case, electron masses are balanced, i.e., cancelled out in equation.

$Q$ for this equation $= [M({}^{A}_{Z}\mathrm{X}) - M({}^{A}_{Z-1}\mathrm{D})]c^2$

[Electron capture]

# Exercises

## Single Correct Answer Type

1. Consider two arbitrary decay equations and mark the correct alternative(s) given below.

(i) $^{230}_{92}U \rightarrow n + {}^{229}_{92}U$ (ii) $^{230}_{92}U \rightarrow p + {}^{229}_{91}Pa$

Given: $M(^{230}_{92}U) = 230.033927$ u,

$M(^{229}_{92}U) = 229.03349$ u, $m_n = 1.008665$ u,

$M(^{229}_{91}Pa) = 229.032089$, $m_p = 1.007825$, 1 amu = 931.5 MeV.

(1) Only decay (i) is possible.

(2) Only decay (ii) is possible.

(3) Both the decays are possible.

(4) Neither of the two decays is possible.

2. In a sample of rock, the ratio of $^{206}Pb$ to $^{238}U$ nuclei is found to be 0.5. The age of the rock is (given half-life of $U^{238}$ is $4.5 \times 10^9$ years)

(1) $2.25 \times 10^9$ year (2) $4.5 \times 10^9 \ln 3$ year

(3) $4.5 \times 10^9 \dfrac{\ln\left(\frac{3}{2}\right)}{\ln 2}$ year (4) $2.25 \times 10^9 \ln\left(\frac{3}{2}\right)$ year

3. A radioactive sample decays by 63% of its initial value in 10 s. It would have decayed by 50% of its initial value in

(1) 7 s (2) 14 s

(3) 0.7 s (4) 1.4 s

4. A nucleus moving with velocity $\vec{v}$ emits an $\alpha$-particle. Let the velocities of the $\alpha$-particle and the remaining nucleus be $\vec{v}_1$ and $\vec{v}_2$ and their masses be $m_1$ and $m_2$, then

(1) $\vec{v}$, $\vec{v}_1$, and $\vec{v}_2$ must be parallel to each other

(2) none of the two of $\vec{v}$, $\vec{v}_1$, and $\vec{v}_2$ should be parallel to each other

(3) $\vec{v}_1 + \vec{v}_2$ must be parallel to $\vec{v}$

(4) $m_1\vec{v}_1 + m_2\vec{v}_2$ must be parallel to $\vec{v}$

5. A certain radioactive material can undergo three different types of decay, each with a different decay constant $\lambda$, $2\lambda$, and $3\lambda$. Then, the effective decay constant $\lambda_{eff}$ is

(1) $6\lambda$ (2) $4\lambda$

(3) $2\lambda$ (4) $3\lambda$

6. In an $\alpha$-decay, the kinetic energy of $\alpha$-particle is 48 MeV and $Q$ value of the reaction is 50 MeV. The mass number of the mother nucleus is (assume that daughter nucleus is in ground state)

(1) 96 (2) 100

(3) 104 (4) none of these

7. A sample of radioactive material decays simultaneously by two processes A and B with half-lives $\frac{1}{2}$ and $\frac{1}{4}$ h, respectively. For the first half hour it decays with the process A, next one hour with the process B, and for further half an hour with both A and B. If, originally, there were $N_0$ nuclei, find the number of nuclei after 2 h of such decay.

(1) $\dfrac{N_0}{(2)^8}$ (2) $\dfrac{N_0}{(2)^4}$

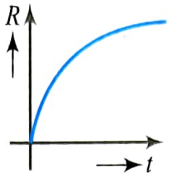
(3) $\dfrac{N_0}{(2)^6}$ (4) $\dfrac{N_0}{(2)^5}$

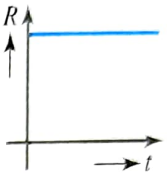
8. In which of the following processes, the number of protons in the nucleus increase?

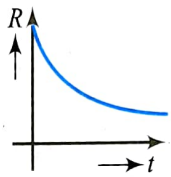
(1) $\alpha$-decay (2) $\beta^-$-decay

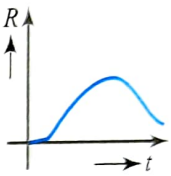
(3) $\beta^+$-decay (4) k-capture

9. A radioactive nucleus X decays to a stable nucleus Y. Then, time graph of rate of formation of Y against time $t$ will be

(1) 


(2) 


(3) 


(4) 


10. 90% of a radioactive sample is left undecayed after time $t$ has elapsed. What percentage of the initial sample will decay in a total time $2t$?

(1) 20% (2) 19%

(3) 40% (4) 38%

11. A radioactive element X converts into another stable element Y. Half-life of X is 2 h. Initially, only X is present. After time $t$, the ratio of atoms of X and Y is found to be 1:4. Then $t$ in hours is

(1) 2 (2) 4

(3) between 4 and 6 (4) 6

12. $A \xrightarrow{\lambda} B \xrightarrow{2\lambda} C$

| | | | |
|---|---|---|---|
| $T = 0$ | $N_0$ | 0 | 0 |
| $T$ | $N_1$ | $N_2$ | $N_3$ |

The ratio of $N_1$ to $N_2$ when $N_2$ is maximum is

(1) at no time this is possible (2) 2

(3) 1/2 (4) $\dfrac{\ln 2}{2}$

13. The binding energy of an electron in the ground state of He-atom is $E_0 = 24.6$ eV. The energy required to remove both the electrons from the atom is

(1) 24.6 eV (2) 79.0 eV

(3) 54.4 eV (4) none of these

14. The mean life time of a radionuclide, if its activity decreases by 4% for every 1 h, would be (product is non-radioactive, i.e., stable)

(1) 25 h (2) 1.042 h

(3) 2 h (4) 30 h

**15.** On an average, a neutron loses half of its energy per collision with a quasi-free proton. To reduce a 2 MeV neutron to a thermal neutron having energy 0.04 eV, the number of collisions required is nearly

(1) 50 (2) 52

(3) 26 (4) 15

**16.** Atomic masses of two isobars $^{64}_{29}Cu$ and $^{64}_{30}Zn$ are 63.9298 u and 63.9292 u, respectively. It can be concluded from this data that

(1) both the isobars are stable

(2) $^{64}Zn$ is radioactive, decaying to $^{64}Cu$ through $\beta$-decay

(3) $^{64}Cu$ is radioactive, decaying to $^{64}Zn$ through $\beta$-decay

(4) $^{64}Cu$ is radioactive, decaying to $^{64}Zn$ through $\gamma$-decay

**17.** If a nucleus such as $^{226}Ra$ that is initially at rest undergoes $\alpha$-decay, then which of the following statements is true?

(1) The alpha particle has more kinetic energy than the daughter nucleus.

(2) The alpha particle has less kinetic energy than the daughter nucleus.

(3) The alpha particle and daughter nucleus both have same kinetic energy

(4) We cannot say anything about kinetic energy of alpha particle and daughter nucleus.

**18.** If the $Q$ value of an endothermic reaction is 11.32 MeV, then the minimum energy of the reactant nuclei to carry out the reaction is (in laboratory frame of reference)

(1) 11.32 MeV (2) less than 11.32 MeV

(3) greater than 11.32 MeV (4) Data is insufficient

**19.** 1.00 kg of $^{235}U$ undergoes fission process. If energy released per event is 200 MeV, then the total energy released is

(1) $5.12 \times 10^{24}$ MeV (2) $6.02 \times 10^{23}$ MeV

(3) $5.12 \times 10^{26}$ MeV (4) $6.02 \times 10^{26}$ MeV

**20.** Mark out the incorrect statement.

(1) A free neutron can transform itself into photon.

(2) A free proton can transform itself into neutron.

(3) In beta minus decay, the electron originates from nucleus.

(4) All of the above.

**21.** U-235 can decay by many ways, let us here consider only two ways, A and B. In decay of U-235 by means of A, the energy released per fission is 210 MeV while in B it is 186 MeV. Then, the uranium 235 sample is more likely to decay by

(1) scheme A

(2) scheme B

(3) equally likely for both schemes

(4) it depends on half-life of schemes A and B

**22.** $^{49}_{19}K$ isotope of potassium has a half-life of $1.4 \times 10^9$ yr and decays to form stable argon, $^{40}_{18}Ar$. A sample of rock has been taken which contains both potassium and argon in the ratio 1:7, i.e.,

$$\frac{\text{Number of potassium-40 atoms}}{\text{Number of argon-40 atoms}} = \frac{1}{7}$$

Assuming that when the rock was formed no argon-40 was present in the sample and none has escaped subsequently, determine the age of the rock.

(1) $4.2 \times 10^9$ years (2) $9.8 \times 10^9$ years

(3) $1.4 \times 10^9$ years (4) $10 \times 10^9$ years

**23.** The probability of survival of a radioactive nucleus for one mean life is

(1) $\frac{1}{e}$ (2) $1 - \frac{1}{e}$

(3) $\frac{\ln 2}{e}$ (4) $1 - \frac{\ln 2}{e}$

**24.** Consider one of fission reactions of $^{235}U$ by thermal neutrons $^{235}_{92}U + n \rightarrow {}^{94}_{38}Sr + {}^{140}_{54}Xe + 2n$. The fission fragments are however unstable and they undergo successive $\beta$-decay until $^{94}_{38}Sr$ becomes $^{94}_{40}Zr$ and $^{140}_{54}Xe$ becomes $^{140}_{58}Ce$. The energy released in this process is

[Given: $m(^{235}U) = 235.439$ u, $m$ (n) = 1.00866 u, $m$ ($^{94}Zr$) = 93.9064 u, $m(^{140}Ce) = 139.9055$ u, 1 u = 931 MeV]

(1) 156 MeV (2) 208 MeV

(3) 456 MeV (4) cannot be computed

**25.** A star initially has $10^{40}$ deuterons. It produces energy via the processes $^2_1H + {}^2_1H \rightarrow {}^3_1He + p$ and $^2_1H + {}^3_1H \rightarrow {}^4_2He + n$. If the average power radiated by the star is $10^{16}$ W, the deuteron supply of the star is exhausted in a time of the order of

[Given: $M(^2H) = 2.014$ u, $M$(n) = 1.008 u, $M$(p) = 1.008 u, and $M(^4He) = 4.001$ u]

(1) $10^6$ s (2) $10^8$ s

(3) $10^{12}$ s (4) $10^{16}$ s

**26.** Two radioactive materials $X_1$ and $X_2$ have decay constants $10\lambda$ and $\lambda$, respectively. If initially they have the same number of nuclei, the ratio of the number of nuclei of $X_1$ to that of $X_2$ will be $1/e$ after a time

(1) $\frac{1}{10\lambda}$ (2) $\frac{1}{11\lambda}$

(3) $\frac{11}{10\lambda}$ (4) $\frac{1}{9\lambda}$

**27.** A radioactive substance is being consumed at a constant rate of 1 $s^{-1}$. After what time will the number of radioactive nuclei become 100. Initially, there were 200 nuclei present.

(1) 1 s (2) $\frac{1}{\ln(2)}$ s

(3) ln(2) s (4) 2 s

**28.** A radioactive isotope is being produced at a constant rate X. Half-life of the radioactive substance is Y. After some time, the number of radioactive nuclei become constant. The value of this constant is

(1) $\frac{XY}{\ln(2)}$ (2) $XY$

(3) (XY) ln (2) (4) $\frac{X}{Y}$

29. A radioactive substance X decays into another radioactive substance Y. Initially, only X was present. $\lambda_x$ and $\lambda_y$ are the disintegration constants of X and Y. $N_y$ will be maximum when

(1) $\frac{N_y}{N_x - N_y} = \frac{\lambda_y}{\lambda_x - \lambda_y}$ (2) $\frac{N_x}{N_x - N_y} = \frac{\lambda_x}{\lambda_x - \lambda_y}$

(3) $\lambda_y N_y = \lambda_x N_x$ (4) $\lambda_y N_x = \lambda_x N_y$

30. There are two radio nuclei A and B. A is an alpha emitter and B a beta emitter. Their disintegration constants are in the ratio of 1:2. What should be the ratio of number of atoms of A and B at any time $t$ so that probabilities of getting alpha and beta particles are same at that instant?

(1) 2:1 (2) 1:2

(3) $e$ (4) $e^{-1}$

31. Half-life of a radioactive substance A is two times the half-life of another radioactive substance B. Initially, the number of A and B are $N_A$ and $N_B$, respectively. After three half-lives of A, number of nuclei of both are equal. Then, the ratio $N_A/N_B$ is

(1) 1/4 (2) 1/8

(3) 1/3 (4) 1/6

32. There are two radioactive substances A and B. Decay constant of B is two times that of A. Initially, both have equal number of nuclei. After $n$ half-lives of A, rates of disintegration of both are equal. The value of $n$ is

(1) 1 (2) 2

(3) 4 (4) all of these

33. A radioactive nucleus is being produced at a constant rate $\alpha$ per second. Its decay constant is $\lambda$. If $N_0$ are the number of nuclei at time $t = 0$, then maximum number of nuclei possible are

(1) $\frac{\alpha}{\lambda}$ (2) $N_0 \frac{\alpha}{\lambda}$

(3) $N_0$ (4) $\frac{\lambda}{\alpha} + N_0$

34. In a sample of a radioactive substance, what fraction of the initial nuclei will remain undecayed after a time $t = T/2$, where $T$ = half-life of radioactive substance?

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{4}$ (4) $\frac{1}{\sqrt{2}-1}$

35. The activity of a radioactive substance is $R_1$ at time $t_1$ and $R_2$ at time $t_2$ ($> t_1$). Its decay constant is $\lambda$. Then

(1) $R_1 t_1$ (2) $R_2 = R_1 e^{\lambda(t_1 - t_2)}$

(3) $\frac{R_1 - R_2}{t_2 - t_1}$ = constant (4) $R_2 = R_1 e^{\lambda(t_2 - t_1)}$

36. In problem 35, number of atoms decayed between time interval $t_1$ and $t_2$ are

(1) $\frac{\ln(2)}{\lambda}(R_1 R_2)$ (2) $R_1 e^{-\lambda t_2} - R_2 e^{-\lambda t_2}$

(3) $\lambda(R_1 - R_2)$ (4) $\left(\frac{R_1 - R_2}{\lambda}\right)$

37. The ratio of molecular mass of two radioactive substances is 3/2 and the ratio of their decay constants is 4/3. Then, the ratio of their initial activity per mole will be

(1) 2 (2) $\frac{8}{9}$

(3) $\frac{4}{3}$ (4) $\frac{9}{8}$

38. $N_1$ atoms of a radioactive element emit $N_2$ beta particles per second. The decay constant of the element is (in $s^{-1}$)

(1) $\frac{N_1}{N_2}$ (2) $\frac{N_2}{N_1}$

(3) $N_1 \ln(2)$ (4) $N_2 \ln(2)$

39. The binding energies of nuclei X and Y are $E_1$ and $E_2$, respectively. Two atoms of X fuse to give one atom of Y and an energy $Q$ is released. Then,

(1) $Q = 2E_1 - E_2$ (2) $Q = E_2 - 2E_1$

(3) $Q < 2E_1 - E_2$ (4) $Q > E_2 - 2E_1$

40. Binding energy per nucleon of ${}_1H^2$ and ${}_2He^4$ are 1.1 MeV and 7.0 MeV, respectively. Energy released in the process ${}_1H^2 + {}_1H^2 = {}_2He^4$ is

(1) 20.8 MeV (2) 16.6 MeV

(3) 25.2 MeV (4) 23.6 MeV

41. ${}_{92}U^{238}$ absorbs a neutron. The product emits an electron. This product further emits an electron. The result is

(1) ${}_{94}Pu^{239}$ (2) ${}_{90}Pu^{239}$

(3) ${}_{93}Pu^{237}$ (4) ${}_{94}Pu^{237}$

42. The activity of a radioactive element decreases to one-third of the original activity $A_0$ in a period of 9 years. After a further lapse of 9 years, its activity will be

(1) $A_0$ (2) $\frac{2}{3}A_0$

(3) $\frac{A_0}{9}$ (4) $\frac{A_0}{6}$

43. The half-life of a radioactive decay is $x$ times its mean life. The value of $x$ is

(1) 0.3010 (2) 0.6930

(3) 0.6020 (4) $\frac{1}{0.6930}$

44. Neutron decay in the free space is given as follows: ${}_0n^1 \rightarrow {}_1H^1 + {}_{-1}e^0 + [\,]$
Then, the parenthesis represents

(1) photon (2) graviton

(3) neutrino (4) antineutrino

45. A nucleus ${}^A_Z X$ emits an $\alpha$-particle. The resultant nucleus emits a $\beta^+$ particle. The respective atomic and mass numbers of the final nucleus will be

(1) $Z-3, A-4$ (2) $Z-1, A-4$

(3) $Z-2, A-4$ (4) $Z, A-2$

**46.** Certain radioactive substance reduces to 25% of its value in 16 days. Its half-life is

(1) 32 days (2) 8 days

(3) 64 days (4) 28 day

**47.** In the disintegration series

$$^{238}_{92}U \xrightarrow{\alpha} X \xrightarrow{\beta^-} {}^{A}_{Z}Y$$

the values of $Z$ and $A$, respectively, will be

(1) 92, 326 (2) 88, 230

(3) 90, 234 (4) 91, 234

**48.** In the case of thorium ($A = 232$ and $Z = 90$), we obtain an isotope of lead ($A = 208$ and $Z = 82$) after some radioactive disintegrations. The number of $\alpha$- and $\beta$-particles emitted are, respectively,

(1) 6, 3 (2) 6, 4

(3) 5, 5 (4) 4, 6

**49.** The initial activity of a certain radioactive isotope was measured as 16000 counts $min^{-1}$. Given that the only activity measured was due to this isotope and that its activity after 12 h was 2100 counts $min^{-1}$, its half-life, in hours, is nearest to [Given $\log_e(7.2) = 2$]

(1) 9.0 (2) 6.0

(3) 4.0 (4) 3.0

**50.** The minimum frequency of a $\gamma$-ray that causes a deuteron to disintegrate into a proton and a neutron is ($m_d = 2.0141$ amu, $m_p = 1.0078$ amu, $m_n = 1.0087$ amu.)

(1) $2.7 \times 10^{20}$ Hz (2) $5.4 \times 10^{20}$ Hz

(3) $10.8 \times 10^{20}$ Hz (4) $21.6 \times 10^{20}$ Hz

**51.** The fission of a heavy nucleus gives, in general, two smaller nuclei, two or three neutrons, some $\beta$-particles, and some $\gamma$-radiation. It is always true that the nuclei produced

(1) have a total rest-mass that is greater than that of the original nucleus

(2) have large kinetic energies that carry off the greater part of the energy released

(3) travel in exactly opposite directions

(4) have neutron-to-proton ratios that are too low for stability

**52.** The rest mass of a deuteron is equivalent to an energy of 1876 MeV, that of a proton to 939 MeV, and that of a neutron to 940 MeV. A deuteron may disintegrate to a proton and a neutron if it

(1) emits an $\gamma$-ray photon of energy 2 MeV

(2) captures an $\gamma$-ray photon of energy 2 MeV

(3) emits an $\gamma$-ray photon of energy 3 MeV

(4) captures an $\gamma$-ray photon of energy 3 MeV

**53.** A newly prepared radioactive nuclide has a decay constant $\lambda$ of $10^{-6}$ $s^{-1}$. What is the approximate half-life of the nuclide?

(1) 1 hour (2) 1 day

(3) 1 week (4) 1 month

**54.** Samples of two radioactive nuclides, X and Y, each have equal activity $A$ at time $t = 0$. X has a half-life of 24 years and Y a half-life of 16 years. The samples are mixed together. What will be the total activity of the mixture at $t = 48$ years?

(1) $\frac{1}{2}A_0$ (2) $\frac{1}{4}A_0$

(3) $\frac{3}{16}A_0$ (4) $\frac{3}{8}A_0$

**55.** A sample of a radioactive element has a mass of 10 g at an instant $t = 0$. The approximate mass of this element in the sample after two mean lives is

(1) 1.35 g (2) 2.50 g

(3) 3.70 g (4) 6.30 g

**56.** After an interval of one day, 1/16th initial amount of a radioactive material remains in a sample. Then, its half-life is

(1) 6 h (2) 12 h

(3) 1.5 h (4) 3 h

**57.** The half-life of At is 100 μs.The time taken for the radioactivity of a sample of At to decay to 1/16th of its initial value is

(1) 400 μs (2) 6.3 μs

(3) 40 μs (4) 300 μs

**58.** A stationary thorium nucleus ($A = 220$, $Z = 90$) emits an alpha particle with kinetic energy $E_\alpha$. What is the kinetic energy of the recoiling nucleus?

(1) $\frac{E_\alpha}{108}$ (2) $\frac{E_\alpha}{110}$

(3) $\frac{E_\alpha}{55}$ (4) $\frac{E_\alpha}{54}$

**59.** The fraction of a radioactive material which remains active after time $t$ is 9/16. The fraction which remains active after time $t/2$ will be

(1) $\frac{4}{5}$ (2) $\frac{7}{8}$

(3) $\frac{3}{5}$ (4) $\frac{3}{4}$

**60.** The rate of decay of a radioactive element at any instant is $10^3$ disintegrations $s^{-1}$. If the half-life of the elements is 1 s then the rate of decay after 1 s will be

(1) 500 $s^{-1}$ (2) 1000 $s^{-1}$

(3) 250 $s^{-1}$ (4) 2000 $s^{-1}$

**61.** If 10% of a radioactive substance decays in every 5 yea then the percentage of the substance that will have decaye in 20 years will be

(1) 40% (2) 50%

(3) 65.6% (4) 34.4%

**62.** Stationary nucleus $^{238}U$ decays by a emission generating total kinetic energy $T$:

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}\alpha$$

What is the kinetic energy of the $\alpha$-particle?

(1) Slightly less than $T/2$

(2) $T/2$

(3) Slightly less than $T$

(4) Slightly greater than $T$

63. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with the source is
(1) 6 h (2) 12 h
(3) 24 h (4) 128 h

64. Uranium ores contain one radium-226 atom for every $2.8 \times 10^6$ uranium-238 atoms. Calculate the half-life of $_{92}U^{238}$, given that the half-life of $_{88}Ra^{226}$ is 1600 years ($_{88}Ra^{226}$ is a decay product of $_{92}U^{238}$).
(1) $1.75 \times 10^3$ years (2) $1600 \times \frac{238}{92}$ years
(3) $4.5 \times 10^9$ years (4) $1600 \times 238$ years

65. Plutonium has atomic mass 210 and a decay constant equal to $5.8 \times 10^{-8}$ $s^{-1}$. The number of $\alpha$-particles emitted per second by 1 mg plutonium is
(Avogadro's constant = $6.0 \times 10^{23}$)
(1) $1.7 \times 10^9$ (2) $1.7 \times 10^{11}$
(3) $2.9 \times 10^{11}$ (4) $3.4 \times 10^9$

66. At any instant, the ratio of the amounts of two radioactive substances is 2:1. If their half-lives be, respectively, 12 h and 16 h, then after two days, what will be the ratio of the substances?
(1) 1:1 (2) 2:1
(3) 1:2 (4) 1:4

67. The radioactivity of a sample is $R_1$ at a time $T_1$ and $R_2$ at a time $T_2$. If the half-life of the specimen is $T$, the number of atoms that have disintegrated in the time $(T_2 - T_1)$ is proportional to
(1) $R_1 T_1 = R_2 T_2$ (2) $R_1 - R_2$
(3) $\frac{(R_1 - R_2)}{T}$ (4) $(R_1 - R_2)T$

68. Half-lives of two radioactive substances A and B are, respectively, 20 min and 40 min. Initially, the samples of A and B have equal number of nuclei. After 80 min, the ratio of the remaining number of A and B nuclei is
(1) 1:16 (2) 4:1
(3) 1:4 (4) 1:1

69. A radioactive nucleus can decay by two different processes. The mean value period for the first process is $t_1$ and that for the second process is $t_2$. The effective mean value period for the two processes is
(1) $\frac{t_1 + t_2}{2}$ (2) $t_1 + t_2$
(3) $\sqrt{t_1 t_2}$ (4) $\frac{t_1 t_2}{t_1 + t_2}$

70. The half-life of radium is 1620 years and its atomic weight is 226. The number of atoms that will decay from its 1 g sample per second will be
(1) $3.6 \times 10^{10}$ (2) $3.6 \times 10^{12}$
(3) $3.1 \times 10^{15}$ (4) $31.1 \times 10^{15}$

71. The nuclear radius of a nucleus with nucleon number 16 is $3 \times 10^{-15}$ m. Then, the nuclear radius of a nucleus with nucleon number 128 is
(1) $3 \times 10^{-15}$ m (2) $1.5 \times 10^{-15}$ m
(3) $6 \times 10^{-15}$ m (4) $4.5 \times 10^{-15}$ m

72. The nuclear radius of $_8O^{16}$ is $3 \times 10^{-15}$ m. If an atomic mass unit is $1.67 \times 10^{-27}$ kg, then the nuclear density is approximately
(1) $2.35 \times 10^{17}$ g $cm^{-3}$ (2) $2.35 \times 10^{17}$ kg $m^{-3}$
(3) $2.35 \times 10^{17}$ g $m^{-3}$ (4) $2.35 \times 10^{17}$ kg $mm^{-3}$

73. What would be the energy required to dissociate completely 1 g of Ca-40 into its constituent particles?
Given: Mass of proton = 1.007277 amu,
Mass of neutron = 1.00866 amu,
Mass of Ca-40 = 39.97545 amu
(take 1 amu = 931 MeV)
(1) $4.813 \times 10^{24}$ MeV (2) $4.813 \times 10^{24}$ eV
(3) $4.813 \times 10^{23}$ MeV (4) None of the above

74. In fission, the percentage of mass converted into energy is about
(1) 10% (2) 1%
(3) 0.1% (4) 0.01%

75. In the nuclear reaction $_1H^2 + _1H^2 \rightarrow _2He^3 + _0n^1$ if the mass of the deuterium atom = 2.014741 amu, mass of $_2He^3$ atom = 3.016977 amu, and mass of neutron = 1.008987 amu, then the $Q$ value of the reaction is nearly
(1) 0.00352 MeV (2) 3.27 MeV
(3) 0.82 MeV (4) 2.45 MeV

76. Assuming that about 20 MeV of energy is released per fusion reaction
$_1H^2 + _1H^2 \rightarrow + _2He^4 + E$ + other particles
then the mass of $_1H^2$ consumed per day in a fusion reactor of power 1 megawatt will approximately be
(1) 0.001 g (2) 0.1 g
(3) 10.0 g (4) 1000 g

77. Assuming that about 200 MeV of energy is released per fission of $_{92}U^{235}$ nuclei, the mass of $U^{235}$ consumed per day in a fission reactor of power 1 megawatt will be approximately
(1) $10^{-2}$ g (2) 1 g
(3) 100 g (4) 10,000 g

78. If mass of $U^{235}$ = 235.12142 amu, mass of $U^{236}$ = 236.1205 amu, and mass of neutron = 1.008665 amu, then the energy required to remove one neutron from the nucleus of $U^{236}$ is nearly about
(1) 75 MeV (2) 6.5 MeV
(3) 1 eV (4) zero

79. The binding energies per nucleon of deuteron ($_1H^2$) and helium ($_2He^4$) atoms are 1.1 MeV and 7 MeV. If two deuteron atoms react to form a single helium atom, then the energy released is
(1) 13.9 MeV (2) 26.9 MeV
(3) 23.9 MeV (4) 19.2 MeV

80. In the fusion reaction $_1^2He + _1^2H \rightarrow _2^3He + _0^1n$, the masses of deuteron, helium, and neutron expressed in amu are

and 1.009, respectively. If 1 kg of deuterium undergoes complete fusion, find the amount of total energy released. (1 amu = 931.5 meV/$c^2$)

(1) $\approx 6.02 \times 10^{13}$ J (2) $\approx 5.6 \times 10^{13}$ J

(3) $\approx 9.0 \times 10^{13}$ J (4) $\approx 0.9 \times 10^{13}$ J

**81.** The half-life of radium is 1500 years. In how many years will 1 g of pure radium be reduced to one centigram?

(1) $3.927 \times 10^2$ years

(2) $9.972 \times 10^2$ years

(3) $99.927 \times 10^2$ years

(4) $0.927 \times 10^2$ years

**82.** A proton and a neutron are both shot at 100 m $s^{-1}$ towards a $^{12}_{6}C$ nucleus. Which particle, if either, is more likely to be absorbed by the nucleus?

(1) The proton.

(2) The neutron.

(3) Both particles are about equally likely to be absorbed.

(4) Neither particle will be absorbed.

**83.** A container is filled with a radioactive substance for which the half-life is 2 days. A week later, when the container is opened, it contains 5 g of the substance. Approximately how many grams of the substances were initially placed in the container?

(1) 40 (2) 60

(3) 80 (4) 100

**84.** The half-life of $^{131}I$ is 8 days. Given a sample of $^{131}I$ at time $t = 0$, we can assert that

(1) no nucleus will decay before $t = 4$ days

(2) no nucleus will decay before $t = 8$ days

(3) all nuclei will decay before $t = 16$ days

(4) a given nucleus may decay at any time after $t = 0$

**85.** When an atom undergoes $\beta^+$ decay,

(1) a neutron changes into a proton

(2) a proton changes into a neutron

(3) a neutron changes into an antiproton

(4) a proton changes into an antineutron

**86.** Calculate the binding energy of a deuteron atom, which consists of a proton and a neutron, given that the atomic mass of the deuteron is 2.014102 u.

(1) 0.002388 MeV (2) 2.014102 MeV

(3) 2.16490 MeV (4) 2.224 MeV

**87.** The compound unstable nucleus $^{236}_{92}U$ often decays in accordance with the following reaction $^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr +$ other particles

In the nuclear reaction presented above, the "other particle" might be

(1) an alpha particle, which consists of two protons and two neutrons

(2) two protons

(3) one proton and one neutron

(4) two neutrons

**88.** Why is a $^{4}_{2}He$ nucleus more stable than a $^{4}_{3}Li$ nucleus?

(1) The strong nuclear force is larger when the neutron to proton ratio is higher.

(2) The laws of nuclear physics forbid a nucleus from containing more protons than neutrons.

(3) Forces other than the strong nuclear force make the lithium nucleus less stable.

(4) None of the above.

**89.** What is the power output of $_{92}U^{235}$ reactor if it takes 30 days to use up 2 kg of fuel and if each fission gives 185 MeV of usable energy? Avogadro's number = $6.02 \times 10^{26}$ per kilomole.

(1) 45 megawatt (2) 58.46 megawatt

(3) 72 megawatt (4) 92 megawatt

**90.** Consider the following reaction

$$^{1}H_2 + {}^{1}H_2 \rightarrow {}_{1}He^4 + Q$$

If $m(_1H^2) = 2.0141$ u: $m(_2He^4) = 4.0024$ u, the energy $Q$ released (in MeV) in this fusion reaction is

(1) 12 (2) 6

(3) 24 (4) 48

**91.** A radioactive nuclide is produced at the constant rate of $n$ per second (say, by bombarding a target with neutrons). The expected number $N$ of nuclei in existence $t$ s after the number is $N_0$ is given by

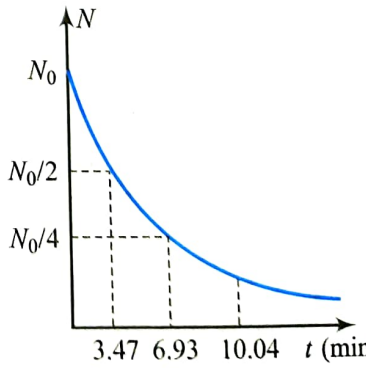
(1) $N = N_0 e^{-\lambda t}$

(2) $N = \frac{n}{\lambda} + N_0 e^{-\lambda t}$

(3) $N = \frac{n}{\lambda} + \left(N_0 - \frac{n}{\lambda}\right) e^{-\lambda t}$

(4) $N = \frac{n}{\lambda} + \left(N_0 + \frac{n}{\lambda}\right) e^{-\lambda t}$

**92.** A radioactive sample undergoes decay as per the following graph. At time $t = 0$, the number of undecayed nuclei is $N_0$. Calculate the number of nuclei left after 1 h.




(1) $N_0/e^8$ (2) $N_0/e^{10}$

(3) $N_0/e^{12}$ (4) $N_0/e^{14}$

**93.** The luminous dials of watches are usually made by mixing a zinc sulphide phosphor with an $\alpha$-particle emitter. The mass of radium (mass number 226, half-life 1620 years) that is

needed to produce an average of 10 $\alpha$-particles per second for this purpose is

(1) 2.77 mg (2) 2.77 g

(3) $2.77 \times 10^{-23}$ g (4) $2.77 \times 10^{-13}$ kg

44. The following deuterium reactions and corresponding reaction energies are found to occur.

$^{14}N$ (d, p) $^{15}N$, $Q = 8.53$ MeV

$^{14}N$ (d, $\alpha$) $^{12}C$, $Q = 7.58$ MeV

$^{12}C$ (d, $\alpha$) $^{11}B$, $Q = 5.16$ MeV

The notation $^{14}N$ (d, p) $^{15}N$ represents the reaction

$^{14}N + d \rightarrow {}^{15}N + p$

$^4_2He = 4.0026$ amu, $^2_1He = 2.014$ amu, $^1_1H = 1.0078$ amu, $n = 1.0087$ amu (1 amu = 931 MeV)

The $Q$ values of the reaction $^{11}B(\alpha, n)\ ^{14}N$ is

(1) 0.5 eV (2) 0.5 MeV

(3) 0.05 MeV (4) 0.05 eV

45. A neutron of energy 1 MeV and mass $1.6 \times 10^{-27}$ kg passes a proton at such a distance that the angular momentum of the neutron relative to the proton approximately equals $10^{-33}$ J s. The distance of closest approach neglecting the interaction between particles is

(1) 0.44 nm (2) 0.44 mm

(3) 0.44 Å (4) 0.44 fm

46. Rank the following nuclei in order from largest to smallest value of the binding energy per nucleon: (i) $^4_2He$, (ii) $^{52}_{24}Cr$, (iii) $^{152}_{62}Sm$, (iv) $^{200}_{80}Hg$, (v) $^{252}_{92}Cf$.

(1) $E_{(v)} > E_{(iv)} > E_{(iii)} > E_{(ii)} > E_{(i)}$

(2) $E_{(i)} > E_{(ii)} > E_{(iii)} > E_{(iv)} > E_{(v)}$

(3) $E_{(ii)} > E_{(iii)} > E_{(iv)} > E_{(v)} > E_{(i)}$

(4) $E_{(i)} = E_{(ii)} = E_{(iii)} = E_{(iv)} = E_{(v)}$

47. A nucleus with atomic number $Z$ and neutron number $N$ undergoes two decay processes. The result is a nucleus with atomic number $Z - 3$ and neutron number $N - 1$. Which decay processes took place?

(1) Two $\beta^-$ decays

(2) Two $\beta^+$ decays

(3) An $\alpha$-decay and a $\beta^-$ decay

(4) An $\alpha$-decay and a $\beta^+$ decay

48. Gold $^{198}_{79}Au$ undergoes $\beta^-$ decay to an excited state of $^{198}_{80}Hg$. If the excited state decays by emission of a $\gamma$-photon with energy 0.412 MeV, the maximum kinetic energy of the electron emitted in the decay is (This maximum occurs when the antineutrino has negligible energy. The recoil energy of the $^{198}_{80}Hg$ nucleus can be ignored. The masses of the neutral atoms in their ground states are 197.968225 u for $^{198}_{79}Au$ and 197.966752 u for $^{198}_{79}Hg$.)

(1) 0.412 MeV (2) 1.371 MeV

(3) 0.959 MeV (4) 1.473 MeV

## Multiple Correct Answers Type

1. For a certain radioactive substance, it is observed that after 4 h, only 6.25% of the original sample is left undecayed. It follows that

(1) the half-life of the sample is 1 h

(2) the mean life of the sample is $\frac{1}{\ln 2}$ h

(3) the decay constant of the sample is ln (2) $h^{-1}$

(4) after a further 4 h, the amount of the substance left over would by only 0.39% of the original amount

2. Mark out the correct statement(s).

(1) Higher binding energy per nucleon means the nucleus is more stable.

(2) If the binding energy of nucleus were zero, then it would spontaneously break apart.

(3) Binding energy of a nucleus can be negative.

(4) Binding energy of a nucleus is always positive.

3. Mark out the correct statement(s).

(1) In alpha decay, the energy released is shared between alpha particle and daughter nucleus in the form of kinetic energy and share of alpha particle is more than that of the daughter nucleus.

(2) In beta decay, the energy released is in the form of kinetic energy of beta particles.

(4) In beta minus decay, the energy released is shared between electron and antineutrino.

(4) In gamma decay, the energy released is in the form of energy carried by photons termed as gamma rays.

4. Mark the correct statement(s).

(1) For an exothermic reaction, if $Q$ value is +12.56 MeV and the KE of incident particle is 2.44 MeV, then the total KE of products of reaction is 15.00 MeV.

(2) For an exothermic reaction, if $Q$ value is +12.56 MeV and the KE of incident particle is 2.44 MeV, then the total KE of products of reaction is 12.56 MeV.

(3) For an endothermic reaction, if we give the energy equal to $|Q|$ value of reaction, then the reaction will be carried out.

(4) For an exothermic reaction, the BE per nucleon of products should be greater than the BE per nucleon of reactants.

5. Mark out the correct statement(s).

(1) In both fission and fusion processes, the mass of reactant nuclide is greater than the mass of product nuclide.

(2) In fission process, BE per nucleon of reactant nuclide is less than the binding energy per nucleon of product nuclide.

(3) In fusion process, BE per nucleon of reactant nuclide is less than the binding energy per nucleon of product nuclide.

(4) In fusion process, BE per nucleon of reactant nuclide is greater than the binding energy per nucleon of product nuclide.

**6.** During $\beta$-decay (beta minus), the emission of antineutrino particle is supported by which of the following statement(s)?
(1) Angular momentum conservation holds good in any nuclear reaction.
(2) Linear momentum conservation holds good in any nuclear reaction.
(3) The KE of emitted $\beta$-particle is varying continuously to a maximum value.
(4) None of the above.

**7.** Two samples A and B of same radioactive nuclide are prepared. Sample A has twice the initial activity of sample B. For this situation, mark out the correct statement(s).
(1) The half-lives of both the samples would be same.
(2) The half-lives of the samples are different.
(3) After each has passed through 5 half-lives, the ratio of activity of A to B is 2:1.
(4) After each has passed through 5 half-lives, ratio of activities of A to B is 64:1.

**8.** The decay constant of a radioactive substance is 0.173 $year^{-1}$. Therefore,
(1) nearly 63% of the radioactive substance will decay in (1/0.173) year
(2) half-life of the radioactive substance is (1/0.173) year
(3) one-fourth of the radioactive substance will be left after 8 years
(4) all of the above

**9.** A nuclide A undergoes $\alpha$-decay and another nuclide B undergoes $\beta$-decay. Then,
(1) all the $\alpha$-particles emitted by A will have almost the same speed
(2) the $\alpha$-particles emitted by B will have widely different speeds
(3) all the $\beta$-particles emitted by B will have almost the same speed
(4) the $\beta$-particles emitted by B may have widely different speeds

**10.** If $A$, $Z$, and $N$ denote the mass number, the atomic number, and the neutron number for a given nucleus, we can say that
(1) $N = Z + A$
(2) isobars have the same $A$ but different $Z$ and $N$
(3) isotopes have the same $Z$ but different $N$ and $A$
(4) isotopes have the same $N$ but different $A$ and $Z$

**11.** It has been found that nuclides with 2, 8, 20, 50, 82, and 126 protons or neutrons are exceptionally stable. These numbers are referred to as the magic numbers and their existence has led us to
(1) the idea of periodicity in nuclear properties similar to the periodicity of chemical elements in periodic table
(2) the so-called "liquid drop model of the nucleus"
(3) the so-called "shell model of the nucleus"
(4) have a convenient explanation of "nuclear fission"

**12.** The phenomenon of nuclear fission can be carried out both in a controlled and in an uncontrolled way. Out of the following, the correct statements vis-à-vis these phenomena are:
(1) The fission energy released per reaction is much more than conventional nuclear reactions and one of the products of the reaction is that very particle which initiates the reaction.
(2) It is the "surface to volume" ratio of the sample of nuclear fuel used which determines whether or not the reaction would sustain itself as a "chain reaction".
(3) The "control rods" in a nuclear reactor must be made of a material that absorbs neutrons effectively.
(4) The energy released per fission as well as energy released per unit mass of the fuel in nuclear fission are both greater than the corresponding quantities for nuclear fusion.

**13.** Choose the correct statements from the following:
(1) Like other light nuclei, the $_2He^4$ nuclei also have a low value of the binding energy per nucleon.
(2) The binding energy per nucleon decreases for nuclei with small as well as large atomic number.
(3) The energy required to remove one neutron from $_3Li^7$ to transform it into the isotope $_3Li^6$ is 5.6 MeV, which is the same as the binding energy per nucleon of $_3Li^6$
(4) When two deuterium nuclei fuse together, they give rise to a tritium nucleus accompanied by a release of energy.

**14.** It is observed that only 0.39% of the original radioactive sample remains undecayed after eight hours. Hence,
(1) the half-life of that substance is 1 h
(2) the mean-life of the substance is [1/(log 2)] h
(3) decay constant of the substance is (log 2) $h^{-1}$
(4) if the number of radioactive nuclei of this substance at a given instant is 10, then the number left after 30 min would be 7.5

**15.** In a nuclear reactor
(1) the chain reaction is kept under control by rods of cadmium, which reduces the rate
(2) the thick concrete shield is used to slow down the speed of fast neutrons
(3) heavy water (or graphite) moderate the activity of the reactor
(4) out of $U^{238}$ and $U^{235}$ natural uranium has less than 1% of $U^{235}$

**16.** A radioactive sample has initial concentration $N_0$ of nuclei. Then,
(1) the number of undecayed nuclei present in the sample decays exponentially with time
(2) the activity ($R$) of the sample at any instant is directly proportional to the number of undecayed nuclei present in the sample at that time
(3) the number of decayed nuclei grows exponentially with time
(4) the number of decayed nuclei grows linearly with time

**17.** An $O^{16}$ nucleus is spherical and has a radius $R$ and a volume $V = \frac{4}{3}\pi R^3$. According to the empirical observations, the

volume of the $_{54}X^{128}$ nucleus assumed to be spherical is $V'$ and radius is $R'$. Then

(1) $V' = 8V$ (2) $V' = 2V$

(3) $R' = 2R$ (4) $R' = 8R$

18. Which of the following statements) is(are) correct?

(1) The rest mass of a stable nucleus is greater than the sum of the rest masses of its separated nucleons.

(2) The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons.

(3) In nuclear fission, energy is released by fusing two nuclei of medium mass (approximately 100 amu).

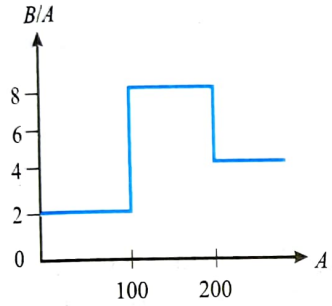
(4) In nuclear fission, energy is released by fragmentation of a very heavy nucleus.

19. Let $m_p$ be the mass of proton, $m_n$ the mass of a neutron, $M_1$ the mass of a $^{20}_{10}Ne$ nucleus, and $M_2$ the mass of a $^{40}_{20}Ca$ nucleus. Then,

(1) $M_2 = 2M_1$ (2) $M_2 > 2M_1$

(3) $M_2 < 2M_1$ (4) $M_1 < 10(m_p + m_n)$

20. Assume that the nuclear binding energy per nucleon ($B/A$) versus mass number ($A$) is as shown in figure. Use this plot to choose the correct choice(s) given below:




(1) Fusion of two nuclei with mass numbers lying in the range of $1 < A < 50$ will release energy.

(2) Fusion of two nuclei with mass numbers lying in the range of $51 < A < 100$ will release energy.

(3) Fission of a nucleus lying in the mass range of $100 < A < 200$ will release energy when broken into equal fragments.

(4) Fission of a nucleus lying in the mass range of $200 < A < 260$ will release energy when broken into equal fragments.

21. Two radioactive materials $A$ and $B$ decay constant $3\lambda$ and $2\lambda$ respectively. At $t = 0$, the numbers of nuclei of A and B are $4N_0$ and $2N_0$ respectively, then

(1) their number of radioactive nuclei will be equal at $t = \frac{1}{\lambda}\ln 2$

(2) their decay rate will be equal at $t = \frac{1}{\lambda}\ln 4$

(3) their decay rate will be equal at $t = \frac{1}{\lambda}\ln 3$

(4) At $t = \frac{1}{\lambda}\ln 4$, the decay rate of $A$ will be greater than that of $B$.

22. A nucleus $X$ of mass $M$, initially at rest, undergoes alpha decay according the equation $^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}He$

The alpha particle emitted in the above process is found to move in a circular track of radius $r$ in a uniform magnetic field $B$. Then (mass and charge of $\alpha$-particle are $m$ and $q$ respectively)

(1) The ratio of kinetic energies of the alpha particle and the daughter nucleus Y approximately equals $\frac{M-m}{m}$

(2) The ratio of kinetic energies of the alpha particle and the daughter nucleus Y approximately equals $\frac{m}{M}$

(3) The energy released in the process approximately equals $\frac{M}{M-m}\left(\frac{r^2q^2B^2}{2m}\right)$

(4) The energy released in the process approximately equals $\frac{M-m}{M}\left(\frac{r^2q^2B^2}{2m}\right)$

23. The half life of a radioactive sample can be experimentally round by the following method. A pure radioactive sample $A$ decays into $B$ by a decay.

$$A \rightarrow B + \alpha$$

$N_A$ and $N_B$ are measured at different times and $\frac{N_B}{N_A}$ is plotted against time. If time interval is much less than half life,

(1) graph is straight line with –ve slope.

(2) graph is straight line with +ve slope.

(3) graph has slope whose magnitude is the disintegration constant.

(4) graph has slope whose magnitude is the reciprocal of disintegration constant.

## Linked Comprehension Type

**For Problems 1–3**

Nuclei of a radioactive element X are being produced at a constant rate $K$ and this element decays to a stable nucleus Y with a decay constant $\lambda$ and half-life $T_{1/2}$. At time $t = 0$, there are $N_0$ nuclei of the element X.

**1.** The number $N_X$ of nuclei of X at time $t = T_{1/2}$ is

(1) $\frac{K + \lambda N_0}{2\lambda}$ (2) $(2\lambda N_0 - K)\frac{1}{\lambda}$

(3) $\left[\lambda N_0 + \frac{K}{2}\right]\frac{1}{\lambda}$ (4) Data insufficient

**2.** The number $N_Y$ of nuclei of Y at time $t$ is

(1) $Kt - \left(\frac{K - \lambda N_0}{\lambda}\right)e^{-\lambda t} + \frac{K - \lambda N_0}{\lambda}$

(2) $Kt + \left(\frac{K - \lambda N_0}{\lambda}\right)e^{-\lambda t} - \frac{K - \lambda N_0}{\lambda}$

(3) $Kt + \left(\frac{K - \lambda N_0}{\lambda}\right)e^{-\lambda t}$

(4) $Kt - \left(\frac{K - \lambda N_0}{\lambda}\right) e^{-\lambda t}$

**3.** The number $N_Y$ of nuclei of Y at $t = T_{1/2}$ is

(1) $K\frac{\ln 2}{\lambda} + \frac{3}{2}\left(\frac{K - \lambda N_0}{\lambda}\right)$ (2) $K\frac{\ln 2}{\lambda} + \frac{1}{2}\left(\frac{K - \lambda N_0}{\lambda}\right)$

(3) $K\frac{\ln 2}{\lambda} - \frac{1}{2}\left(\frac{K - \lambda N_0}{\lambda}\right)$ (4) $K\frac{\ln 2}{\lambda} - 2\left(\frac{K - \lambda N_0}{\lambda}\right)$

**For Problems 4–6**

A radionuclide with decay constant $\lambda$ is being produced in a nuclear reactor at a rate $q_0 t$ per second, where $q_0$ is a positive constant and $t$ is the time. During each decay, $E_0$ energy is released. The production of radionuclide starts at time $t = 0$.

**4.** Which differential equation correctly represents the above process?

(1) $\frac{dN}{dt} + \lambda N = q_0 t$ (2) $\frac{dN}{dt} - \lambda N = q_0 t$

(3) $\frac{dN}{dt} + q_0 t = \lambda N$ (4) $\frac{dN}{dt} + q_0 t = -\lambda N$

**5.** Instantaneous power developed at time $t$ due to the decay of the radionuclide is

(1) $\left(q_0 t - \frac{q_0}{\lambda} + \frac{q_0}{\lambda} e^{-\lambda t}\right) E_0$ (2) $\left(q_0 t + \frac{q_0}{\lambda} - \frac{q_0}{\lambda} e^{-\lambda t}\right) E_0$

(3) $\left(q_0 t + \frac{q_0}{\lambda} + \frac{q_0}{\lambda} e^{-\lambda t}\right) E_0$ (4) $\left(q_0 t - \frac{q_0}{\lambda} - \frac{q_0}{\lambda} e^{-\lambda t}\right) E_0$

**6.** Average power developed in time $t$ due to the decay of the radionuclide is

(1) $\left(\frac{q_0 t}{2} - \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} - \frac{q_0}{\lambda^2 t} e^{-\lambda t}\right) E_0$

(2) $\left(\frac{q_0 t}{2} + \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} - \frac{q_0}{\lambda^2 t} e^{-\lambda t}\right) E_0$

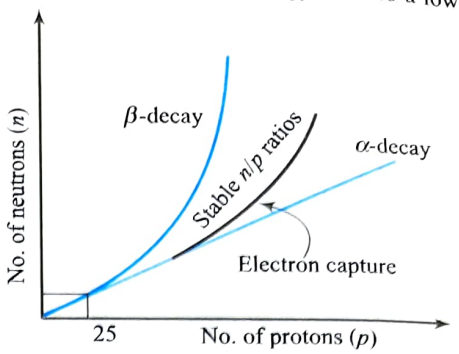
(3) $\left(\frac{q_0 t}{2} - \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} + \frac{q_0}{\lambda^2 t} e^{-\lambda t}\right) E_0$

(4) $\left(\frac{q_0 t}{2} + \frac{q_0}{\lambda} + \frac{q_0}{\lambda^2 t} + \frac{q_0}{\lambda^2 t} e^{-\lambda t}\right) E_0$

**For Problems 7–9**

Various rules of thumb have been proposed by the scientific community to explain the mode of radioactive decay by various radioisotopes. One of the major rules is called the $n/p$ ratio. If all the known isotopes of the elements are plotted on a graph of number of neutrons ($n$) versus number of protons ($p$), it is observed that all isotopes lying outside of a "stable" $n/p$ ratio region are radioactive as shown in figure.

The graph exhibits straight line behavior with unit slope up to $p = 25$. Above $p = 25$, those isotopes with an $n/p$ ratio lying below the stable region usually undergo electron capture while those with $n/p$ ratios lying above the stable region usually undergo beta decay. Very heavy isotopes ($p > 83$) are unstable because of their relatively large nuclei and they undergo alpha decay. Gamma ray emission does not involve the release of a particle. It represents a change in an atom from a higher energy level to a lower energy level.




**7.** How would the radioisotope of magnesium with atomic mass 27 undergo radioactive decay?

(1) Electron capture (2) Alpha decay

(3) Beta decay (4) Gamma ray emission

**8.** Th-230 undergoes a series of radioactive decay processes resulting in Bi-214 being the final product. What was the sequence of the processes that occurred?

(1) $\alpha, \alpha, \alpha, \gamma, \beta$ (2) $\alpha, \alpha, \alpha, \alpha, \beta$

(3) $\alpha, \alpha, \beta, \beta$ (4) $\alpha, \beta, \beta, \beta, \gamma$

**9.** Which of the following represents the relative penetrating power of the three types of radioactive emission in decreasing order?

(1) $\beta > \alpha > \gamma$ (2) $\beta > \gamma > \alpha$

(3) $\gamma > \alpha > \beta$ (4) $\gamma > \beta > \alpha$

**For Problems 10–12**

The radionuclide $^{56}$Mn is being produced in a cyclotron at a constant rate $P$ by bombarding a manganese target with deuterons. $^{56}$Mn has a half-life of 2.5 h and the target contains large number of only the stable manganese isotopes $^{56}$Mn. The reaction that produces $^{56}$Mn is

$$^{56}\text{Mn} + \text{d} \rightarrow {}^{56}\text{Mn} + \text{p}$$

After being bombarded for a long time, the activity of $^{56}$Mn becomes constant, equal to $13.86 \times 10^{10}$ s$^{-1}$. (Use ln 2 = 0.693; Avagardo number = $6 \times 10^2$; atomic weight of $^{56}$Mn = 56 g mol$^{-1}$.)

**10.** At what constant rate $P$, $^{56}$Mn nuclei are being produced in the cyclotron during the bombardment?

(1) $2 \times 10^{11}$ nuclei s$^{-1}$ (2) $13.86 \times 10^{10}$ nuclei s$^{-1}$

(3) $9.6 \times 10^{10}$ nuclei s$^{-1}$ (4) $6.93 \times 10^{10}$ nuclei s$^{-1}$

**11.** After the activity of $^{56}$Mn becomes constant, number of $^{56}$Mn nuclei present in the target is equal to

(1) $5 \times 10^{11}$ (2) $20 \times 10^{11}$

(3) $1.2 \times 10^{14}$ (4) $1.8 \times 10^{15}$

**12.** After a long time bombardment, number of $^{56}$Mn nucle present in the target depends upon

(i) the number of $^{56}$Mn nuclei present at the start of the process

(ii) half-life of the $^{56}$Mn

(iii) the constant rate of production $P$

(1) All (i), (ii), and (iii) are correct.

(2) Only (i) and (ii) are correct.

(3) Only (ii) and (iii) are correct.

(4) Only (i) and (iii) are correct.

**For Problems 13–15**

Many unstable nuclei can decay spontaneously to a nucleus of lower mass but different combination of nucleons. The process of spontaneous emission of radiation is called radioactivity. Three types of radiations are emitted by radioactive substance. Radioactive decay is a statistical process. Radioactivity is independent of all external conditions. The number of decays per unit time or decay rate is called activity. Activity exponentially decreases with time. Mean lifetime is always greater than half-life time.

13. Choose the correct statement about radioactivity:
(1) Radioactivity is a statistical process.
(2) Radioactivity is independent of high temperature and high pressure.
(3) When a nucleus undergoes $\alpha$- or $\beta$-decay, its atomic number changes.
(4) All of these.

14. If $T_H$ is the half-life and $T_M$ is the mean life. Which of the following statement is correct.
(1) $T_M > T_H$
(2) $T_M < T_H$
(3) Both are directly proportional to square of the decay constant.
(4) $T_M \propto \lambda_0$.

15. $n$ number of $\alpha$-particles per second are being emitted by $N$ atoms of a radioactive element. The half-life of element will be
(1) $\frac{n}{N}$ s
(2) $\frac{N}{n}$ s
(3) $\frac{0.693N}{n}$ s
(4) $\frac{0.693n}{N}$ s

**For Problems 16–18**

All nuclei consist of two types of particles—protons and neutrons. Nuclear force is the strongest force. Stability of nucleus is determined by the neutron–proton ratio or mass defect or binding energy per nucleus or packing fraction. Shape of nucleus is calculated by quadrupole moment. Spin of nucleus depends on even or odd mass number. Volume of nucleus depends on the mass number. Whole mass of the atom (nearly 99%) is centered at the nucleus. Magnetic moment of the nucleus is measured in terms of the nuclear magnetons.

16. The correct statements about nuclear force is/are
(1) charge independent
(2) short-range force
(3) non-conservative force
(4) spin-dependent force

17. Binding energy per nucleon is maximum
(1) for lighter order element (low mass number)
(2) for heavier order elements (high mass number)
(3) for middle order elements
(4) equal for all order elements

18. Volume ($V$) of the nucleus is related to mass number ($A$) as
(1) $V \propto A^2$
(2) $V \propto A^{1/3}$
(3) $V \propto A^{2/3}$
(4) $V \propto A$

**For Problems 19–22**

When subatomic particles undergo reactions, energy is conserved, but mass is not necessarily conserved. However, a particle's mass "contributes" to its total energy, in accordance with Einstein's famous equation, $E = mc^2$.

In this equation, $E$ denotes the energy a particle carries because of its mass. The particle can also have additional energy due to its motion and its interactions with other particles.

Consider a neutron at rest, and well separated from other particles. It decays into a proton, an electron, and an undetected third particle:

Neutron → proton + electron + ???

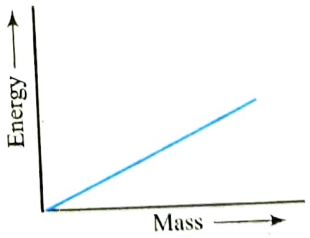
The table below summarizes some data from a single neutron decay. An MeV (mega electron volt) is a unit of energy. Column 2 shows the rest mass of the particle times the speed of light squared.

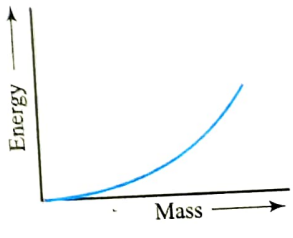
| Particle | Mass × $c^2$ (MeV) | Kinetic energy (MeV) |
|---|---|---|
| Neutron | 940.97 | 0.00 |
| Proton | 939.67 | 0.01 |
| Electron | 0.51 | 0.39 |

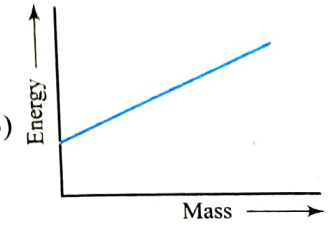
19. Assuming the table contains no major errors, what can we conclude about the (mass × $c^2$) of the undetected third particle?
(1) It is 0.79 MeV.
(2) It is 0.39 MeV.
(3) It is less than or equal to 0.79 MeV; but we cannot be more precise.
(4) It is less than or equal to 0.39 MeV; but we cannot be more precise.

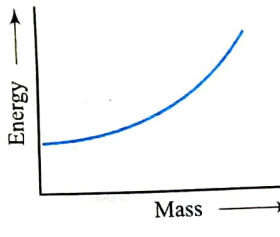
20. From the given table, which properties of the undetected third particle can we calculate?
(1) Total energy, but not kinetic energy.
(2) Kinetic energy, but not total energy.
(3) Both total energy and kinetic energy.
(4) Neither total energy nor kinetic energy.

21. Consider an ensemble of particles, all of which have the same positive kinetic energy but different masses. For this ensemble, which graph best represents the relationship between the particle's mass and its total energy?

(1) 


(2) 


(3) 


(4) 

**22.** Could this reaction occur?

Proton → neutron + other particles

(1) Yes, if the other particles have much more kinetic energy than mass energy.

(2) Yes, but only if the proton has potential energy (due to interactions with other particles).

(3) No, because a neutron is more massive than a proton.

(4) No, because a proton is positively charged while a neutron is electrically neutral.

**For Problems 23–26**

The compound unstable nucleus $^{236}_{92}U$ often decays in accordance with the following reaction:

$$^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + \text{other particles}$$

During the reaction, the uranium nucleus "fissions" (splits) into the two smaller nuclei. The reaction is energetically favorable because the small nuclei have higher nuclear binding energy per nucleon (although the lighter nuclei have lower total nuclear binding energies, because they contain fewer nucleons).

Inside a nucleus, the nucleons (protons and neutrons) attract each other with a "strong nuclear" force. All nucleons exert approximately the same strong nuclear force on each other. This force holds the nucleus together. Importantly, the strong nuclear force becomes important only when the protons and neutrons are very close together at intranuclear distances.

**23.** In the nuclear reaction presented above, the "other particles" might be

(1) an alpha particle, which consists of two protons and neutrons

(2) two protons

(3) one proton and one neutron

(4) two neutrons

**24.** Why is a $^{4}_{2}He$ nucleus more stable than a $^{4}_{3}Li$ nucleus?

(1) The strong nuclear force is larger when the neutron-to-proton ratio is higher.

(2) The laws of nuclear physics forbid a nucleus from containing more protons than neutrons.

(3) Forces other than the strong nuclear force make the lithium nucleus less stable.

(4) None of the above.

**25.** A proton and a neutron are both shot at 100 m $s^{-1}$ toward a $^{12}_{6}C$ nucleus. Which particle, if either, is more likely to be absorbed by the nucleus?

(1) The proton.

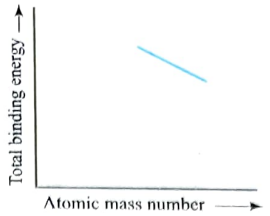
(2) The neutron.

(3) Both particles are about equally likely to be absorbed.

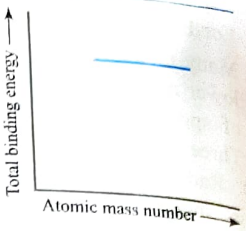
(4) Neither particle will be absorbed.

**26.** Which of the following graphs might represent the relationship between atomic number (i.e., "atomic weight") and the total binding energy of the nucleus, for nuclei heavier than $^{94}_{38}Sr$?

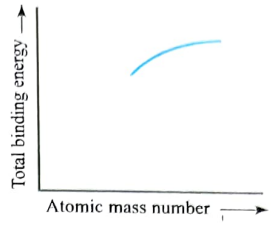
(1)



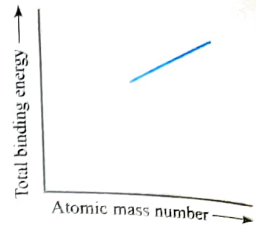

(2)




(3)




(4)




**For Problems 27–29**

A beam of alpha particles is incident on a target of lead. A particular alpha particle comes in 'head-on' to a particular lead nucleus and stops $6.50 \times 10^{-14}$ m away from the center of the nucleus. (This point is well outside the nucleus.) Assume that the lead nucleus, which has 82 protons, remains at rest. The mass of alpha particle is $6.64 \times 10^{-27}$ kg.

**27.** Calculate the electrostatic potential energy at the instant when the alpha particle stops?

(1) 36.3 MeV (2) 45.0 MeV

(3) 3.63 MeV (4) 40.0 MeV

**28.** What initial kinetic energy (in joule and in MeV) did the alpha particle have?

(1) 36.3 (2) 0.36

(3) 3.63 (4) 2.63

**29.** What was the initial speed of the alpha particle?

(1) $132 \times 10^{2}$ m $s^{-1}$ (2) $1.32 \times 10^{7}$ m $s^{-1}$

(3) $13.2 \times 10^{2}$ m $s^{-1}$ (4) $0.13 \times 10^{7}$ m $s^{-1}$

**For Problems 30–32**

A nucleus, kept at rest in free space, break up into two smaller nuclei of masses $m$ and $2m$. Total energy generated in this fission is $E$. The bigger part is radioactive, emits five gamma ray photons in the direction opposite to its velocity, and finally comes to rest. Now, answer the following questions: (given: $h = 6.6 \times 10^{-34}$ J s, $m = 1.00 \times 10^{-26}$ kg, $E = 3.63 \times 10^{-8}\ mc^2$, $c = 3 \times 10^{8}$ m $s^{-1}$)

**30.** Fractional loss of mass in the fission is

(1) $1.21 \times 10^{-8}$ (2) $2.56 \times 10^{-8}$

(3) $1.73 \times 10^{-8}$ (4) $3.52 \times 10^{-8}$

**31.** Velocity of smaller daughter nucleus is

(1) $5.6 \times 10^{4}$ m $s^{-1}$

(2) $6.6 \times 10^{4}$ m $s^{-1}$

(3) $7.6 \times 10^{4}$ m $s^{-1}$

(4) $8.6 \times 10^{4}$ m $s^{-1}$

**32.** The wavelength of the gamma ray is

(1) 0.02 Å (2) 0.03 Å

(3) 0.04 Å (4) 0.05 Å

**For Problems 33–35**

The results of activity measurements on a radioactive sample are given in the table below.

| Time (h) | Decays ($s^{-1}$) |
|---|---|
| 0 | 20000 |
| 0.5 | 14800 |
| 1.0 | 11000 |
| 1.5 | 8130 |
| 2.0 | 6020 |
| 2.5 | 4460 |
| 3.0 | 3300 |
| 4.0 | 1810 |
| 5.0 | 1000 |
| 6.0 | 550 |
| 7.0 | 300 |

33. The half-life of the radioactive nuclei is nearly ($\ln 2 = 0.693$, $\ln 3 = 1.0986$)
(1) 2.5 h (2) 7 h
(3) 5 h (4) 1.2 h

34. The number of radioactive nuclei that were present in the sample at $t = 0$ is
(1) $2 \times 10^4$ (2) $2 \times 10^8$
(3) $1.25 \times 10^8$ (4) $1.25 \times 10^4$

35. The number of radioactive nuclei that were present after 7 h is
(1) $2 \times 10^4$ (2) $1.25 \times 10^8$
(3) $1.875 \times 10^6$ (4) $1.25 \times 10^4$

## Matrix Match Type

1. Four physical quantities are given in Column I and their order of values in Column II. Match appropriately.

| Column I | Column II |
|---|---|
| i. Thermal energy of air molecules at room temperature | a. 0.02 eV |
| ii. Binding energy of heavy nuclei per nucleon | b. 2 eV |
| iii. X-ray photon energy | c. 10 keV |
| iv. Photon energy of visible light | d. 7 MeV |

2. Match the Column I of properties with Column II of reactions

| Column I | Column II |
|---|---|
| i. Mass of products formed is less than the original mass of the system in | a. $\alpha$-decay |
| ii. Binding energy per nucleon increase in | b. $\beta$-decay |
| iii. Mass number is conserved in | c. Nuclear fission |
| iv. Charge number is conserved in | d. Nuclear fusion |

3. In Column I consider each process just before and just after it occurs. Initial system is isolated from all other bodies. Consider all product particles (even those having rest mass zero) in the system. Match the system in Column I with the result they produce in Column II:

| Column I | Column II |
|---|---|
| i. Spontaneous radioactive decay of a uranium nucleus initially at rest as given by reaction ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He + \cdots$ | a. Number of protons is increased |
| ii. Fusion reaction of two hydrogen nuclei as given by reaction ${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + \cdots$ | b. Momentum is conserved |
| iii. Fission of $U^{235}$ nucleus initiated by a thermal neutron as given by reaction ${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}K + 3{}^{1}_{0}n + \cdots$ | c. Mass is converted to energy or vice versa |
| iv. $\beta$-decay (negative beta decay) | d. Charge is conserved |

4.

| Column I | Column II |
|---|---|
| i. Photoelectric effect | a. Photon |
| ii. Wave | b. Frequency |
| iii. X-rays | c. K capture |
| iv. Nucleus | d. $\gamma$-rays |

5.

| Column I | Column II |
|---|---|
| i. Binding energy per nucleon for middle order of element is | a. Shell model |
| ii. Nuclear force depends on | b. 8.8 MeV |
| iii. For nuclear fission, $\frac{Z^2}{A}$ is | c. 2.5 eV |
| iv. Magic numbers 2, 8, 20, 28, 50, 82, 126 are explained by | d. Spin of nucleons |
| | e. Greater than 15 |

6.

| Column I | Column II |
|---|---|
| i. Stability of nucleus decided by | a. –ve |
| ii. Four radioactive substance spontaneously decays because its | b. Binding energy per nucleon is minimum |
| iii. For the stable orbit or bound orbit, total energy is | c. Neutron–proton ratio |
| iv. Stopping potential | d. Packing fraction |
| | e. Mass defect |

7.

| Column I | Column II |
|---|---|
| i. Nuclear fusion | a. Satisfies $E = mc^2$ |
| ii. Nuclear fission | b. Generally possible for nuclei with low atomic number |
| iii. $\beta$-decay | c. Generally possible for nuclei with higher atomic number and unstable |
| iv. Exothermic nuclear reaction | d. Essentially proceeds by weak nuclear forces |
| | e. Significant momentum conservation |

8. In Column I some of the nuclear reactions are given. Match this with the energy involved in these reactions in Column II.

| Column I | Column II |
|---|---|
| i. ${}_1^2H+{}_1^2H\rightarrow{}_1^3H+{}_1^1H+E_1$ | a. 3.3 MeV |
| ii. ${}_1^3H+{}_1^2H\rightarrow{}_2^4He+{}_0^1n+E_2$ | b. 18.3 MeV |
| iii. ${}_1^2H+{}_1^2H\rightarrow{}_2^3He+{}_0^1n+E_3$ | c. 4 MeV |
| iv. ${}_2^3H+{}_1^2H\rightarrow{}_2^4He+{}_1^1H+E_4$ | d. 17.6 MeV |
| | e. 200 MeV |

9. Before the neutrino hypothesis, the beta decay process was thought to be the transition: $n \rightarrow p + e^-$

If the neutron was at rest, the proton and electron would emerge with fixed energies. Given $p$ is the magnitude of the momentum of the electron. ($m_pc^2$ = 936 MeV, $m_nc^2$ = 938 MeV, $m_ec^2$ = 0.51 MeV). Now match the List-I and List-II.

| Column I | Column II |
|---|---|
| i. $m_pc^2+\frac{p^2c^2}{2m_p^2c^4}+pc$ | a. 936 MeV |
| ii. $E_p$ (energy of the proton) | b. 2.06 MeV |
| iii. $E_e$ (energy of the electron) | c. 2.00 MeV |
| iv. $pc$ | d. 938 MeV |

**Codes:**

| | i. | ii. | iii. | iv. |
|---|---|---|---|---|
| (1) | d | b | c | a |
| (2) | c | a | b | d |
| (3) | d | a | b | c |
| (4) | c | d | b | a |

## Numerical Value Type

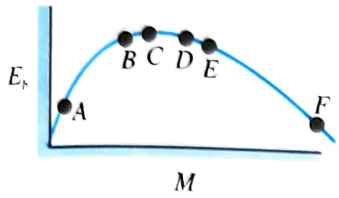
1. Nuclei $A$ and $B$ convert into a stable nucleus $C$. Nucleus $A$ is converted into $C$ by emitting 2 $\alpha$-particles and 3$\beta$-particles. Nucleus $B$ is converted into $C$ by emitting one $\alpha$-particle and 5 $\beta$-particles. At time $t = 0$, nuclei of $A$ are $4N_0$ and nuclei of $B$ are $N_0$. Initially, number of nuclei of $C$ are zero. Half-life of $A$ (into conversion of $C$) is 1 min and that of $B$ is 2 min. Find the time (in minutes) at which rate of disintegration of $A$ and $B$ are equal.
2. The half-life of a radioactive nuclide is 20 h. It is found that the fraction ($1/x$) of original activity remains after 40 hours? What is the value of $x$?
3. A radioactive sample has $8.0 \times 10^{18}$ active nuclei at a certain instant. How many of these nuclei will still be in the active state after two half-lives (in $\times 10^{18}$)?
4. A radioactive sample decays with an average-life of 20 ms. A capacitor of capacitance 100 $\mu$F is charged to some potential and then the plates are connected through a resistance $R$. What should be the value of $R$ (in $\times 10^2\ \Omega$) so that the ratio of the charge on the capacitor to the activity of the radioactive sample remains constant in time?
5. A radioactive sample decays through two different decay processes $\alpha$-decay and $\beta$-decay. Half-life time for $\alpha$-decay is 3 h and half-life time for $\beta$-decay is 6 h. What will be the ratio of number of initial radioactive nuclei to the number of radioactive nuclei present after 6 h?
6. ${}_{92}U^{238}$ changes to ${}_{85}At^{210}$ by a series of $\alpha$-and $\beta$-decays. Find the number of $\alpha$-decays undergone (an integer).
7. A certain radioactive material can undergo three different types of decay, each with a different decay constant $\lambda$, $2\lambda$, and $3\lambda$. Then, the effective decay constant $\lambda_{eff}$ is equal to $n\lambda$. What is the value of $n$?
8. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. Find the minimum time (in hrs) after which it would be possible to work safely.
9. A radioactive sample $S_1$ having an activity of 5 $\mu$Ci has twice the number of nuclei as another sample $S_2$ which has an activity of 10 $\mu$Ci. Find the ratio of the half lives of $S_1$ and $S_2$.
10. Nuclei X and Y convert into a stable nucleus Z. At $t = 0$ the number of nuclei of X is 8 times that of Y. Half life of X is 1 hour and half life of Y is 2 hour. Nucleus X is converted into Z by emitting 4$\alpha$ and 8$\beta$ particles while nucleus Y is converted into Z by emitting 2$\alpha$ and 4$\beta$ particles. Find the time (in hour) at which rate of disintegration of X and Y are equal.
11. A sample of $^{18}F$ is used internally as a medical diagnostic tool to look for the effects of the positron decay ($T_{1/2}$ =110 min). How long(in hr) does it take for 99% of the $^{18}F$ to decay? (Give answer in nearest integer)
12. A neutron is scattered through (≡ deviation from its old direction) $\theta$ degree in an elastic collision with an initially stationary deutron. If the neutron loses 2/3 of its initial KE to the deutron then find the value of $\theta$ (in degree) (In atomic mass unit, the mass of a neutron is 1u and mass of a deutron is 2u).

# Archives

## JEE MAIN

### Single Correct Answer Type

1. The figure given below is a plot of binding energy per nucleon, $E_b$, against the nuclear mass $M$; $A$, $B$, $C$, $D$, $E$, and $F$ correspond to different nuclei. Consider four reactions:




(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$

(iii) $D + E \rightarrow F + \varepsilon$ (iv) $F \rightarrow D + E + \varepsilon$

Here $\varepsilon$ is the energy released. In which reaction $\varepsilon$ is positive?

(1) (i) and (iv) (2) (i) and (iii)

(3) (ii) and (iv) (4) (ii) and (iii)

**(AIEEE 2009)**

**Direction for Questions 2 and 3:**

These questions are based on the following paragraph.

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal masses $M/2$ each. The speed of light is $c$.

**(AIEEE 2010)**

2. The binding energy per nucleon for the parent nucleus is $E_1$ and that for the daughter nuclei is $E_2$. Then

(1) $E_2 = 2E_1$ (2) $E_1 > E_2$

(3) $E_2 > E_1$ (4) $E_1 = 2E_2$

3. The speed of daughter nuclei is

(1) $c\dfrac{\Delta m}{M + \Delta m}$ (2) $c\sqrt{\dfrac{2\Delta m}{M}}$

(3) $c\sqrt{\dfrac{\Delta m}{M}}$ (4) $c\sqrt{\dfrac{m}{\Delta M}}$

4. A radioactive nucleus (initial mass number $A$ and atomic number $Z$) emits three $\alpha$-particles and two positrons. The ratio of the number of neutrons to that of protons in the final nucleus will be

(1) $\dfrac{A - Z - 8}{Z - 4}$ (2) $\dfrac{A - Z - 4}{Z - 8}$

(3) $\dfrac{A - Z - 12}{Z - 4}$ (4) $\dfrac{A - Z - 4}{Z - 2}$

**(AIEEE 2010)**

5. The half-life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time $t_2$ when 2/3 of it has decayed and time $t_1$ when 1/3 of it had decayed is

(1) 7 min (2) 14 min

(3) 20 min (4) 28 min **(AIEEE 2011)**

6. Assume that a neutron breaks into a proton and an electron. The energy released during this process is: (mass of neutron $= 1.6725 \times 10^{-27}$ kg, mass of proton $= 1.6725 \times 10^{-27}$ kg, mass of electron $= 9 \times 10^{-31}$ kg)

(1) 0.73 MeV (2) 7.10 MeV

(3) 6.30 MeV (4) 5.4 MeV

**(AIEEE 2012)**

7. Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionised Helium $(_2He^4)^+$ and doubly ionised lithium $(_3Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wave lengths of emitted radiation are $\lambda_1$, $\lambda_2$, $\lambda_3$ and $\lambda_4$ respectively then approximately which one of the following is correct?

(1) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (2) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

(3) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (4) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = \lambda_4$

**(JEE Main 2014)**

8. Half-lives of two radioactive elements $A$ and $B$ are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of $A$ and $B$ nuclei will be

(1) 1 : 16 (2) 4 : 1

(3) 1 : 4 (4) 5 : 4

**(JEE Main 2016)**

9. A radioactive nucleus $A$ with a half-life $T$, decays into a nucleus $B$. At $t = 0$, there is no nucleus $B$. At some time $t$, the ratio of the number of $B$ to that of $A$ is 0.3. Then, $t$ is given by

(1) $t = T \log(1.3)$ (2) $t = \dfrac{T}{\log(1.3)}$

(3) $= -\dfrac{\log 2}{2 \log 1.3}$ (4) $t = T\dfrac{\log 1.3}{\log 2}$

**(JEE Main 2017)**

## JEE ADVANCED

### Single Correct Answer Type

1. An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use ?

(1) 64 (2) 90

(3) 108 (4) 120

**(JEE Advanced 2016)**

2. The electrostatic energy of $Z$ protons uniformly distributed throughout a spherical nucleus of radius $R$ is given by

$$E = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

The measured masses of the neutron, $^1_1H$, $^{15}_7N$, and $^{15}_8O$ are 1.008665 u, 1.007825 u, 15.000109 u and 15.003065 u, respectively. Given that the radii of both the $^{15}_7N$ and $^{15}_8O$ nuclei are same, 1 u = 931.5 MeV/$c^2$ ($c$ is the speed of light)

and $e^2/(4\pi\varepsilon_0) = 1.44$ MeV fm. Assuming that the difference between the binding energies of ${}^{15}_{7}\text{N}$ and ${}^{15}_{8}\text{O}$ is purely due to the electrostatic energy, the radius of either of the nuclei is (1 fm = $10^{-15}$ m)

(1) 2.85 fm (2) 3.03 fm
(3) 3.42 fm (4) 3.80 fm

**(JEE Advanced 2016)**

## Multiple Correct Answers Type

1. In a radioactive decay chain, ${}^{232}_{90}\text{Th}$ nucleus decays to ${}^{212}_{82}\text{Pb}$ nucleus. Let $N_\alpha$ and $N_\beta$ be the number of $\alpha$ and $\beta$ particles, respectively, emitted in this decay process. Which of the following statements is (are) true ?

(1) $N_\alpha = 5$ (2) $N_\alpha = 6$
(3) $N_\beta = 2$ (4) $N_\beta = 4$

**(JEE Advanced 2018)**

## Linked Comprehension Type

### For Problems 1–3

Scientists are working hard to develop nuclear fusion reactor. Nuclei of heavy hydrogen, ${}^{2}_{1}\text{H}$, known as deuteron and denoted by D can be thought of as a candidate for fusion reactor. The D–D reaction is ${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + n +$ energy. In the core of fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}^{2}_{1}\text{H}$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time $t_0$ before the particles fly away from the core. If $n$ is the density (number/volume) of deuterons, the product $nt_0$ is called Lawson number. In one of the criteria, a reactor is termed successful if Lawson number is greater than $5 \times 10^{14}$ s cm$^{-3}$.

It may be helpful to use the following: Boltzmann constant, $k = 8.6 \times 10^{-5}$ eV K$^{-1}$; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eV m.

**(IIT-JEE 2009)**

1. In the core of nuclear fusion reactor, the gas becomes plasma because of
   (1) strong nuclear force acting between the deuterons
   (2) Coulomb force acting between the deuterons
   (3) Coulomb force acting between deuteron–electron pairs
   (4) the high temperature maintained inside the reactor core
2. Assume that two deuteron nuclei in the core of fusion reactor at temperature $T$ are moving toward each other, each with kinetic energy $1.5kT$, when the separation between them is large enough to neglect Coulomb potential energy. Also, neglect any interaction from other particles in the core. The minimum temperature $T$ required for them to reach a separation of $4 \times 10^{-15}$ m is in the range
   (1) $1.0 \times 10^9$ K $< T < 2.0 \times 10^9$ K
   (2) $2.0 \times 10^9$ K $< T < 3.0 \times 10^9$ K
   (3) $3.0 \times 10^9$ K $< T < 4.0 \times 10^9$ K
   (4) $4.0 \times 10^9$ K $< T < 5.0 \times 10^9$ K
3. Results of calculations for four different designs of a fusion reactor using D–D reaction are given below. Which of these is most promising based on Lawson criterion?
   (1) Deuteron density = $2.0 \times 10^{12}$ cm$^{-3}$, confinement time = $5.0 \times 10^{-3}$ s
   (2) Deuteron density = $8.0 \times 10^{14}$ cm$^{-3}$, confinement time = $9.0 \times 10^{-1}$ s
   (3) Deuteron density = $4.0 \times 10^{23}$ cm$^{-3}$, confinement time = $1.0 \times 10^{-11}$ s
   (4) Deuteron density = $1.0 \times 10^{24}$ cm$^{-3}$, confinement time = $4.0 \times 10^{-12}$ s

### For Problems 4 and 5

The $\beta$-decay process, discovered around 1900, is basically the decay of a neutron ($n$). In the laboratory, a proton ($p$) and an electron ($e^-$) are observed as the decay products of the neutron. Therefore, considering the decay of a netron as a two body decay process, it was predicated theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has continuous spectrum. Considering a three-body decay process, i.e., $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is $0.8 \times 10^6$ eV. The kinetic energy carried by the proton is only the recoil energy. **(IIT-JEE 2012)**

4. If the anti-neutrino had a mass of 3 eV/$c^2$ (where $c$ is the speed of light) instead of zero mass, what should be the range of the kinetic energy, $K$ of the electron?
   (1) $0 \le K \le 0.8 \times 10^6$ eV
   (2) 3.0 eV $\le K \le 0.8 \times 10^6$ eV
   (3) 3.0 eV $\le K < 0.8 \times 10^6$ eV
   (4) $0 \le K < 0.8 \times 10^6$ eV
5. What is the maximum energy of the anti-neutrino?
   (1) Zero
   (2) Much less than $0.8 \times 10^6$ eV
   (3) Nearly $0.8 \times 10^6$ eV
   (4) Much larger than $0.8 \times 10^6$ eV

### For Problems 6 and 7

The mass of nucleus ${}^{A}_{Z}X$ is less than the sum of the masses of ($A$–$Z$) number of neutrons and $Z$ number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass $M$ can break into two light nuclei of mass $m_1$ and $m_2$ only if $(m) < M$. Also two light nuclei of masses $m_3$ and $m_4$ can undergo complete fusion and form a heavy nucleus of mass $M$ "only if $(m_3 + m_4) > M$". The masses of some neutral atoms are given in the table below: **(JEE Advanced 2013)**

| | | | |
|---|---|---|---|
| ${}^{1}_{1}\text{H}$ | 1.007825 u | ${}^{2}_{1}\text{H}$ | 2.014102 u |
| ${}^{3}_{1}\text{H}$ | 3.016050 u | He | 4.002603 u |
| ${}^{6}_{3}\text{Li}$ | 6.015123 u | ${}^{7}_{3}\text{Li}$ | 7.016004 u |

| | | | |
|---|---|---|---|
| $^{70}_{30}Zn$ | 69.925325 u | $^{82}_{34}Se$ | 81.916709 u |
| $^{152}_{64}Gd$ | 151.919803 u | $^{206}_{82}Pb$ | 205.974455 u |
| $^{209}_{83}Bi$ | 208.980388 u | $^{210}_{84}Po$ | 209.982876 u |

6. The correct statement is
(1) The nucleus $^{6}_{3}Li$ can emit an alpha particle
(2) The nucleus $^{210}_{84}Po$ can emit a proton
(3) Deuteron and alpha particle can undergo complete fusion
(4) The nuclei $^{70}_{30}Zn$ and $^{82}_{34}Se$ can undergo complete fusion

7. The kinetic energy (in keV) of the alpha particle, when the nucleus at rest undergoes alpha decay, is
(1) 5319 (2) 5422
(3) 5707 (4) 5818

## Matrix Match Type

1. Column II gives certain systems undergoing a process. Column I suggests changes in some of the parameters related to the system. Match the statements in Column I to the appropriate process(es) from Column II.

| Column I | Column II |
|---|---|
| a. The energy of the system is increased. | p. System: A capacitor, initially uncharged Process: It is connected to a battery |
| b. Mechanical energy is provided to the system, which is converted into energy of random motion of its parts. | q. System: A gas in an adiabatic container fitted with an adiabatic piston Process: The gas is compressed by pushing the piston |
| c. Internal energy of the system is converted into its mechanical energy. | r. System: A gas in a rigid container Process: The gas gets cooled due to colder atmosphere surrounding it |
| d. Mass of the system is decreased. | s. System: A heavy nucleus, initially at rest Process: The nucleus fissions into two fragments of nearly equal masses and some neutrons are emitted |
| | t. System: A resistive wire loop Process: The loop is placed in a time-varying magnetic field perpendicular to its plane |

**(IIT-JEE 2009)**

2. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists:

| List I | List II |
|---|---|
| a. Alpha decay | p. $^{15}_{8}O \rightarrow {}^{15}_{7}N + ....$ |
| b. $\beta+$ decay | q. $^{238}_{92}U \rightarrow {}^{234}_{90}Th + ....$ |
| c. Fission | r. $^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + ....$ |
| d. Proton-emission | s. $^{239}_{94}Pu \rightarrow {}^{140}_{57}La + ....$ |

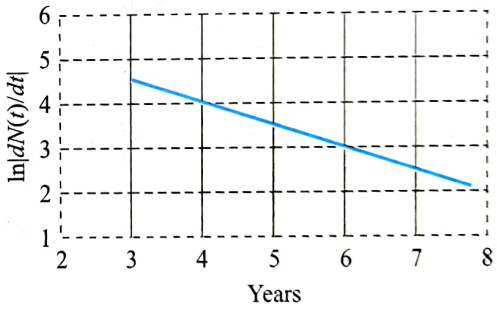
**Codes:**

| | a | b | c | d |
|---|---|---|---|---|
| (1) | s | q | p | r |
| (2) | p | r | q | s |
| (3) | q | p | s | r |
| (4) | s | r | q | p |

**(JEE Advanced 2013)**

## Numerical Value Type

1. To determine the half life of a radioactive element, student plots a graph of $\ln|dN(t)/dt|$ versus $t$. Here $dN(t)/dt$ is the rate of radioactive decay at time $t$. If the number of radioactive nuclei of this element decreases by a factor of $p$ after 4.16 years, the value of $p$ is _____. **(IIT-JEE 2010)**




2. The activity of a freshly prepared radioactive sample is $10^{10}$ disintegrations per second, whose mean life is $10^{9}$ s. The mass of an atom of this radioisotope is $10^{-25}$ kg. The mass (in mg) of the radioactive sample is _____. **(IIT-JEE 2011)**

3. A freshly prepared sample of a radioisotope of half-life 1386 s has activity $10^{3}$ disintegrations per second. Given that $\ln 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is

**(JEE Advanced 2013)**

4. The isotope $^{12}_{5}B$ having a mass 12.014 u undergoes $\beta$-decay to $^{12}_{6}C$. $^{12}_{6}C$ has an excited state of the nucleus ($^{12}_{6}C^{*}$) at 4.041 MeV above its ground state. If $^{12}_{5}B$ decays to $^{12}_{6}C^{*}$, the maximum kinetic energy of the $\beta$-particle in units of MeV is ($1u = 931.5\ MeV/c^2$, where $c$ is the speed of light in vacuum). **(JEE Advanced 2016)**

**5.** $^{131}I$ is an isotope of Iodine that β decays to an isotope of Xenon with a half-life of 8 days. A small amount of a serum labelled with $^{131}I$ is injected into the blood of a person. The activity of the amount of $^{131}I$ injected was $2.4 \times 10^5$ becquerel (Bq). It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After 11.5 hours, 2.5 ml of blood is drawn from the person's body, and gives an activity of 115 Bq. The total volume of blood in the person's body, in liters is approximately (you may use $e^x \approx 1 + x$ for $|x| << 1$ and $\ln 2 \approx 0.7$). **(JEE Advanced 2017)**

# Answers Key

## EXERCISES

### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| **1.** (4) | **2.** (3) | **3.** (1) | **4.** (4) | **5.** (1) |
| **6.** (2) | **7.** (1) | **8.** (2) | **9.** (3) | **10.** (2) |
| **11.** (3) | **12.** (2) | **13.** (2) | **14.** (1) | **15.** (3) |
| **16.** (3) | **17.** (1) | **18.** (3) | **19.** (3) | **20.** (1) |
| **21.** (1) | **22.** (1) | **23.** (1) | **24.** (2) | **25.** (3) |
| **26.** (4) | **27.** (3) | **28.** (1) | **29.** (3) | **30.** (1) |
| **31.** (2) | **32.** (1) | **33.** (1) | **34.** (1) | **35.** (2) |
| **36.** (4) | **37.** (3) | **38.** (2) | **39.** (2) | **40.** (4) |
| **41.** (1) | **42.** (3) | **43.** (2) | **44.** (4) | **45.** (1) |
| **46.** (2) | **47.** (4) | **48.** (2) | **49.** (3) | **50.** (2) |
| **51.** (2) | **52.** (4) | **53.** (3) | **54.** (4) | **55.** (1) |
| **56.** (1) | **57.** (1) | **58.** (4) | **59.** (4) | **60.** (1) |
| **61.** (4) | **62.** (3) | **63.** (2) | **64.** (3) | **65.** (2) |
| **66.** (1) | **67.** (4) | **68.** (3) | **69.** (4) | **70.** (1) |
| **71.** (3) | **72.** (2) | **73.** (1) | **74.** (3) | **75.** (2) |
| **76.** (2) | **77.** (2) | **78.** (2) | **79.** (3) | **80.** (3) |
| **81.** (2) | **82.** (2) | **83.** (3) | **84.** (4) | **85.** (2) |
| **86.** (4) | **87.** (4) | **88.** (3) | **89.** (2) | **90.** (3) |
| **91.** (3) | **92.** (3) | **93.** (4) | **94.** (3) | **95.** (4) |
| **96.** (3) | **97.** (4) | **98.** (3) | | |

### Multiple Correct Answers Type

| | | |
|---|---|---|
| **1.** (1),(2),(3),(4) | **2.** (1),(2),(4) | **3.** (1),(3),(4) |
| **4.** (1),(4) | **5.** (1),(2),(3) | **6.** (1),(2),(3) |
| **7.** (1),(3) | **8.** (1),(3) | **9.** (1),(4) |
| **10.** (2),(3),(4) | **11.** (1),(3) | **12.** (1),(2),(3) |
| **13.** (2),(3) | **14.** (1),(2),(3) | **15.** (1),(4) |
| **16.** (1),(2),(3) | **17.** (1),(3) | **18.** (1),(4) |
| **19.** (3),(4) | **20.** (2),(4) | **21.** (1),(3) |
| **22.** (1),(3) | **23.** (2),(3) | |

### Linked Comprehension Type

| | | | | |
|---|---|---|---|---|
| **1.** (1) | **2.** (2) | **3.** (3) | **4.** (1) | **5.** (1) |
| **6.** (1) | **7.** (3) | **8.** (2) | **9.** (4) | **10.** (2) |
| **11.** (4) | **12.** (3) | **13.** (4) | **14.** (1) | **15.** (3) |
| **16.** (1),(2),(3),(4) | | **17.** (3) | **18.** (4) | **19.** (4) |
| **20.** (1) | **21.** (3) | **22.** (2) | **23.** (4) | **24.** (1) |
| **25.** (2) | **26.** (3) | **27.** (3) | **28.** (3) | **29.** (2) |
| **30.** (1) | **31.** (2) | **32.** (4) | **33.** (4) | **34.** (3) |
| **35.** (3) | | | | |

### Matrix Match Type

**1.** i. → a.; ii. → d.; iii. → c.; iv. → b.
**2.** i.→ a., b., c., d.; ii.→ a., b., c., d.; iii. → a., b., c., d.; iv. → a., b., c., d.
**3.** i. → b., c., d.; ii. → b., c., d.; iii. → b., c., d.; iv. → a., c., d.
**4.** i. → a.; ii. → b.; iii. → c.; iv. → d.
**5.** i. → a.; ii. → d.; iii. → e.; iv. → b.
**6.** i. → c., d., e.; ii. → b.; iii. → a.; iv. → a.
**7.** i. → a., b., e.; ii. → a., c.; iii. → d.; iv. → b., d.
**8.** i. → c.; ii. → d.; iii. → a.; iv. → b.
**9.** (3)

### Numerical Value Type

| | | | | |
|---|---|---|---|---|
| **1.** (6) | **2.** (4) | **3.** (2) | **4.** (2) | **5.** (8) |
| **6.** (7) | **7.** (6) | **8.** (12) | **9.** (4) | **10.** (8) |
| **11.** (12) | **12.** (90) | | | |

## ARCHIVES

### JEE Main

#### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| **1.** (1) | **2.** (3) | **3.** (2) | **4.** (2) | **5.** (3) |
| **6.** (None) | **7.** (1) | **8.** (4) | **9.** (4) | |

### JEE Advanced

#### Single Correct Answer Type

**1.** (3) **2.** (3)

#### Multiple Correct Answers Type

**1.** (1),(3)

#### Linked Comprehension Type

| | | | | |
|---|---|---|---|---|
| **1.** (4) | **2.** (1) | **3.** (2) | **4.** (4) | **5.** (3) |
| **6.** (3) | **7.** (1) | | | |

#### Matrix Match Type

**1.** a. → p., q., t.; b. → q.; c. → s.; d. → s. **2.** (3)

#### Numerical Value Type

| | | | | |
|---|---|---|---|---|
| **1.** (8) | **2.** (1) | **3.** (4) | **4.** (9) | **5.** (5) |

# 8 Semiconductor Devices and Digital Circuits

## INTRODUCTION

Electronics is the discipline which deals with the development and application of devices and systems involving the flow of electrons in a vacuum, in gaseous media, and in semiconductors. Electronic devices are components for controlling the flow of electrical currents for the purpose of information processing and system control. Prominent examples include transistors and diodes. Electronic devices are usually small and can be grouped together into packages called integrated circuits. This miniaturization is central to the modern electronics boom. Electronics-based techniques have advanced our knowledge about the properties of many substances existing in nature, provided a deeper insight into the structure of matter, and have brought us closer to a proper understanding of the laws that govern the material world.

## BAND THEORY OF SOLIDS

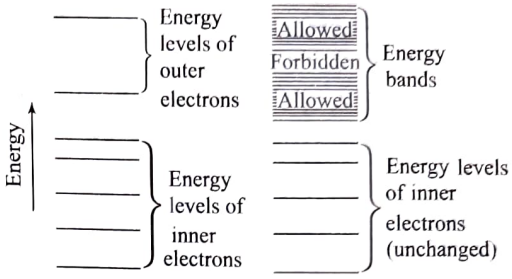
Based on electrical conductivities of materials, we classified different materials into three different categories: conductors, insulators and semiconductors. **Conductors** such as gold, copper, silver and other metals are good conductors of electricity. **Insulators** such as wood, diamond and quartz are poor conductors of electricity. **Semiconductors** are those which have electrical conductivities between conductors and insulators. Two typical examples are silicon (Si) and germanium (Ge). How this classification comes about can be understood according to the **band theory of solids**.

### ENERGY BANDS IN SOLIDS

In an isolated free atom, electrons have well defined energy levels. If we talk about crystal, the atoms do not have well defined energy levels. It is because, in a crystal, there are many atoms and each atom in the crystal is surrounded by neighboring atoms and they interact with one another through their electric fields. Due to the electric interaction, the energy levels of these outer electrons are slightly modified. However, the so-called modification in the energy levels is not appreciable in case of the electrons in the inner shells but it is considerable in case of electrons in the outermost shell. It is because, electrons in the outermost shell are shared by more than one atom in the crystal.

Due to the electric interaction, the energy levels of these outer electrons are slightly altered. Consequently, in place of sharply defined energy levels, we obtain a large number of levels which are lying very close to each other as shown in figure.

A group of large number of closely spaced energy level is called the **energy band**.

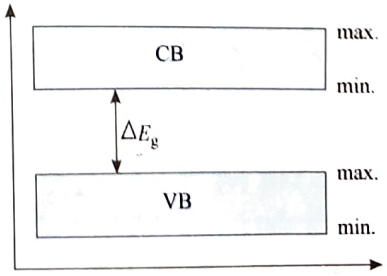



Energy bands are of the following types:

(1) **Valence band:** The energy band formed by a series of energy levels containing valence electrons is known as valence band. At 0 K, the electrons start filling the energy levels in valence band starting from the lowest one.
The highest energy level, which an electron can occupy in the valence band at 0 K, is called **Fermi level**.
   (i) This band is always filled with electrons.
   (ii) This is the band of maximum energy.
   (iii) Electrons are not capable of gaining energy from external electric field.
   (iv) No flow of current due to electrons present in this band.

(2) **Conduction band:** The lowest unfilled energy band formed just above the valence band is called **conduction band**.
   (i) This band is partially filled by the electrons.
   (ii) In this band the electrons can gain energy from external electric field.
   (iii) The electrons in the conduction band are called the free electrons. They are able to move any where within the volume of the solid.
   (iv) Current flows due to such electrons.

(3) **Forbidden energy gap ($\Delta E_g$):** Energy gap between conduction band and valence band $\Delta E_g = (CB)_{min} - (VB)_{max}$




   (i) No free electron is present in forbidden energy gap.
   (ii) Width of forbidden energy gap depends upon the nature of substance.
   (iii) As temperature increases (↑), forbidden energy gap decreases (↓) very slightly.

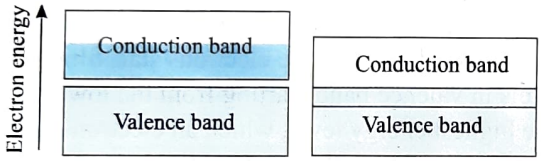
# CLASSIFICATION OF SOLIDS ON THE BASIS OF BAND THEORY

The electrical behaviour of conductors, insulators and semiconductors can be explained by concept of energy band. A material can be classified as a conductor, insulator or a semiconductor on the basis of its energy band diagram.

At 0 K, the Fermi level as well as all the lower energy levels are completely occupied by the electrons. As the temperature rises, the electrons absorb energy and get excited. The excited electrons jump to the higher energy levels. These electrons in the higher energy levels are comparatively at larger distances from the nucleus and are freer as compared to the electrons in the lower energy levels. Depending upon the energy gap between valence band and the conduction band, the solids behave as conductors, insulators and semiconductors as explained below.

(a) **Conductors:** The energy band structure in solids have two possibilities:

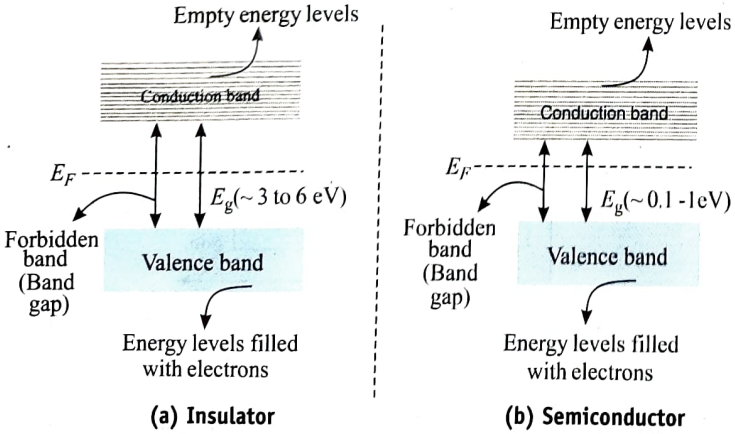
(i) The valence band may be completely filled and the conduction band partially filled with an extremely small energy gap between them [Fig. (a)]. For example, in sodium, the conduction band is partially filled, while the valence band is completely filled.




(a) (b)

(ii) The valence band is completely filled and the conduction band is empty but the two overlap each other [Fig. (b)]. Zinc is an example of band overlap metals. In both the situations, it can be assumed that there is a single energy band, which is partially filled. Therefore, on applying even a small electric field, the metals conduct electricity.

(b) **Insulators:** In insulators (or a dielectric), the forbidden energy gap is quite large (3eV to 6eV) [Fig. (a)]. For example, the forbidden energy gap for diamond is 6 eV, it means that a minimum of 6 eV energy is required to make the electron jump from the completely filled valence band to the conduction band. When electric field is applied across such a solid, the electrons find it difficult to acquire such a large amount of energy and so the conduction band continues to be almost empty. No electron flow occurs i.e., no current flows through such solids. So they behave as insulators.




(a) Insulator (b) Semiconductor

(c) **Semiconductors:** A solid is a semiconductor if the energy gap is much smaller as compared to that in the case of insulators. Its typical value is about 1 eV.

For example, the forbidden band for silicon is 1.1 eV. As a result of this small energy gap [Fig. (b)], the electrons in the valence band may acquire thermal energy and shift to the conduction band. Therefore, semiconductors also carry current but not as easily as the conductors do.

## CURRENT CARRIERS IN SEMICONDUCTORS

In case of a pure semiconductor, each atom acts as if there are 8 electrons in its valence shell (due to formation of covalent bonds), it means entire material behaves as an insulator at low temperatures.

As we know a semiconductor atom needs energy of the order of 1.1 eV to shake off the valence electron and this energy becomes available to the semiconductor even at room temperature. Due to thermal agitation of the crystal structure, when an electron is removed from a covalent bond, it leaves a vacancy behind. An electron from a neighboring atom can move into this vacancy, leaving the neighbor with a vacancy. In this way the vacancy formed is called hole and can travel through the material and serve as an additional current carrier.

This hole can be filled by some other electron in a covalent bond. As an electron from a covalent bond moves to fill the hole, the hole is created in the covalent bond from which the electron has moved. In other words one can say that the hole shifts from one covalent bond to another in a similar way as an electron does in an attempt to fill the hole. Since the direction of movement of the hole is opposite to that of the negative electron, a hole behaves as a positive charge carrier. In an applied electric field, the electrons move against the electric field while the holes move with the field. The electrical conduction of a semiconductor due to the motion of holes is called the intrinsic carriers.

**Important Points:**

- A hole is considered as a seat of positive charge, having magnitude of charge equal to that of an electron.
- Hole acts as virtual charge, although there is no physical charge on it.
- Effective mass of hole is more than electron.
- Mobility of hole is less than electron.

### ILLUSTRATION 8.1

Find the maximum wavelength of electromagnetic radiation, which can create a hole–electron pair in germanium. Given that forbidden energy gap in germanium is 0.72 eV.

**Sol.** Forbidden energy gap in germanium,

$$E_g = 0.72 \text{ eV} = 0.72 \times 1.6 \times 10^{-19} \text{ J}$$

The maximum wavelength of radiation, which can create a hole-electron pair in germanium is given by

$$E_g = \frac{hc}{\lambda}$$

or $$\lambda = \frac{hc}{E_g} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{0.72 \times 1.6 \times 10^{-19}} = 1.724 \times 10^{-6} \text{m}$$

**ILLUSTRATION 8.2**

A semiconductor object has acceptor energy levels 54 meV (millielectron volt) above the valence band. Find the maximum wavelength of the radiation, which can create a hole?

**Sol.** Energy required to create a hole = energy required to transfer an electron from the valence to conduction band

$= 54\ \text{meV} = 54\times10^{-3}\ \text{eV} = 54\times10^{-3}\times1.6\times10^{-19}\ \text{J}$

If $\lambda$ is the wavelength of the radiation which can create a hole,

then $\frac{hc}{\lambda} = 54\times10^{-3}\times1.6\times10^{-19}$

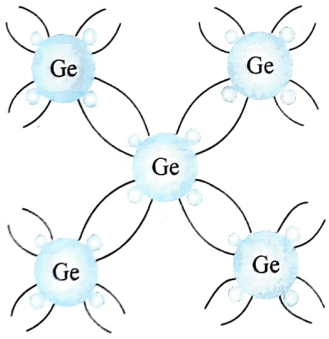
$\lambda = \frac{hc}{54\times10^{-3}\times1.6\times10^{-19}}$

$\lambda = \frac{6.62\times10^{-34}\times3\times10^{8}}{54\times10^{-3}\times1.6\times10^{-19}} = 2.3\times10^{-5}\ \text{m}$

## INTRINSIC SEMICONDUCTORS

A semiconductor free from impurities, i.e., a pure semiconductor is called an **intrinsic** or **i-type semiconductor**. It has thermally generated current carriers.

Germanium and silicon are the most widely used semiconductors. The four valence electrons in Ge (as well as in Si) form covalent bonds with the electrons of the neighboring atoms in the crystal lattice as shown in figure. Obviously, no free electron is left in the crystal. As a result of this, pure Ge (or Si) is a good insulator at absolute zero. But it has been experimentally shown that the electrical conductivity of pure semiconductors increases with temperature. This is a major distinction between a semiconductor and a metal because in the case of metals, conductivity decreases with temperature. Band theory can easily explain this increase in conductivity of semiconductors with temperature.




An intrinsic semiconductor has equal concentration of electrons and holes under conditions of thermal equilibrium, i.e., $n_e^{(o)} = n_h^{(o)} = n_i$; where $n_e^{(o)}$ = Electron density in conduction band, $n_h^{(o)}$ = Hole density in valence band, $n_i$ = Density of intrinsic carriers.

### INCREASING THE CONDUCTIVITY OF A SEMICONDUCTOR (DOPING)

In room temperature, a pure semiconductor possesses free electrons and holes but their number is so small that conductivity offered by the pure semiconductor cannot be made of any practical use.

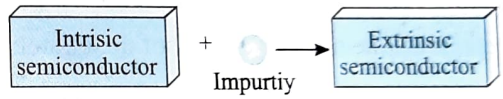
The conductivity of some semiconductors such as Si-crystal (or Ge-crystal) can be remarkably improved by adding impurities in a very small ratio (1:$10^6$). This process of adding impurity to a pure semiconductor crystal to improve its conductivity, is called doping. We can add two types of impurity atoms:

(i) **Trivalent impurity atoms:** These atoms have 3 valence electrons such as indium (In) or gallium (Ga), on being added to a pure semiconductor, instead of producing free electrons, accept electrons from the semiconductor. That is why, trivalent impurity atoms are called acceptor impurity atoms.

(ii) **Pentavalent impurity atoms:** These atoms having 5 valence electrons such as antimony (Sb) or arsenic (As). Such atoms, on being added to a pure semiconductor, produce excess of free electrons i.e., donate electrons to the semiconductor. For this reason, pentavalent impurity atoms are called donor impurity atoms.

## EXTRINSIC SEMICONDUCTORS

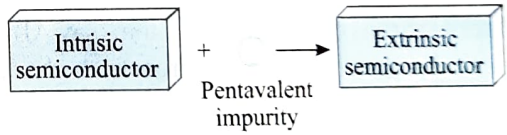
When an impurity is added to a semiconductor i.e., it is doped, it strongly affects its electrical properties. The impurity atoms may supply excess electrons or excess holes. The type of semiconductors whose conductivity is mainly due to impurities are called the extrinsic semiconductors.



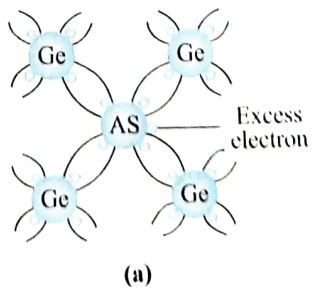

Extrinsic semiconductors are of two types.

### n-TYPE EXTRINSIC SEMICONDUCTOR

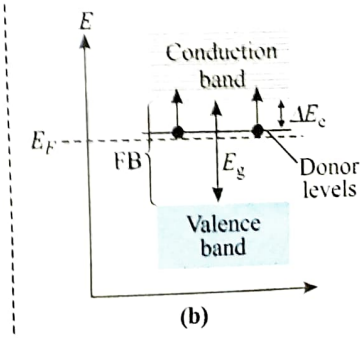
An n-type extrinsic semiconductor is obtained by adding a small amount of pentavalent impurity to a pure sample of semiconductor. This may be phosphorous, arsenic or antimony to the Ge (or Si) lattice.




Four electrons of the impurity atoms (say As) form covalent bonds with the electrons of the adjacent Ge atoms while the fifth electron cannot do so and as such is surplus as shown in Fig. (a). This extra free electrons created in the crystal will be as many as the number of the pentavalent impurity atoms added. As the pentavalent impurity increases the number of free electrons, it is called **donor impurity**.




(a)




(b)

This excess electron is only loosely bound by its atom and can be moved into conduction band of the semiconductor with relative

ease. The energy of such excess electrons is somewhat less than the energy corresponding to the bottom of the conduction band. These energy levels are filled with electrons and are called the **donor levels** as shown in Fig. (b). A small amount of energy ($\Delta E_e$) is required to push electrons from the donor levels into the conduction band. No holes are produced in the donor atoms as a result of the process. This energy can be supplied by heating. For Si with As as impurity, $\Delta E_e = 0.054$ eV. Since the conductivity of the extrinsic semiconductor is primarily due to electrons, it is called the ***n*-type semiconductor** (*n* stands for negative charge on electrons).

In case of a doped or extrinsic semiconductor, the number density of electrons in the conduction band ($n_g$) and the number density of holes in the valence band ($n_h$) differ from that in a pure semiconductor. If $n_i$ is the number density of electrons in conduction band or the number density of holes in valence band in a pure semiconductor, then relation can be expressed as

$$n_e n_h = n_i^2 \qquad \ldots(i)$$

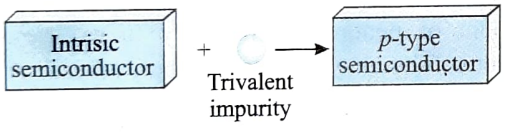
In *n*-type (extrinsic) semiconductor, the number density of electrons in conduction band is approximately equal to that of donor atoms and very large as compared to number density of holes in valence band. Thus,

$$n_e \approx N_d >> n_h \qquad \ldots(ii)$$

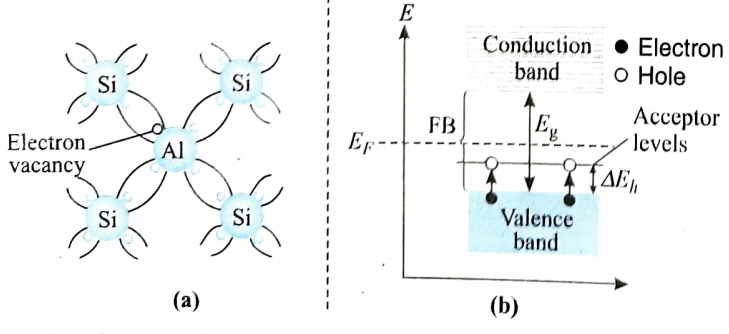
where $N_d$ represents the number density of donor atoms.

## *p*-TYPE EXTRINSIC SEMICONDUCTOR

If we add a trivalent impurity (this may be boron, aluminum or indium) to the Ge or Si crystal lattice we get a *p*-type extrinsic semiconductor.




All the three electrons of the impurity atom (say Al) form three covalent bonds by borrowing an electron from the nearest Ge atom. As a result of this borrowing, a hole appears at the site of borrowed electron as shown in Fig. (a). Thus, for every trivalent impurity atoms added, an extra hole will be created. As the trivalent impurity atoms accept electrons from the silicon crystal, it is called **acceptor impurity**. The Si-crystal so obtained is called ***p*-type** as it contains free holes. Each hole is equivalent to positive charge. The holes so created are **extrinsic carriers** and the *p*-type Si-crystal so obtained is called ***p*-type extrinsic semiconductor**.




The electrons from valence band can raise themselves to the acceptor energy level by absorbing thermal energy at room temperature and in turn create holes in the valence band.

Number density of valence band holes ($n_h$) in p-type semiconductor is approximately equal to that of the acceptor atoms ($N_a$) and is very large as compared to the number density of conduction band electrons ($n_e$). Thus,

$$n_h \approx N_a >> n_e \qquad \ldots(ii)$$

**ILLUSTRATION 8.3**

Suppose a pure Si crystal has $5 \times 10^{28}$ atoms/m$^3$ at room temperature. It is doped by 1 ppm concentration of pentavalent As. Calculate the number of electrons and holes. Given $n_i = 1.5 \times 10^{16}/\text{m}^3$.

**Sol.** Here the number density of electrons in conduction band; $n_i = 1.5 \times 10^{16}/\text{m}^3$, The number density of donor atoms,

$$N_d = \frac{5 \times 10^{28}\,\text{atom/m}^3}{10^6} = 5 \times 10^{22}\,\text{atom/m}^3$$

(as 1 ppm = 1 part per million = $1/10^6$)

Clearly, Since $n_e = N_d = 5 \times 10^{22}$ atoms/m$^3$

Since $n_e n_h = n_i^2$, $n_h = n_i^2 / n_e = \dfrac{(1.5 \times 10^{16}/\text{m}^3)^2}{5 \times 10^{22}/\text{m}^3} = 4.5 \times 10^9/\text{m}^3$

**ILLUSTRATION 8.4**

Pure Si at 300 K has equal electron ($n_e$) and hole ($n_h$) concentration of $1.5 \times 10^{16}$ m$^{-3}$. Doping by indium increases $n_h$ to $4.5 \times 10^{22}$ m$^{-3}$. Calculate $n_e$ in the doped silicon.

**Sol.** Here the number density of electrons in conduction band; $n_i = 1.5 \times 10^{16}/\text{m}^3$

After doping Si by indium (a trivalent atom), the doped silicon becomes a p-type semiconductor in which

$$n_h >> n_e$$

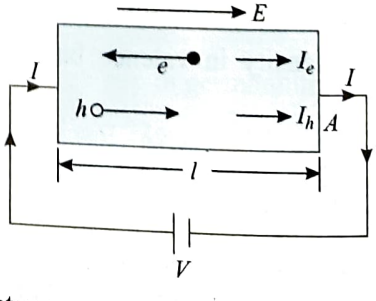
After doping hole concentration, $n_h = 4.5 \times 10^{22}\,\text{m}^{-3}$

Now, $n_e n_h = n_i^2$

$n_e$ in the doped silicon, $n_e = \dfrac{n_i^2}{n_h} = \dfrac{(1.5 \times 10^{16})^2}{4.5 \times 10^{22}} = 5 \times 10^9\,\text{m}^{-3}$

# ELECTRICAL CONDUCTIVITY OF A SEMICONDUCTOR

Let us take a block of semiconductor of length *l*, area of cross-section *A* and having number density of electrons and holes as $n_e$ and $n_h$ respectively. If we apply a potential difference, say *V*, a current *I* flows through it as shown in figure.




If $I_e$ is the electron current and $I_h$ is the hole current, the total current in the semiconductor is given by

$$I = I_e + I_h \qquad \ldots(i)$$

Further, if $v_e$ and $v_h$ be their corresponding drift velocities, then

$$I_e = en_e Av_e \quad \ldots(ii)$$

$$I_h = en_h Av_h \quad \ldots(iii)$$

and $e$ is the magnitude of charge on an electron or a hole.

From Eqs. (i), (ii) and (iii)

$$I = I_e + I_h = en_e Av_e + en_h Av_h \quad \ldots(iv)$$

$$I = e\left(n_e v_e + n_h v_h\right) A$$

Or we can write, $R = \dfrac{V}{I}$ $\quad \ldots(v)$

where $R$ is the resistance of the semiconductor block.

We know that $R = \rho \dfrac{l}{A}$ $\quad \ldots(vi)$

where $\rho$ is the resistivity of the semiconductor.

From Eqs. (v) and (vi), $\rho \dfrac{l}{A} = \dfrac{V}{I}$

Or $\rho = \dfrac{VA}{Il} = \dfrac{EA}{I}$ $\quad$ (as $V/l = E$)

The conductivity of the semiconductor,

$$\sigma = \frac{1}{\rho} = \frac{I}{EA} \quad \ldots(vii)$$

From Eqs. (iv) and (vii), $\sigma = \dfrac{e\left(n_e v_e + n_h v_h\right) A}{EA}$

Or $\sigma = e\left[n_e\left(v_e / E\right) + n_h\left(v_h / E\right)\right]$

On applying electric field, the drift velocity acquired by the electrons (or holes) per unit strength of electric field is called mobility of electrons (or holes).

Since $v_e / E = \mu_e$ (electron mobility) and $v_h / E = \mu_h$ (hole mobility), and

$$\sigma = e\left[n_e \mu_e + n_h \mu_h\right] \quad \ldots(viii)$$

Thus, we find that the conductivity (and consequently, the resistivity) of a semiconductor depends upon:

(i) the electron and hole number densities and

(ii) the electron and hole mobilities.

It is found that $\mu_e > \mu_h$. Further, $\mu_e$ and $\mu_h$ both decrease with rise in temperature whereas $n_e$ and $n_h$ both increase with temperature. But the increase in $n_e$ and $n_h$ is larger than the corresponding decrease in $\mu_e$ and $\mu_h$ and as a result of this, in case of a semiconductor, conductivity increases with temperature. (Recall that the conductivity of metals decreases with temperature.)

**Note:**

1. For an instrinsic semiconductor, $n_e = n_h = n_i$ and $\sigma_i = en_i\left(\mu_e + \mu_h\right)$
2. For an n-type semiconductor, $n_e >> n_h$, $\sigma_e \approx en_e\mu_e$
3. For a p-type semiconductor, $n_h >> n_e$ $\sigma_h \approx en_h\mu_h$

**Important Points:**

- A pure semiconductor behaves as an insulator at low temperature, it becomes slightly conducting at room temperature. It is because, a few electrons in the valence band acquire energy greater than the forbidden energy gap and move to the conduction band.
- Electron mobility is greater than the hole mobility in semiconductors.
- The semiconductors are insulators at low temperature but become slightly conducting at room temperature. Thus, unlike metals, the resistance of semiconductors decreases with increase of temperature.
- If the energy of radiation (light) greater than forbidden energy gap is incident on an intrinsic semiconductor, the electrons from the valence band move to the conduction band. It means electron and hole pairs are created. It increases the concentration of carriers therefore conductivity of semiconductor increases. This property of semiconductors is called photoconductivity. Semiconductors possessing large photoconductivity are used as light dependent resistors (LDR). Such resistors are used in automatic light control circuits.

## ILLUSTRATION 8.5

A semiconductor is known to have an electron concentration of $8 \times 10^{13}$ cm$^{-3}$ and a hole concentration of $5 \times 10^{12}$ cm$^{-3}$.

**(a)** Is the semiconductor *n*-type or *p*-type?

**(b)** What is the resistivity of the sample, if the electron mobility is 23,000 cm$^2$ V$^{-1}$ s$^{-1}$ and hole mobility is 100 cm$^2$ V$^{-1}$s$^{-1}$? Take charge on electron, $e = 1.6 \times 10^{-19}$ C.

**Sol.** We are given, $n_e = 8 \times 10^{13}\,\text{cm}^{-3} = 8 \times 10^{19}\,\text{m}^{-3}$;

$n_h = 5 \times 10^{12}\,\text{cm}^{-3} = 5 \times 10^{18}\,\text{m}^{-2}$;

$\mu_e = 23{,}000\,\text{cm}^2\,\text{V}^{-1}\text{s}^{-1} = 2.3\,\text{m}^2\,\text{V}^{-1}\text{s}^{-1}$;

$\mu_h = 100\,\text{cm}^2\text{V}^{-1}\text{s}^{-1} = 0.01\,\text{m}^2\,\text{V}^{-1}\text{s}^{-1}$

**(a)** Since the semiconductor has greater electron concentration, it is n-type semiconductor.

**(b)** Now, $\dfrac{1}{\rho} = e\left(n_e\mu_e + n_h\mu_h\right)$

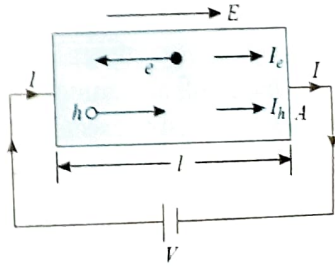
$$= 1.6 \times 10^{-19}\left(8 \times 10^{19} \times 2.3 + 5 \times 10^{18} \times 0.01\right)$$

$$= 1.6 \times 10^{-19}\left(1.84 \times 10^{20} + 5 \times 10^{16}\right) \approx 29.45\ \text{mho m}^{-1}$$

or $\rho = \dfrac{1}{29.45} = 3.396 \times 10^{-2}\ \Omega\text{m}$

## ILLUSTRATION 8.6

In figure, a battery of emf 2 V is used. The length of the block is 0.1 m and the area is $1 \times 10^{-4}$ m$^2$. If the block is of intrinsic silicon at 300 K, find the electron and hole currents. What will be magnitude of the total current? What will be magnitude of the total current if germanium is used instead of silicon?




Given for silicon, $n = 1.5 \times 10^{16}/\text{m}^3$, $\mu_e = 0.135\,\text{m}^2/\text{Vs}$, $\mu_h = 0.048\,\text{m}^2/\text{V}_\text{s}$ and for germanium, $n = 2.4 \times 10^{19}/\text{m}^3$ $\mu_e = 0.39\,\text{m}^2/\text{V}_\text{s}$ and $\mu_h = 0.19\ \text{m}^2/\text{V}_\text{s}$

**Sol.** We are given that

e.m.f of the battery $V = 2$ V, $l = 0.1$ m, $A = 1\times10^{4}$ m$^2$

In case of silicon, $n_e = n_h = n = 1.5\times10^{16}/\text{m}^3$

Electric field, $E = \dfrac{V}{l}$ where, e.m.f of the battery $V = 2$ V, and length of the block $l = 0.1$ m

$$\Rightarrow \quad E = \frac{2\text{V}}{0.1\text{m}} = 20\,\text{V/m}$$

Clearly, $v_e = E\mu_e = 20\times0.135\ \text{m/s} = 2.70\ \text{m/s}$

$v_h = E\mu_h = 20\times0.048\ \text{m/s} = 0.96\ \text{m/s}$

Electron current, $I_e = eAn_ev_e = (1.6\times10^{-19})(10^{-4})(1.5\times10^{16})2.70$

$= 6.48\times10^{-7}\,\text{A}$

Hole current, $I_h = eAn_hv_h = (1.6\times10^{-19})(10^{-4})(1.5\times10^{16})0.96$

$= 2.304\times10^{-7}\,\text{A}$

Total current, $I = I_e + I_h = (6.48\times10^{-7}\,\text{A}) + (2.304\times10^{-7}\,\text{A})$

$= 8.784\times10^{-7}\,\text{A}$

In case of germanium, $n_e = n_h = n = 2.4\times10^{19}/\text{m}^3$

$v_e = E\mu_e = 20\times0.39 = 7.8\ \text{m/s}$

$v_h = E\mu_h = 20\times0.19 = 3.8\ \text{m/s}$

Total current, $I = eA(n_ev_e + n_hv_h)$

$I = (1.6\times10^{-19})(10^{-4})\times[(2.4\times10^{19})(7.8) + (2.4\times10^{19})(3.8)]$

$\Rightarrow \quad I = 4.45\times10^{-3}\,\text{A}$

## CONCEPT APPLICATION EXERCISE 8.1

1. Suppose that the energy liberated in the combination of a hole–electron pair is converted into electromagnetic radiation. If the maximum wavelength of the radiation emitted is 630 mm, what is the width of forbidden energy gap?
2. A doped semiconductor has impurity levels 40 meV (millielectron volt) below the conduction band, **(a)** Is the material n-type or p-type? **(b)** In a thermal collision, an amount of energy $kT$ is given to the extra electron loosely bound to the impurity ion and this electron is just able to jump into the conduction band. Calculate the temperature $T$. Given, Boltzmann's constant, $k = 8.62\times10^{-5}\,\text{eV K}^{-1}$.
3. Energy of the photon of sodium light of wavelength 5,890 Å equals the energy gap between valence and conduction band of a semiconductor. Find the minimum energy $E$ required to create a hole–electron pair.

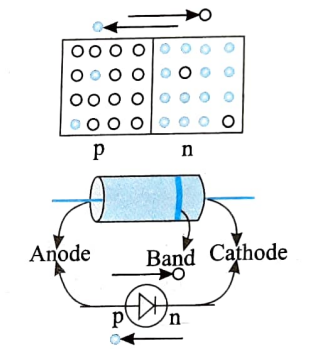
   Given, Boltzmann's constant, $k = 1.38\times10^{-23}\,\text{J K}^{-1}$.
4. Determine the number density of donor atoms which have to be added to an intrinsic germanium semiconductor to produce an n-type semiconductor of conductivity $5\,\Omega^{-1}\,\text{cm}^{-1}$. Given that the mobility of electrons in n-type Ge is $3{,}900\,\text{cm}^2\,\text{V}^{-1}\text{s}^{-1}$. Neglect the contribution of holes to conductivity. Take charge on electron, $e = 1.6\times10^{-19}\,\text{C}$.

**ANSWERS**

| | |
|---|---|
| 1. 1.97 eV | 2. (a) $n$-type (b) 464 K |
| 3. 2.11 eV | 4. $8.013\times10^{21}$ m$^{-3}$ |

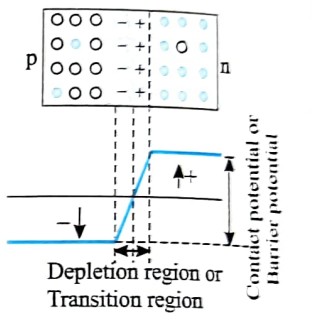
## P-N JUNCTION, DIODE: SEMICONDUCTOR DIODE

When a p-type semiconductor is suitably joined to an n-type semiconductor, then resulting arrangement is called p-n junction or p-n junction diode. The surface of contact of p-and n-type crystals is called junction. In the p-section, holes are the majority carriers; while in n-section, the majority carriers are electrons. Due to the high concentration of different types of charge carriers in the two sections, holes from p-region diffuse into n-region and electrons from n-region diffuse into p-region Due to diffusion, neutrality of both n-and p-type semiconductor is disturbed, a layer of negative charged ions appear near the junction in the p-crystal and a layer of positive ions appears near the junction in n-crystal. This layer is called depletion layer




Ajunction diode with its symbol

(a)




(b)

Due to the diffusion of holes and electrons, the two sections of the junction diode no longer remain neutral. The p-section of the junction diode becomes slightly negative, while the n-section is rendered positive. It appears as if some fictitious battery is connected across the junction with its negative pole connected to p-region and positive pole connected to n-region. The potential difference created across the $p$-$n$ junction due to the diffusion of electron and holes is called potential barrier.

For $Ge$, $V_B = 0.3$ V and for silicon, $V_B = 0.7$ V

On the average the potential barrier in $p$-$n$ junction is ~ 0.5 V and the width of depletion region ~$10^{-6}$m.

It may be pointed out that across the junction, a very large electric field is set up due to potential difference developed across it.

The barrier electric field, $E = \dfrac{V}{d} = \dfrac{0.5}{10^{-6}} = 5\times10^{5}\,\text{V/m}$

**Important Points:**

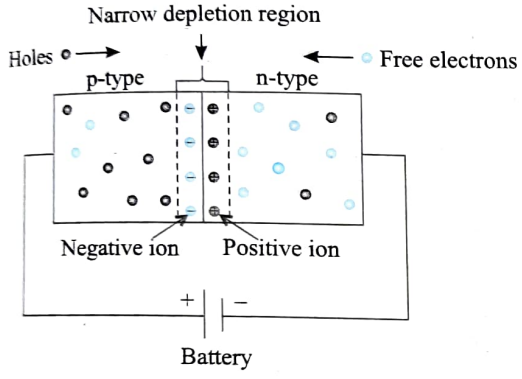
- The thickness of depletion layer is 1 micron = $10^{-6}$ m.
- Width of depletion layer $\propto \dfrac{1}{\text{Dopping}}$
- Depletion is directly proportional to temperature.
- The $P$-$N$ junction diode is equivalent to capacitor in which the depletion layer acts as a dielectric.

## CURRENT-VOLTAGE (I-V) CHARACTERISTIC OF SEMICONDUCTOR

Biasing means the way of connecting emf source to p-n junction diode. There are two ways in which a p-n junction diode can be biased.

### Forward Bias

If we connect an external d.c. source to the junction diode with p-section connected to positive pole and n-section to the negative pole, the junction diode is said to be forward biased. If a battery be connected with its positive pole to p-type and negative pole to n-type semiconductor, all the holes will tend to move to the negative pole and all the electrons to the positive pole. Thus, most of the holes and free electrons tend to move towards the junction between the semiconductors. Because of the attraction between the opposite charges, the electrons and the holes rush towards each other and recombine. The effect of the applied voltage is to produce more free electrons in the n-type semiconductor and holes in the p-type semiconductor.



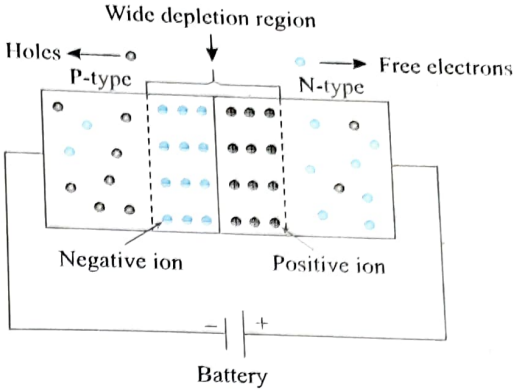

It means there is a continuous movement of electrons and holes through the junction. This recombination around the junction continues as long as the external battery is connected. Because of this, a relatively large current, called forward current flows through the junction. This point is called **knee point** and the corresponding voltage is called the **knee voltage ($V_k$)**. The current in the external circuit is due to the electrons and is from negative terminal of battery to positive terminal through the junction diode.

Hence during the forward bias, the applied d.c. voltage opposes the fictitious battery developed across the p-n junction. Due to this, the potential drop across the junction decreases and as a result, the diffusion of holes and electrons across the junction increases. It makes the depletion layer thin and as such, the junction diode offers low resistance during forward bias.

### Reverse Bias

If we connect a battery to junction diode with p-section connected to negative pole and n-section connected to the positive pole, the junction diode is said to be reverse biased.

In this case the free electrons flow towards the positive and the holes to the negative terminal of the battery. But at the junction, there are relatively few electrons and holes (minority carriers). Hence, there is very little current flowing across the junction. This current is called the **leakage current** or the **reverse current ($I_R$)**. The circuit in this case is said to possess reverse bias.



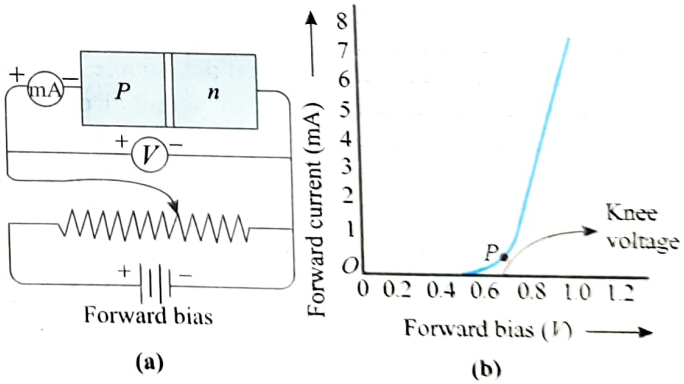

During the reverse bias, the applied d.c. voltage aids the fictitious battery developed across the junction. Due to this, the potential drop across the junction increases and as a result, the diffusion of holes and electrons across the junction decreases. It makes the depletion layer thick and the junction diode offers high resistance during reverse bias.

It may be noted that the potential barrier opposes the forward current, while it aids the reverse current.

## CHARACTERISTICS OF A P-N JUNCTION

Now we are going to learn how the current flows during forward and reverse bias of a junction diode and then draw the characteristics in the two cases.

**Forward bias characteristic:** The circuit diagram in case of forward-bias connections of a p-n junction is shown in Fig. (a).



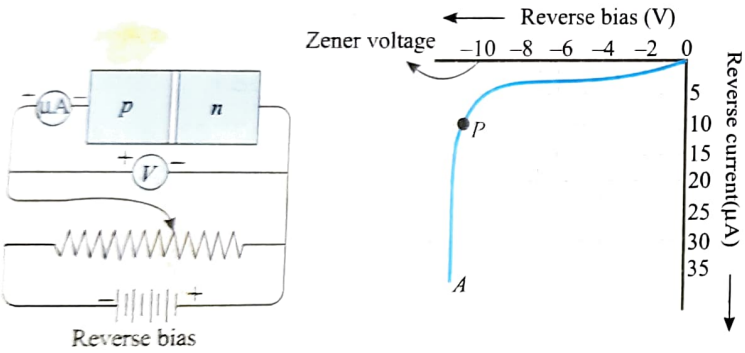

When we connect the battery with rheostat as shown in figure, in the beginning, when the applied forward bias is low, practically no current flows through the junction diode. Initially the potential barrier opposes the applied voltage, which permits a small forward current flows, till the applied forward bias does not become greater than the potential barrier voltage. It is represented by portion $OP$ of the graph between forward bias and the forward current [Fig. (b)]. But when the forward bias becomes greater than the potential barrier, the forward current increases almost linearly. The point $P$ in the forward characteristic corresponding to the potential barrier appears

like a knee and the forward voltage corresponding to knee point $P$ is called knee voltage.

Thus, when a p-n junction is forward biased, the current increases linearly and rapidly above the knee voltage and the variation in current with increase of forward bias is extremely slow below the knee voltage.

When the p-n junction is forward biased, the depletion layer becomes thin. It is because, the polarity of the external d.c. source opposes the fictitious battery developed across the junction. As a result, the potential drop across the junction decreases making the depletion layer thin. It leads to the low resistance of the junction diode during forward bias.

**Reverse bias characteristic:** In case of the p-n junction is reverse biased, the current flows due to minority charge carriers and hence a microammeter is used to measure the small current that flows during reverse bias. The reverse bias opposes the majority carriers but makes the minority carriers to cross the p-n junction. Therefore, a current of about 1–2 μA is obtained, on applying reverse bias to the p-n junction. It remains almost constant till the Zener voltage is reached and then it increases suddenly as shown in Fig. (b).




(a) (b)

In case of the p-n junction is reverse biased, the depletion layer becomes thick. It is because, the external d.c. source in this case aids the fictitious battery. It increases of potential drop across the junction and the depletion layer appears thick. This increased thickness of the depletion layer, the p-n junction offers high resistance during reverse bias.

## Dynamic Resistance of a Diode

In case of both forward bias and reverse bias characteristics of the p-n junction have knees and hence Ohm's law is not obeyed. Therefore, the resistance offered by the junction diode depends upon the applied voltage.

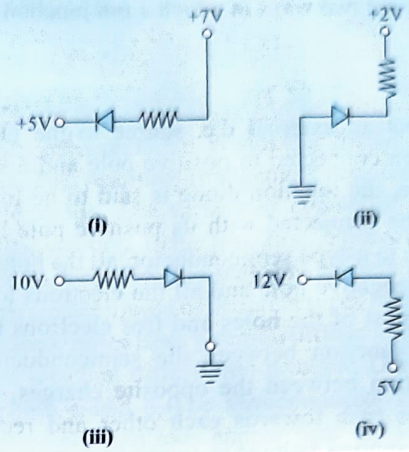
The dynamic resistance of a junction diode is defined as the ratio of small change in voltage to the small change in current produced. It is also called dynamic resistance of the junction diode and is denoted by $r_d$.

Thus, $$r_d = \frac{\Delta V}{\Delta I} \qquad ...(ii)$$

where $\Delta V$ is the small change in applied voltage and $\Delta I$ is the corresponding small change in current.

### Illustration 8.7

In the following diagrams, indicate which of the diodes are forward biased and which are reverse biased.




**Sol.**

**(i)** Forward biased, because $p$-side is at higher potential (+7 V) than n-side (+5 V).

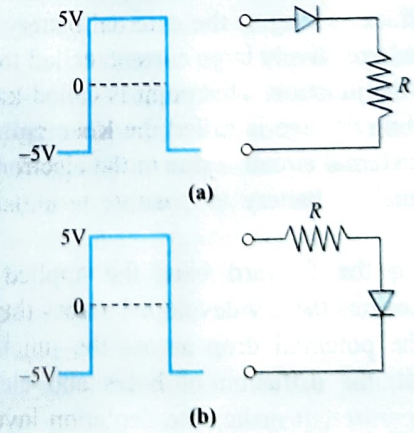
**(ii)** Reverse biased, because $p$-side is at lower potential (0 V) than $n$-side (+2 V).

**(iii)** Reverse biased, because $p$-side is at lower potential (−10 V) than $n$-side (0 V).

**(iv)** Forward biased, because $p$-side is at higher potential (−5 V) than $n$-side (−12 V).

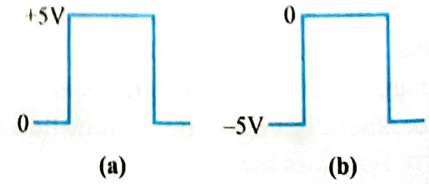
### Illustration 8.8

In the following circuits, if the input waveform is as shown in the figure, what will be the output waveform, **(i)** across $R$ in Fig. (a) and **(ii)** across the diode in Fig. (b)? Assume that the diode is ideal.




**Sol.**

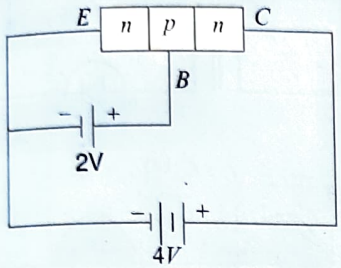
**(i)** When the input level is −5 V, the diode gets reverse biased. No output is obtained across $R$. When the input level becomes +5 V, the diode gets forward biased and current flows through $R$. As the diode is ideal, the output across $R$ will be exactly 5 V, as shown in Fig. (a).



(ii) When the input level is –5V, the diode remains reverse biased. It does not conduct current. This part of input wave appears across the diode, as shown in Fig. (b). When the input level is +5 V, the diode gets forward biased and conducts currents. As the diode is ideal, no voltage appears across it.

## ILLUSTRATION 8.9

In the n-p-n transistor circuit shown in figure, what is the base to collector voltage? What are the biasings (forward or reverse) of the emitter-base and the base-collector junctions?




**Sol.** Base to collector voltage = 4 – 2 = 2 V

$V_{EB} = 2\text{V}$ (Forward)

$V_{BC} = 2\text{V}$ (Reverse)

## ILLUSTRATION 8.10

The following table provides a set of values of $V$ and $I$ obtained for a given diode.

| | $V$ | $I$ |
|---|---|---|
| **Forward biasing** | 2.0 V<br>2.4 V | 60 mA<br>80 mA |
| **Reverse biasing** | 0 V<br>– 2 V | 0 μA<br>– 0.25 μA |

Assuming the characteristics to be linear over the range, calculate the forward and reverse bias resistance of the given diode.

**Sol.** Resistance, $r$ (forward biasing) $= \dfrac{\Delta V}{\Delta I}$

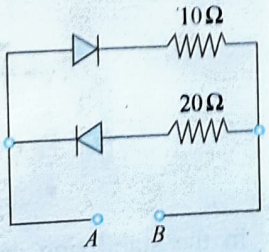
$$= \frac{2.4\text{V} - 2.0\text{V}}{80\text{ mA} - 20\text{ mA}} = \frac{0.4\text{V}}{20\times10^{-3}\text{A}} = 20\,\Omega$$

Resistance, $r$ (reverse biasing) $= \dfrac{\Delta V}{\Delta I}$

$$= \frac{-2\text{V} - 0\text{V}}{-0.25\,\mu\text{A} - 0\,\mu\text{A}} = \frac{2\text{V}}{0.25\,\mu\text{A}} = \frac{2\text{V}}{0.25\times10^{-6}\text{A}} = 8\times10^{6}\,\Omega$$

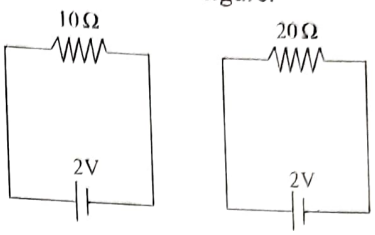
## ILLUSTRATION 8.11

A battery of 2 V may be connected across the points $A$ and $B$, as shown in figure. Find the current drawn from the battery if the positive terminal is connected to **(i)** the point $A$ and **(ii)** the point $B$. Assume that the resistance of each diode is zero in forward bias and infinity in reverse bias.




**Sol.**

**(i)** When positive terminal of the battery is connected to point $A$, diode $D_1$ gets forward biased and offers zero resistance and diode $D_2$ gets reverse biased and offers infinite resistance. The given circuit reduces to the equivalent circuit shown in figure.



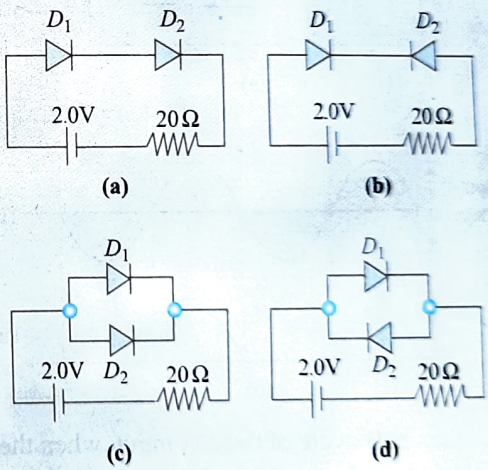

∴ Current drawn from the battery, $I = \dfrac{2\text{V}}{10\,\Omega} = 0.2\text{A}$

**(ii)** When positive terminal of the battery is connected to point $B$, the diode $D_1$ gets reverse biased and offers infinite resistance and diode $D_2$ gets forward biased and offers zero resistance. The given circuit reduces to the equivalent circuit shown in figure.

∴ Current drawn from the battery, $I = \dfrac{2\text{V}}{20\,\Omega} = 0.1\text{A}$

## ILLUSTRATION 8.12

Determine the currents through the resistances of the circuits shown in figure.




**Sol.**

**(i)** In Fig. (a), both the diodes $D_1$ and $D_2$ are forward biased and offer no resistance.

∴ Current in the circuit $= \dfrac{2.0\text{V}}{20\,\Omega} = 0.1\text{A}$

**(ii)** In Fig. (b), diode $D_2$ being reverse biased offers infinite resistance, so current through the series circuit = zero.

**(iii)** In Fig. (c), $D_1$ and $D_2$ are forward biased and offer zero resistance.

∴ Current in the circuit $= \dfrac{2.0\text{V}}{20\,\Omega} = 0.1\text{A}$

**(iv)** In Fig. (d), no current flows through $D_2$ as it is reverse biased.

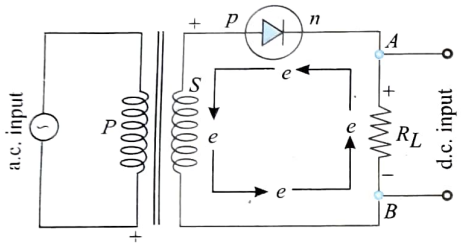
∴ Current in the remaining circuit $= \dfrac{2.0\text{V}}{20\,\Omega} = 0.1\text{A}$

# P-N JUNCTION DIODE AS A RECTIFIER

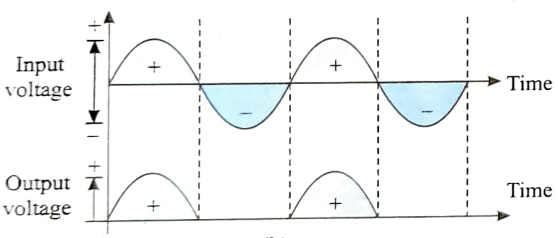
A **rectifier** is a device which converts a.c. to unidirectional pulsating output. In other words it converts a.c. to d.c. The process of converting a.c. into 'd.c. is called **rectification**. It is of the following two types: (a) half-wave rectifier and (b) full-wave rectifier.

## HALF-WAVE RECTIFIER

The working of a p-n junction diode as a half-wave rectifier can be understood from Fig. (a). The a.c. supply is fed across the primary coil $P$ of a step-down transformer. The secondary coil $S$ of the transformer is connected to the junction diode and a load resistance $R_L$ as shown in figure. The output d.c. voltage is obtained across the load resistance $R_L$. The p-section is positive and n-section is negative the junction is forward biased and the electronic current will flow in the direction shown in the figure. The conventional current will flow in the opposite direction, making the terminal $A$ of the load resistance positive and the terminal $B$ as negative.




(a)




(b)

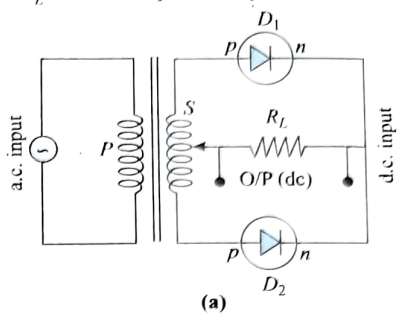
During the next half cycle of the a.c. input, when the p-section becomes negative and n-section becomes positive, the p-n junction is reverse biased and almost no current flows in the load.

The input and the output waveforms have been shown in Fig. (b). We find that only half the wave is present the output and as such a lot of power is wasted in half wave rectification.

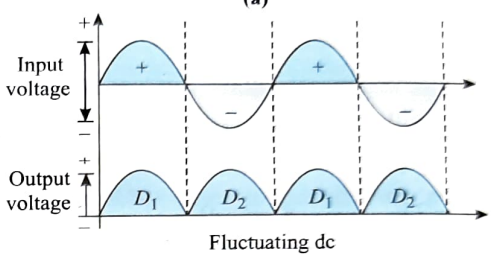
## FULL-WAVE RECTIFIER

A rectifier which rectifies both halves of each a.c. input cycle is called a full wave rectifier. To make use of both the halves of input cycle, two junction diodes are used. In the circuit diagram we have got two p-n junction diodes, $D_1$ and $D_2$, which are connected as shown in figure. Suppose that during first half of the input cycle, upper end of coil $S$ is at positive potential and the lower end is at negative potential. The junction diode $D_1$ will get forward biased, while the diode $D_2$ reverse biased. The conventional current due to the diode $D_1$ will flow along the path of full arrows.

When the second half of the input cycle comes, the situation will be exactly reverse. Now, the junction diode $D_2$ will conduct and the conventional current will flow along the path of the dotted arrows. Thus, the fulll wave is rectified and appears across the load. Since current during both the half cycles' flows from right to left through the load resistance $R_L$, the output during both the half cycles will be of the same nature. The right end of the load resistance $R_L$ will be at positive potential w.r.t. its left end.



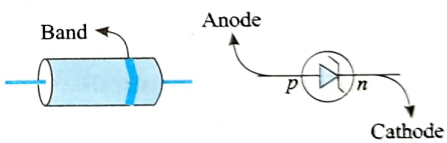

(a)




(b)

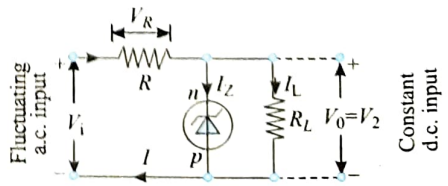
## SPECIAL PURPOSE DIODES

1. **Zener diode:** It is a highly doped $p$-$n$ junction which is not damaged by high reverse current. It can operate continuously, without being damaged in the region of reverse background voltage. In the forward bias, the Zener diode acts as ordinary diode. It can be used as voltage regulator




**A Zener diode with its symbol**

Consider that the Zener diode and a resistor $R$, called dropping resistor are connected to a fluctuating d.c. supply (say of a rectifier), such that the Zener diode is reverse biased. Whenever voltage across the diode tends to increase, the current through the diode rises out of proportion and causes a sufficient increase in voltage drop across the dropping resistor. As a result, the output voltage lowers back to the normal value. Similarly, when the voltage across the diode tends to decrease, the current through the diode goes down out of proportion, so that the voltage drop across the dropping resistor is much less and now the output voltage is raised to normal.




In the stabilizing circuit shown in figure, a Zener diode is reversed biased and is in parallel with the load ($R_L$). The input

voltage ($V_i$) is taken from an unstabilized supply, e.g., a battery, and is greater than the output voltage ($V_o$) developed across the load.

If $I_L$ and $I_Z$ are the currents in load and diode respectively, the total current ($I$) supplied is given by

$$I = I_L + I_Z \quad \ldots(i)$$

If $I_L$ rises (or falls), $I_Z$ is found to fall (or rise) by the same amount so keeping $I$ and therefore $V_0 = I_L R_L = V_Z$ (constant). This is because in the breakdown region, Zener voltage remains constant even through the current through Zener diode changes. The resistance $R$ of current-limiting resistor is given by

$$R = \frac{V_R}{I} = \frac{(V_i - V_Z)}{I} \quad \ldots(ii)$$

**ILLUSTRATION 8.13**

The output of a Zener diode is 5 V with an input voltage of 9 V. If $I_L$ and $I_Z$ are 40 mA and 10 mA respectively, find: **(i)** resistance of the current limiting resistor, **(ii)** power dissipated in the resistor, **(iii)** power dissipated in the diode. **(iv)** If the load is disconnected, how much power must the diode be able to dissipate?

**Sol.** Here, $V_i = 9\text{ V}, V_0 = V_Z = 5\text{ V}, I_L = 40\text{ mA}, I_Z = 10\text{ mA}$

(i) Resistance of the limiting resistor,

$$R = \frac{V_i - V_Z}{I} = \frac{9\text{ V} - 5\text{ V}}{50\text{ mA}} = \frac{4\text{V}}{0.05\text{ A}} = 80\,\Omega$$

(as $I = I_L + I_Z = 40\text{ mA} + 10\text{ mA} = 50\text{ mA}$)

(ii) Power dissipated in $R$, i.e.,

$$P_R = I^2R = \left(50\times10^{-3}\text{ A}\right)^2 \times 80\,\Omega = 200\text{ mW}$$

(iii) Power dissipated in the diode, i.e.,

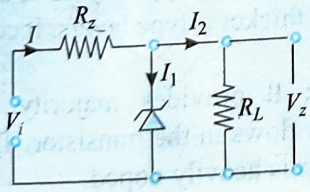
$$P_Z = V_Z I_Z = (5\text{V})(10\text{mA}) = 150\text{mW}$$

(iv) When the load is disconnected, $I_L = 0$ and $I_Z = 5$ mA.

Thus, $P_Z = V_Z I_Z = (5\text{V})(50\text{ mA}) = 250\text{ mW}$

**ILLUSTRATION 8.14**

A 10 V Zener diode along with a series resistance is connected across a 40 V supply. Calculate the minimum value of the resistance required, if the maximum Zener current is 50 mA.




**Sol.** In figure, $V_i = 40\text{ V}, I_1 = 50\text{ mA}$

$\therefore$ Maximum current, $I = I_1 + I_2 = 50 = 0 = 50\text{ mA} = 50\times10^{-3}\text{ A}$

This is because maximum current flows through Zener diode, so $I_2 = 0$.

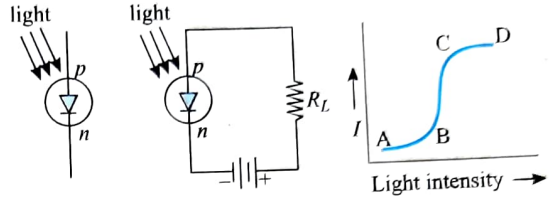
$V_z$ = Voltage drop across Zener diode = 10 V

As $I$ is maximum, so maximum value of $R$ is

$$R = \frac{V_i - V_z}{I} = \frac{40 - 10}{50\times10^{-3}} = 600\,\Omega$$

2. **Photodiode:** Photodiode is a special type of photo-detector. In case of semiconductors, electrons are released from the valence band, when energy is supplied to such electrons. This energy can be supplied in the form of heat or through an accelerating voltage. Light energy can also be used to produce the current in semiconductors. Suppose an optical photon of frequency $\nu$ is incident on a semiconductor, such that its energy is greater than the band gap of the semiconductor (i.e. $h\nu > E_g$) This photon will excite an electron from the valence band to the conduction band leaving a vacancy or hole in the valence band.

It is represented by the symbol as shown in Fig. (a).

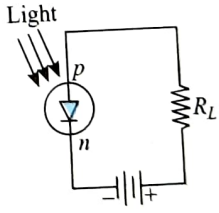



(a) (b) (c)

If the photodiode is forward biased [Fig. (b)], a certain current flows in the circuit as depicted by portion AB of the graph in Fig. (c). The current AB, that flows when no light is incident, is called dark current. When photons of light having energy $h\nu$ falls on the photodiode, more electrons from valence band move to the conduction band, provided $h\nu$ is greater than forbidden energy gap. Due to this, current in the circuit increases. As the intensity of light is increased, the current goes on increasing as indicated by part BC of the graph. A stage comes, when current does not increase with increase in intensity of light and the photodiode is said to be saturated. Portion CD of the graph represents saturated current.

3. **Light Emitting Diodes (LED):** Light emitting diodes, commonly called LEDs, which emits spontaneous radiation. In this type of diode, doping concentration is very high and it is used in forward bias.

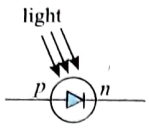
The p-n junction acts as a barrier to the flow of electrons between the p- and the n-regions. Only when sufficient voltage is applied to the LED, the electrons cross the junction into the p-region and a current flows through it.




If sufficient voltage is applied to the LED, electrons move across the junction into the p-region. Once in the p-region, the electrons are immediately attracted to the positive holes. When an electron moves sufficiently close to the hole in the p-region, they recombine. Each time an electron recombines with a positive hole, electric potential energy is converted into electromagnetic energy and a photon of light with a frequency characteristic of the semiconductor material is emitted. In GaAsP, the forbidden energy gap is 1.90 eV. The wavelength of radiation emitted is given by

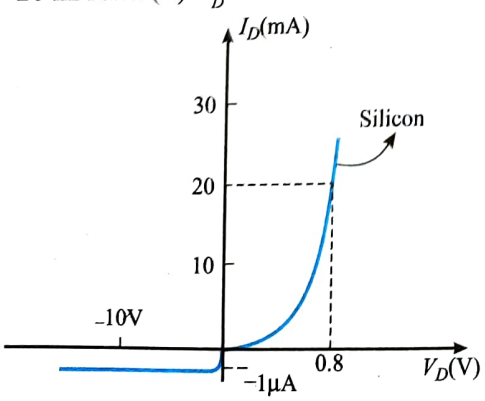
$$\lambda = \frac{hc}{E_g} = \frac{6.62\times10^{-34}\times3\times10^{8}}{1.90\times1.6\times10^{-19}} = 6{,}500\,\text{Å}$$

4. **Solar cell:** It is based on the photovoltaic effect. One of the semiconductor region is made so thin that the light incident on it reaches the p-n junction and gets absorbed. It converts solar energy into electrical energy.

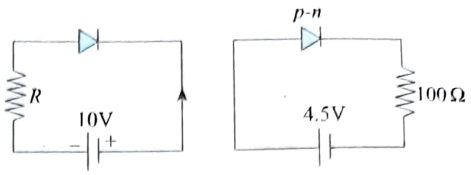



## CONCEPT APPLICATION EXERCISE 8.2

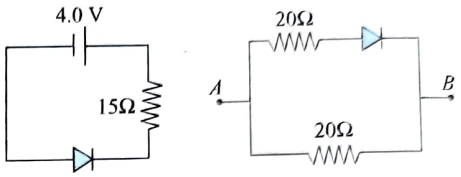
1. In an n-type silicon, which of the following statement is true?
   (a) Electrons are majority carriers and trivalent atoms are the dopants.
   (b) Electrons are minority carriers and pentavalent atoms are the dopants.
   (c) Holes are minority carriers and pentavalent atoms are the dopants.
   (d) Holes are majority carriers and trivalent atoms are the dopants.
2. Which of the statements given in previous problem is true for p-type semiconductors.
3. Carbon, silicon and germanium have four valence electrons each. These are characterized by valence and conduction bands separated by energy band gap respectively equal to $(E_g)_C$, $(E_g)_{Si}$ and $(E_g)_{Ge}$. Which of the following statements is true?
   (a) $(E_g)_{Si} < (E_g)_{Ge} < (E_g)_C$
   (b) $(E_g)_C < (E_g)_{Ge} < (E_g)_{Si}$
   (c) $(E_g)_C > (E_g)_{Si} > (E_g)_{Ge}$
   (d) $(E_g)_C = (E_g)_{Si} = (E_g)_{Ge}$
4. In an unbiased p-n junction, holes diffuse from the p-region to n-region because
   (a) free electrons in the n-region attract them.
   (b) they move across the junction by the potential difference.
   (c) hole concentration in p-region is more as compared to n-region.
   (d) all the above.
5. When a forward bias is applied to a p-n junction, it:
   (a) raises the potential barrier
   (b) reduces the majority carrier current to zero
   (c) lowers the potential barrier
   (d) none of the above
6. The volt–ampere characteristic of a silicon diode is shown in figure. Determine the resistance of the diode at (i) $I_D = 20$ mA and (ii) $V_D = -10$ V.



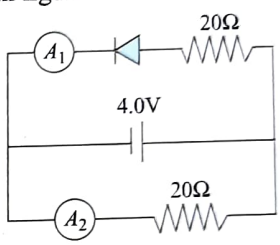

7. The silicon diode shown in figure is rated for a maximum current of 10 mA. Calculate the minimum value of resistor $R$. Assume the forward voltage drop across the diode to be 0.7 V.




8. Find the current through the circuit and the potential difference across the diode shown in figure. The drift current for the diode is 20 μA.




9. Assuming that the resistances of the meters are negligible, what will be the readings of the ammeters $A_1$ and $A_2$ in the circuit shown in figure?




### ANSWERS

**1.** (c) **2.** (d) **3.** (c) **4.** (c) **5.** (c)
**6.** (i) 40 Ω (ii) 10 MΩ **7.** 100 Ω **8.** 20 μA, 4.0 V
**9.** 0, 0.2 A

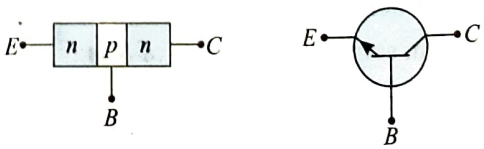
# TRANSISTORS

A junction diode cannot be used for amplifying a signal. For amplification, another type of semiconductor device called transistor is used. A **transistor** is a semiconductor device obtained by growing either a very thin layer of n-type crystal between two much thicker p-type layers or a thin layer of p-type between two much thicker n-type layers. It consists of three main regions

(i) **Emitter (*E*):** It provides majority charge carriers by which current flows in the transistor. Therefore the emitter semiconductor is heavily doped.
(ii) **Base (*B*):** The base region is lightly doped and thin.
(iii) **Collector (*C*):** The size of collector region is larger than the two other regions.

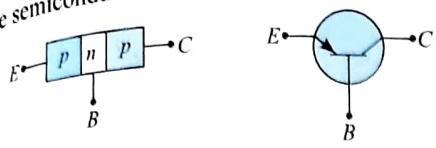
Junction transistor are of two types:

(i) *npn* transistor: It is formed by sandwiching a thin layer of *p*-type semiconductor between two *n*-type semiconductors.



In *npn* transistor electrons are majority charge carriers and flow from emitter to base.

(ii) *pnp* transistor: It is formed by sandwiching a thin layer of *n*-type semiconductor between two *p*-type semiconductors.




In *pnp* transistor holes are majority charge carriers and flow from emitter to base.

In the symbols of both *npn* and *pnp* transistor, the arrow-head on the emitter points in the direction of the flow of conventional current (positive charge or hole movement). Thus, the arrow-head points inwards for a p-n-p transistor and outwards for an n-p-n transistor.

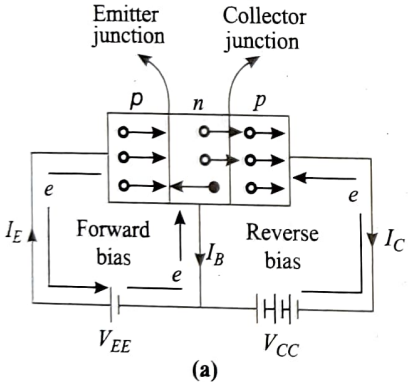
When a transistor is used in a circuit, the base-emitter junction is always forward biased and the base-collector junction is always reverse biased.

## ACTION OF TRANSISTOR

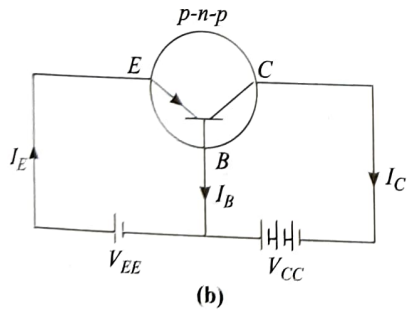
The action of both the types of transistors i.e. n-p-n and p-n-p is similar, except that the majority and minority carriers in the two cases are of opposite nature.

### The p-n-p Transistor

The p-type emitter of a p-n-p transistor is forward biased by connecting it to positive pole of emitter-base battery $V_{EE}$ and the n-type collector is reverse biased by connecting it to the negative pole of the collector-base battery $V_{CC}$ as shown in figure.




(a)




(b)

In this case, majority carriers in emitter are holes and they are repelled towards the base due to the forward bias. With the emitter junction forward biased, the holes in the emitter and the electrons in the base, begin to move towards this junction, the holes being repelled by the positive terminal and the electrons by the negative terminal of the battery $V_{EE}$ (emitter voltage). On reaching the emitter junction, a small fraction (about 5%) of the total number of holes combine with electrons in the base to get neutralized. As the base region is extremely thin and only slightly doped, the collector which is at very high negative potential, will collect almost all the holes. As each hole reaches the collector electrode, an electron is emitted from the battery $V_{CC}$ (collector voltage) and neutralizes the hole.

For each hole that is lost by combination with an electron in the collector and base region, a covalent bond near the emitter electrode breaks down and an electron is liberated which enters the positive terminal of the buttery $V_{EE}$. Thus:

**The current is carried by holes in the crystal and by the electrons in the external circuit.**

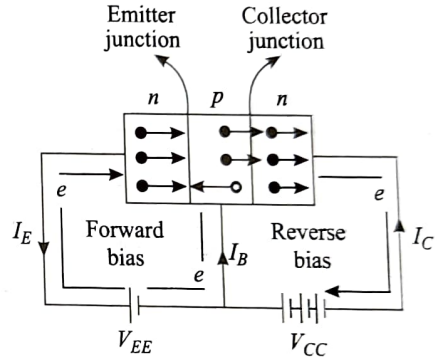
It is clear that $I_E = I_C + I_B$ ...(i)

where $I_E$ = emitter current, $I_C$ = collector current, $I_B$ = base current.

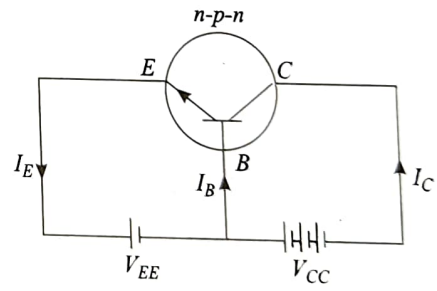
It should be noted that $I_C$ and $I_E$ flow in opposite directions in the base as shown in Fig. (b).

### The n-p-n Transistor

Figure shows the proper biasing of an n-p-n transistor. The n-type emitter is forward biased by connecting it to negative pole of the battery $V_{EE}$ (emitter-base battery ) and n-type collector is reverse biased by connecting it to the positive pole of the battery $V_{CC}$ (collector-base battery).




(a)




(b)

The electrons in the emitter and the holes in the base move towards the emitter junction, the holes being repelled by the positive terminal and the electrons by the negative terminal of the battery $V_{EE}$. Obviously, the emitter junction will have low impedance. The collector-base circuit is reverse biased and as such the collector junction will have high impedance. On reaching the emitter junction, a small fraction (about 5%) of the total number of electrons combine with the holes in the base to get neutralized. As the base layer is extremely thin and only slightly doped, the collector, which is at a very high positive potential will collect almost all the electrons. These electrons will flow to the positive terminal of the battery $V_{CC}$. The deficiency of the electrons in the emitter is made up by the electrons from the negative terminal of the battery $V_{EE}$. Thus,

**the current is carried in the crystal as well as in the external circuit by the electrons.** Further,

$$I_E = I_C + I_B \qquad \ldots(ii)$$

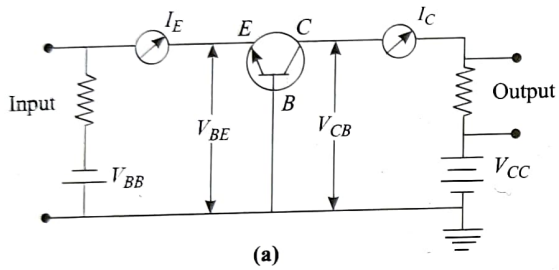
where the letters have their usual meanings. Here, also $I_E$ and $I_C$ flow in the opposite directions in the base as shown in Fig. (b).

## THREE CONFIGURATIONS OF A TRANSISTOR

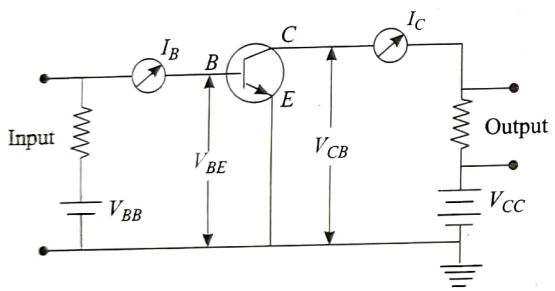
A transistor is a three element device. One terminal has to be always common to the input and the output circuits. This terminal is connected to the ground and serves as a reference point for the entire circuit. So a transistor can be used in one of the following three configurations:

1. Common-base (CB) circuit
2. Common-emitter (CE) circuit
3. Common-collector (CC) circuit

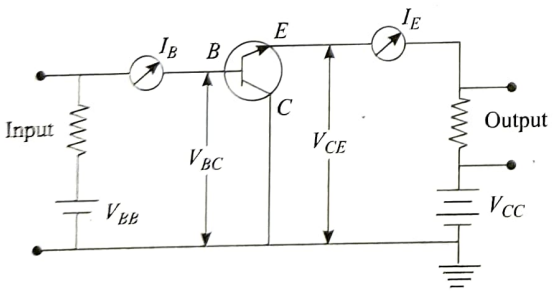
Figure shows the three types of circuit arrangements for an n-p-n transistor. In each case, the emitter-base junction is forward biased while the collector-base junction is reverse biased.




(a)




(b)




(c)

## CURRENT GAINS IN A TRANSISTOR

Usually two types of current gains are defined for a transistor:

1. **Common base current amplification factor or a.c. current gain**: It is defined as the ratio of the small change in the collector current to the small change in the emitter current when the collector-base voltage is kept constant.

Thus $$\alpha = \left[\frac{\Delta I_C}{\Delta I_E}\right]_{V_{CE}=\text{constant}}$$

2. **Common emitter current amplification factor or a.c. current gain**: It is defined as the ratio of the small change in the collector current to the small change in the base current when the collector-emitter voltage is kept constant. Thus

$$\beta = \left[\frac{\Delta I_C}{\Delta I_B}\right]_{V_{CE}=\text{constant}}$$

**Relation between $\alpha$ and $\beta$.** For both *n-p-n* and *p-n-p* transistors, we have $I_E = I_B + I_C$

For small changes, we can write $\Delta I_E = \Delta I_B + \Delta I_C$

Dividing both sides by $\Delta I_C$, $\frac{\Delta I_E}{\Delta I_C} = \frac{\Delta I_B}{\Delta I_C} + 1$

or $$\frac{1}{\alpha} = \frac{1}{\beta} + 1 \qquad \left[\because \frac{\Delta I_C}{\Delta I_E} = \alpha, \frac{\Delta I_C}{\Delta I_B} = \beta\right]$$

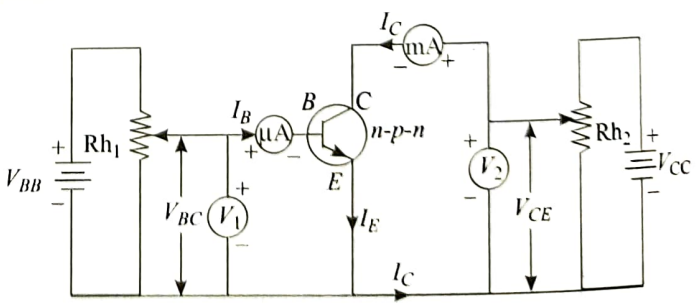
or $$\alpha = \frac{\beta}{1+\beta} \quad \text{or} \quad \beta = \frac{\alpha}{1-\alpha}$$

As the value of $I_B$ is about 1–5% of $I_E$ or $I_C$ is 95–99% of $I_E$, $\alpha$ is about 0.95 to 0.99 and $\beta$ is about 20 to 100. The *CE* configuration is frequently used as it gives high current gain as well as voltage gain.

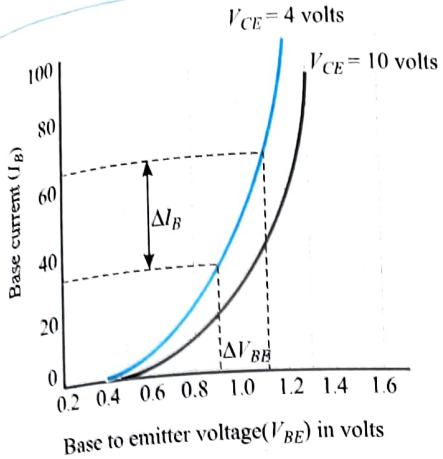
## COMMON EMITTER CHARACTERISTICS

The common-emitter characteristics are the graphs drawn between appropriate voltages and currents for a transistor when its emitter is taken as the common terminal and grounded (zero potential), base is the input terminal and collector is the output terminal.

Figure shows the circuit diagram for studying the common emitter characteristics of an n-p-n transistor. The emitter-base junction is forward biased by means of battery $V_{BB}$ through rheostat $Rh_1$. The emitter-collector circuit is reverse biased by means of battery $V_{CC}$ through rheostat $Rh_2$. The base-emitter voltage $V_{BE}$ and the collector-emitter-voltage $V_{CE}$ are measured by high resistance voltmeters. The base current $I_B$ is measured by a microammeter and the collector current $I_C$ by a milliammeter. Three types of characteristic curves are studied.




1. **Input characteristics:** A graph showing the variation of base current $I_B$ with base-emitter voltage $V_{BE}$ at constant collector-emitter voltage $V_{CE}$ is called the input characteristic of the transistor. Two such curves for two different collector-emitter voltages have been plotted in figure.




A study of the these curves reveals the following facts:

(i) As long as $V_{BE}$ is less than the barrier voltage, the base current $I_B$ is small as in the case of forward biased diode (ii). When the base-emitter voltage $V_{BE}$ exceeds the barrier voltage, the base current $I_B$ increases sharply with a small increase in $V_{BE}$ as in the case of a forward biased diode.

(iii) The value of $I_B$ is much smaller than that in a normal diode, more than 95% majority emitter carriers (electrons in *n-p-n* and holes in *p-n-p* transistor) go to the collector to constitute the collector current $I_C$.

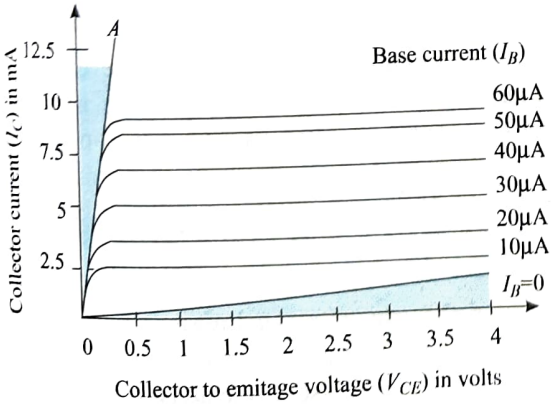
Since the increase in $V_{CE}$ appears as the increase in $V_{CB}$, its effect on $I_B$ is negligible. As a result, input characteristics for various values of $V_{CE}$ give almost identical curves. Hence, it is enough to determine only one input characteristic.

**The input resistance** $r_i$ of the transistor in CE configuration is defined as the ratio of the small change in base-emitter voltage to the corresponding small change in the base current, when the collector-emitter voltage is kept fixed. Thus

$$r_i = \left[\frac{\Delta V_{BE}}{\Delta V_B}\right]_{V_{CE}=\text{constant}}$$

As input characteristic is non-linear, so $r$ varies. At any point of the curve, $r_i$ is equal to the slope of the tangent to the curve.

2. **Output characteristics:** A graph showing the variation of collector current $I_c$ with collector-emitter voltage $V_{CE}$ at constant base current $I_B$ is called the output characteristic of the transistor. Figure shows such curves for different values of $I_B$.




A study of these curves reveals the following features:

(i) When the voltage $V_{CE}$ increases from 0 V to about 0.5 V, the collector current $I_C$ increases rapidly. The value of $V_{CE}$ increases rapidly is called **knee voltage**.

(ii) Once the voltage $V_{CE}$ exceeds the voltage $V_{BE}$ (so that the collector-base junction is reverse biased), the output current $I_C$ varies *very slowly* but *linearly* with $V_{CE}$ for a given base current $I_B$, i.e., beyond the knee voltage the output resistance of the transistor is high.

(iii) Larger the value of $I_B$, larger is the value of $I_c$ for a given $V_{CE}$.

**Three regions of the output characteristic**:

(a) The shaded region towards the left of line *OA* is called *saturation region* and the line *OA* is called *saturation line*. Here $V_{CE} < V_{BE}$. Both the junctions are forward biased. Here $I_C$ does not depend on the input current $I_B$.

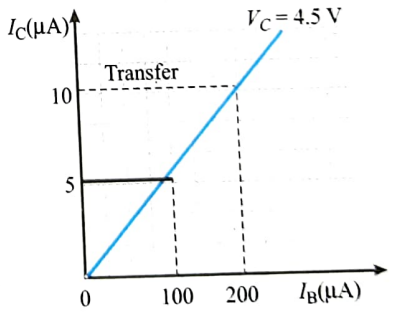
(b) The shaded region lying below the curve for $I_B = 0$ is called *cut-off region*. In this region, both the junctions are reverse biased. Here $I_C = 0$. In the shaded regions, the transistor works as switch, it turns over rapidly from OFF state for which $I_C = 0$ (cut-off) to the ON state for which $I_C$ is maximum (saturation state).

(c) The non-shaded central region of the output characteristic is called *active region*. In this region, the emitter-base junction is forward biased and the collector-base junction is reverse biased. A transistor works as an *audio amplifier* in this region.

The output resistance $r_0$ of a transistor in CE configuration is defined as the ratio of the small change in the collector-emitter voltage to the corresponding change in the collector current when the base current is kept constant. Thus

$$r_0 = \left[\frac{\Delta V_{CE}}{\Delta V_C}\right]_{I_B=\text{constant}}$$

3. **Transfer characteristics**: It is a graph showing the variation of collector current $I_C$ with base current $I_B$ at constant collector-emitter voltage $V_{CE}$. As shown in figure, the transfer characteristic of a transistor is almost a straight line.




**Current amplification factor** ($\beta$): It is defined as the ratio of the change in collector current to the small change in base current at constant collector-emitter voltage ($V_{CE}$) when the transistor is in the active state.

$$\beta = \left[\frac{\Delta I_C}{\Delta I_B}\right]_{V_{CE}=\text{constant}}$$

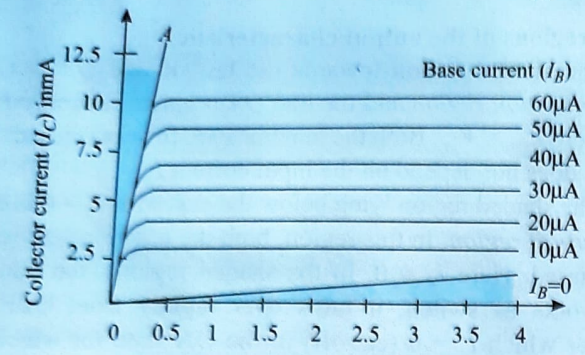
This is also known as small signal current gain and its value is very large. The direct ratio of $I_C$ and $I_B$ gives the d.c. current gain ($\beta_{dc}$) of the transistor.

Hence, $\beta_{dc} = \dfrac{I_C}{I_B}$

Since $I_C$ increases with $I_B$ almost linearly and $I_C = 0$ when $I_B = 0$, the values of both $\beta_{ac}$ and $\beta_{dc}$ are nearly equal.

**ILLUSTRATION 8.15**

From the output characteristics shown in figure, calculate the value amplification factor of the transistor when $V_{CE}$ is 2 V.




**Sol.** Consider two output characteristics corresponding to $I_B = 10\ \mu A$ and $I_B = 60\ \mu A$. Then for $V_{CE} = 2$ V, from these curves, we get

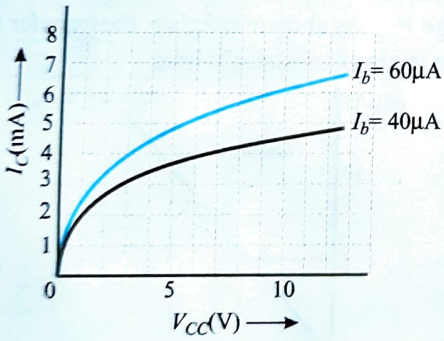
$$\Delta I_B = (60-10)\,\mu A = 50\mu A = 50\times10^{-6}\,A$$

$$\Delta I_C = (10-2.5)\,mA = 7.5\,mA = 7.5\times10^{-3}\,A$$

$$\therefore \quad \beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}} = \left(\frac{7.5\times10^{-3}\,A}{50\times10^{-6}\,A}\right) = 150$$

**ILLUSTRATION 8.16**

A certain *n-p-n* transistor has the common emitter output characteristics as shown in figure.




**(a)** Find the emitter current at $V_{CC} = 10$ V and $I_B = 60\ \mu A$

**(b)** Find $\beta$ at this point.

**Sol.**

**(a)** For $V_{CC} = 10$ V and $I_B = 60\ \mu A$

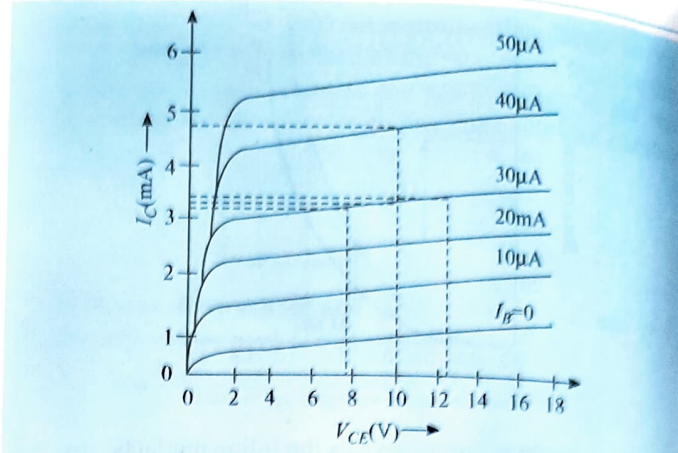
$$I_C = 6\,mA$$

$$I_E = I_C + I_B = 6\,mA + 60\,\mu A = 6.06\,mA$$

**(b)** $$\beta = \frac{I_C}{I_B} = \frac{6\,mA}{6\,\mu A} = 1000$$

**ILLUSTRATION 8.17**

Figure shows the output characteristic of an n-p-n transistor in CE configuration. For this transistor, determine:




**(i)** the dynamic output resistance,

**(ii)** the d.c. current gain and

**(iii)** the a.c. current gain.

**Sol.** Draw a vertical line at $V_{CE} = 10$ V. The point of intersection of this line with the characteristic curve for $I_B = 30\ \mu A$ gives the operating point. At this point, $I_C = 3.6$ mA.

**(i)** To calculate the dynamic output resistance of the transistor, we consider a small change in $V_{CE}$ around the operating point.

$$R_{out} = \left.\frac{\Delta V_{CE}}{\Delta I_C}\right|_{I_E=30\mu A} = \frac{125-7.5}{(3.7-3.5)\times10^{-3}} = 25\times10^3\,\Omega = 25\,k\Omega$$

**(ii)** From the output characteristic, when $I_B = 30\ \mu A$, we have $I_C = 3.6$ mA. Therefore, the d.c. current gain is

$$\beta_{dc} = \frac{I_C}{I_B} = \frac{3.6\,mA}{30\,\mu A} = 120$$

**(iii)** To determine a.c. current gain, we draw a vertical line corresponding to $V_{CE} = 10$ V. Then from the output characteristic, the a.c. current gain is

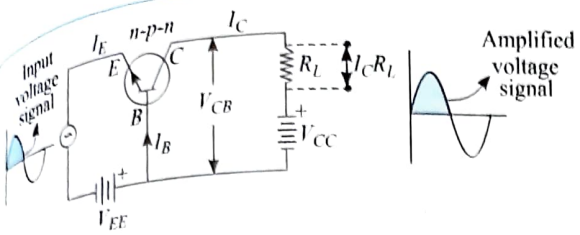
$$R_{ac} = \left.\frac{\Delta I_C}{\Delta I_B}\right|_{V_{CE}=10V} = \frac{(4.7-3.6)\,mA}{(40-30)\,\mu A} = \frac{1.1\times10^{-3}}{10\times10^{-6}} = 110$$

## TRANSISTOR AS A COMMON BASE AMPLIFIER

In case of a common base amplifier, base of the transistor is common to emitter and collector. Let us now discuss the common base amplifier circuits using an n-p-n and a p-n-p transistor.

### *n-p-n* TRANSISTOR AS A COMMON-BASE AMPLIFIER

Figure shows the use of *n-p-n* transistor as a common base amplifier. The base is common to both input and output circuits. The emitter is forward biased by battery $V_{CE}$ and the collector is reverse biased by battery $V_{CC}$, and as a result, the resistance of output circuit ($R_{out}$) is quite large. Thus, $R_{out} > R_{in}$. The low input voltage signal is applied across emitter-base circuit (input circuit) and the amplified output is obtained across the collector and base (output circuit).




Let us assume that $I_E$, $I_B$ and $I_C$ represent emitter current, base current and collector current respectively, when the applied input signal voltage is zero. The arrows represent the direction of flow of hole current i.e. conventional current. According to Kirchhoffs law,

$$I_E = I_B + I_C$$

When current $I_C$ flows in collector circuit, a potential drop equal to $I_C R_L$ will occur across the resistance connected in collector-base circuit and therefore, collector voltage i.e. voltage across the collector and base will be

$$V_{CB} = V_{CC} - I_C R_L$$

When the input signal is applied in emitter-base circuit, it will change the emitter voltage. Due to this, emitter current will change, which in turn will change collector current and then in accordance with the above equation, the collector voltage will vary. The variation in collector voltage, when input signal is applied, appears as the amplified output.

### Phase Relationship Between Input and Output Signals

When a.c. signal is fed to the input circuit, its positive half cycle decreases the forward bias. This decreases the emitter current and the collector current. The potential drop $I_C R_L$ across load resistance decreases. The output signal $V_0$ becomes more positive. So as the input signal goes through its positive half cycle, the output signal also goes through a positive half cycle. Similarly, as the input signal goes through its negative half cycle, the output signal also goes through its negative half cycle. Hence in a common base amplifier, the input and output voltages are in same phase.

### Current, Voltage and Power Gains of a Common Base Amplifier

**a.c. current gain:** It is defined as the ratio of change in collector current to the change in emitter current at constant collector voltage. It is denoted by $\alpha_{ac}$.

$$\therefore \quad \alpha_{AC} = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{V_{CB}} \qquad ...(i)$$

**a.c. voltage gain:** It is defined as the ratio of change in output voltage to the change in input voltage. It is denoted by $A_v$.

Suppose that on applying an a.c. input signal, the input current changes by $\Delta I_E$ and correspondingly, the output current changes by $\Delta I_C$. If $R_{out}$ and $R_{in}$ are the resistances of output and input circuits respectively, then

$$A_v = \frac{\text{change in output voltage}}{\text{change in input voltage}} = \frac{\Delta I_C \times R_{out}}{\Delta I_E \times R_{in}}$$

or $A_v = \alpha_{ac} \times$ resistance gain, ...(ii)

where $R_{out}/R_{in}$ is called resistance gain.

**a.c. power gain:** It is defined as the ratio of change in output power to the change in input power.

$$\text{a.c. power gain} = \frac{\text{change in output power}}{\text{change in input power}}$$

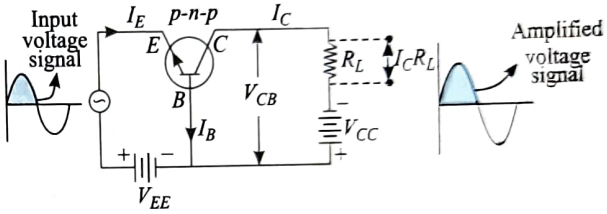
$$= \frac{(\Delta I_C)^2 \times R_{out}}{(\Delta I_E)^2 \times R_{in}} = \left(\frac{\Delta I_C}{\Delta I_E}\right)^2 \times \frac{R_{out}}{R_{in}}$$

or a.c. power gain $= \alpha_{ac}^2 \times$ resistance gain ...(iii)

### p-n-p Transistor as a Common Base Amplifier

Figure shows the use of p-n-p transistor as a common base amplifier. The emitter is common to both input and output circuits. The emitter is forward biased by battery $V_{EE}$ and the collector is reverse biased by battery $V_{CC}$.

The basic criteria of the circuit making use of a p-n-p transistor remains the same as in case of an n-p-n transistor circuit. In this case also, the output voltage signal obtained across the collector and the base is in phase with the input voltage signal as explained below:




In the first positive half cycle of the input signal increases the forward bias of the emitter. This will increase emitter current and hence the collector current. The increase in collector current will increase the potential drop across $R_L$ and hence the collector voltage will decrease. As collector is connected to negative pole of the battery $V_{CC}$, the decrease in collector voltage means that it has become less negative i.e. positive output signal will be produced. Thus, corresponding to the positive output half cycle of a.c. input, positive output half cycle will be produced.

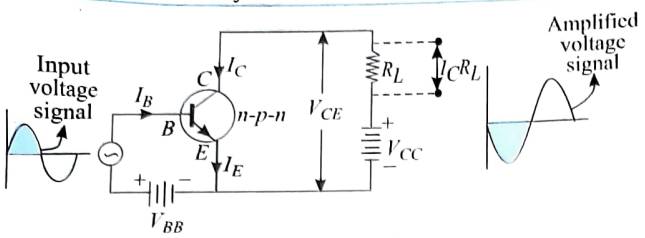
Similarly, we can prove that corresponding to the negative half cycle of a.c. input, the negative output half cycle will be produced. Hence, input and output voltage signals are in phase with each other as shown in figure, when output is obtained across collector and base in a common base amplifier.

## COMMON EMITTER AMPLIFIER

In a common emitter amplifier, the emitter of the transistor is common to base and collector. Let us now discuss common emitter amplifier using an n-p-n and a p-n-p transistor.

### n-p-n TRANSISTOR AS A COMMON EMITTER AMPLIFIER

Figure shows the use of n-p-n transistor as a common emitter amplifier. The emitter is common to both input and output circuits. The emitter is forward biased by battery $V_{BB}$ and the collector is reverse biased by battery $V_{CC}$. This decreases the resistance $R_{in}$ of the input circuit and increases the resistance $R_{out}$ of the output circuit. The low a.c. input signal $V_i$ is superimposed on the forward bias $V_{BE}$. A load resistance $R_L$ is connected between the collector and the d.c. supply and the amplified output is obtained between the collector and the ground.




The emitter current, base current and collector current are related to each other by the equation,

$I_E = I_B + I_C$ (according to Kirchhoff's first law)

Also, corresponding to collector current $I_{CC}$, the collector voltage i.e. voltage across the collector and emitter will be

$$V_{CE} = V_{CC} - I_C R_L$$

The variation in input signal voltage causes variation in emitter current, which in turn produces change in collector current and hence in collector voltage. These variations in collector voltage appear as amplified output voltage.

## Phase Relationship Between Input and Output Signals

When an a.c. signal is fed to the input circuit, its positive half cycle increases the forward bias of the circuit which, in turn, increases the emitter current and hence the collector current. The increase in collector current increases the potential drop across $R_L$, which makes the output voltage $V_Q$ less positive or more negative. So as the input signal goes through its positive half cycle, the amplified output signal goes through a negative half cycle. Similarly, as the input signal goes through its negative half cycle, the amplified output signal goes through its positive half cycle. Hence in a common emitter amplifier, the input and output voltages are 180° out of phase or in opposite phases.

**a.c. current gain:** It is defined as the ratio of the change in collector current to the change in base current at constant collector voltage. It is denoted by $\beta_{ac}$.

$$\therefore \quad \beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}} \qquad ...(i)$$

Its value is quite large as compared to 1 and lies between 15 to 50,

**Transconductance:** It is defined as the ratio of change in collector current to the change in base voltage at constant collector voltage. It is denoted by $g_m$.

$$\text{Therefore, } g_m = \left(\frac{\Delta I_C}{\Delta I_{BE}}\right)_{V_{CE}} \qquad ...(ii)$$

It can be expressed in another form as given below:

$$g_m = \frac{\Delta I_C}{\Delta I_{BE}} \times \frac{\Delta I_B}{\Delta I_B} = \frac{\Delta I_C}{\Delta I_B} \times \frac{\Delta I_B}{\Delta I_{BE}}$$

$$\text{Or} \quad g_m = \beta_{ac} = \frac{1}{R_{in}} \qquad ...(iii)$$

**a.c. voltage gain:** It is the ratio of the change in output voltage to the change in input voltage. It is denoted by $A_v$.

Suppose that on applying an a.c. input signal, the input current changes by $\Delta I_b$ and correspondingly, the output current changes by $\Delta I_c$. Then,

$$A_v = \frac{\text{change in output voltage}}{\text{change in input voltage}} = \frac{\Delta I_C \times R_{out}}{\Delta I_B \times R_{in}}$$

$$\text{or} \quad A_v = \beta_{ac} \times \text{resistance gain} \qquad ...(iv)$$

Since $\beta_{ac} > \alpha_{ac}$ a.c. voltage gain in common emitter amplifier is very large as compared to that in common base amplifier. From the Eqs. (iii) and (iv), it follows that

$$A_v = g_m R_{out} \qquad ...(v)$$

**a.c. power gain:** It is the ratio of the change in output power to the change in input power.

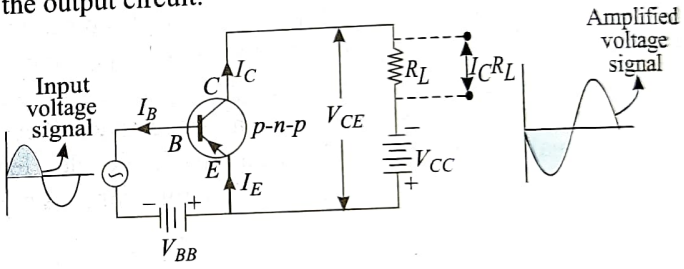
$$\therefore \text{ a.c. power gain } \frac{\text{change in output power}}{\text{change in input power}} = \frac{(\Delta I_C)^2 \times R_{out}}{(\Delta I_B)^2 \times R_{in}}$$

$$\text{or a.c. power gain } = \beta_{ac}^2 \times \text{resistance gain} \qquad ...(vi)$$

As $\beta_{ac}^2 > \alpha_{ac}^2$, a.c. power gain in common emitter amplifier is extremely large as compared to that in case of common base amplifier.

## p-n-p Transistor as a Common Emitter Amplifier

Figure shows the use of p-n-p transistor as a common emitter amplifier. The emitter is common to both input and output circuits. The emitter is forward biased by battery $V_{BB}$ while the collector is reverse biased by battery $V_{CC}$. This decreases the resistance $R_{in}$ of the input circuit and increases the resistance $R_{out}$ of the output circuit.




The a.c. input signal $V_i$ (to be amplified) is superimposed on the forward bias $V_{BE}$. A load $R_L$ is connected between the collector and the d.c. supply and the amplified output is obtained between the collector and the ground. When current $I_C$ flows in the output circuit, the potential drop across load resistance is $I_C R_L$. Hence the output voltage is

$$V_0 = V_{CE} = V_{CC} - I_C R_L$$

When the input signal fed to the base-emitter circuit, the base-emitter voltage changes. This changes the emitter current $I_E$ and hence the collector current $I_C$. The output voltage $V_0$ varies in accordance with the above relation. These variations in the collector voltage appear as amplified output.

## Phase Relationship Between Input and Output Signals

When a.c. is fed to the input circuit, its positive half cycle oppose the forward bias of emitter-base circuit. This decreases th emitter current and hence the collector current $I_C$. As a result, th collector voltage $V_{CE}$ increases. As the collector is connected t the negative terminal of the battery $V_{CC}$, the increase in collecto

voltage means that the collector will become more negative. So as the input signal goes through its positive half cycle, the output signal goes through its negative half cycle. Similarly, as the input signal goes through its negative half cycle, the amplified output signal goes through its positive half cycle. So in a common emitter amplifier, the input and output voltages are 180° out of phase.

## RELATION BETWEEN $\alpha$ AND $\beta$

For both the types of amplifier, we have $I_C = I_B + I_C$

Dividing both sides of the above equation by $I_C$, we get

$$\frac{I_E}{I_C} = \frac{I_B}{I_C} + 1$$

But $\alpha = I_C / I_E$ and $\beta = I_C / I_B$

$$\therefore \quad \frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\text{or} \quad \frac{1}{\beta} = \frac{1}{\alpha} - 1 = \frac{1-\alpha}{\alpha}$$

$$\text{or} \quad \beta = \frac{\alpha}{1-\alpha} \qquad \ldots(i)$$

Equation (i) is the relation between the current gains of common base and common emitter amplifiers.

### ILLUSTRATION 8.18

The input resistance of a transistor is 1000 Ω. On changing, its base current by 10 μA, the collector current increases by 2 mA. If a load resistance of 5 kΩ is used in the circuit, calculate **(i)** the current gain **(ii)** voltage gain of the amplifier.

**Sol.** Here, $R_{in} = 1000\,\Omega$, $\Delta I_B = 10\mu A = 10^{-5}$ A

$R_{out} = 5\text{k}\Omega = 5\times10^3\,\Omega$, $\Delta I_C = 2\text{mA} = 2\times10^{-3}$ A

(i) Current gain, $\beta = \dfrac{\Delta I_C}{\Delta I_B} = \dfrac{2\times10^{-3}}{10^{-5}} = 200$

(ii) Voltage gain, $A_v = \beta\dfrac{R_{out}}{R_{in}} = \dfrac{200\times5\times10^3}{1000} = 1000$

### ILLUSTRATION 8.19

For a CE-transistor amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 2V. Suppose the current amplification factor of the transistor is 100. Find the input signal voltage and base current, if the base resistance is 1 kΩ.

**Sol.** Given: $V_0 = 2$ V, $\beta = 100$

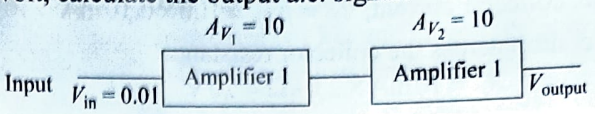
$R_C$ (collector resistance) = 2 kΩ, $R_B$ = 1 kΩ

As $\dfrac{V_0}{V_i} = \beta\left(\dfrac{R_C}{R_B}\right)$

$$V_i = V_0\left(\frac{R_B}{R_C}\right)\left(\frac{1}{\beta}\right) = (2\text{V})\left(\frac{1\text{k}\Omega}{2\text{k}\Omega}\right)\left(\frac{1}{100}\right) = 0.01\text{V}$$

### ILLUSTRATION 8.20

Two amplifiers are connected one after the other in series (cascaded). The first amplifier has a voltage gain of 10 and the second has a voltage gain of 20. If the input signal is 0.01 volt, calculate the output a.c. signal.



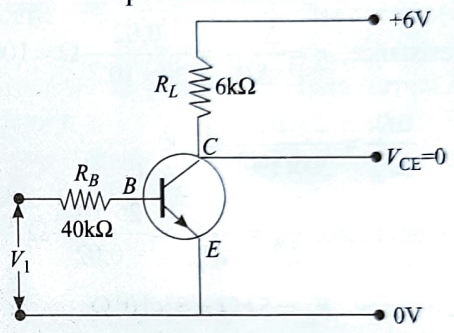

**Sol.** Given: $\left(\dfrac{V_0}{V_i}\right)_1 = 10$ and $\left(\dfrac{V_0}{V_i}\right)_2 = 20$, $V_i = 0.01$ V

Total voltage gain $\left(\dfrac{V_0}{V_i}\right) = \left(\dfrac{V_0}{V_i}\right)_1 \times \left(\dfrac{V_0}{V_i}\right)_2 = 10 \times 20 = 200$

Thus, $V_0 = 200\ V_i = 200 \times 0.01$ V = 2 V

### ILLUSTRATION 8.21

In figure the transistor is just saturated with an input d.c. voltage $V_i$. If the gain is 100, calculate $V_i$. Neglect the base-emitter resistance and potential difference.




**Sol.** Since the transistor is just saturated, $V_{CE} = 0$,

Thus, from $V_{CE} = V_{CC} - I_C R_L$, $I_C R_L = V_{CC}$

or $\quad I_C = \dfrac{V_{CC}}{R_L} = \dfrac{6\text{V}}{6\text{ k}\Omega} = \dfrac{6\text{V}}{6000\Omega} = 1\text{mA}$

As $\quad \beta = \dfrac{I_C}{I_B}, I_B = \dfrac{I_C}{\beta} = \dfrac{1\text{ mA}}{100} = 0.01\text{ mA}$

Further, $V_i = I_B R_B = (0.01\text{ mA})(40\text{k}\Omega)$

$= (0.01\times10^{-3}\text{A})(40\times10^3\,\Omega) = 0.4\text{V}$

### ILLUSTRATION 8.22

For a CE-transistor amplifier, the audio signal voltage across the collector resistance of 2.0 kΩ is 2.0 V. Suppose the current amplification factor of the transistor is 100, what should be the value of $R_B$ in series with $V_{BB}$ supply of 2.0 V if the d.c. base current has to be 10 times the signal current? Also calculate the d.c. drop across the collector resistance. Take $V_{BE} = 0.6$ V.

**Sol.** Here, $R_C = 2.0\text{k}\Omega$, $\beta = 100$, $V_{BB} = 2.0\text{V}$ or $V_{BE} = 0.6\text{V}$

The output a.c. voltage = 2.0 V

$\therefore$ The a.c. collector current, $i_C = \dfrac{2.0\text{V}}{R_C} = \dfrac{2.0\text{V}}{2.0\times10^3\,\Omega} = 1.0\text{mA}$

The signal current through the base,

$$i_B = \frac{i_C}{\beta} = \frac{1.0\text{mA}}{100} = 0.010\text{ mA}$$

The d.c. base current, $I_g = 10 \times$ signal current
$= 10 \times 0.010 = 0.10\,\text{mA}$

$$R_B = \frac{V_{BB} - V_{EB}}{I_B} = \frac{(2.0 - 0.6)\,\text{V}}{0.10\,\text{mA}} = 14\,\text{k}\Omega$$

The d.c. collector current, $I_C = \beta I_B = 100 \times 0.10\,\text{mA} = 10\,\text{mA}$

The d.c. drop across the collector resistance
$= I_C R_C = 10\,\text{mA} \times 2.0\,\text{k}\Omega = 20\,\text{V}$

### ILLUSTRATION 8.23

In a silicon transistor, the base current is changed by 20 μA. This results in a change of 0.02 V in base to emitter voltage and a change of 2 mA in the collector current.

**(a)** Find the input resistance $\beta_{ac}$ and transconductance of the transistor.

**(b)** This transistor is used as a CE amplifier with the load resistance 5 kΩ. What is the voltage gain of the amplifier?

**Sol.** Here, $\Delta I_B = 20\mu\text{A} = 20 \times 10^{-6}\,\text{A}$

$\Delta I_C = 2\,\text{mA} = 20 \times 10^{-3}\,\text{A}$, $\Delta V_{BE} = 0.02\,\text{V}$

**(a)** Input resistance, $r_i = \dfrac{\Delta V_{BE}}{\Delta I_B} = \dfrac{0.02}{20 \times 10^{-6}}\,\Omega = 1000\,\Omega$

$$\beta_{ac} = \frac{\Delta I_C}{\Delta I_B} = \frac{2 \times 10^{-3}}{20 \times 10^{-6}} = 100$$

Transconductance, $g_m = \dfrac{\Delta I_C}{\Delta V_{BE}} = \dfrac{2 \times 10^{-3}}{0.02}\,\Omega^{-1} = 0.1\,\Omega^{-1}$

**(b)** Load resistance, $R_L = 5k\Omega = 5 \times 10^3\,\Omega$

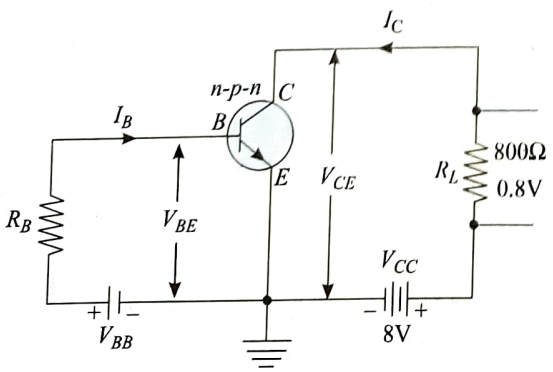
$\therefore$ Voltage gain, $A_v = \dfrac{\Delta V_{CE}}{\Delta V_{BE}} = \dfrac{R_L \cdot \Delta I_C}{\Delta V_{BE}}$

$$A_v = \frac{5 \times 10^3 \times 2 \times 10^{-3}}{0.02} = 500$$

### ILLUSTRATION 8.24

An n-p-n transistor is connected in common-emitter configuration in which collector supply is 8 V and the voltage drop across the load resistance of 800 Ω connected in the collector circuit is 0.8 V. If current amplification factor is 25, determine collector emitter voltage and base current. If the internal resistance of the transistor is 200 Ω, calculate the voltage gain and power gain.

**Sol.** A common emitter *n-p-n* amplifier circuit is shown in figure. Here $V_{CC}$ and $V_{BB}$ are the collector supply and base supply voltages respectively and $R_L$ is the load resistance in the collector circuit.




Collector circuit, $I_C = \dfrac{\text{Voltage drop across } R_L}{R_L}$

$$= \frac{0.8}{800} = 1.0 \times 10^{-3}\,\text{A}$$

Collector-emitter voltage, $V_{CE} = V_{CC} - I_C R_L = 8 - 0.8 = 7.2\,\text{V}$

As, $\beta = \dfrac{I_C}{I_B}$

$\therefore \quad I_B = \dfrac{I_C}{\beta} = \dfrac{1.0 \times 10^{-3}\,\text{A}}{25} = 0.04 \times 10^{-3}\,\text{A} = 40\,\mu\text{A} \quad [\because \beta = 25]$

Voltage gain, $A_v = \beta . \dfrac{R_L}{R_B} = 25 \times \dfrac{800}{200} = 100$

Power gain, $A_p = \beta^2 . \dfrac{R_L}{R_B} = (25)^2 \times \dfrac{800}{200} = 2500$

## TRANSISTOR AS AN OSCILLATOR

An **oscillator** is a generator of an alternating voltage, usually of high frequency and constant amplitude. The energy required for this purpose is taken from a d.c. source.

An oscillator can, therefore, be taken as: a device for converting energy of d.c. source into an alternating voltage of high frequency.

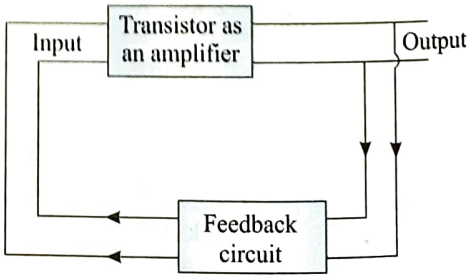
The basic oscillating circuit consists of an inductance $L$ connected in parallel to a capacitor $C$. This circuit is called the tank circuit. The frequency of oscillations ($f$) set up in the tank circuit is given by

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad \ldots(i)$$

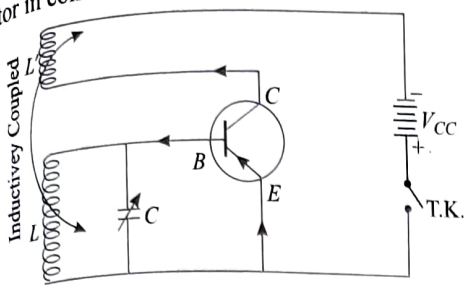
where $L$ is inductance of the inductor in henry and $C$ is capacitance of the capacitor in **farad**. The frequency of the oscillations can be varied by using a variable capacitor.

**Transistor as oscillator:** In order to produce undamped oscillations, a transistor may be used. It does so by supplying a suitable amount of electric energy to the LC-circuit in proper phase, so that it just compensates the loss of energy during each oscillation. This process is called feedback and the arrangement as feedback oscillator.

The block diagram of a feedback oscillator circuit using a transistor is as shown in figure. In a feedback oscillator, the amplifier circuit is inductively coupled to the LC-circuit. Since the LC-circuit produces damped oscillations due to loss of energy in the form of heat, the feedback should be of the nature so that it will increase the input of the amplifier. Such a feedback is called positive feedback. The feedback circuit supplies an appropriate part of the output of the amplifier to the LC-circuit in proper phase, so as to compensate for the the loss of energy during each oscillation. As such, the oscillator produces undamped oscillations of the frequency given by the Eq. (i).



**Circuit diagram:** Figure shows the circuit for p-n-p transistor as an oscillator in common emitter configuration.




The battery is connected in series with the high impedance coil $L'$ in collector-emitter circuit and a low impedance coil $L$ is connected in the emitter-base circuit. A variable capacitor $C$ of suitable range connected in parallel to the coil $L$ may be used to vary the frequency of the oscillations produced.

**ILLUSTRATION 8.25**

All India Radio broadcasts at 710 kcps, whereas BBC broadcasts at 1,380 kcps on the same medium wave band. For changing over from All India Radio to BBC, then capacitance of the variable capacitor has to be decreased. Give reasons in support of your answer.

**Sol.** The frequency of oscillations is given by $f = \dfrac{1}{2\pi\sqrt{LC}}$

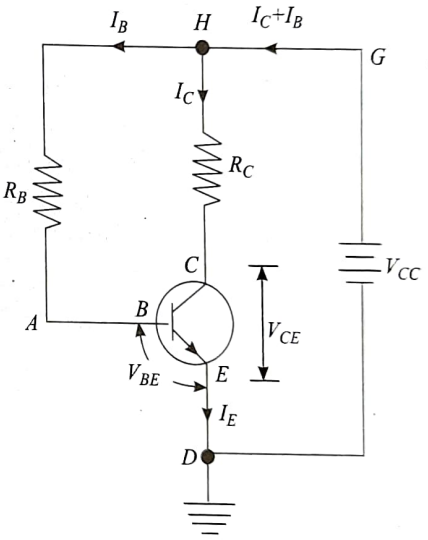
As both the stations are tuned on the same band (medium wave band), the value of $L$ is same in both the cases and therefore,

$$f \propto \frac{1}{\sqrt{LC}}$$

Since in changing over from All India Radio (710 kcps) to BBC (1,380 kcps), frequency increases, the capacitance of variable capacitor has to be decreased.

## CONCEPT APPLICATION EXERCISE 8.3

1. For transistor action, which of the following statements are correct.
   (a) Base, emitter and collector regions should have similar size and doping concentrations.
   (b) The base region must be very thin and lightly doped.
   (c) The emitter junction is forward biased and collector junction is reverse biased.
   (d) Both the emitter junction as well as the collector junction are forward biased.
2. For a transistor amplifier, the voltage gain
   (a) remains constant for all frequencies.
   (b) is high at high and low frequencies and constant in the middle frequency range.
   (c) is low at high and low frequencies and constant at mid frequencies.
   (d) none of the above
3. In a common emitter transistor circuit, if the collector-emitter voltage changes by 0.2 V, the collector current changes by 0.004 mA. What is the output resistance of the circuits.
4. A transistor has a current gain of 30. If the collector resistance is 6 kΩ, input resistance is 1 kΩ, calculate its voltage gain.
5. A change of 0.2 mA in the base current causes a change of 5 mA in the collector current for a common emitter amplifier. **(i)** Find the a.c. current gain of the transistor. **(ii)** If the input resistance is 2 kΩ and its voltage gain is 75, calculate the load resistance used in the circuit.
6. The power gain for CB amplifier is 800, and the voltage amplification factor is 840. Find the collector current when the base current is 1.2 mA.
7. In a common-emitter transistor amplifier, an increase of 50 μA in the base current causes an increase of 1.0 mA in the collector current. Calculate gain $\beta$. What will be the change in emitter current? Also calculate current gain $\alpha$.
8. The potential difference across the collector of a transistor, used in common emitter mode is 1.5 V, with the collector resistance of 3 kΩ. Find **(i)** the emitter and **(ii)** the base current, if the d.c. gain of the transistor is 50.
9. The current gain for common emitter amplifier is 59. If the emitter current is 6.0 mA, find **(i)** base current **(ii)** collector current.
10. In the circuit shown in figure, the base current $I_B = 5.0$ μA, base resistor $R_B = 1.0$ MΩ, collector resistance $R_C = 1.1$ kΩ, the collector current $I_C = 5.0$ mA and the d.c. voltage in the collector circuit $V_{CC} = 6.0$ V.



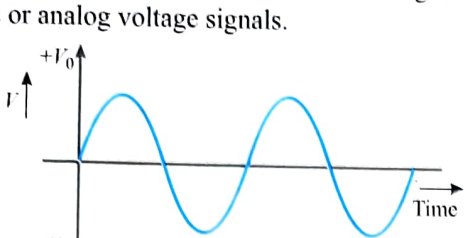

**(i)** Can this circuit be used in an amplifier?
**(ii)** What happen if the resistance $R_C$ is 400 Ω and $I_B$, $I_C$ and $R_B$ remain same as above?

### ANSWERS

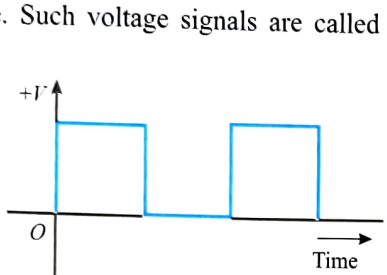
**1.** (b), (c) **2.** (c) **3.** 50 kΩ **4.** 180
**5.** (i) 25 (ii) 6 kΩ **6.** 24 mA
**7.** 20, 1050 μA, $\dfrac{20}{21}$ **8.** (i) 0.49 mA (ii) 0.10 mA
**9.** (i) 0.1 mA (ii) 5.9 mA
**10.** (i) No (ii) can be used as amplifier

# DIGITAL ELECTRONICS: ANALOG AND DIGITAL CIRCUITS

**Analog circuits:** In analog electronic circuits, the voltage signals vary continuously with time and such signals are called continuous or analog voltage signals.




**Digital circuits:** The digital electronic circuits make use of entirely different type of voltage signals. Such signals have pulse wave form, which do not vary continuously with the time, in the manner the analog voltage signals do. Instead, such a signal has only two voltage levels i.e. either zero or some constant value of the voltage. Such voltage signals are called digital voltage signals.

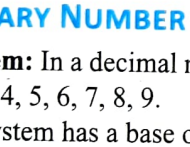



By representing these two voltage levels by binary numbers 0 and 1, the digital electronics has been developed. The counters, computers, etc. are the outcome of digital electronics. We now first learn some elementary ideas about binary number system and then the logic gates, which are the basic building blocks of digital electronics.

## DECIMAL AND BINARY NUMBER SYSTEM

**Decimal number system:** In a decimal number system, we have ten digits i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

A decimal number system has a base of ten (10)




e.g., $1971 = 1000 + 900 + 70 + 1$
$= 1 \times 10^3 + 9 \times 10^2 + 7 \times 10^1 + 1 \times 10^0$

LSD = Least significant digit
MSD = Most significant digit

**Binary number system:** A number system which has only two digits i.e. 0 (Low) and 1 (High) is known as binary system. The base of binary number system is 2.

(i) Each digit in binary system is known as a bit and a group of bits is known as a byte.

(ii) The electrical circuit which operates only in these two state i.e. 1 (On or High) and 0 (i.e. Off or Low) are known as digital circuits.

### Different names for the digital signals

| State Code | 1 | 0 |
|---|---|---|
| Name for the State | On | Off |
| | Up | Down |
| | Close | Open |
| | Excited | Unexcited |
| | True | False |
| | Pulse | No pulse |
| | High | Low |
| | Yes | No |

## DECIMAL TO BINARY CONVERSION

The binary representation of any decimal digit or number can be easily obtained by double dabble method. To represent a decimal number in binary number system by double dabble method, the following steps are involved:

(i) Divide the given decimal number by 2 and the successive quotients by 2 till the quotient becomes zero.

(ii) The sequence of remainders obtained during divisions gives the binary equivalent of decimal number.

(iii) Then, the remainders taken in reverse order (obtained in successive divisions) give the binary representation of the given decimal number.

**For example:** Binary equivalence of 61

| 2 | 61 | Remainder |
|---|---|---|
| 2 | 30 | 1 |
| 2 | 15 | 0 |
| 2 | 7 | 1 |
| 2 | 3 | 1 |
| 2 | 1 | 1 |
| | 0 | 1 |

$\Rightarrow (61)_{10} = (111101)_2$

## BINARY TO DECIMAL CONVERSION

The least significant digit in the binary number is the coefficient of 2 with power zero. As we move towards the left side of LSD, the power of 2 goes on increasing.

**For example:** $(11111100101)_2 = 1 \times 2^{10} + 1 \times 2^9 + 1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 2021$

## BOOLEAN ALGEBRA

(1) In Boolean algebra only two states of variables (0 and 1) are allowed.

(2) The variables (A, B, C,…) of Boolean algebra are subjected to three operations.

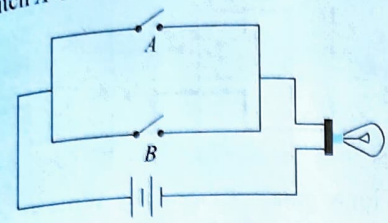
Boolean algebra uses only three basic operations on its variables:

(i) OR (ii) AND (iii) NOT

**OR Operation:** It is represented by the symbol '+'. In Boolean algebra, '+', does not denote addition as in ordinary algebra. Here in symbolic logic, it denotes the OR operation. In logic algebra $A + B = Y$ means that when $A$ is true OR $B$ is true OR both are true, then $Y$ will be true. The laws of OR operation may be expressed as

$0+0=0$ ⋮ $1+0=1$
$0+1=1$ ⋮ $1+1=1$

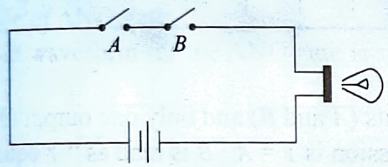
**Important Point:**
Boolean expression for OR, $Y = A + B$
When switch $A$ or $B$ is closed – Bulb glows




**AND operation:** It is represented by the symbol '×' or by '·'. In Boolean algebra '·' does not denote multiplication as in ordinary algebra. Here in symbolic logic it denotes the AND operation. In logic algebra $A \cdot B = Y$ means that output $C$ is true only when both $A$ and $B$ are true. The laws of AND operation may be expressed as

$0 \cdot 0 = 0$ ⋮ $1 \cdot 0 = 0$
$0 \cdot 1 = 0$ ⋮ $1 \cdot 1 = 1$

**Important Point:**
Boolean expression for AND, $Y = A \cdot B$
When switches $A$ and $B$ both are closed – Bulb glows

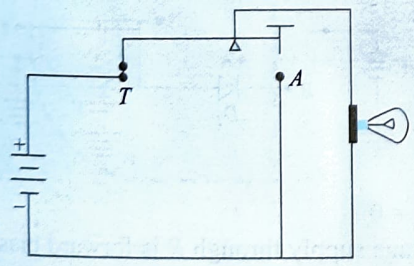



**NOT operation:** It is a complementary operation and it is represented by bar over the variables.

In Boolean algebra $Y = \overline{A}$ mean if $A$ is true then $Y$ is not true. The laws of NOT operation may be expressed as

$\overline{0} = 1$ ⋮ $\overline{1} = 0$

**Important Point:**




a OFF → Lamp ON
a ON → Contact at $T$ is broken
→ Lamp OFF

**Basic Boolean postulates and laws**

(i) Boolean postulates : $0 + A = A$, $1 \cdot A = A$, $1 + A = 1$, $0 \cdot A = 0$, $A + \overline{A} = 1$

(ii) Identity law : $A + A = A$, $A \cdot A = A$

(iii) Negation law : $\overline{\overline{A}} = A$

(iv) Commutative law : $A + B = B + A$, $A \cdot B = B \cdot A$

(v) Associative law : $(A+B) + C = A + (B+C)$, $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

(vi) Distributive law : $A \cdot (B+C) = A \cdot B + A \cdot C$
$(A + B) \cdot (A + C) = A + BC$

(vii) Absorption laws : $A + A \cdot B = A$, $A \cdot (A + B) = A$
$\overline{A} \cdot (A + B) = \overline{A} \cdot B$

(viii) Boolean identities : $A + \overline{A}B = A + B$, $A(\overline{A} + B) = AB$,
$A + BC = (A + B)(A + C)$, $(\overline{A} + B) \cdot (A + C) = \overline{A}C + AB$

(ix) **De Morgan's theorem**: It states that the complement of the whole sum is equal to the product of individual complements and vice versa i.e. $\overline{A + B} = \overline{A} \cdot \overline{B}$ and $\overline{A \cdot B} = \overline{A} + \overline{B}$

## ILLUSTRATION 8.26

Perform binary addition on the following sets of numbers:
**(a)** 110010 and 111101
**(b)** 101010 and 010101

**Sol.**

**(a)** To find the sum of the given binary numbers 110010 and 111101, we first find their values in the decimal number system.

$(110010)_2 = 1\times2^5 + 1\times2^4 + 0\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$
$= 32 + 16 + 0 + 0 + 2 + 0 = 50$

$(111101)_2 = 1\times2^5 + 1\times2^4 + 1\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$
$= 32 + 16 + 8 + 4 + 0 + 1 = 61$

Therefore, sum of the given two numbers in decimal system is 50 + 61 i.e. 111.

The binary representation of the number 111 in the decimal system can be found by double dabble method as below:

| | |
|---|---|
| 2 | 111 |
| 2 | 55 – 1 |
| 2 | 27 – 1 |
| 2 | 13 – 1 |
| 2 | 6 – 1 |
| 2 | 3 – 0 |
| 2 | 1 – 1 |
| | 0 – 1 |

Writing the remainders in the inverse order, it follows that $(111)_{10} = 1101111$ i.e. addition of binary numbers 110010 and 111101 is equal to 1101111.

**(b)** $(101010)_2 = 1\times2^5 + 0\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$
$= 32 + 0 + 8 + 0 + 2 + 0 = 42$

$(010101)_2 = 0\times2^5 + 1\times2^4 + 0\times2^3 + 1\times2^2 + 0\times2^1 + 1\times2^0$
$= 0 + 16 + 0 + 4 + 0 + 1 = 21$

The sum of the given two numbers in the decimal number system is 42 + 21 i.e. 63. By proceeding as above, it can be shown that binary representation of 63 i.e. $(63)_{10} = 111111$. Therefore, addition of binary numbers 101010 and 010101 is equal to 111111.

## LOGIC GATES AND TRUTH TABLE

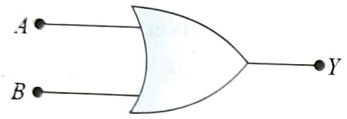
**Logic gate:** The digital circuit that can be analysed with the help of Boolean algebra is called logic gate or logic circuit. A logic gate has two or more inputs but only one output.

There are primarily three logic gates namely the OR gate, the AND gate and the NOT gate.

**Truth table:** The operation of a logic gate or circuit can be represented in a table which contains all possible inputs and their corresponding outputs is called the truth table. To write the truth table we use binary digits 1 and 0.

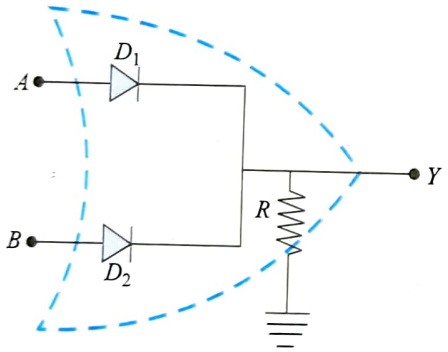
### 'OR' Gate

It has two inputs ($A$ and $B$) and only one output ($Y$).
Boolean expression is $Y = A + B$ and is read as "$Y$ equals $A$ **OR** $B$"




**Logical symbol of OR gate**

*Realization of OR Gate*




(i) $A = 0, B = 0$

Both diodes $D_1$ and $D_2$ do not conduct and hence $Y = 0$

ii) $A = 0, B = 1$

$D_1$ = Does not conduct, $D_2$ = Conducts, hence $Y = 1$

ii) $A = 1, B = 0$

$D_1$ = Conducts, $D_2$ = Does not conduct, hence $Y = 1$

v) $A = 1, B = 1$

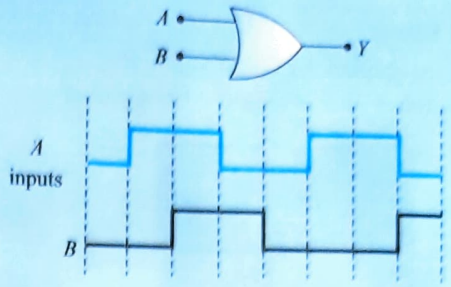
Both $D_1$ and $D_2$ conduct, hence $Y = 1$

*ruth Table for 'OR' Gate*

| A | B | Y = A + B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

**ILLUSTRATION 8.27**

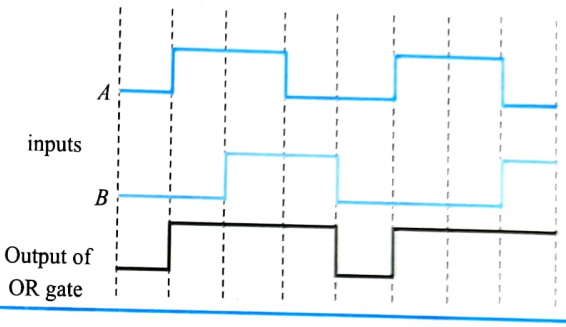
In figure below, circuit symbol of a logic gate and two input waveforms '$A$' and '$B$' are shown.




**(a)** Name the logic gate. **(b)** Write its truth table
**(c)** Give the output waveform.

**Sol.**

**(a)** The given logic gate is an OR gate.
**(b)** Truth table of OR gate
**(c)** The output waveform for the OR gate is shown below.

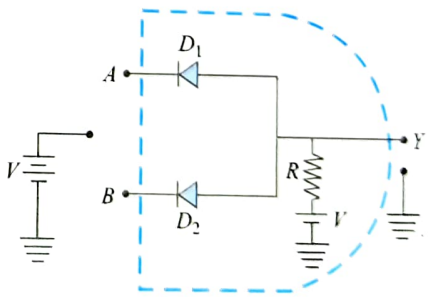



### 'AND' Gate

It has two inputs ($A$ and $B$) and only one output ($Y$).
Boolean expression is $Y = A \cdot B$ is read as "$Y$ equals $A$ **AND** $B$"

**Logical symbol of AND gate**

*Realization of AND Gate*




(i) $A = 0, B = 0$

The voltage supply through $R$ is forward biasing diodes $D_1$ and $D_2$ (offers low resistance) the voltage $V$ would drop across $R$

The output voltage at $Y$ = the voltage across diode = 0

(ii) $A = 0, B = 1$

$D_1$ = conducts, $D_2$ = Not conducts

the output voltage at $Y$= The voltage across the diode ($D_1$) =0

(iii) $A = 1, B = 0$

$D_1$ = conducts, $D_2$ = Not conducts

the output voltage at $Y$= The voltage across the diode ($D_2$) =0

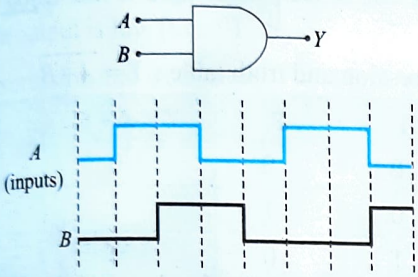
(iv) $A = 1, B = 1$
None of the diode conducts
the output voltage at $Y$ = Battery voltage = 1

*Truth Table for 'AND' Gate*

| A | B | Y = A · B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

**ILLUSTRATION 8.28**

In figure below, circuit symbol of a logic gate and two input waveforms '$A$' and '$B$' are shown.




(a) Name the logic gate. **(b)** Write its truth table.
(c) Give the output waveform.

**Sol.**

(a) The given logic gate is an AND gate.
(b) Truth table of AND gate.
(c) The output waveform for the AND gate is shown below.

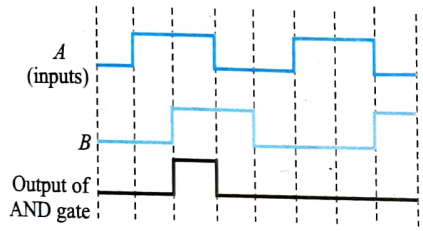
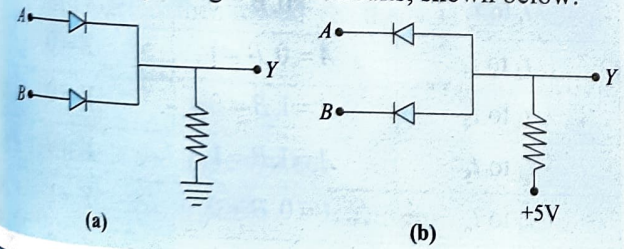


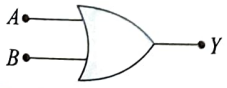

**ILLUSTRATION 8.29**

Given the symbol, and the truth table of each of the two logic gates, obtained by using the two circuits, shown below.



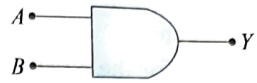

**Sol.**

(a) The circuit of Fig. (a) represents OR gate.
Truth table of OR gate




| Input A | Input B | Output Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

**(b)** The circuit of Fig. (b) represents AND gate.
Truth table of AND gate

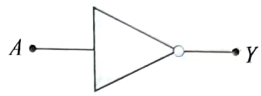



| Input A | Input B | Output Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

## 'NOT' Gate

It has only one input and only one output.

Boolean expression is $Y = \bar{A}$ and is read as "$Y$ equals not $A$"



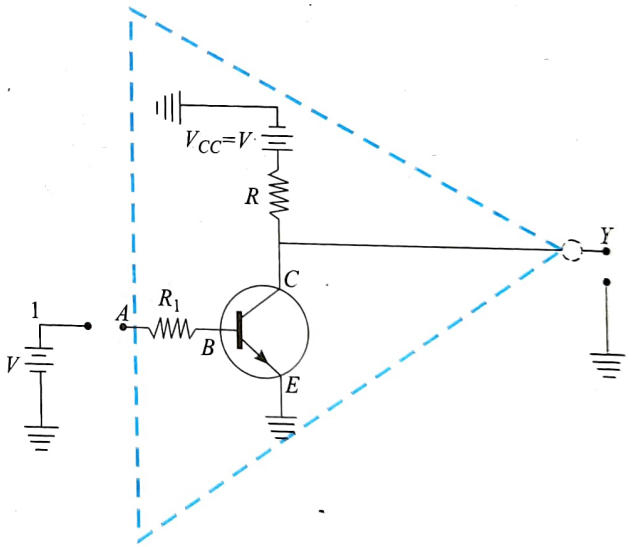

**Logical symbol of NOT gate**

### *Realization of NOT Gate*

The transistor is so biased that the collector voltage $V_{CC} = V$ (Voltage corresponding to 1 state)

The resistors $R$ and $R_1$ are so chosen that if the input is low i.e. $O$, the transistor is in the cut off and hence the voltage appearing at the output will be the same as applied $V$. Hence $Y = V$ (or state 1).

If the input is high, the transistor current is in saturation and the net voltage at the output $Y$ is 0 (in state 0).

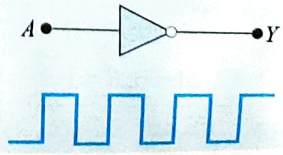



### *Truth Table for NOT Gate*

| A | $Y = \bar{A}$ |
|---|---|
| 0 | 1 |
| 1 | 0 |

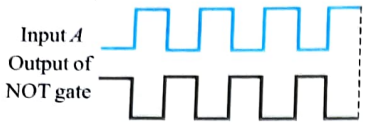
**ILLUSTRATION 8.30**

In the figure below, circuit of a logic gate and input waveform is shown. **(a)** Name the logic gate, **(b)** write its truth table and **(c)** give the output waveform.



**Sol.**

**(a)** The logic gate is a NOT gate.
**(b)** Truth table of NOT gate
**(c)** The output waveform for the NOT gate is shown below.




## COMBINATION OF LOGIC GATES

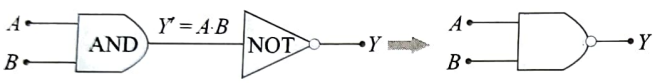
Various combinations of three basic gates i.e. OR, AND and NOT give rise to complicated digital circuits. We now discuss a few combinations of these basic gates using their symbols.

### 'NAND' Gate

It is a logic circuit in which AND gate is followed by a NOT gate.

If the ouput ($Y'$) of AND gate is connected to the input of NOT gate, the gate so obtained is called NAND gate. The logic symbol of the NAND gate is as shown in figure.

From 'AND' and 'NOT' gate

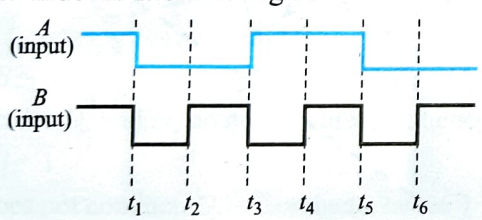



Boolean expression and truth table: $Y = \overline{A \cdot B}$

| A | B | Y' = A · B | Y |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

### ILLUSTRATION 8.31

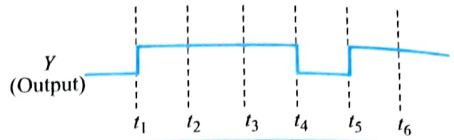
Sketch the output waveform obtained from NAND gate for the inputs $A$ and $B$ as shown in figure.




**Sol.** The output of NAND gate is low when both the inputs are high otherwise it is high.

| Time interval | Inputs | Outputs |
|---|---|---|
| $t < t_1$ | $A = 1, B = 1$ | $Y = 0$ |
| $t_1$ to $t_2$ | $A = 0, B = 0$ | $Y = 1$ |
| $t_2$ to $t_3$ | $A = 0, B = 1$ | $Y = 1$ |
| $t_3$ to $t_4$ | $A = 1, B = 0$ | $Y = 1$ |
| $t_4$ to $t_5$ | $A = 1, B = 1$ | $Y = 0$ |
| $t_5$ to $t_6$ | $A = 0, B = 0$ | $Y = 1$ |
| $t > t_6$ | $A = 0, B = 1$ | $Y = 1$ |

Therefore, the output waveform for NAND gate will be as shown in figure.

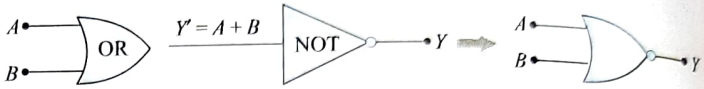



### 'NOR' Gate

It is a logic circuit in which OR gate is followed by a NOT gate.

If the output ($Y'$) of OR gate is connected to the input of a NOT gate, the gate so obtained is called the NOR gate. The logic symbol of the NOR gate is shown in figure.

From 'OR' and 'NOT' gate

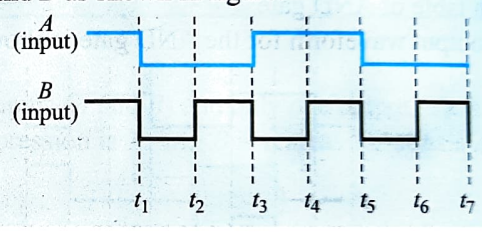



Boolean expression and truth table : $Y = \overline{A + B}$

| A | B | Y' = A + B | Y |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

### ILLUSTRATION 8.32

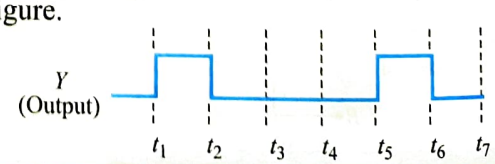
Sketch the output waveform obtained from NOR gate for the inputs $A$ and $B$ as shown in figure.




**Sol.** The output of NOR gate is high when both the inputs are low otherwise it is low.

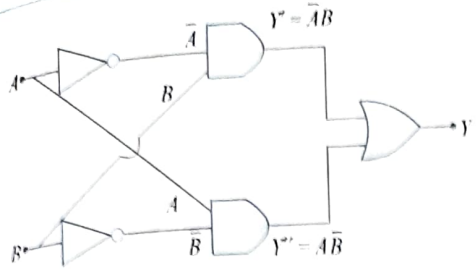
| Time interval | Inputs | Outputs |
|---|---|---|
| $t < t_1$ | $A = 1, B = 1$ | $Y = 0$ |
| $t_1$ to $t_2$ | $A = 0, B = 0$ | $Y = 1$ |
| $t_2$ to $t_3$ | $A = 0, B = 1$ | $Y = 0$ |
| $t_3$ to $t_4$ | $A = 1, B = 0$ | $Y = 0$ |
| $t_4$ to $t_5$ | $A = 1, B = 1$ | $Y = 0$ |
| $t_5$ to $t_6$ | $A = 0, B = 0$ | $Y = 1$ |
| $t_6$ to $t_7$ | $A = 0, B = 1$ | $Y = 0$ |

Therefore, the output waveform for NAND gate will be as shown in figure.




### 'XOR' Gate

An XOR gate is obtained by using OR, AND and NOT gates . It is also called exclusive OR gate and its logic symbol is as shown in figure.



The logic gate which gives high output (i.e., 1) if either input or input $B$ but not both are high (i.e., 1) is called exclusive OR gate or the XOR gate.

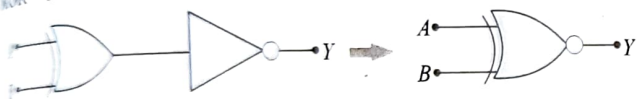
It may be noted that if both the inputs of the XOR gate are high, then the output is low (i.e., 0).

Boolean expression and truth table: $Y = A \oplus B = \bar{A}B + A\bar{B}$

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

### Exclusive NOR (XNOR) Gate

XOR - NOT → XNOR



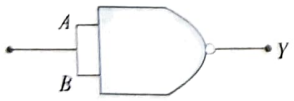

Boolean expression: $Y = A \odot B = \bar{A}\bar{B} + AB$

## LOGIC GATES USING 'NAND' GATE

The NAND gate is the building block of the digital electronics. All the logic gates like the OR, the AND and the NOT can be constructed from the NAND gates.

### Construction of the 'NOT' Gate from the 'NAND' Gate

(i) When both the inputs ($A$ and $B$) of the NAND gate are joined together then it works as the NOT gate.

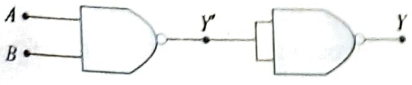



(ii) Truth table and logic symbol

| Input | Output |
|---|---|
| $A = B$ | $Y$ |
| 0 | 1 |
| 1 | 0 |

### Construction of the 'AND' Gate from the 'NAND' Gate

(i) When the output of the NAND gate is given to the input of the NOT gate (made from the NAND gate), then the resultant logic gate works as the AND gate

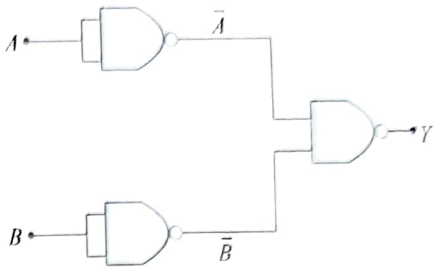



(ii) Truth table and logic symbol

| A | B | Y' | Y |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

### Construction of the 'OR' Gate by the 'NAND' Gate

(i) When the outputs of two NOT gates (obtained from the NAND gate) is given to the inputs of the NAND gate, the resultant logic gate works as the OR gate

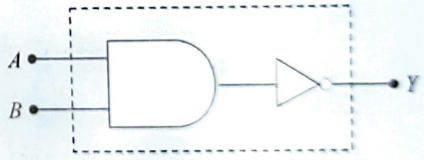



(ii) Truth table and logic symbol

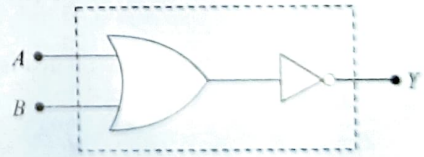
| A | B | $\bar{A}$ | $\bar{B}$ | Y |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |

### ILLUSTRATION 8.33

Give the symbol, and the truth table of each of the two logic gates, obtained by using the two gate combinations, shown in figure.



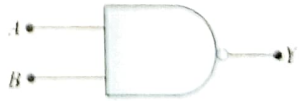

(a)




(b)

**Sol.**

(a) The gate combination of Fig. (a) represents a NAND gate.



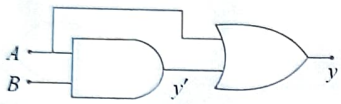

| Input A | Input B | Output Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**(b)** The gate combination of Fig. (b) represents a NOR gate.

| Input A | Input B | Output Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

## ILLUSTRATION 8.34

The given figure represents a combination of gates, write the truth table for this.



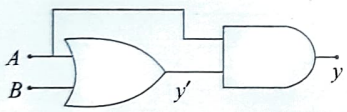

**Sol.** In the combination of gates given in figure, input ($A$) and output (say $y'$) of the AND gate form the two inputs of the OR gate.

Following is the truth table, which gives the final output ($y$) of the combination of two gates.

| A | B | y' | y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

## ILLUSTRATION 8.35

The given figure represents a combination of gates, write the truth table.



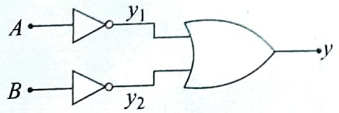

**Sol.** In the combination of gates shown in figure, the input ($A$) and output (say $y$) of the OR gate form the two inputs of the AND gate.

The output ($y$) of the combination of two gates is given by the truth table as given in table.

| A | B | y' | y |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

## ILLUSTRATION 8.36

Express by a truth table, the output $y$ for all possible inputs $A$ and $B$ for the combination of gates shown in figure. Hence, show that it behaves as a NAND gate.



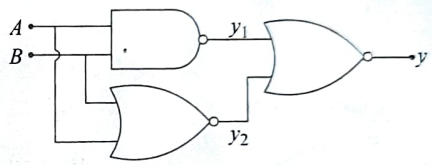

**Sol.** In given situation, outputs $y_1$ and $y_2$ of two NOT gates form the two inputs of the OR gate. Let us consider that the inputs $A$, $B$ have values 0, 0, then 0, 1, then 1, 0 and finally 1, 1. Because of the NOT gate, the column $y_1$, contains ones, where $A = 0$ and 0, where $A = 1$. Similarly, $y_2$ can be obtained for different values of $B$. The output $y$ of OR gate is 1, if $y_1$ or $y_2$ is 1 or both are 1. Hence, the truth table for the given combination of gates is as shown in table.

| A | B | $y_1$ | $y_2$ | y | | A | B | y |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | ≡ | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | | 1 | 1 | 1 |

From the completed table showing the relationship between output $y$ and the inputs $A$ and $B$, it follows that it is the truth table for a single NAND gate. Hence, the combination of gates as shown in figure, behaves as NAND gate.

## ILLUSTRATION 8.37

Produce the truth table for the combination of gates shown in figure and hence show that it behaves as XOR gate.




**Sol.** Here, the outputs $y_1$ and $y_2$ of AND and NOR gates respectively form the two inputs of another NOR gate. Because of AND gate, the column $y_1$ contains 1, when both $A$ and $B$ are 1. On the other hand, because of NOR gate, the column $y_2$ contains 1, when both $A$ and $B$ are zero. Again, the column $y$ contains 1, when both $y_1$ and $y_2$ are 0. Hence, the truth table for the given combination of gates is as shown in table.

| A | B | $y_1$ | $y_2$ | y | | A | B | y |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | ≡ | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | | 1 | 1 | 1 |

From the completed table showing the relationship between output $y$ and the inputs $A$ and $B$, it follows that it is the truth table for a single OR gate. Hence, the combination of gates shown in figure behaves as OR gate.

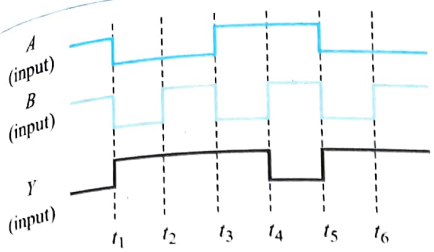
## CONCEPT APPLICATION EXERCISE 8.4

**1.** The Boolean expressions for the logic circuits are as given below:

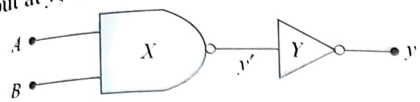
**(a)** $y = A \cdot \bar{B} + A \cdot B$ **(b)** $y = A \cdot B + \overline{A \cdot B}$

Find the output $y$ in each case, if the inputs are $A = 0$, $B = 0$; $A = 1$, $B = 0$; $A = 0$, $B = 1$ and $A = 1$, $B = 1$.

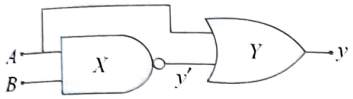
**2.** The following figure shows the input waveforms ($A$, $B$) and the output waveform ($Y$) of a gate. Identify the gate.



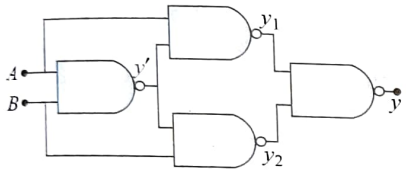

3. Identify the logic gates marked $X$, $Y$ in figure. Write down the output at $y$, when $A = 1$, $B = 1$ and $A = 0$ and $B = 0$.



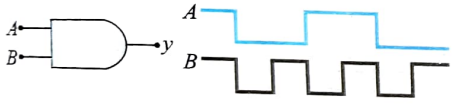

4. Identify the logic gates marked, $X$, $Y$ in figure. Write down the output at $y$, when $A = 1$, $B = 1$ and $A = 0$ and $B = 1$.



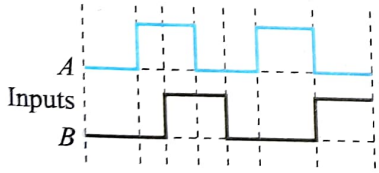

5. Produce the truth table of the combination of four NAND gates arranged as shown in figure. Hence, show that it behaves as XOR gate.



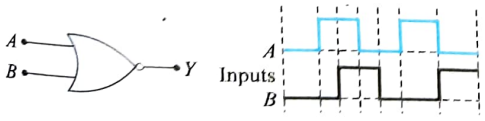

6. In the given figure, circuit symbol of a logic gate and two input waveforms $A$ and $B$ are shown. Write its truth table and give the output waveform.



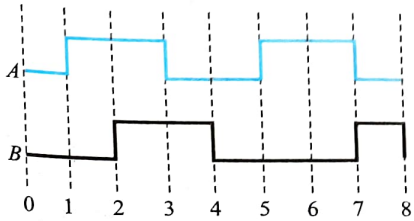

7. Draw the output waveform for input waveforms $A$ and $B$ shown in figure for OR gate




8. In the given figure, circuit symbol of a logic gate and two input waveforms $A$ and $B$ are shown. Write its truth table and give the output waveform.




9. (i) Sketch the output waveform from an AND gate for the inputs $A$ and $B$ shown in figure.




**(ii)** If the output of the above AND gate is fed to a NOT gate, name the gate of the combination so formed.

**ANSWERS**

**1.** (a) 0, 1, 0, 1; (b) 1, 1, 1, 1 **2.** NAND gate

## Solved Examples

### EXAMPLE 8.1

In an intrinsic semiconductor, the energy gap $E_g$ is 1.2 eV. Its hole mobility is much smaller than electron mobility and independent of temperature. What is the ratio between conductivity at 600 K and that at 300 K? Assume that the temperature dependence of intrinsic carrier concentration $n_i$ is given by

$$n_i = n_0 \exp\left(-\frac{E_g}{2k_BT}\right)$$

where $n_0$ is a constant.

**Sol.** Given: $E_g = 1.2$ eV, $T_1 = 300$ K, $T_2 = 600$ K

$k_B = 8.62 \times 10^{-5}$ eV/K

Conductivity of an intrinsic semiconductor,

$$\sigma = e(n_e\mu_e + n_h\mu_h) = en_i(\mu_e + \mu_h) = en_i\mu_e$$

(as $n_e = n_h = n_i$ and $\mu_e >> \mu_h$)

$$= e\mu_e n_0 \exp\left(\frac{-E_g}{2k_BT}\right)$$

Thus, $$\frac{\sigma_{600}}{\sigma_{300}} = \frac{\exp(-E_g/2k_B \times 600\text{K})}{\exp(-E_g/2k_B \times 300\text{K})}$$

$$= \exp\left(\frac{E_g}{1200k_B}\right) = \exp\left[\frac{1.2\,\text{eV}}{1200(8.62\times10^{-5}\,\text{eV/K})\,\text{K}}\right]$$

$$= \exp(11.6) \approx 10^5$$

Conductivity increases rapidly with the rise of temperature.

### EXAMPLE 8.2

In a p-n junction diode, the current $I$ can be expressed as,

$$I = I_0\left[\exp\left(\frac{eV}{k_BT} - 1\right)\right]$$

where $I_0$ is called the reverse saturation current, $V$ is the voltage across the diode and is positive for forward bias and negative for reverse bias, and $I$ is the current through the diode, $k_B$ is the Boltzmann constant ($8.6 \times 10^{-5}$ eV/K) and $T$ is the absolute temperature. If for a diode, $I_0 = 5 \times 10^{-12}$ A and $T = 300$ K, then

**(a)** What will be the forward current at a forward voltage of 0.6 V?

**(b)** What will be the increase in the current if the voltage across the diode is increased to 0.7 V?

**(c)** What is the dynamic resistance?

**(d)** What will be the current if reverse bias voltage changes from 1 V to 2 V?

**Sol.** The current $I$ through a junction diode is given as

$$I = I_0\left[\exp\left(\frac{eV}{k_BT}\right)-1\right]$$

where $I_0 = 5\times10^{-12}$ A, $T = 300$ K

$$k_B = 8.6\times10^{-5}\text{ eV K}^{-1} = 8.6\times10^{-5}\times1.6\times10^{-19}\text{ JK}^{-1}$$

**(a)** As $\frac{eV}{k_BT} = \frac{e\times0.6\text{ V}}{(8.6\times10^{-5}\text{ eV/K})300\text{ K}} = 23.26$

$$\exp\left(\frac{eV}{k_BT}\right) = \exp(23.26) = 1.259\times10^{10}$$

Thus, $I = I_0[1.259\times10^{10}-1]$

$$= (5\times10^{-12}\text{ A})(1.259\times10^{10}) = 0.063\text{ A}$$

**(b)** When $V = 0.7$ V, $\frac{eV}{k_BT} = 27.13$ and

$$\exp\left(\frac{eV}{k_BT}\right) = \exp(27.13) = 6.07\times10^{11}$$

Thus, $I = (5\times10^{-12})(6.07\times10^{11}-1) = 3.035\text{ A}$

Hence, $\Delta I = 3.035\text{ A} - 0.063\text{ A} = 2.972\text{ A}$

**(c)** For $\Delta V = 0.7 - 0.6 = 0.1$ V, $\Delta I = 2.972$ A

Dynamic resistance, $r_{dc} = \frac{\Delta V}{\Delta I} = \frac{(0.7-0.6)\text{ V}}{2.972\text{ A}} = 0.0336\,\Omega$

**(d)** The reverse current is given by $I = I_0\left[\exp\left(\frac{-eV}{k_BT}\right)-1\right]$

When $V = 1$ V, $\frac{eV}{k_BT} = 38.76$ and $I = I_0[\exp(-38.76)-1] = -I_0$

When $V = 2$ V, $\frac{eV}{k_BT} = 77.52$ and $I = I_0[\exp(-77.52)-1] = -I_0$

Thus, the reverse current remains almost constant (i.e., $I_0 = 5\times10^{-2}$ A) with change in voltage. This shows that in the reverse bias, the dynamic resistance is infinite.

## EXAMPLE 8.3

A junction diode of negligible forward resistance is used as a half-wave rectifier. A sinusoidal voltage supply having peak value 50 V is fed to the junction diode and output is obtained across the load resistance $R_L$. Calculate **(a)** the r.m.s. voltage across $R_L$ and **(b)** reading of a d.c. voltmeter connected across $R_L$.

**Sol.** Here, $E_0 = 50$ V

$$\therefore\quad E_{\text{r.m.s.}} = \frac{50}{\sqrt{2}}\text{ V}$$

**(a)** A half-wave rectifier gives output corresponding to alternate half cycles of the a.c. input. Since junction diode is of negligible forward resistance, the r.m.s. value of voltage across $R_L$ during half cycle of conduction is $50/\sqrt{2}$ V. Therefore, r.m.s. value of voltage for a complete cycle (including the non-conducting half cycle) is

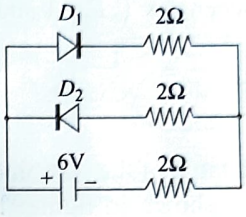
$$V_{\text{r.m.s.}} = \sqrt{\frac{\left(50/\sqrt{2}\right)^2+(0)^2}{2}} = \sqrt{\frac{1250+0}{2}} = \sqrt{625} = 25\text{ V}$$

**(b)** The d.c. voltmeter connected across $R_L$ would read the average value of voltage across $R_L$ during a complete cycle. Therefore, reading of the d.c. voltmeter connected across $R_L$,

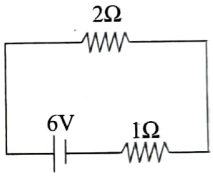
$$V_{\text{d.c.}} = \frac{2E_0/\pi+0}{2} = \frac{E_0}{\pi} = \frac{50}{\pi} = 15.92\text{ V}$$

## EXAMPLE 8.4

For the circuit shown in figure, find the current through the 1 Ω resistor. Assume that the two diodes, $D_1$ and $D_2$ are ideal diodes.




**Sol.** Diode $D_1$ is forward biased and offers zero resistance. Diode $D_2$ is reversed biased and offers infinite resistance. The given circuit reduces to the equivalent circuit shown in figure.



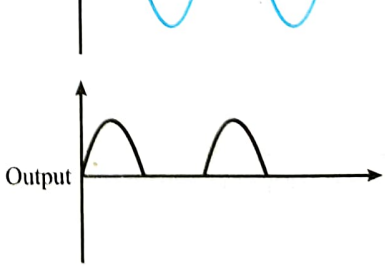

By Ohm's law, the current through the 1 Ω resistor is

$$I = \frac{6}{2+1}\text{ A} = 2\text{ A}$$

## EXAMPLE 8.5

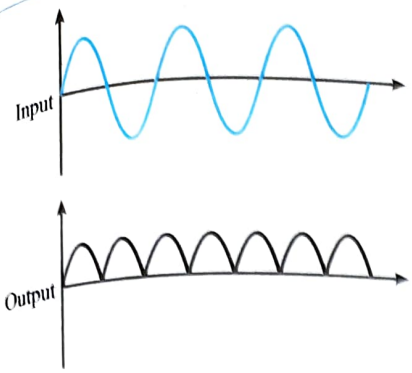
In half-wave rectification, what is the output frequency if the input frequency is 50 Hz? What is the output frequency of a full-wave rectifier for the same input frequency?

**Sol.** Since in half-wave rectification, only one ripple i obtained per cycle at the output.



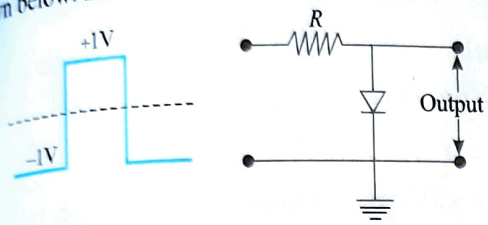

Output frequency of the half-wave rectifier = input frequen = 50 Hz.

Further, in full-wave rectification, as two ripples are obtain per cycle at the output, output frequency of the full-wa rectifier = 2 × input frequency = 2 × 50 Hz = 100 Hz.

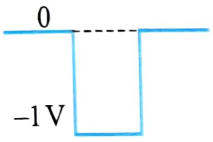



## EXAMPLE 8.6

A square wave (–1 V to 1 V) is applied to p-n junction diode as shown below. Draw the output waveform.

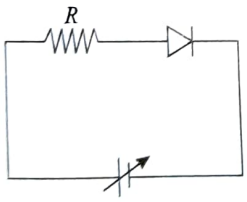



**Sol.** The *p*-side of the diode is earthed, it is at zero potential. So the diode conducts current when input level is –1 V and does not conduct when the input level is +1 V. As the diode is ideal, the output across it will be either 0 or –1 V, as shown in figure.




## EXAMPLE 8.7

Assume that the silicon diode in the circuit shown in figure requires a minimum current of 1 mA to be above the knee point (0.7 V) of its I-V characteristics. Also assume that the voltage across the diode is independent of current above the knee point.




(a) If $V_B = 5$ V, what should be the maximum value of $R$ so that the voltage is above the knee point?

(b) If $V_B = 5$ V, what should be the value of $R$ to establish the current of 5 mA in the circuit?

(c) What is the power dissipated in the resistance $R$ and in the diode, when a current of 5 mA flows in the circuit at $V_B = 6$ V?

**Sol.** When the minimum current of 1 mA flows through the circuit, the potential drop of 0.7 V occurs across the diode for a forward bias of 5 V.

(a) Here, $V_B = 5\text{V}, V_D = 0.7\text{V}, I_{min} = 1\text{mA} = 10^{-3}\text{A}$

Now $V_B = V_R + V_D$

or $5 = I_{min} \times R_{max} + 0.7$

or $5 - 0.7 = 10^{-3} \times R_{max}$

or $R_{max} = \dfrac{4.3}{10^{-3}} = 4.3 \times 10^3\,\Omega$

(b) To establish a current of 5 mA in the circuit, we have

$V_B = 5\text{V}, I = 5\text{mA} = 5 \times 10^{-3}\text{A}, V_D = 0.7\text{V}$

(which does not depend on current above knee point)

As $V_B = V_R + V_D = IR + V_D$

$\therefore \quad 5 = 5 \times 10^{-3} \times R + 0.7$

or $R = \dfrac{5 - 0.7}{5 \times 10^{-3}} = \dfrac{4.3}{5 \times 10^{-3}}\,\Omega = 860\,\Omega$

(c) Here $V_B = 6\text{V}, I = 5\text{mA} = 5 \times 10^{-3}\text{A}, V_D = 0.7\text{V}$

$\therefore \quad V_R = V_B - V_D = 6 - 0.7 = 5.3\text{V}$

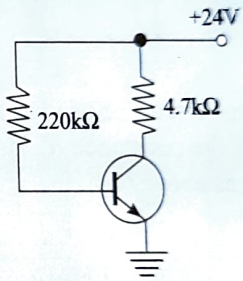
As power dissipated, $P = VI$

$\therefore$ Power dissipated in resistance $R$

$= V_R I = 5 \times 10^{-3} \times 5.3\,\text{W} = 26.5 \times 10^{-3}\,\text{W} = 26.5\,\text{mW}$

## EXAMPLE 8.8

In the circuit shown in figure, the value of β is 100. Find $I_B$, $V_{CE}$, $V_{BE}$ and $V_{BC}$, when $I_C = 1.5$ mA. The transistor is in active, cut off or saturation state?

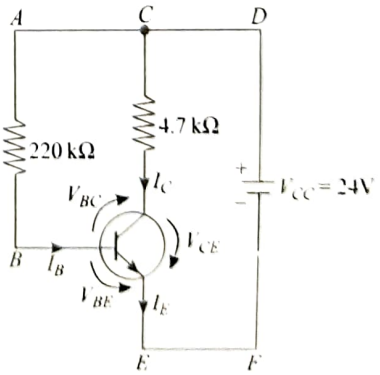



**Sol.** Figure shows the equivalent circuit of the given circuit.

Here, $\beta = 100, I_C = 1.5\text{mA} = 1.5 \times 10^{-3}\text{A}, V_{CC} = 24\text{V}$

As $\beta = \dfrac{I_C}{I_B}$

$\therefore \quad I_B = \dfrac{I_C}{\beta} = \dfrac{1.5 \times 10^{-3}}{100}\text{A} = 15\,\mu\text{A}$




To calculate $V_{CE}$, we apply Kirchhoff's rule to loop CEFDC, therefore

$V_{CC} = I_C \times 4.7\,\text{k}\Omega + V_{CE}$

or $24 = 1.5 \times 10^{-3} \times 4.7 \times 10^3 + V_{CE}$

or $V_{CE} = 24 - 7.05 = 16.95\,\text{V}$

Again, applying Kirchhoff's rule to loop ABEFDCA we get

$$V_{CC} = I_B \times 220\text{k}\Omega + V_{BE}$$

$$\therefore \quad V_{BE} = 24 - 3.3 = 20.7 \text{ V}$$

Going along loop ABCA, we get

$$I_B \times 220\,\text{k}\Omega + V_{BC} = I_C \times 4.7\,\text{k}\Omega$$

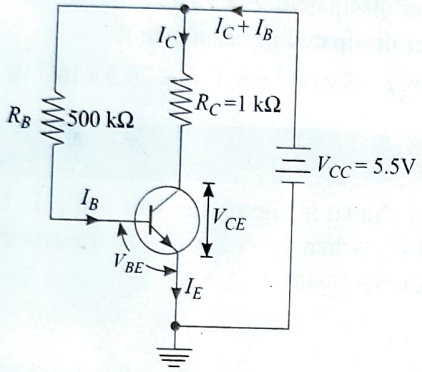
or $$15\times10^{-6}\times220\times10^{3} + V_{BC} = 1.5\times10^{-3}\times4.7\times10^{3}$$

$$V_{BC} = 7.05 - 3.3 = 3.75\,\text{V}$$

As $V_{CE} < V_{BE}$, both the junctions are forward biased, so the transistor is in the saturation state.

## EXAMPLE 8.9

In the circuit shown in figure, the base current $I_B$ is 10 μA and the collector current is 5.2 mA.




**(a)** Can this transistor circuit be used as an amplifier?

**(b)** What happens if the resistance $R_C$ is 500 Ω and $I_B$, $I_C$ and $R_B$ remain same as above?

**Sol.**

**(a)** To use a transistor as an amplifier, the emitter base junction must be forward biased and the collector base junction must be reverse biased. we now determine $V_{BE}$ and $V_{CE}$.

Applying Kirchhoff's law to the loop containing the base resistor and emitter base junction, we get

$$V_{CC} = V_{BE} + I_B R_B$$

i.e., $$V_{BE} = V_{CC} - I_B R_B$$

$$= 5.5\,\text{V} - 10\times10^{-6}\times5\times10^{-5}\,\text{V} = 0.5\,\text{V}$$

Applying Kirchhoff's law to the collector-emitter loop, we get

$$V_{CC} = V_{CE} + I_C R_C$$

i.e., $$V_{CE} = V_{CC} - I_C R_C = (5.5 - 5.2\times10^{-3}\times1\times10^{3})\,\text{V}$$

$$= (5.5 - 5.2)\,\text{V} = 0.3\,\text{V}$$

The collector is found to be at +0.3 V with respect to the emitter and the base is at +0.5 V with respect to the emitter, so the base is at +0.2 V with respect to the collector. Thus, both the emitter-base and collector-base junctions are forward biased. Hence, the circuit cannot be used as an amplifier.

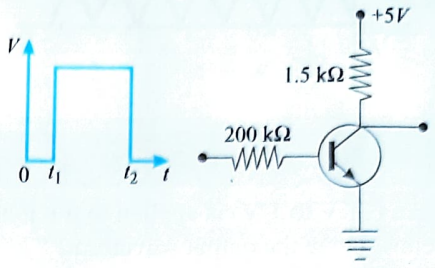
**(b)** When the resistance $R_C$ of 500 Ω is used, the voltage of collector with respect to emitter is

$$V_{CC} = (5.5 - 5.2\times10^{3}\times500)\,\text{V} = (5.5 - 2.60)\,\text{V} = 2.9\,\text{V}$$

Now the collector is at 2.9 V with respect to emitter and base is at 0.5 V with respect to emitter. So the base collector junction is reverse biased by –2.4 V. Hence the circuit can now work as an amplifier.

## EXAMPLE 8.10

In the circuit shown in figure, if we assume that when the input voltage at the base resistance is 5 V, $V_{BE}$ is zero and $V_{CE}$ is also zero; what is $I_B$, $I_C$ and β? When the input is zero, the $I_B$ is zero. What will be the output waveform if the input waveform is as shown in figure?

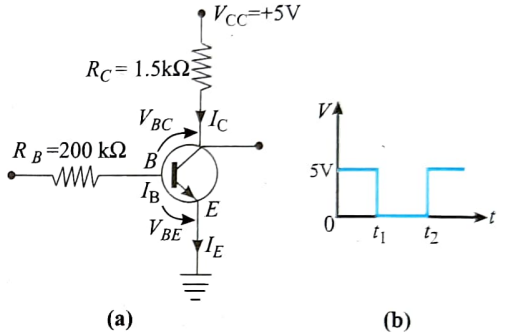



**Sol.** Various voltages and resistances are shown in figure.

Here $R_B = 200\,\text{k}\Omega$, $V_{BE} = 0$,

$V_{CE} = 0$, $V_{BB} = +5\text{V}$,

$V_{CC} = +5\text{V}$, $R_C = 1.5\,\text{k}\Omega$




(a) (b)

Using Kirchhoff's second rule, $V_{CE} + I_C R_C - V_{CC} = 0$

$$\therefore \quad I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{5-0}{1.5\times10^{3}} = 3.3\times10^{-3}\,\text{A} = 3.3\,\text{mA}$$

$$I_B = \frac{\text{Base voltage}}{\text{Base resistance}} = \frac{V_{BB}}{R_B} = \frac{5}{200\times10^{3}}\,\text{A}$$

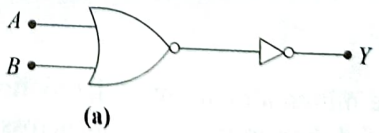
$$= 25\times10^{-6}\,\text{A} = 25\,\mu\text{A}$$

$$\beta = \frac{I_C}{I_B} = \frac{3.3\times10^{-3}}{25\times10^{-6}} = 133.3$$

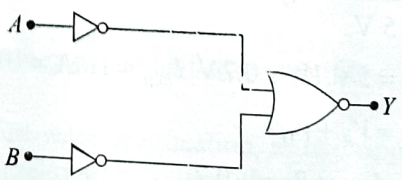
Since a CE configuration of a transistor inverts the input signal, so the output waveform is of the type shown in Fig. (b).

## EXAMPLE 8.11

You are given the two circuits as shown in figure. Show that circuit **(a)** acts as OR gate while the circuit **(b)** acts as AND gate.




(a)




(b)

**Sol.** (a) The circuit consists of NOR gate followed by a NOT gate. The truth table is as follows:

| A | B | A + B | $Y' = \overline{A+B}$ | $Y = \overline{Y'}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 |

Since $Y = A + B$, the circuit acts as an OR gate.

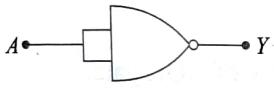
(b) The circuit consists of two NOT gates followed by a NOR gate. The truth table is as follows.

| A | B | $\overline{A}$ | $\overline{B}$ | $\overline{A}+\overline{B}$ | $Y = \overline{\overline{A}+\overline{B}}$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 |

Since $Y = A.B$, the circuit acts as an AND gate.

## EXAMPLE 8.12

Write the truth table for a NAND gate connected as given in figure.




**Sol.** The truth table of a NAND gate is as given below:

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 0 |

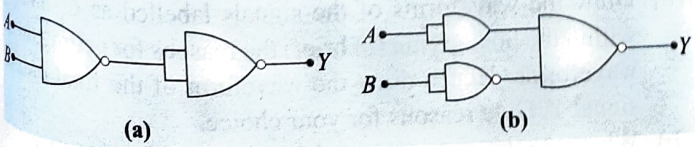
When the two imputs of a NAND gates are joined as shown in figure, i.e., when $A = B = 0$ or $A = B = 1$, the above truth table of the NAND gate reduces to the one as given below:

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 0 |

It is the truth table of a NOT gate. Therefore, the given circuit carries out the logic operation of a NOT gate.

## EXAMPLE 8.13

You are given two circuits as shown in figure, which consist of NAND gates. Identify the logic operation carried out by the two circuits.




(a) (b)

**Sol.** Refer to Fig. (a): Here, the output of NAND gate is connected to NOT gate (obtained from NAND gate). Let $y'$ be the output of NAND gate and the final output of the combination of two gates be $y$. The output of a NAND gate is 0 only when both the inputs are zero, while in NOT gate, the input gets inverted. Using these facts, the truth table for the given arrangement can be written as below:

| A | B | Y | Y' |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 |

=

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 1 |

It is the truth table of AND gate. Therefore, the given circuit acts as AND gate.

Refer to Fig. (b): Here, the outputs of two NOT gates (obtained from NAND gates) are connected NAND gate. Let $y_1$ and $y_2$ be the outputs of the NOT gates and the final output of the combination of three gates be $y$. In a NOT gate, the input gets inverted, while the output of a NAND gate is 0 only when both the inputs are zero. Using these facts, the truth table for the given arrangement can be written as below:

| A | B | $Y_1$ | $Y_2$ | Y |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |

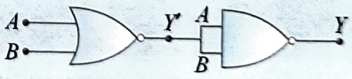
=

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |

It is the truth table of OR gate. Therefore, the given circuit acts as OR gate.

## EXAMPLE 8.14

Write the truth table for circuit given in figure consisting of NOR gate and identify the logic operation (OR, AND, NOT) which this circuit is performing.



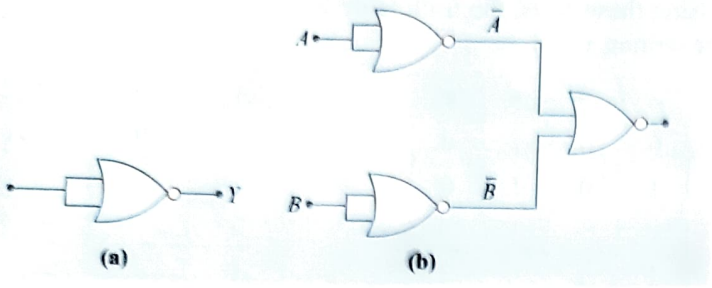

**Sol.** The first gate is a NOR gate. The second gate is also a NOR gate, with both the input terminals connected together. The truth table for the above circuit is as follows.

| A | B | A + B | $Y = \overline{A+B}$ | A = Y' | B = Y' | $Y = \overline{A+B}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |

Since in this case $Y = \overline{A+B} = A+B$, the given circuit performs the function of an OR gate.

## EXAMPLE 8.15

Write the truth table for the circuits given in figure consisting of NOR gates only. Identify the logic operations (OR, AND, NOT) performed by the two circuits.




**Sol.**

a) The circuit of Fig. (a) is a NOR gate with its input terminals connected together. The truth table for this circuit is as follows.

| A | B = A | A + B | $Y = \overline{A+B}$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Since $Y = \overline{A+B} = \bar{A}$, a NOR gate with both its terminals connected together, performs the NOT operation.

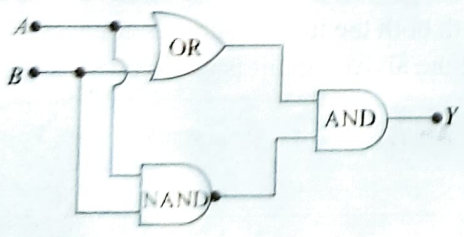
The inputs $A$ and $B$ are inverted by the two NOT gates [obtained from NOR gates as detailed in (a)]. These outputs $\bar{A}$ and $\bar{B}$ are fed to a NOR gate and the truth table is as follows:

| A | B | $\bar{A}$ | $\bar{B}$ | $\bar{A}+\bar{B}$ | $Y = \overline{\bar{A}+\bar{B}}$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 |

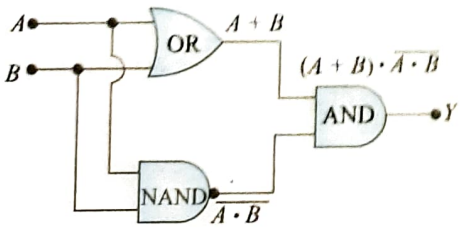
Since in the case $Y = \overline{\bar{A}+\bar{B}} = A \cdot B$, the circuit [Fig. (b)] performs the function of an AND gate.

## AMPLE 8.16

te the truth table for the circuit shown in figure. Show that presents a XOR gate.




The given circuit contains an OR, a NAND and an AND s shown in figure.




The inputs $A$ and $B$ are fed to OR gate so that its output is

$$Y' = A + B$$

Also, the inputs $A$ and $B$ are fed to NAND gate so that its output is

$$Y'' = \overline{A \cdot B}$$

Then $Y'$ and $Y''$ are fed to AND gate so that the output is

$$Y = Y' \cdot Y'' = (A + B) \cdot (\overline{A \cdot B})$$

The logic table for the given circuit is

| OR gate | | | NAND gate | | | AND gate | | |
|---|---|---|---|---|---|---|---|---|
| Inputs | | Output | Inputs | | Output | Inputs | | Output |
| A | B | $Y' = A+B$ | A | B | $Y'' = \overline{A \cdot B}$ | $Y'$ | $Y''$ | $Y = Y' \times Y''$ |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |

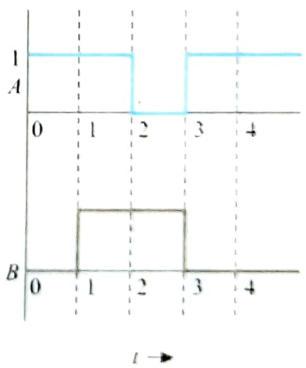
Hence the truth table of XOR gate.

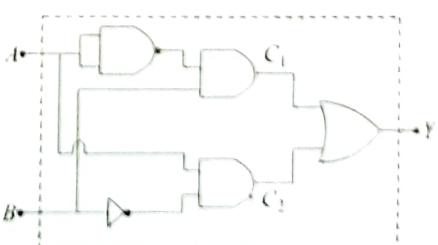
| Inputs | | Output |
|---|---|---|
| A | B | Y |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

which is the truth table of XOR gate.

## EXAMPLE 8.17

Input signals $A$ and $B$ are applied to the input terminals of the 'dotted box' set-up shown here. Let $Y$ be the final output signal from the box.



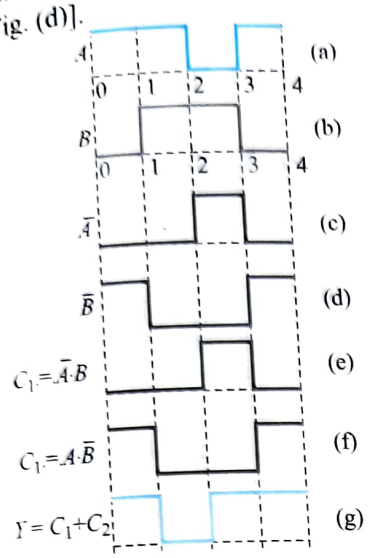




**(a)** Draw the waveforms of the signals labelled as $C_1$ and $C_2$ within the box, giving (in brief) the reasons for getting these waveforms. Hence draw the waveform of the final output signal $Y$. Give reasons for your choice.

**(b)** What can we state (in words) as the relation between the final output signal $Y$ and the input signals $A$ and $B$?

(i) The NAND gate with both its terminals connected together acts as a NOT gate. The input $A$ given to it appears $\overline{A}$ as its output [Fig. (c)].

(ii) The input $B$ given to the NOT get appears $\overline{B}$ as the output [Fig. (d)].




(iii) For the upper AND gate, $\overline{A}$ and $B$ are the inputs. Its output is $C_1 = \overline{A} \cdot B$.

The waveform of $C_1$ is shown in Fig. (e).

(iv) For the lower AND gate, the inputs are $A$ and $\overline{B}$.

Its output is $C_2 = A \cdot \overline{B}$.

The waveform of $C_2$ is shown in Fig. (f).

(v) For the last OR gate, $C_1$ and $C_2$ are the inputs. Its output is $Y = C_1 + C_2$.

The waveform of $Y$ is shown in Fig. (g).

On comparing the output $Y$ with the inputs $A$ and $B$, we conclude that

**(b)** (i) The output $Y$ is low (0) when both the inputs $A$ and $B$ are high (1).

(ii) The output $Y$ is high (1) when either of the inputs $A$ and $B$ is low and the other is high.

# Exercises

## Single Correct Answer Type

1. A piece of copper and the other of germanium are cooled from the room temperature to 80 K. Then which of the following would be a correct statement?
   (1) Resistance of each increases
   (2) Resistance of each decreases
   (3) Resistance of copper increases while that of germanium decreases
   (4) Resistance of copper decreases while that of germanium increases

2. A pure semiconductor has equal electron and hole concentration of $10^{16}$ m$^{-3}$. Doping by Indium increases $n_h$ to $4.5 \times 10^{22}$ m$^{-3}$. What is $n_e$ in the doped semiconductor?
   (1) $10^{6}$ m$^{-3}$ (2) $10^{22}$ m$^{-3}$
   (3) $\frac{10^{32}}{4.5\times10^{22}}$ m$^{-3}$ (4) $4 \times 10^{22}$ m$^{-3}$

3. A semiconductor is known to have an electron concentration of $8 \times 10^{13}$/cm$^3$ and hole concentration of $5 \times 10^{12}$/cm$^3$. The semiconductor is
   (1) $n$-type (2) $p$-type
   (3) intrinsic (4) insulator

4. If $n_e$ and $n_h$ are the number of electrons and holes in a semiconductor heavily doped with phosphorus, then
   (1) $n_e >> n_h$ (2) $n_e << n_h$
   (3) $n_e \leq n_h$ (4) $n_e = n_h$

5. The forbidden energy band gap in conductors, semiconductors and insulators is $EG_1$, $EG_2$ and $EG_3$, respectively. The relation among them is
   (1) $EG_1 = EG_2 = EG_3$ (2) $EG_1 < EG_2 < EG_3$
   (3) $EG_1 > EG_2 > EG_3$ (4) $EG_1 < EG_2 > EG_3$

6. In a semiconductor, the separation between conduction band and valence band is of the order of
   (1) 100 eV (2) 10 eV
   (3) 1 eV (4) 0 eV

7. To a germanium sample, traces of gallium are added as an impurity. The resultant sample would behave like
   (1) a conductor
   (2) a $p$-type semiconductor
   (3) an $n$-type semiconductor
   (4) an insulator

8. When $n$-type of semiconductor is heated
   (1) number of electrons increases while that of holes decreases
   (2) number of holes increases while that of electrons decreases
   (3) number of electrons and holes remains same
   (4) number of electrons and holes increases equally

9. $p$-type semiconductor is formed when
   A. As impurity is mixed in Si
   B. Al impurity is mixed in Si
   C. B impurity is mixed in Ge
   D. P impurity is mixed in Ge
   (1) A and C (2) A and D
   (3) B and C (4) B and D

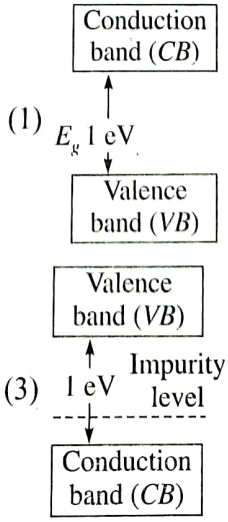
10. In case of a semiconductor, which of the following statement is wrong?
    (1) Doping increases conductivity
    (2) Temperature coefficient of resistance is negative
    (3) Resistivity is in between that of a conductor and insulator
    (4) At absolute zero temperature, it behaves like a conductor

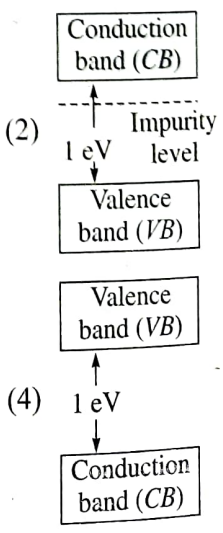
11. Electronic configuration of germanium is 2, 8, 18 and 4. To make it extrinsic semiconductor, small quantity of antimony is added. Then
    (1) the material obtained will be $n$-type germanium in which electrons and holes are equal in number
    (2) the material obtained will be $p$-type germanium
    (3) the material obtained will be $n$-type germanium which has more electrons than holes at room temperature
    (4) the material obtained will be $n$-type germanium which has less electrons than holes at room temperature

12. Wires $P$ and $Q$ have the same resistance at ordinary (room) temperature. When heated, resistance of $P$ increases and that of $Q$ decreases. We conclude that
    (1) $P$ and $Q$ are conductors of different materials
    (2) $P$ is $n$-type semiconductor and $Q$ is $p$-type semiconductor
    (3) $P$ is semiconductor and $Q$ is conductor
    (4) $P$ is conductor and $Q$ is semiconductor

13. Let $n_P$ and $n_e$ be the number of holes and conduction electrons, respectively, in a semiconductor. Then
    (1) $n_P > n_e$ in an intrinsic semiconductor
    (2) $n_P = n_e$ in an extrinsic semiconductor
    (3) $n_P = n_e$ in an intrinsic semiconductor
    (4) $n_e > n_P$ in an intrinsic semiconductor

14. Which of the following energy band diagram shows the $n$-type semiconductor?

(1)



(2)


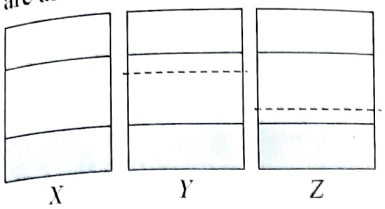

(3)



(4)

16. In a semiconducting material, the mobilities of electrons and holes are $\mu_e$ and $\mu_h$, respectively. Which of the following is true?
(1) $\mu_e > \mu_h$ (2) $\mu_e < \mu_h$
(3) $\mu_e = \mu_h$ (4) $\mu_e < 0; \mu_h > 0$

17. The energy band diagrams for three semiconductor samples of silicon are as shown. We can then assert that




(1) sample $X$ is undoped, while samples $Y$ and $Z$ have been doped with a third group and a fifth group impurity, respectively.
(2) sample $X$ is undoped, while both samples $Y$ and $Z$ have been doped with a fifth group impurity.
(3) sample $X$ has been doped with equal amounts of third and fifth group impurities, while samples $Y$ and $Z$ are undoped.
(4) sample $X$ is undoped, while samples $Y$ and $Z$ have been doped with a fifth group and a third group impurity, respectively.

18. Carbon, silicon and germanium atoms have four valence electrons each. Their valence and conduction bands are separated by energy band gaps represented by $(E_g)_C$, $(E_g)_{Si}$ and $(E_g)_{Ge}$, respectively. Which one of the following relationship is true in their case?
(1) $(E_g)_C > (E_g)_{Si}$ (2) $(E_g)_C = (E_g)_{Si}$
(3) $(E_g)_C < (E_g)_{Ge}$ (4) $(E_g)_C < (E_g)_{Si}$

18. If $N_p$ and $N_e$ are the numbers of holes and conduction electrons in an extrinsic semiconductor, then
(1) $N_p > N_e$
(2) $N_p = N_e$
(3) $N_p < N_e$
(4) $N_p < N_e$ or $N_p < N_e$ depending on the nature of impurity

19. A silicon specimen is made into a $P$-type semiconductor by doping, on an average, one indium atom per $5 \times 10^7$ silicon atoms. If the number density of atoms in the silicon specimen is $5 \times 10^{28}$ atoms/m$^3$, then the number of acceptor atoms in silicon per cubic centimetre will be
(1) $2.5 \times 10^{30}$ atoms/cm$^3$ (2) $1.0 \times 10^{13}$ atoms/cm$^3$
(3) $1.0 \times 10^{15}$ atoms/cm$^3$ (4) $2.5 \times 10^{36}$ atoms/cm$^3$

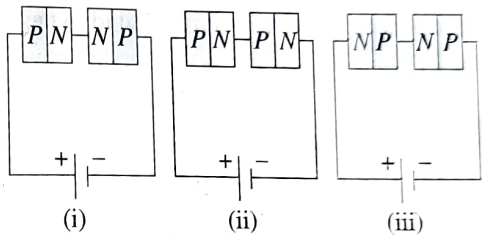
20. In a pure silicon ($n_i = 10^{16}$/m$^3$) crystal at 300 K, $10^{21}$ atoms of phosphorus are added per cubic metre. The new hole concentration will be
(1) $10^{21}$ per m$^3$ (2) $10^{19}$ per m$^3$
(3) $10^{11}$ per m$^3$ (4) $10^5$ per m$^3$

21. A *pn* junction diode cannot be used
(1) for converting light energy to electrical energy
(2) for converting electrical energy to light energy
(3) for getting light radiation
(4) for increasing the amplitude of an a.c. signal

22. When a *pn* junction diode is forward biased, energy is released at the junction due to the recombination of electrons and holes. This energy is in
(1) visible region (2) infrared region
(3) UV region (4) X-ray region

23. In a *pn* junction diode,
(1) the current in the reverse-biased condition is generally very small.
(2) the current in the reverse-biased condition is small but the forward-biased current is strongly dependent on the applied bias voltage.
(3) the reverse-biased current is strongly dependent on the applied bias voltage.
(4) the forward-biased current is very small in comparison to reverse-biased current.

24. Two *pn* junctions can be connected in series by three different methods as shown in the figure. If the potential difference in the junctions is the same, then the correct connections will be




(1) in circuits (i) and (ii) (2) in circuits (ii) and (iii)
(3) in circuits (i) and (iii) (4) only in circuit (i)

25. The reverse biasing in a *pn* junction diode
(1) decreases the potential barrier
(2) increases the potential barrier
(3) increases the number of minority charge carriers
(4) increases the number of majority charge carriers

26. In a *pn* junction diode, if $p$ region is heavily doped than $n$ region, then the depletion layer is
(1) greater in $p$ region
(2) greater in $n$ region
(3) equal in both region
(4) no depletion layer is formed in this case

27. Zener breakdown in a semiconductor diode occurs when
(1) forward current exceeds certain value
(2) reverse bias exceeds certain value
(3) forward bias exceeds certain value
(4) potential barrier is reduced to zero

28. The depletion layer in silicon diode is 1 μm wide and the knee potential is 0.6 V, then the electric field in the depletion layer will be
(1) zero (2) 0.6 V/m
(3) $6 \times 10^4$ V/m (4) $6 \times 10^5$ V/m

**29.** In a *pn* junction diode not connected to any circuit

(1) The potential is the same everywhere

(2) The *p*-type is a higher potential than the *n*-type side

(3) There is an electric field at the junction directed from the *n*-type side to the *p*-type side

(4) There is an electric field at the junction directed from the *p*-type side to the *n*-type side

**30.** The dominant mechanisms for motion of charge carriers in forward and reverse biased silicon *p-n* junctions are

(1) drift in forward bias, diffusion in reverse bias

(2) diffusion in forward bias, drift in reverse bias

(3) diffusion in both forward and reverse bias

(4) drift in both forward and reverse bias

**31.** The depletion layer in the *p-n* junction region is caused by

(1) drift of holes

(2) diffusion of charge carriers

(3) migration of impurity ions

(4) drift of electrons

**32.** A semiconductor device is connected in a series circuit with a battery and a resistance. A current is found to pass through the circuit. If the polarity of the battery is reversed, the current drops almost to zero. The device may be

(1) a *p*-type semiconductor

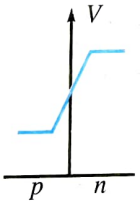
(2) an *n*-type semiconductor

(3) a *p-n* junction

(4) an intrinsic semiconductor

**33.** In a forward-biased *pn* junction diode, the potential barrier in the depletion region is of the form

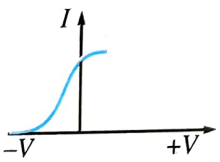
(1)

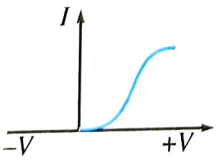

(2) 

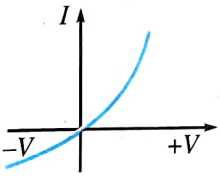

(3)

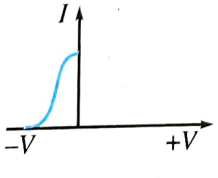

(4)


**34.** Different voltages are applied across a *p-n* junction and the currents are measured for each value. Which of the following graphs is obtained between voltage and current?

(1) 


(2) 


(3) 

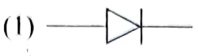

(4) 

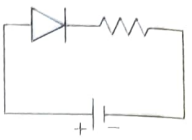

**35.** A *p*-type semiconductor has acceptor levels 57 meV above the valence band. The maximum wavelength of light required to create a hole is (Planck's constant $h = 6.6 \times 10^{-34}$ J-s)

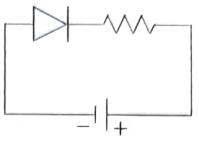
(1) 57 Å (2) $57 \times 10^{-3}$ Å

(3) 217100 Å (4) $11.61 \times 10^{-33}$ Å

**36.** Which one is in forward bias?

(1) 

(2) 

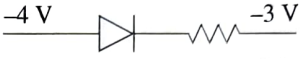

(3) 

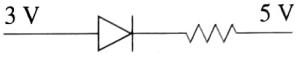

(4) None of these

**37.** The forward biased diode is

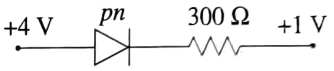
(1)


(2)


(3) 


(4) 

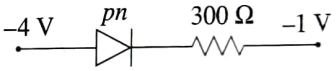

**38.** In the circuit given below, the value of the current is




(1) 0 A (2) $10^{-2}$ A

(3) $10^{2}$ A (4) $10^{-3}$ A

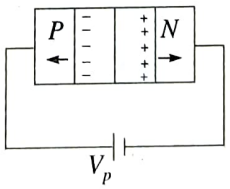
**39.** What is the current in the circuit shown below?

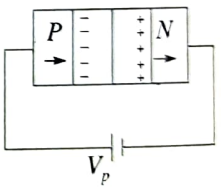


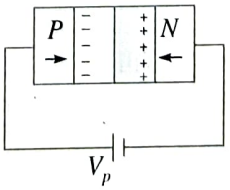

(1) 0 A (2) $10^{-2}$ A

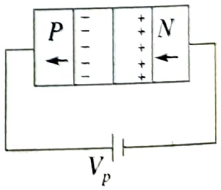
(3) 1 A (4) 0.10 A

**40.** In the case of forward biasing of *p-n* junction, which one of the following figures correctly depicts the direction of flow of carriers?

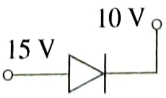
(1) 

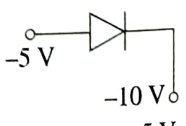

(2) 

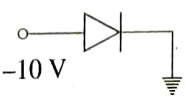

(3) 

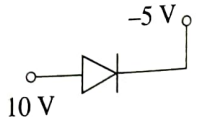

(4) 


**41.** Which one is reverse-biased?

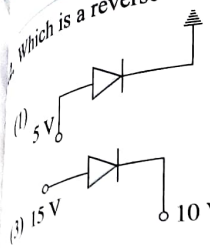
(1) 

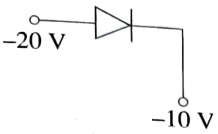

(2) 

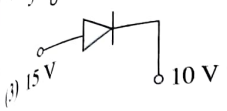

(3) 

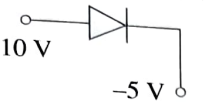

(4) 

Which is a reverse-biased diode?

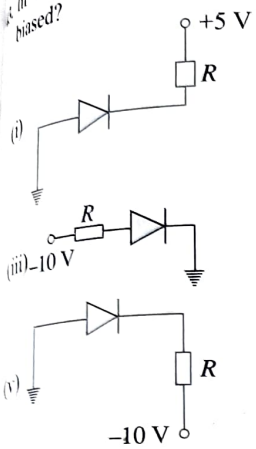
(1) 

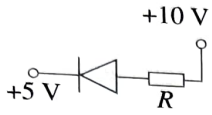

(2) 

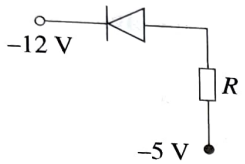

(3) 


(4) 


In the given figure, which of the diodes are forward biased?

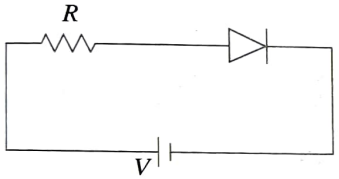



(ii) 


(iv) 

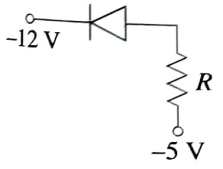

(1) i, ii, iii
(2) ii, iv, v
(3) i, iii, iv
(4) ii, iii, iv

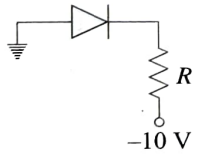
For the given circuit of $p$-$n$ junction diode, which of the following statement is correct?

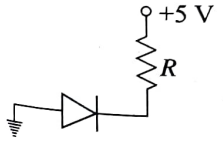


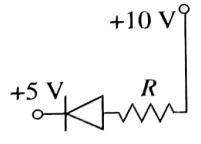

(1) In forward biasing, the voltage across $R$ is $V$.
(2) In forward biasing, the voltage across $R$ is 2 $V$.
(3) In reverse biasing, the voltage across $R$ is $V$.
(4) In reverse biasing, the voltage across $R$ is 2 $V$.

Of the diodes shown in the following diagrams, which one is reverse biased?

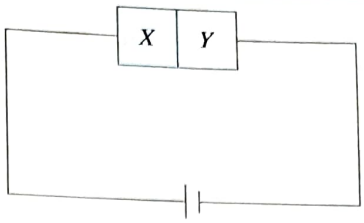
(1) 


(2) 


(3) 

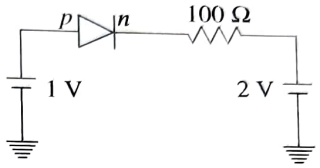

(4) 


A semiconductor $X$ is made by doping a germanium crystal with arsenic ($Z = 33$). A second semiconductor $Y$ is made by doping germanium with indium ($Z = 49$). The two are joined end to end and connected to a battery as shown. Which of the following statements is correct?




(1) $X$ is $p$-type, $Y$ is $n$-type and the junction is forward biased
(2) $X$ is $n$-type, $Y$ is $p$-type and the junction is forward biased
(3) $X$ is $p$-type, $Y$ is $n$-type and the junction is reverse biased
(4) $X$ is $n$-type, $Y$ is $p$-type and the junction is reverse biased

**47.** The current through an ideal $p$-$n$ junction shown in the following circuit diagram will be

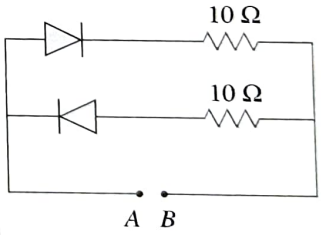



(1) zero
(2) 1 mA
(3) 10 mA
(4) 30 mA

**48.** A diode having potential difference 0.5 V across its junction which does not depend on current is connected in series with resistance of 20 Ω across source. If 0.1 A passes through resistance, then what is the voltage of the source?

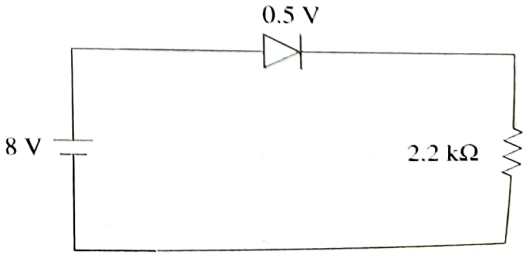
(1) 1.5 V
(2) 2.0 V
(3) 2.5 V
(4) 5 V

**49.** A 2 V battery is connected across the points $A$ and $B$ as shown in the figure given below. Assuming that the resistance of each diode is zero in forward bias and infinity in reverse bias, the current supplied by the battery when its positive terminal is connected to $A$ is



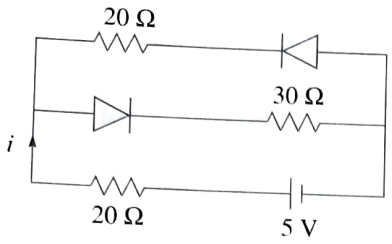

(1) 0.2 A
(2) 0.4 A
(3) Zero
(4) 0.1 A

**50.** In the circuit, if the forward voltage drop for the diode is 0.5 V, the current will be




(1) 3.4 mA
(2) 2 mA
(3) 2.5 mA
(4) 3 mA

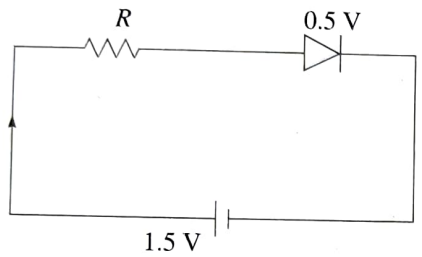
current in the circuit will be




(1) $\frac{5}{40}$ A (2) $\frac{5}{50}$ A

(3) $\frac{5}{10}$ A (4) $\frac{5}{20}$ A

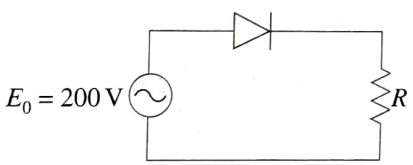
**52.** The diode used in the circuit shown in the figure has a constant voltage drop of 0.5 V at all currents and a maximum power rating of 100 milliwatts. What should be the value of the resistor $R$ connected in series with the diode for obtaining maximum current?




(1) 1.5 Ω (2) 5 Ω

(3) 6.67 Ω (4) 200 Ω

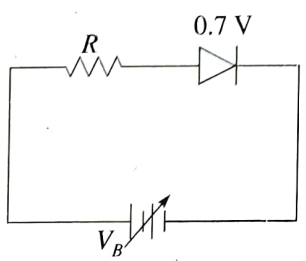
**53.** A sinusoidal voltage of peak value 200 V is connected to a diode and resistor $R$ in the circuit shown so that half wave rectification occurs. If the forward resistance of the diode is negligible compared to $R$, the rms voltage (in volt) across $R$ is approximately




(1) 200 (2) 100

(3) $\frac{200}{\sqrt{2}}$ (4) 280

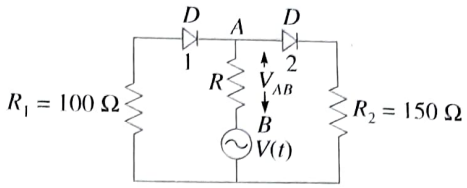
**54.** The junction diode in the following circuit requires a minimum current of 1 mA to be above the knee point (0.7 V) of its I-V characteristic curve. The voltage across the diode is independent of current above the knee point. If $V_B = 5$ V, then the maximum value of $R$ so that the voltage is above the knee point, will be




(1) 4.3 kΩ (2) 860 kΩ

(3) 4.3 Ω (4) 860 Ω

**55.** In the circuit given below, $V(t)$ is the sinusoidal voltage source, voltage drop $V_{AB}(t)$ across the resistance $R$




(1) is half wave rectified

(2) is full wave rectified

(3) has the same peak value in the positive and negative half cycles

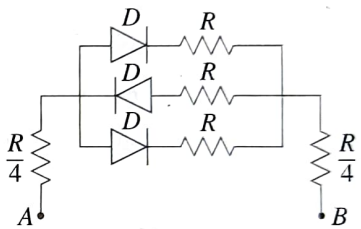
(4) has different peak values during positive and negative half cycle

**56.** The peak voltage in the output of a half-wave diode rectifier fed with a sinusoidal signal without filter is 10 V. The d.c. component of the output voltage is

(1) $10/\sqrt{2}$ V (2) $10/\pi$ V

(3) 10 V (4) $20/\pi$ V

**57.** In the following circuits, *pn* junction, diodes $D_1$, $D_2$ and $D_3$ are ideal for the following potential of $A$ and $B$, the correct increasing order of resistance between $A$ and $B$ will be



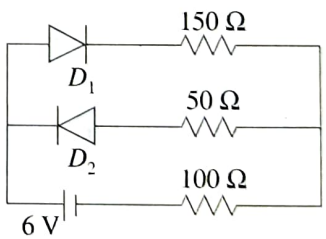

(i) −10 V, −5 V (ii) −5 V, −10 V

(iii) −4 V, −12 V

(1) (i) < (ii) < (iii) (2) (iii) < (ii) < (i)

(3) (ii) = (iii) < (i) (4) (i) = (iii) < (ii)

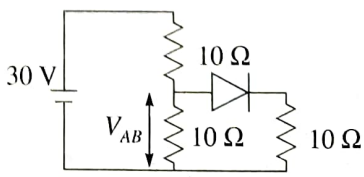
**58.** The circuit shown in the following figure contains two diodes $D_1$ and $D_2$ each with a forward resistance of 50 ohms and with infinite backward resistance. If the battery voltage is 6 V, the current through 100 ohms resistance (in amperes) is




(1) zero (2) 0.02

(3) 0.03 (4) 0.036

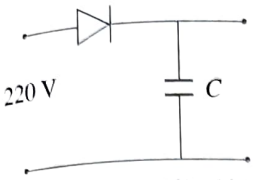
**59.** Find $V_{AB}$.




(1) 10 V (2) 20 V

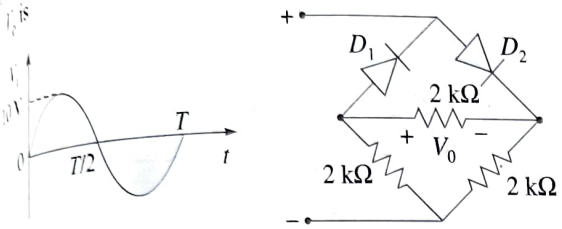
(3) 30 V (4) None of these

A diode is connected to 220 V (rms) a.c. in series with a capacitor as shown in figure. The voltage across the capacitor is



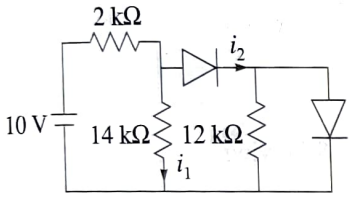

(1) 220 V
(2) 110 V
(3) 311.1 V
(4) $\frac{220}{\sqrt{2}}$ V

In the circuit shown in figure, the maximum output voltage $V_0$ is



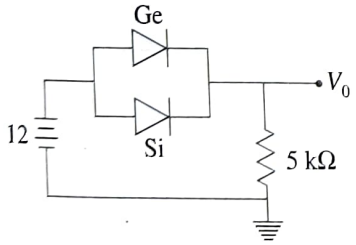

(1) 0 V
(2) 5 V
(3) 10 V
(4) $\frac{5}{\sqrt{2}}$ V

In the following circuit, find $I_1$ and $I_2$.




(1) 0, 0
(2) 5 mA, 5 mA
(3) 5 mA, 0
(4) 0, 5 mA

Ge and Si diodes conduct at 0.3 V and 0.7 V, respectively. In the following figure, if Ge diode connection are reversed, the value of $V_0$ changes by

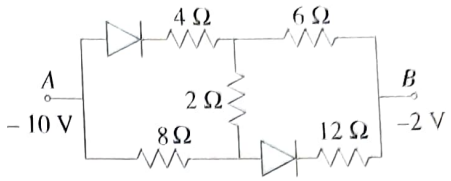



(1) 0.2 V
(2) 0.4 V
(3) 0.6 V
(4) 0.8 V

In space charge limited region, the plate current in a diode is 10 mA for plate voltage 150 V. If the plate voltage is increased to 600 V, then the plate current will be
(1) 10 mA
(2) 40 mA
(3) 80 mA
(4) 160 mA

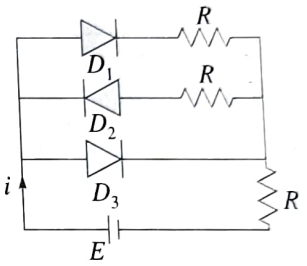
**65.** When a silicon *pn* junction is in forward-biased condition with series resistance, it has knee voltage of 0.6 V. Current flow in it is 5 mA, when *pn* junction is connected with 2.6 V battery, the value of series resistance is
(1) 100 Ω
(2) 200 Ω
(3) 400 Ω
(4) 500 Ω

**66.** In the following circuit, the equivalent resistance between *A* and *B* is



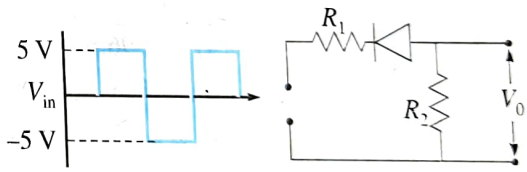

(1) $\frac{20}{3}$ Ω
(2) 10 Ω
(3) 16 Ω
(4) 20 Ω

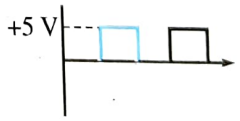
**67.** In the following circuit of *pn* junction diodes, $D_1$, $D_2$ and $D_3$ are ideal, then *i* is

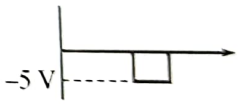


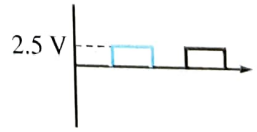

(1) *E*/*R*
(2) *E*/2*R*
(3) 2*E*/3*R*
(4) Zero

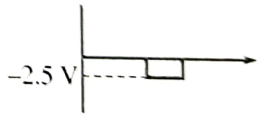
**68.** A waveform when applied to the following circuit will produce which of the following output waveform. Assuming ideal diode configuration and $R_1 = R_2$.



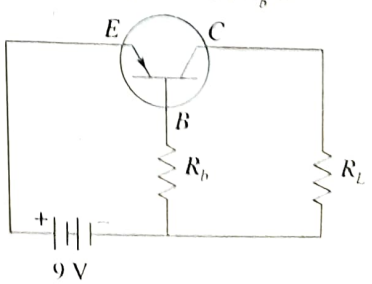

(1) 


(2) 


(3) 


(4) 

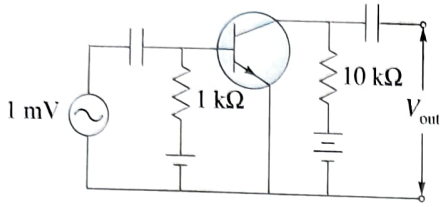

**69.** In a transistor circuit shown here the base current is 35 μA. The value of the resistor $R_b$ is



(1) 123.5 kΩ (2) 257 kΩ
(3) 380.05 kΩ (4) None of these

**70.** For a transistor amplifier in common emitter configuration for load impedance of 1 kΩ ($h_{fe} = 50$ and $h_{oe} = 25\ \mu A/V$), the current gain is
(1) –5.2 (2) –15.7
(3) –24.8 (4) –48.78

**71.** In the following common emitter configuration, an *npn* transistor with current gain $\beta = 100$ is used. The output voltage of the amplifier will be

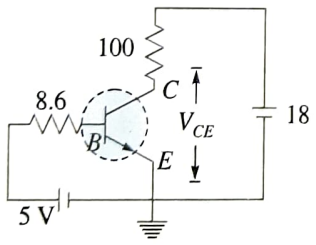



(1) 10 mV (2) 0.1 V
(3) 1.0 V (4) 10 V

**72.** In semiconductor, the concentrations of electrons and holes are $8 \times 10^{18}/m^3$ and $5 \times 10^{18}/m$, respectively. If the mobilities of electrons and hole are 2.3 $m^2$/volt-sec and 0.01 $m^2$/volt-sec, respectively, then semiconductor is
(1) *n*-type and its resistivity is 0.34 ohm-metre
(2) *p*-type and its resistivity is 0.034 ohm-metre
(3) *n*-type and its resistivity is 0.034 ohm-metre
(4) *p*-type and its resistivity is 3.40 ohm-metre

**73.** A transistor is used as an amplifier in *CB* mode with a load resistance of 5 kΩ. The current gain of amplifier is 0.98 and the input resistance is 70 Ω. The voltage gain and power gain, respectively, are
(1) 70, 68.6 (2) 80, 75.6
(3) 60, 66.6 (4) 90, 96.6

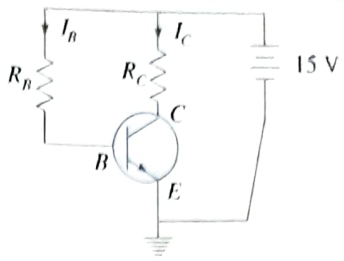
**74.** For the transistor circuit shown below, if $\beta = 100$, voltage drop between emitter and base is 0.7 V, then value of $V_{CE}$ will be




(1) 10 V (2) 5 V
(3) 13 V (4) 0 V

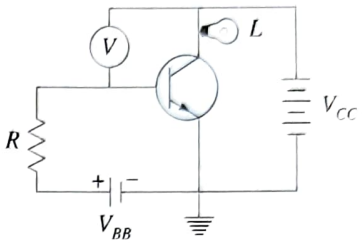
**75.** In *npn* transistor, $10^{10}$ electrons enter in emitter region in $10^{-6}$ sec. If 2% electrons are lost in base region, then collector current and current amplification factor ($\beta$), respectively, are
(1) 1.57 mA, 49
(2) 1.92 mA, 70
(3) 2 mA, 25
(4) 2.25 mA, 100

**76.** In the following common emitter circuit, if $\beta = 100$, $V_{CE} = 7$ V, $V_{BE}$ = negligible, $R_C = 2$ kΩ, then $I_B = ?$



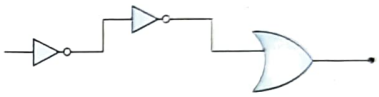

(1) 0.01 mA (2) 0.04 mA
(3) 0.02 mA (4) 0.03 mA

**77.** In the following circuit, a voltmeter $V$ is connected across a lamp $L$. What change would occur in voltmeter reading if the resistance $R$ is reduced in value?




(1) Increases (2) Decreases
(3) Remains same (4) None of these

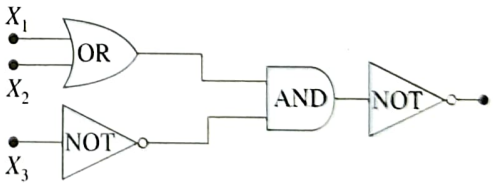
**78.** Identify the gate from the following:



(1) NOT gate (2) AND gate
(3) OR gate (4) None of these

**79.** Which of the following is not correct?
(1) $\overline{\overline{A}.\overline{B}} = A + B$
(2) $\overline{\overline{A}.\overline{B}} = A.B$
(3) $\overline{(A.B)}.\overline{(A.B)} = AB + AB$
(4) $\bar{1} + \bar{1} = 1$

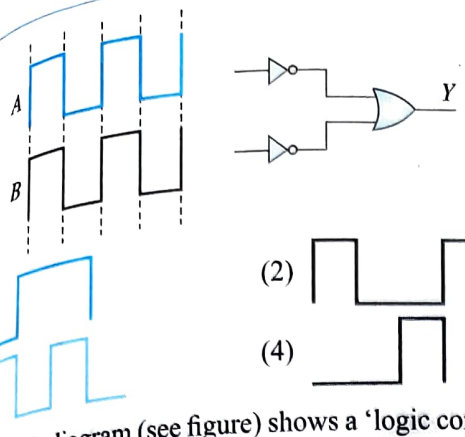
**80.** For the given logic circuit,



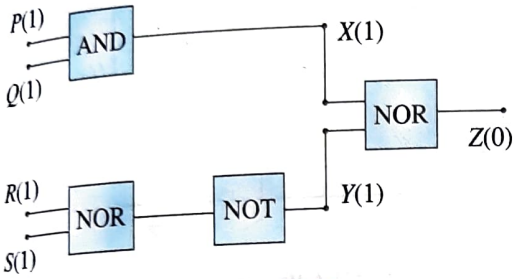

Which of these is correct?
(1) $y = 0$ for $X_1 = X_3 = 0$ and $X_2 = 1$
(2) $y = 0$ for $X_1 = X_2 = X_3 = 0$
(3) $y = 1$ for $X_1 = X_2 = X_3 = 1$
(4) $y = 1$ for $X_1 = X_2 = 1$ and $X_3 = 0$

**81.** In the Boolean algebra, $(\overline{A \cdot B}) \cdot A$ equals to
(1) $\overline{A + B}$ (2) $A$
(3) $\overline{A \cdot B}$ (4) $A + B$

**82.** In a given circuit as shown the two input waveforms $A$ and $B$ are applied simultaneously. The resultant waveform $Y$ is

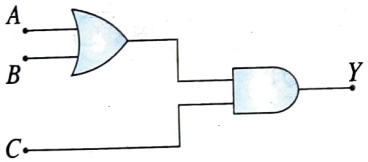



84. The circuit diagram (see figure) shows a 'logic combination' with the states outputs $X$, $Y$ and $Z$ given for inputs $P$, $Q$, $R$ and $S$ all at state 1 (i.e., high). When inputs $P$ and $R$ change to state 0 (i.e. low) with inputs $Q$ and $S$ still at 1, the condition of output $X$, $Y$ and $Z$ changes to



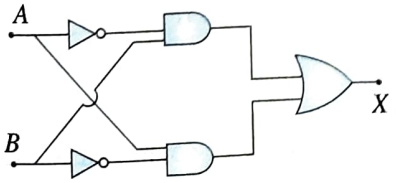

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| X | 1 | 1 | 0 | 0 |
| Y | 0 | 1 | 1 | 0 |
| Z | 0 | 1 | 0 | 1 |

84. In order to obtain an output $Y = 1$ from the circuit of figure, the inputs must be



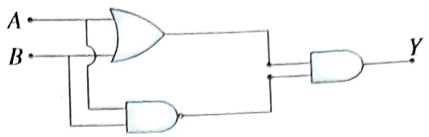

| | A | B | C |
|---|---|---|---|
| (1) | 0 | 1 | 0 |
| (2) | 1 | 0 | 0 |
| (3) | 1 | 0 | 1 |
| (4) | 1 | 1 | 0 |

85. With reference to figure, which of the following is possible?



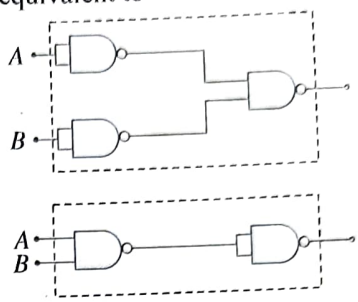

(1) $A = 0, B = 0, X = 1$
(2) $A = 0, B = 1, X = 0$
(3) $A = 0, B = 0, X = 0$
(4) $A = 1, B = 1, X = 0$

**86.** Name the gate represented by the following circuit:




(1) OR gate
(2) XOR gate
(3) AND gate
(4) NAND gate

**87.** The combination of 'NAND' gates shown here under (figure) are equivalent to



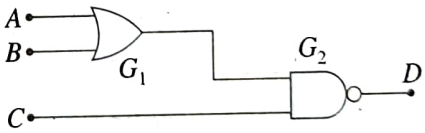

(1) An OR gate and an AND gate respectively
(2) An AND gate and a NOT gate respectively
(3) An AND gate and an OR gate respectively
(4) An OR gate and a NOT gate respectively.

**88.** The following truth table corresponds to the logic gate where $A$ and $B$ represent inputs and $X$ represents output.

| A | B | X |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

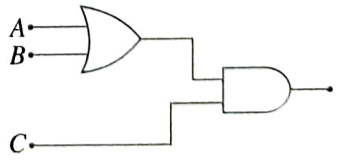
(1) NAND
(2) AND
(3) XOR
(4) OR

**89.** For the given combination of gates, if the logic states of inputs $A$, $B$, $C$ are as follows $A = B = C = 0$ and $A = B = 1$, $C = 0$, then the logic states of output $D$ are



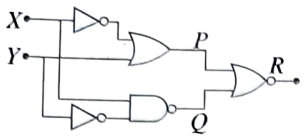

(1) 0, 0
(2) 0, 1
(3) 1, 0
(4) 1, 1

**90.** To get an output 1 from the circuit shown in the figure, the input must be



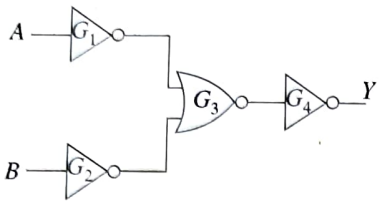

(1) $A = 0, B = 1, C = 0$
(2) $A = 1, B = 0, C = 0$
(3) $A = 1, B = 0, C = 1$
(4) $A = 1, B = 1, C = 0$

**91.** The figure below gives a system of logic gates. From the study of truth table, it can be found that to produce a high output (1) at $R$, we must have



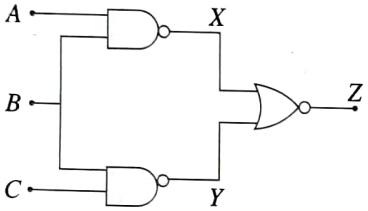

(1) $X = 0, Y = 1$ (2) $X = 1, Y = 1$
(3) $X = 1, Y = 0$ (4) $X = 0, Y = 0$

**92.** The combination of gates shown below produces



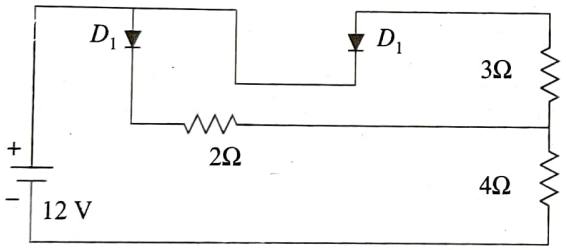

(1) AND gate (2) XOR gate
(3) NOR gate (4) NAND gate

**93.** The figure shows two NAND gates followed by a NOR gate. The system is equivalent to the following logic gate




(1) OR (2) AND
(3) NAND (4) None of these

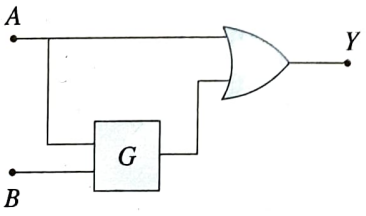
**94.** In the circuit of figure, treat the diodes as ideal. Current in the 4-ohm resistor is




(1) 2 A (2) 3 A
(3) 12/7 A (4) 30/13 A

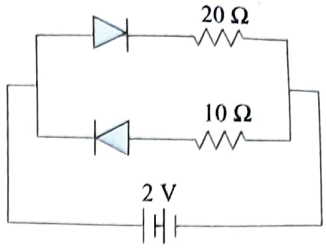
**95.** Identify the logic gate $G$ in the combination of gates shown in figure. The truth table is shown here

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |



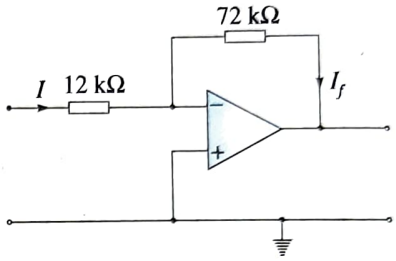

(1) OR gate (2) AND gate
(3) NOT gate (4) XOR gate

**96.** In figure, the current supplied by the battery is



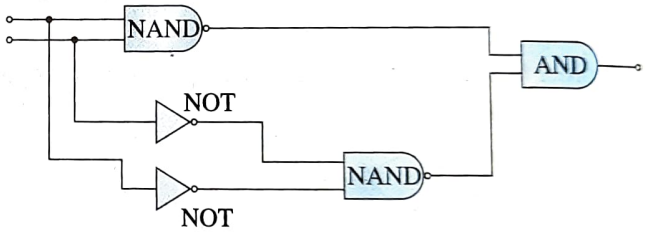

(1) 0.1 A (2) 0.2 A
(3) 0.3 A (4) 0.4 A

**97.** In the circuit shown in the diagram, the operational amplifier may be assumed to be ideal. The current in the 12 kΩ resistor is $I$.



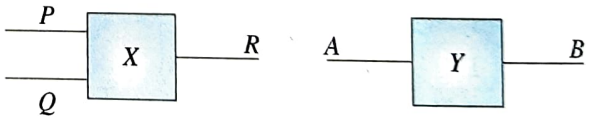

What is the current $I_f$ in the 72 kΩ resistor ?
(1) $-6\,I$ (2) $I$
(3) $-1/6\,I$ (4) $6I$

**98.** The diagram shows a logic network.




Which single gate is equivalent to the network?
(1) EX-NOR (2) NOR
(3) EX-OR (4) OR

**99.** Logic gates $X$ and $Y$ have the truth tables shown below:




| P | Q | R | A | B |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | | |
| 1 | 1 | 1 | | |

When the output of $X$ of connected to the input of $Y$, th resulting combination is equivalent to a single
(1) NOT gate
(2) OR gate
(3) NAND gate
(4) AND gate

# Archives

## JEE MAIN

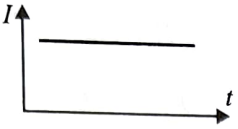
### Single Correct Answer Type

1. A *pn* junction ($D$) shown in the figure can act as a rectifier. An alternating current source ($V$) is connected in the circuit. The current ($I$) in the resistor $R$ can be shown by

(1)

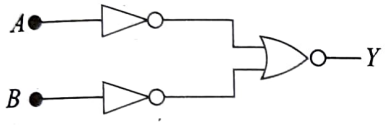

(2)

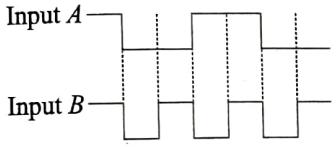

(3)

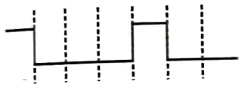

(4) 

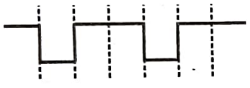

**(AIEEE 2009)**

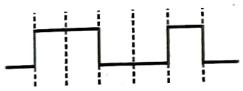
2. The logic circuit below has the input waveforms $A$ and $B$ as shown. Pick out the correct output waveform.

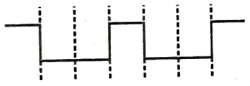



(1) 

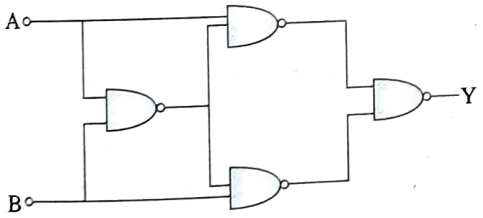
(2) 

(3) 

(4) 

**(AIEEE 2009)**

3. The truth table for system of four NAND gates as shown in figure is




(1)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(2)

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

(3)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

(4)

| A | B | Y |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

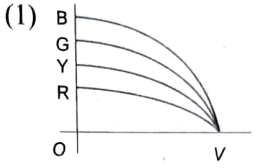
**(AIEEE 2012)**

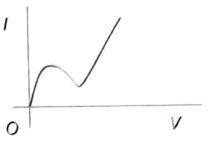
4. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kΩ. Find the maximum modulated frequency which could be detected by it.

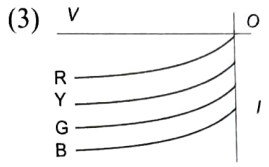
(1) 10.62 kHz (2) 5.31 MHz
(3) 5.31 kHz (4) 10.62 MHz

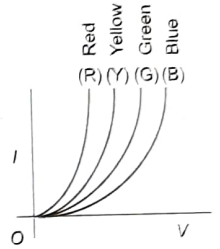
**(JEE Main 2013)**

5. The $I$–$V$ characteristic of an LED is

(1) 


(2) 


(3) 


(4) 

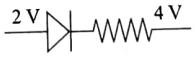

**(JEE Main 2013)**

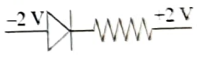
6. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage $V$ is in volts and the temperature $T$ is in degree kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in mA?

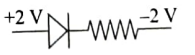
(1) 0.5 mA (2) 0.05 mA
(3) 0.2 mA (4) 0.02 mA

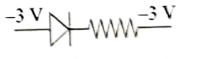
**(JEE Main 2014)**

7. The forward-biased diode connection is

(1) 


(2) 


(c) 

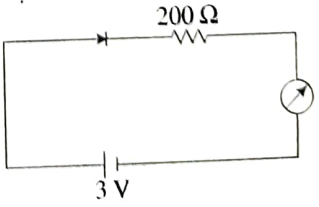

(4) 


**(JEE Main 2014)**

8. In a common emitter amplifier circuit using an *n-p-n* transistor, the phase difference between the input and the output voltages will be

(1) 135° (2) 180°
(3) 45° (4) 90° **(JEE Main 2017)**

9. The reading of the ammeter for a silicon diode in the given circuit is




(1) 13.5 mA (2) 0 (zero)
(3) 15 mA (4) 11.5 mA

**(JEE Main 2018)**

# Answers Key

## XERCISES

### ingle Correct Answer Type

| | | | | |
|---|---|---|---|---|
| **1.** (4) | **2.** (3) | **3.** (1) | **4.** (1) | **5.** (2) |
| **6.** (3) | **7.** (2) | **8.** (4) | **9.** (3) | **10.** (4) |
| **11.** (3) | **12.** (4) | **13.** (3) | **14.** (2) | **15.** (1) |
| **16.** (4) | **17.** (1) | **18.** (4) | **19.** (3) | **20.** (3) |
| **21.** (4) | **22.** (2) | **23.** (1) | **24.** (2) | **25.** (2) |
| **26.** (2) | **27.** (2) | **28.** (4) | **29.** (3) | **30.** (2) |
| **31.** (2) | **32.** (3) | **33.** (2) | **34.** (3) | **35.** (3) |
| **36.** (2) | **37.** (1) | **38.** (2) | **39.** (1) | **40.** (3) |
| **41.** (3) | **42.** (2) | **43.** (2) | **44.** (1) | **45.** (3) |
| **46.** (4) | **47.** (1) | **48.** (3) | **49.** (1) | **50.** (1) |
| **51.** (2) | **52.** (2) | **53.** (2) | **54.** (1) | **55.** (4) |
| **56.** (2) | **57.** (3) | **58.** (2) | **59.** (1) | **60.** (4) |
| **61.** (2) | **62.** (4) | **63.** (2) | **64.** (3) | **65.** (3) |
| **66.** (3) | **67.** (1) | **68.** (4) | **69.** (2) | **70.** (4) |
| **71.** (3) | **72.** (1) | **73.** (1) | **74.** (3) | **75.** (1) |
| **76.** (2) | **77.** (1) | **78.** (4) | **79.** (4) | **80.** (1) |
| **81.** (2) | **82.** (1) | **83.** (3) | **84.** (3) | **85.** (4) |
| **86.** (2) | **87.** (1) | **88.** (4) | **89.** (4) | **90.** (3) |
| **91.** (3) | **92.** (4) | **93.** (2) | **94.** (1) | **95.** (2) |
| **96.** (2) | **97.** (2) | **98.** (3) | **99.** (3) | |

## ARCHIVES

### JEE Main

### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| **1.** (3) | **2.** (1) | **3.** (1) | **4.** (3) | **5.** (4) |
| **6.** (3) | **7.** (3) | **8.** (2) | **9.** (4) | |

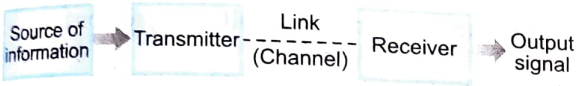
# 9 Communication Systems

## INTRODUCTION

The term communication refers to the transmitting, receiving, and processing of information by electronic means.

## BASIC COMMUNICATION SYSTEM

A basic communication system consists of an information source, a transmitter, a link, and a receiver.




**Information**: The idea/message that is to be conveyed is information. The message may be individual one or a set of messages. It may be a symbol, code, group of words, or any predecided unit.

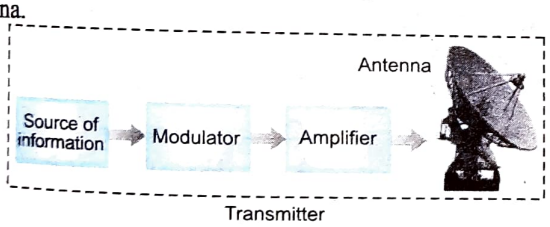
**Transmitter:** In radio transmission, the transmitter consists of a transducer, modulator, amplifier, and transmitting antenna.

**Transducer:** Converts sound signals into electric signal.

**Modulator:** Mixing of audio electric signal with high frequency radio wave.

**Amplifier:** Boosting the power of modulated signal.

**Antenna:** Signal is radiated in the space with the aid of an antenna.




**Communication channel:** The function of communication channel is to carry the modulated signal from transmitter to receiver. The communication channel is also called transmission medium or link.

The term channel refers to the frequency range allocated to a particular service or transmission.

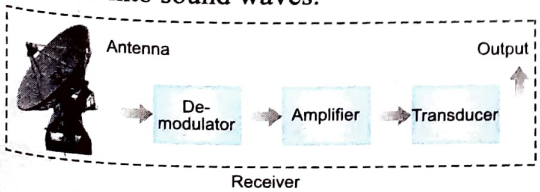
**Receiver:** A receiver consists of the following components:

**Pickup antenna:** To pick the signal

**Demodulator:** To separate out the audio signal from the modulated signal

**Amplifier:** To boost up the weak audio signal

**Transducer:** To convert back audio signal in the form of electrical pulses into sound waves.




## TYPES OF COMMUNICATION SYSTEM

Communication systems can be classified according to the nature of information or mode of transmission or types of transmission channel or types of modulation.

**Classification according to the nature of information source:**

1. Speech transmission
2. Picture transmission
3. Facsimile transmission (FAX): This involves exact reproduction of a document or picture which are static.

**Classification according to the mode of transmission:**

1. **Analog communication:** The communication system which makes use of analog signals is called analog communication system.
2. **Digital communication:** In this system, digital signals are used.

**Classification according to the transmission channel:**

1. Line communication
2. Space communication

**Classification according to the type of modulation:**

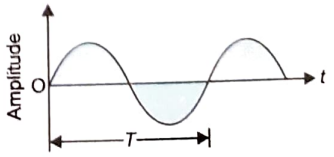
1. Amplitude modulation (AM)
2. Frequency modulation (FM)
3. Phase modulation (PM)
4. Pulse amplitude modulation (PAM)
5. Pulse time modulation (PTM)
6. Pulse code modulation (PCM)

## ANALOG AND DIGITAL SIGNALS

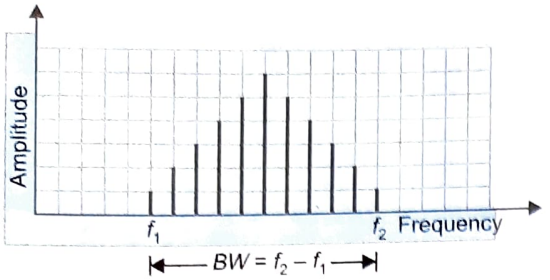
In communication system, a signal means a time, varying electrical signal containing informations.

**Analog signals:** It is a continuous wave form which changes smoothly over time.

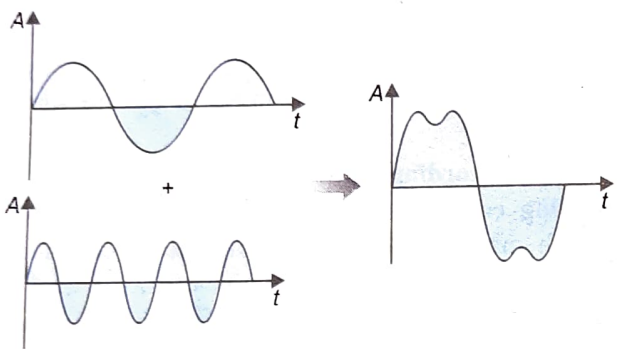
- Such signals can be easily generated from the source of information by using an appropriate transducer, e.g., pressure variations in the sound waves can be converted into corresponding current or voltage pulses with the help of a microphone.
- A simple analog signal is represented by a sine wave.



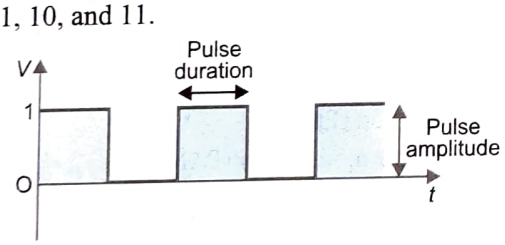
- The frequency of analog signals associated with speed or music varies over a range between 20 Hz to 20 kHz.




- The range over which the frequencies of a signal vary is called bandwidth.
- The term base band designates the band of frequencies representing the signal supplied by the source of information.
- A signal consists of two or more waves of different frequencies known as a complex analog signal.



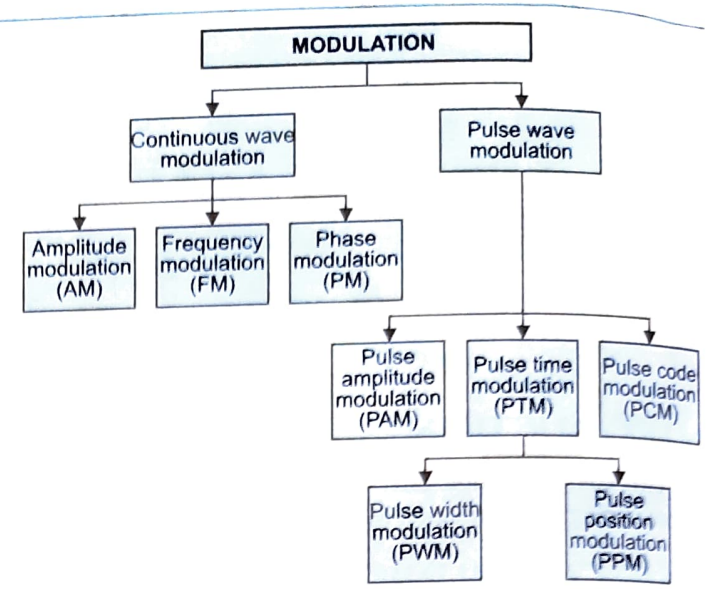

**Digital signals:** A digital signal is a discontinuous function of time. It has only two voltage levels, i.e., either low (0) or high (1). Either of 0 and 1 is known as bit. A group of bit is called byte. A byte comprising 2 bits can give four code combinations, i.e., 00, 01, 10, and 11.




The number of code combination increases with number of bits in a byte is given by $N = 2^x$, where $x$ = number of bits in a byte. The number of binary digits (bits) per second, which describe a digital signal is called its bit rate. Bit rate is expressed in bits per second (bps).

# MODULATION

Digital and analog signals to be transmitted are usually of low frequency and hence cannot be transmitted as such. These signals require some carrier to be transported. These carriers are known as carrier waves or high-frequency signals. The process of placement of a low-frequency (LF) signal over the high-frequency (HF) signal is known as modulation. Various types of modulations are shown in figure.




## NEED FOR MODULATION

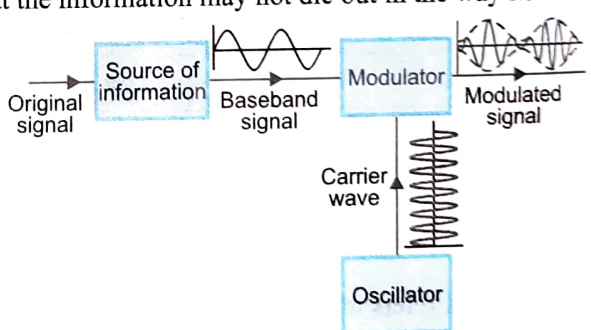
A sound wave (20 Hz to 20 kHz) cannot be transmitted directly from one place to another for the following reasons.

### Height of Antenna

For efficient radiation and reception, the height of transmitting and receiving antennas should be comparable to a quarter of wavelength of the frequency used. For 15 kHz it is 5000 m (too large) and for 1 MHz it is 75 m. The energy radiated from an antenna is practically zero, when the frequency of the signal to be transmitted is below 15 Hz.

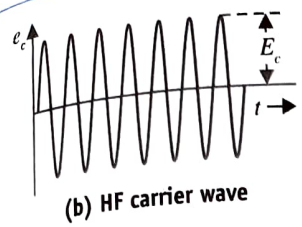
### Detecting Signals

All audible signals are in the range of 20 Hz to 20 kHz. So the signals from all sources remain heavily mixed up in air. It will be very difficult to differentiate or detect the broadcast signal at the receiving station. Thus, modulation is necessary for a low-frequency signal especially when it is to be sent to a distant place, so that the information may not die out in the way itself.

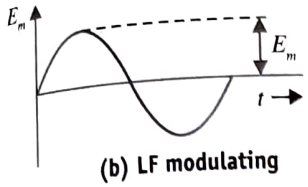



## AMPLITUDE MODULATION

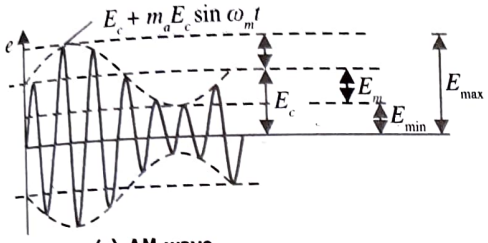
The process of changing the amplitude of a carrier wave in accordance with the amplitude of the audio frequency (AF) signal is known as amplitude modulation (AM). In AM frequency of the carrier wave remains unchanged. The amplitude of modulated wave is varied in accordance with the amplitude of modulating wave.




(b) HF carrier wave




(b) LF modulating




(c) AM wave

**Modulation index:** The ratio of change of amplitude of the carrier wave to the amplitude of original carrier wave is called the modulation factor or degree of modulation or modulation index ($m_a$).

$$m_a = \frac{\text{Change in amplitude of carrier wave}}{\text{Amplitude of original carrier wave}} = \frac{kE_m}{E_c}$$

where $k$ is a factor which determines the maximum change in the amplitude for a given amplitude $E_m$ of the modulating signal. If $k = 1$, then

$$m_a = \frac{E_m}{E_c} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$$

If a carrier wave is modulated by several sine waves, the total modulated index $m_t$ is given by

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + \cdots}$$

**Voltage equation for AM wave:** Suppose voltage equations for carrier wave and modulating wave are $e_c = E_c \cos \omega_c t$ and $e_m = E_m \sin \omega_m t = m_a E_c \sin \omega_m t$, where $e_c$ is the instantaneous voltage of carrier wave, $E_c$ amplitude of carrier wave, $\omega_c (=2\pi f_c)$ angular velocity at carrier frequency $f_c$, $e_m$ instantaneous voltage of modulating, $E_m$ amplitude of modulating wave, and $\omega_m (=2\pi f_m)$ angular velocity of modulating frequency $f_m$.

Voltage equation for AM wave is

$$\begin{aligned} e &= E \sin \omega_c t \\ &= (E_c + e_m) \sin \omega_c t \\ &= (E_c + E_m \sin \omega_m t) \sin \omega_c t \\ &= E_c \sin \omega_c t + \frac{m_a E_c}{2} \cos(\omega_c - \omega_m)t - \frac{m_a E_c}{2} \cos(\omega_c + \omega_m)t \end{aligned}$$

The above AM wave indicated that the AM wave is equivalent to the summation of three sinusoidal waves, one having amplitude $E_c$ and the other two having amplitude $m_a E_c/2$.

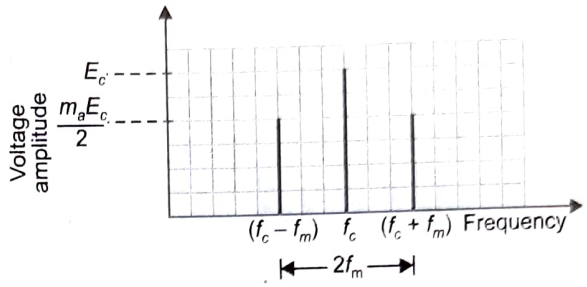
**Side band frequencies:** An AM wave contains three frequencies: $f_c$ (called carrier frequency), $(f_c + f_m)$, and $(f_c - f_m)$ (latter two called side band frequencies).

$(f_c + f_m)$: Upper side band (USB) frequency

$(f_c - f_m)$: Lower side band (LSB) frequency

Side band frequencies are generally close to the carrier frequency.

**Bandwidth in AM wave:** The two side bands lie on either side of the carrier frequency at equal frequency interval $f_m$. So, bandwidth $= (f_c + f_m) - (f_c - f_m) = 2f_m$.




**Power in AM waves:** Power dissipated in any circuit, $P = V^2_{rms}/R$. Hence,

$$\text{Carrier power, } P_c = \frac{(E_c/\sqrt{2})^2}{R} = \frac{E_c^2}{2R}$$

Total power of side bands,

$$P_{sb} = \frac{(m_a E_c/2\sqrt{2})^2}{R} + \frac{(m_a E_c/2\sqrt{2})}{R} = \frac{m_a^2 E_c^2}{4R}$$

Total power of AM wave, $P_t = P_c + P_{sb} = \frac{E_c^2}{2R}\left(1 + \frac{m_a^2}{2}\right)$

$$\frac{P_t}{P_c} = \left(1 + \frac{m_a^2}{2}\right) \text{ and } \frac{P_{sb}}{P_t} = \frac{m_a^2/2}{\left(1 + \frac{m_a^2}{2}\right)}$$

Maximum power in AM (without distortion) will occur when $m_a = 1$, i.e., $P_t = 1.5P_c = 3P_{sb}$

If $I_c$ = unmodulated current and $I_t$ = total or modulated current,

then $\frac{P_t}{P_c} = \frac{I_t^2}{I_c^2} \Rightarrow \frac{I_t}{I_c} = \sqrt{\left(1 + \frac{m_a^2}{2}\right)}$

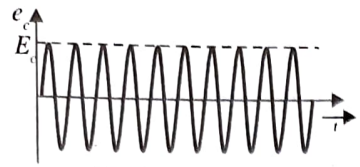
## Limitations of Amplitude Modulation

1. Noisy reception
2. Low efficiency
3. Small operating range
4. Poor audio quality

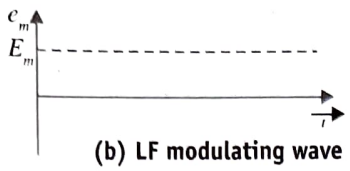
## FREQUENCY MODULATION

The process of changing the frequency of a carrier wave in accordance with the audio frequency signal is known as frequency modulation.

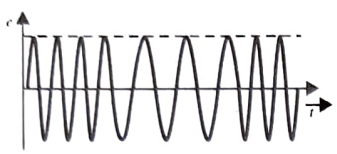
- Audio quality of AM transmission is poor. There is always a need to eliminate amplitude sensitive noise. This is possible if we eliminate amplitude variation. (i.e., a need to keep the amplitude of the carrier constant). This is precisely what we do in FM.
- In FM the overall amplitude of FM wave remains constant at all times.
- In FM, the total transmitted power remains constant.




(a) HF carrier wave




(b) LF modulating wave




(c) FM wave

**Frequency deviation:** The maximum change in frequency from mean value ($f_c$) is known as frequency deviation. This is also the change or shift either above or below the frequency $f_c$ and is called as frequency deviation.

$$\therefore \quad \delta = (f_{max} - f_c) = f_c - f_{min} = k_f \cdot \frac{E_m}{2\pi}$$

where $k_f$ is the constant of proportionality. It determines the maximum variation in frequency of the modulated wave for a given modulating signal.

**Carrier swing (CS):** The total variation in frequency from the lowest to the highest is called the carrier swing,

i.e., $\quad CS = 2 \times \Delta f$

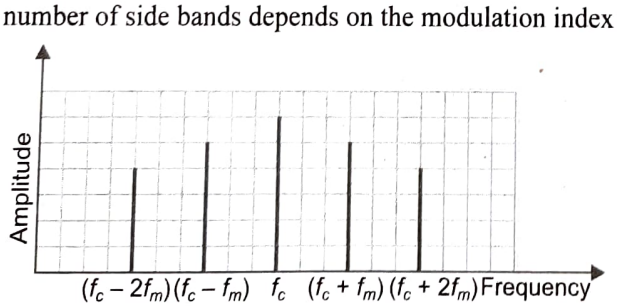
**Frequency modulation index ($m_f$):** The ratio of maximum frequency deviation to the modulating frequency is called modulation index.

$$m_f = \frac{\delta}{f_m} = \frac{f_{max} - f_c}{f_m} = \frac{f_c - f_{min}}{f_m} = \frac{k_f E_m}{f_m}$$

**Frequency spectrum:** FM side band modulated signal consist of infinite number of side bands whose frequencies are

$$(f_c \pm f_m), (f_c \pm 2f_m), (f_c \pm 3f_m), \ldots$$

The number of side bands depends on the modulation index $m_f$.




In FM signal, the information (audio signal) is contained in the side bands. Since the side bands are separated from each other by the frequency of modulating signal $f_m$, so

$$\text{Bandwidth} = 2n \times f_m$$

where $n$ = number of significant side band pairs

**Deviation ratio:** The ratio of maximum permitted frequency deviation to the maximum permitted audio frequency is known as deviation ratio. Thus,

$$\text{Deviation ratio} = \frac{(\Delta f)_{max}}{(f_m)_{max}}$$

## PERCENT MODULATION

The ratio of actual frequency deviation to the maximum allowed frequency deviation is defined as percent modulation. Thus,

$$\text{Percent modulation, } m = \frac{(\Delta f)_{actual}}{(\Delta f)_{max}}$$

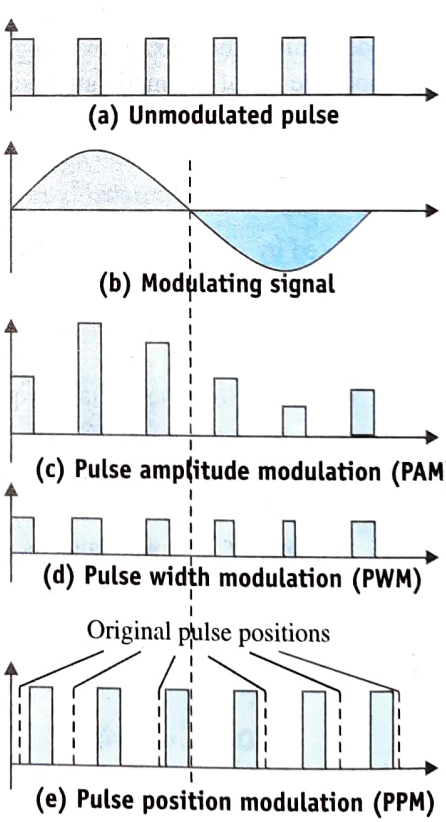
## PULSE MODULATION

In pulse modulation, the carrier wave is in the form of pulses. Mentioned below are the types of pulse modulation.

**Pulse amplitude modulation (PAM):** The amplitude of the pulse varies in accordance with the modulating signal.

**Pulse width modulation (PWM):** The pulse duration varies in accordance with the modulating signal.

**Pulse position modulation (PPM):** In PPM, the position of the pulses of the carrier wave train is varied in accordance with the instantaneous value of the modulating signal.




(a) Unmodulated pulse

(b) Modulating signal

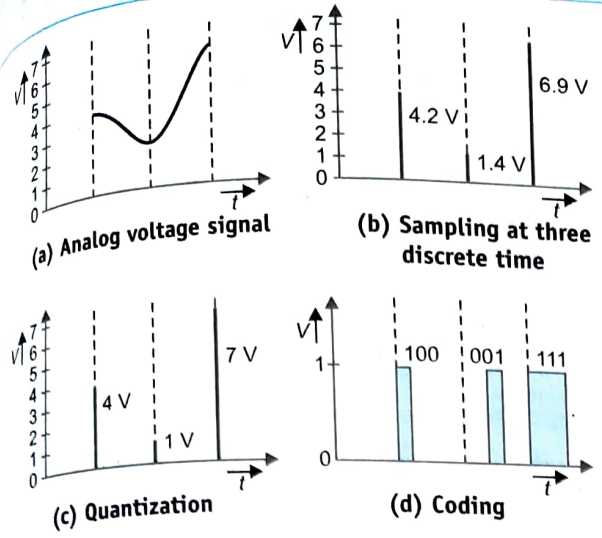
(c) Pulse amplitude modulation (PAM)

(d) Pulse width modulation (PWM)

(e) Pulse position modulation (PPM)

## PULSE CODE MODULATION

The pulse amplitude, pulse width, and pulse position modulations are not completely digital. A completely digital modulation is obtained by pulse code modulation (PCM). An analog signal is pulse code modulated by following three operations:




(a) Analog voltage signal (b) Sampling at three discrete time




(c) Quantization (d) Coding

**Sampling:** It is the process of generating pulses of zero width and of amplitude equal to the instantaneous amplitude of the analog signal. The number of samples taken per second is called sampling rate.

**Quantization:** The process of dividing the maximum amplitude of the analog voltage signal into a fixed number of levels is called quantization. For example, amplitude 5 V of the analog voltage signal divides into six quantization levels, viz., 0, 1, 2, 3, 4, 5.

Pulses having amplitude between –0.5 V and 0.5 V are approximated (quantized) to a value of 0 V, amplitude between 0.5 V and 1.5 V are approximated to a value of 1 V, and so on.

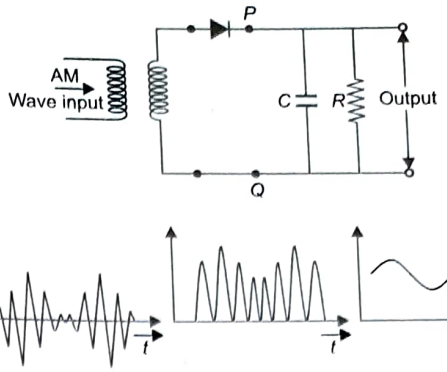
**Coding:** The process of digitizing the quantized pulses according to some code is called coding. For example, consider that voltage amplitude of an analog signal varies between 0 and 7 V.

## DEMODULATION

The process of extracting the audio signal from the modulated wave is known as demodulation or detection. The wireless signals consist of radio frequency (high frequency) carrier wave modulated by audio frequency (low frequency). The diaphragm of a telephone receiver or a loud speaker cannot vibrate with high frequency. So it is necessary to separate the audio frequencies from the radio frequency carrier wave.

**Simple demodulator circuit:** A diode can be used to detect or demodulate an amplitude modulated (AM) wave. A diode basically acts as a rectifier, i.e., it reduces the modulated carrier wave into positive envelope only.

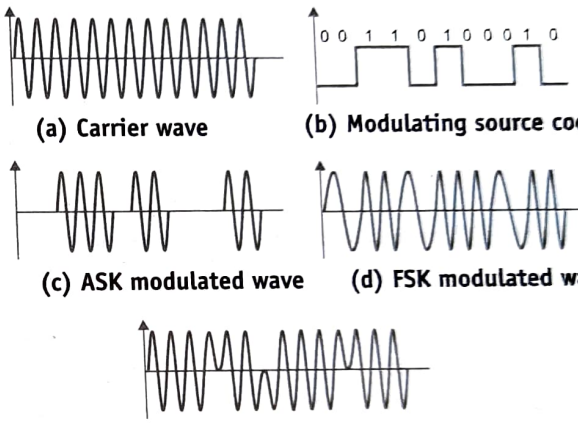
An AM wave input is shown in figure. It appears at the output of the diode across $PQ$ as a rectified wave (since a diode conducts only in the positive half cycle). This rectified wave after passing through the $RC$ network does not contain the radio frequency carrier component. Instead, it has only the envelope of the modulated wave.




In the actual circuit the value of $RC$ is chosen such that $1/f_c << RC$, where $f_c$ is the frequency of carrier signal.

## DATA TRANSMISSION AND RETRIEVAL

The term data is applied to a representation of facts, concepts, or instructions suitable for communication, interpretation, or processing by human beings or by automatic means. Data in most cases consist of pulse type of signals.




(a) Carrier wave (b) Modulating source code

(c) ASK modulated wave (d) FSK modulated wave

(e) PSK modulated wave

The PCM signal is a series of 1s and 0s. Following three modulation techniques are used to transmit a PCM signal:

**Amplitude shift keying (ASK):** Two different amplitudes of the carrier represent the two binary values of the PCM signal. This method is also known as on-off keying (OOK):

**1:** Presence of carrier of same constant amplitude.

**0:** Carrier of zero amplitude.

**Frequency shift keying (FSK):** The binary values of the PCM signal are represented by two frequencies.

**1:** Increase in frequency.

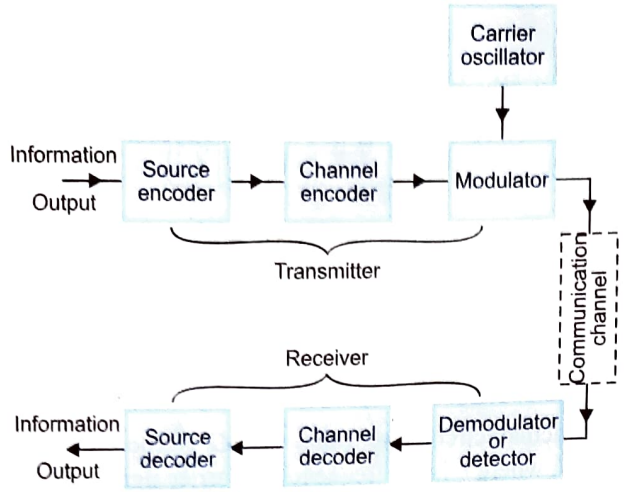
**0:** Frequency unaffected.

**Phase shift keying (PSK):** The phase of the carrier wave is changed in accordance with modulating data signal.

**1:** Phase changed by $\pi$.

**0:** Phase remains unchanged.

The analog signal is sampled by the sampler. The sampled pulses are then quantized. The encoder codes the quantized pulses according to the binary codes. After modulating the PCM signal (by ASK, FSK, or PSK method), the modulated signal is then transmitted into free space in the form of bits.




### ILLUSTRATION 9.1

A TV transmitter has a range of 50 km. What is the height of the TV transmission tower? Radius of earth $R_e = 6.4 \times 10^6$ m.

**Sol.** Here $d = 50 \text{ km} = 50 \times 10^3$ m

$\therefore$ Height of TV transmission tower

$$h = \frac{d^2}{2R_e} = \frac{(50\times10^3)^2}{2\times6.4\times10^6} = 195.3\,\text{m}$$

### ILLUSTRATION 9.2

A TV tower has a height of 100 m. How much population is covered by the TV broadcast if the average population density around the tower is 1000 km$^{-2}$ (radius of earth = 6.4 × 10$^6$ m).

**Sol.** Here $h = 100$ m, $R_e = 6.4 \times 10^6$ m

Average population density

$$= 1000 \text{ km}^{-2} = 1000 \times (1000 \text{ m})^{-2} = 10^{-3} \text{ m}^2$$

The TV transmission range, $d = \sqrt{2hR_e}$

Therefore, area of TV transmission range

$$A = \pi d^2 = \pi 2h R_e = 2\pi hR_e$$

$$= 2 \times 3.14 \times 100 \times 6.4 \times 10^6 \text{ m}^2 = 4.0 \times 10^9 \text{ m}^2$$

Therefore, population covered by TV broadcast

$$= 4.0 \times 10^9 \times 10^{-3} = 4.0 \times 10^6$$

### ILLUSTRATION 9.3

On a particular day, the maximum frequency reflected from the ionosphere is 10 MHz. On another day, it was found to increase to 11 MHz. Calculate the ratio of maximum electron density of the ionosphere on the two days. Point out a plausible explanation for this.

**Sol** $f_c = 9\sqrt{N_{max}}$

$$N_{max} = \frac{f_c^2}{81} \propto f_c^2 \Rightarrow \frac{(N_2)_{max}}{(N_1)_{max}} = \left(\frac{11}{10}\right)^2 = \frac{121}{100} = 1.21$$

The variation may be due to atmospheric disturbances.

### ILLUSTRATION 9.4

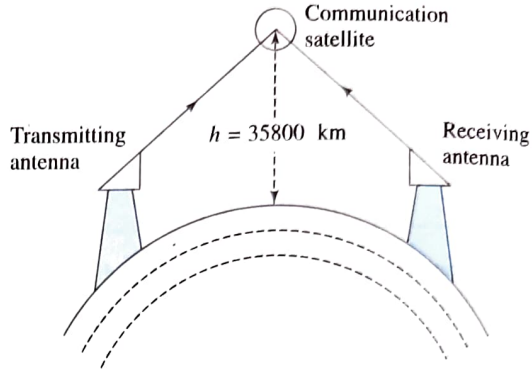
Frequencies higher than 10 MHz are found not to be reflected by the ionosphere on a particular day at a place. Calculate the maximum electron density of the ionosphere.

**Sol.** Given $f_c = 10 \text{ MHz} = 10 \times 10^6$ Hz

If $N_{max}$ is the maximum electron density, then $f_c = 9\sqrt{N_{max}}$

$$N_{max} = \frac{f_c^2}{81} = \frac{(10\times10^6)^2}{81} = 1.2 \times 10^{12} \text{ m}^{-3}$$

Satellite communication




The height of communication satellite from earth's surface is 35,800 km. These communication satellites are required to cover the entire globe of earth.

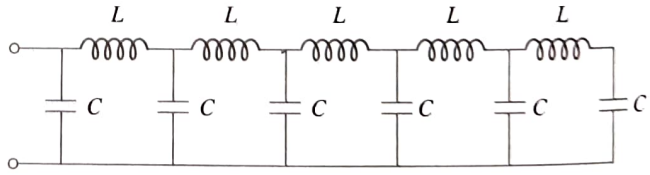
## REMOTE SENSING

Remote sensing is the process of finding information about an object at a distance without going near it. It is achieved by launching satellites in sun synchronous orbits.

## LINE COMMUNICATION

The line communication is conventional and it is used to transmit electricity and telephone signals through a pair of wires. The most used transmission lines are (i) a pair of conducting wires and (ii) a coaxial cable. It consists of two concentric conductors separated by an isolated material. The equivalent circuit of a transmission line consists of a sequence of inductance $L$ and capacitance $C$.

(a) Twisted pair of wires (b) Usual lines




(c) Equivalent circuit

## OPTICAL COMMUNICATION

In optical communication, communication is through carrier optical signals. Optical signals cannot be transmitted through cables. The **essentials** of optical communication are

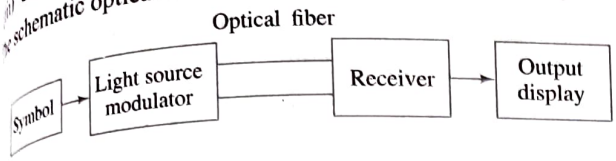
(i) **Light source and modulator:** Light sources are LED, diode lasers.

(ii) **Communication channel:** The optical fibres of small diameters = $10^{-5}$ m based on total internal reflection have been discovered.

(iii) **Optical signal detectors:** These are photodiode, Avalanche photodiode.

vantages of optical communication are:
(i) Wide channel band width has large channel carrying capacity.
(ii) Low transmission losses
(iii) Signal security

y schematic optical communication is shown in the figure.




SER
er is abbreviation of light amplification of stimulated ission of radiation. It is based on stimulated emission. A laser vice consists of a resonant cavity having one totally and one rtially reflecting face. The active substance has metastable ates and most of the atoms are in excited metastable state that population inversion is achieved. When a photon of oper frequency is incident on the active material, it stimulates ission of photons. Finally an intense, monochromatic, highly ectional coherent laser beam is available.

## CONCEPT APPLICATION EXERCISE 9.1

1. A sky wave with a frequency 55 MHz is incident on *D*-region of earth's atmosphere at 45°. The angle of refraction is (electron density for *D*-region is 400 electron/cm³)
(a) 60° (b) 45°
(c) 30° (d) 15°

2. In a diode AM-detector, the output circuit consist of $R$ = 1 kΩ and $C$ = 10 pF. A carrier signal of 100 kHz is to be detected. Is it good?
(a) Yes
(b) No
(c) Information is not sufficient
(d) None of these

3. A photodetector is made from a semiconductor $In_{0.53}Ga_{0.47}As$ with $E_g = 0.73$ eV. What is the maximum wavelength, which it can detect
(a) 1000 nm (b) 1703 nm
(c) 500 nm (d) 173 nm

4. A transmitter supplies 9 kW to the aerial when unmodulated. The power radiated when modulated to 40% is
(a) 5 kW (b) 9.72 kW
(c) 10 kW (d) 12 kW

5. The antenna current of an AM transmitter is 8 A when only carrier is sent but increases to 8.96 A when the carrier is sinusoidally modulated. The percentage modulation is
(a) 50% (b) 60%
(c) 65% (d) 71%

6. The total power content of an AM wave is 1500 W. For 100% modulation, the power transmitted by the carrier is
(a) 500 W (b) 700 W
(c) 750 W (d) 1000 W

7. The total power content of an AM wave is 900 W. For 100% modulation, the power transmitted by each side band is
(a) 50 W (b) 100 W
(c) 150 W (d) 200 W

8. The modulation index of an FM carrier having a carrier swing of 200 kHz and a modulating signal 10 kHz is
(a) 5 (b) 10
(c) 20 (d) 25

9. A 500-Hz modulating voltage fed into an FM generator produces a frequency deviation of 2.25 kHz. If amplitude of the voltage is kept constant but frequency is raised to 6 kHz, then the new deviation will be
(a) 4.5 kHz (b) 54 kHz
(c) 27 kHz (d) 15 kHz

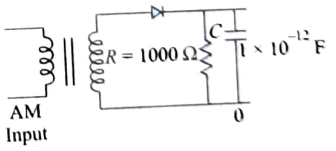
10. The bit rate for a signal, which has a sampling rate of 8 kHz and where 16 quantisation levels have been used is
(a) 32000 bits/sec (b) 16000 bits/sec
(c) 64000 bits/sec (d) 72000 bits/sec

11. An amplitude modulated wave is modulated to 50%. What is the saving in power if carrier as well as one of the side bands are suppressed?
(a) 70% (b) 65.4%
(c) 94.4% (d) 25.5%

12. In a radio receiver, the short wave and medium wave stations are tuned by using the same capacitor but coils of different inductance $L_s$ and $L_m$, respectively, then
(a) $L_s > L_m$ (b) $L_s < L_m$
(c) $L_s = L_m$ (d) None of these

13. A ground receiver station is receiving a signal at 5 MHz and transmitted from a ground transmitter at a height of 300 m, located at a distance of 100 km from the receiver station. The signal is coming via. [Radius of earth = $6.4 \times 10^6$ m, $N_{max}$ of isosphere = $10^{12}$ m$^3$]
(a) Space wave (b) Sky wave propagation
(c) Satellite transponder (d) All of these

14. In the given detector circuit, the suitable value of carrier frequency is




(a) $<< 10^9$ Hz (b) $<< 10^5$ Hz
(c) $>> 10^9$ Hz (d) None of these

### ANSWERS

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (b) | 3. (b) | 4. (b) | 5. (d) |
| 6. (d) | 7. (c) | 8. (b) | 9. (b) | 10. (a) |
| 11. (c) | 12. (b) | 13. (b) | 14. (a) | |

# Exercises

1. In short wave communication waves, which of the following frequencies will be reflected back by the ionospheric layer, having electron density $10^{11}$ per $m^3$?
   (1) 2 MHz (2) 10 MHz
   (3) 12 MHz (4) 18 MHz
2. Range of frequencies allotted for commercial FM radio broadcast is
   (1) 88 to 108 MHz (2) 88 to 108 kHz
   (3) 8 to 88 MHz (4) 88 to 108 GHz
3. The velocity factor of a transmission line $x$. If dielectric constant of the medium is 2.6, the value of $x$ is
   (1) 0.26 (2) 0.62
   (3) 2.6 (4) 6.2
4. A step index fibre has a relative refractive index of 0.88%. What is the critical angle at the corecladding interface?
   (1) 60° (2) 75°
   (3) 45° (4) None of these
5. A laser beam of pulse power $10^{12}$ watt is focussed on an object are $10^{-4}$ $cm^2$. The energy flux in watt/$cm^2$ at the point of focus is
   (1) $10^{20}$ (2) $10^{16}$
   (3) $10^{8}$ (4) $10^{4}$
6. The carrier frequency generated by a tank circuit containing 1 nF capacitor and 10 μH inductor is
   (1) 1592 Hz (2) 1592 MHz
   (3) 1592 kHz (4) 159.2 Hz
7. For television broadcasting, the frequency employed is normally
   (1) 30–300 MHz (2) 30–300 GHz
   (3) 30–300 kHz (4) 30–300 Hz
8. Maximum usable frequency (MUF) in $F$-region layer is $x$, when the critical frequency is 60 MHz and the angle of incidence is 70°. Then $x$ is
   (1) 150 MHz (2) 170 MHz
   (3) 175 MHz (4) 190 MHz
9. Laser beams are used to measure long distances because
   (1) they are monochromatic
   (2) they are highly polarised
   (3) they are coherent
   (4) they have high degree of parallelism
10. An oscillator is producing FM waves of frequency 2 kHz with a variation of 10 kHz. What is the modulating index?
    (1) 0.20 (2) 5.0
    (3) 0.67 (4) 1.5
11. The maximum peak to peak voltage of an AM wire is 24 mV and the minimum peak to peak voltage is 8 mV. The modulation factor is
    (1) 10% (2) 20%
    (3) 25% (4) 50%
12. Sinusoidal carrier voltage of frequency 1.5 MHz and amplitude 50 V is amplitude modulated by sinusoidal voltage of frequency 10 kHz producing 50% modulation. The lower and upper side-band frequencies in kHz are
    (1) 1490, 1510 (2) 1510, 1490
    (3) $\frac{1}{1490}, \frac{1}{1510}$ (4) $\frac{1}{1510}, \frac{1}{1490}$
13. Which of the following is the disadvantage of FM over AM?
    (1) Larger band width requirement
    (2) Larger noise
    (3) Higher modulation power
    (4) Low efficiency
14. An AM wave has 1800 W of total power content, For 100% modulation, the carrier should have power content equal to
    (1) 1000 watt (2) 1200 watt
    (3) 1500 watt (4) 1600 watt
15. An antenna is a device
    (1) that converts electromagnetic energy into radio frequency signal
    (2) that converts radio frequency signal into electromagnetic energy
    (3) that converts guided electromagnetic waves into free space electromagnetic waves and vice-versa
    (4) none of these
16. In an FM system, a 7 kHz signal modulates 108 MHz carrier so that frequency deviation is 50 kHz. The carrier swing is
    (1) 7.143 (2) 8
    (3) 0.71 (4) 350
17. The phenomenon by which light travels in an optical fibres is
    (1) reflection (2) refraction
    (3) total internal reflection (4) transmission
18. Advantage of optical fibre
    (1) high bandwidth and em interference
    (2) low bandwidth and em interference
    (3) high bandwidth, low transmission capacity and no e.m. interference
    (4) high bandwidth, high data transmission capacity and no e.m. interference
19. In frequency modulation
    (1) the amplitude of modulated wave varies as frequency of carrier wave
    (2) the frequency of modulated wave varies as amplitude of modulating wave
    (3) the amplitude of modulated wave varies as amplitude of carrier wave
    (4) the frequency of modulated wave varies as frequency of modulating wave
20. Audio signal cannot be transmitted because
    (1) the signal has more noise
    (2) the signal cannot be amplified for distance communication
    (3) the transmitting antenna length is very small to design
    (4) the transmitting antenna length is very large and impracticable
21. In which of the following remote sensing technique is not used?
    (1) Forest density (2) Pollution
    (3) Wetland mapping (4) Medical treatment

For sky wave propagation of a 10 MHz signal, what should be the minimum electron density in ionosphere?
(1) $\sim 1.2 \times 10^{12}$ m$^{-3}$ (2) $\sim 10^{6}$ m$^{-3}$
(3) $\sim 10^{14}$ m$^{-3}$ (4) $\sim 10^{22}$ m$^{-3}$

What should be the maximum acceptance angle at the aircore interface of an optical fibre if $n_1$ and $n_2$ are the refractive indices of the core and the cladding, respectively?
(1) $\sin^{-1}(n_2/n_1)$ (2) $\sin^{-1}\sqrt{n_1^2 - n_2^2}$
(3) $\left[\tan^{-1}\frac{n_2}{n_1}\right]$ (4) $\left[\tan^{-1}\frac{n_1}{n_2}\right]$

The audio signal used to modulate 60 sin $(2\pi \times 10^6 t)$ is 15 sin 300 $\pi t$. The depth of modulation is
(1) 50% (2) 40%
(3) 25% (4) 15%

The impedance of coaxial cable, when its inductance is 0.40 μH and capacitance is $1 \times 10^{-11}$ F, can be
(1) $2 \times 10^2\ \Omega$ (2) $100\ \Omega$
(3) $3 \times 10^3\ \Omega$ (4) $3 \times 10^{-2}\ \Omega$

An optical fibre communication system works on a wavelength of 1.3 μm. The number of subscribers it can feed if a channel requires 20 kHz are
(1) $2.3 \times 10^{10}$ (2) $1.15 \times 10^{10}$
(3) $1 \times 10^5$ (4) None of these

In an FM system a 7 kHz signal modulates 108 MHz carrier so that frequency deviation is 50 kHz. The carrier swing is
(1) 7.143 (2) 8
(3) 0.71 (4) 350

**28.** The electron density of $E$, $F_1$, $F_2$ layers of ionosphere is $2 \times 10^{11}$, $5 \times 10^{11}$ and $8 \times 10^{11}$ m$^{-3}$ respectively. What is the ratio of critical frequency for reflection of radiowaves?
(1) 2 : 4 : 3 (2) 4 : 3 : 2
(3) 2 : 3 : 4 (4) 3 : 2 : 4

**29.** Mean optical power launched into an 8 km fibre is 120 μW and mean output power is 4 μW, then the overall attenuation is (Given log 30 = 1.477)
(1) 14.77 dB (2) 16.77 dB
(3) 3.01 dB (4) None of these

**30.** An antenna current of an AM broadcast transmitter modulated by 50% is 11 A. The carrier current is
(1) 10.35 A (2) 9.25 A
(3) 10 A (4) 5.5 A

**31.** A transmitter transmits a power of 10 kW when modulation is 50%. Power of carrier wave is
(1) 5 kW (2) 8.89 kW
(3) 14 kW (4) 5.7 kW

**32.** A telephone link operating at a central frequency of 10 GHz is established. If 1% of this is available then how many telephone channel can be simultaneously given when each telephone covering a band width of 5 kHz
(1) $2 \times 10^4$ (2) $2 \times 10^6$
(3) $5 \times 10^4$ (4) $5 \times 10^6$

**33.** In AM, the cent percent modulation is achieved when
(1) carrier amplitude = signal amplitude
(2) carrier amplitude ≠ signal amplitude
(3) carrier frequency = signal frequency
(4) carrier frequency ≠ signal frequency

# Archives

## JEE Main

### Single Correct Answer Type

1. This question has Statement 1 and Statement 2. Of the four choices given after the statements, choose the one that best describes the two statements.
**Statement 1:** Sky wave signals are used for long distance radio communication. These signals are in general, less table than ground wave signals.
**Statement 2:** The state of ionosphere varies from hour to hour, day to day and season to season.
(1) Statement 1 is true, Statement 2 is false.
(2) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.
(3) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(4) Statement 1 is false, Statement 2 is true.
**(AIEEE 2011)**

2. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = $6.4 \times 10^6$ m) is
(1) 80 km (2) 16 km
(3) 40 km (4) 64 km **(AIEEE 2012)**

3. In amplitude modulation, sinusoidal carrier frequency used is denoted by $\omega_c$ and the signal frequency is denoted by $\omega_m$. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?
(1) $\omega_m + \omega_c$ (2) $\omega_c - \omega_m$
(3) $\omega_m$ (4) $\omega_c$
**(JEE Main 2017)**

4. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?
(1) $2 \times 10^6$ (2) $2 \times 10^3$
(3) $2 \times 10^4$ (4) $2 \times 10^5$
**(JEE Main 2018)**

# Answers Key

## EXERCISES

### Single Correct Answer Type

| | | | | |
|---|---|---|---|---|
| **1.** (1) | **2.** (1) | **3.** (2) | **4.** (4) | **5.** (2) |
| **6.** (3) | **7.** (1) | **8.** (3) | **9.** (4) | **10.** (2) |
| **11.** (4) | **12.** (1) | **13.** (1) | **14.** (2) | **15.** (3) |
| **16.** (1) | **17.** (3) | **18.** (4) | **19.** (4) | **20.** (4) |
| **21.** (4) | **22.** (1) | **23.** (2) | **24.** (3) | **25.** (1) |
| **26.** (2) | **27.** (1) | **28.** (3) | **29.** (1) | **30.** (1) |
| **31.** (2) | **32.** (1) | **33.** (1) | | |

## ARCHIVES

### JEE Main

### Single Correct Answer Type

| | | | |
|---|---|---|---|
| **1.** (4) | **2.** (1) | **3.** (3) | **4.** (4) |